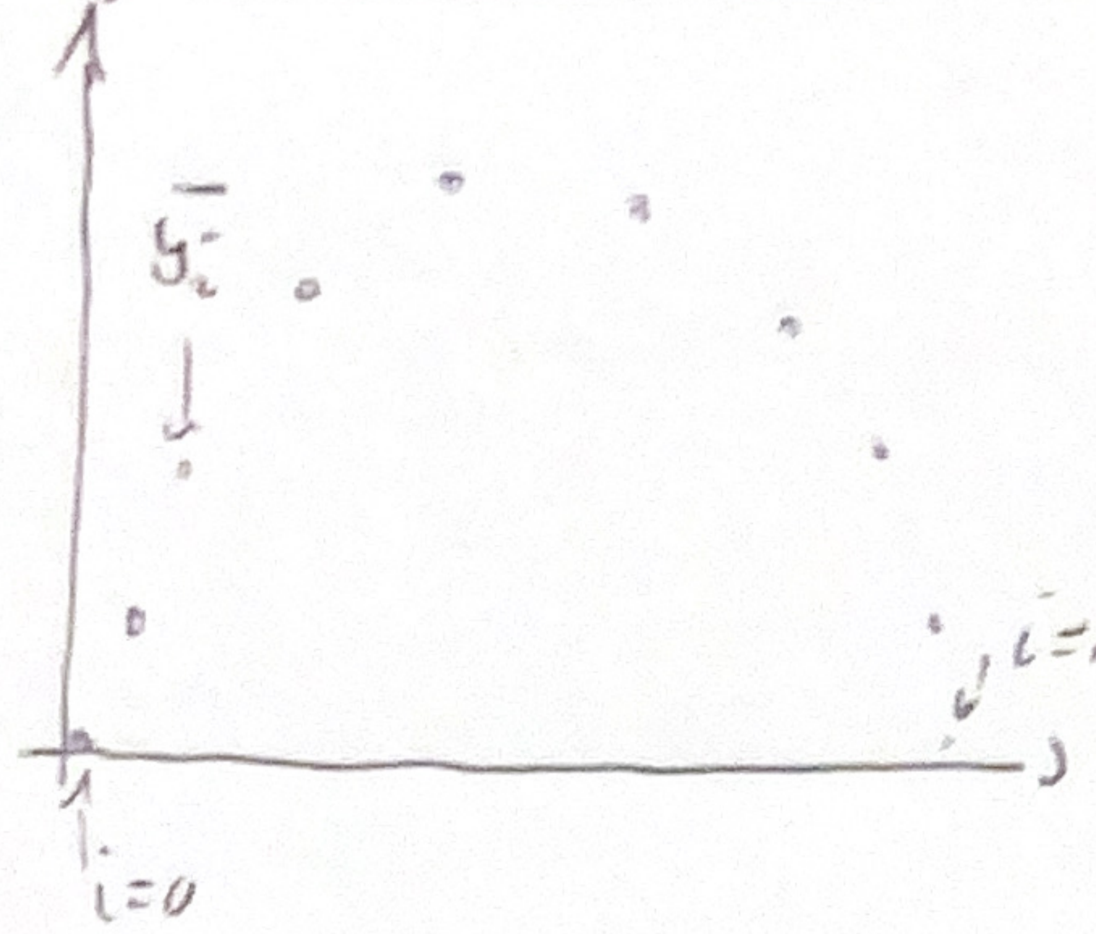
Aufgabe 4 - Least Square Approximation mit Wurzelglied



$i=m-1$ → Summe wird als $\sum^{<m}$ angegeben

Fehlerquadrate

$$F(a_0, a_1, \dots, a_n) = \sum_{i=0}^{m} \left(\bar{y}_i - y(x_i, a_0, a_1, \dots, a_n)\right)^2$$

$$\frac{\partial F}{\partial a_i} \overset{!}{=} 0 \quad \text{für alle } i \in \{0, \dots, n\}$$

n: Anzahl an Koeff
↳ Polynomgrad − 1

$$F(\dots) = \sum_{i=0}^{m} \left(\bar{y}_i - a_0\sqrt{x} + \sum_{i=1}^{n} a_i (x_i)^i\right)$$

$$\frac{\partial F}{\partial a_0} = \cancel{2}\sum^{m} \Big(\bar{y}_i - \underbrace{a_0\sqrt{x} + \sum^{n} a_i (x_i)^i}_{y(x)}\Big) \underbrace{\frac{1}{\cancel{2}\sqrt{x}}}_{\frac{dy(\dots)}{da_0}} \overset{!}{=} 0 \qquad (I)$$

$$\frac{\partial F}{\partial a_k} = \cancel{2}\sum_i^{m} \Big(\bar{y}_i - \underbrace{a_0\sqrt{x} + \sum^{n} a_i (x_i)^i}_{y(x)}\Big) (x_i)^k = 0 \qquad (k)$$

Umstellen für LGS

$$\Big(a_0\sqrt{x} + \sum_i^{n} a_i (x_i)^i\Big) \cdot \cancel{\tfrac{1}{2}}\sqrt{x} = \sum^{m} \bar{y}_i \cdot \cancel{\tfrac{1}{2}}\sqrt{x} \qquad (I)$$

$$\underbrace{\Big(a_0\sqrt{x} + \sum_i^{n} a_i (x_i)^i\Big) \cdot (x_i)^k}_{M} = \underbrace{\sum^{m} \bar{y}_i (x_i)^k}_{B} \qquad (k) \qquad k = 1 \dots n$$

$$M \cdot A = B \qquad A \to \begin{pmatrix} a_0 \\ \vdots \\ a_n \end{pmatrix}$$

Gleichungssystem aufstellen:

kr ↓, kc → ; $kc=0$, $kc=1$, …, $kc=n$

$$\begin{array}{c} kr=0 \\ kr=1 \\ \vdots \\ kr=n \end{array} \begin{bmatrix} \boxed{\sqrt{x}\sqrt{x}} & \sum_i^{n} a_k (x_i)^{kc} \sqrt{x} & \dots \\ \sum_i^{m} \sqrt{x}\,(x_i)^{kr} & \sum (x_i)^{kr} (x_i)^{kc} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_n \end{bmatrix} = \begin{bmatrix} \sum_i^{m} y_i \sqrt{x} \\ \sum y_i (x_i)^{kr} \\ \vdots \end{bmatrix}$$