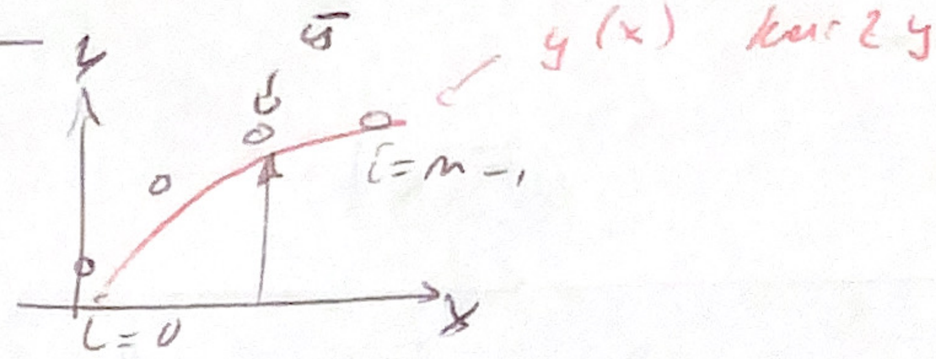
Aufgabe 5 - Approximation via Exponentansatz

$t \to x$; $f \to y$

$y(x) = a x^b + c$



(a) Gleichungssystem aufstellen

$\to$ Abweichende Fehlerquadrate

$$F(a,b,c) = \sum_{i=1}^{m} \left(\bar{y} - y(\bar{x})\right)^2 \to \text{minimieren}$$

$$\frac{\partial F}{\partial a} = 0 \quad , \frac{\partial F}{\partial b} = 0 \quad ; \frac{\partial F}{\partial c} = 0$$

LGS:

$$\left.\begin{aligned}
\frac{\partial F}{\partial a} &= -2\sum^{m} (\bar{y}_i - y_i)^2 \cdot \frac{\partial y}{\partial a} \\
&= -2\sum^{m} (\bar{y}_i - (a x_i^b + c)) \cancel{x^b} \; a && (I) \\
\frac{\partial F}{\partial b} &= -2\sum^{m} (\bar{y}_i - (a x_i^b + c)) \cdot \underbrace{a \cdot x_i^b \ln(x_i)}_{\frac{\partial y}{\partial b}} && (II) \\
\partial F &= -2\sum^{m} (\bar{y}_i - (a x_i^b + c)) \cdot \underbrace{1}_{\frac{\partial y}{\partial x}} && (III)
\end{aligned}\right\} \overset{!}{=} 0 \Rightarrow \text{Gleichungssystem}$$

b willkürlich $\to$ setze $x_i^b = \varphi$ $\to$ Gl I nicht mehr benötigt

$$\frac{\partial F}{\partial a} = \cancel{-2}\left(\sum^{m} \bar{y}\varphi_i - \sum^{m} a\varphi_i \cdot \varphi_i + c\varphi_i\right) = 0$$

$$\Rightarrow \sum^{m} \bar{y}\varphi_i = a\varphi_i\varphi_i + c\varphi_i$$

$$\frac{\partial F}{\partial c} \Rightarrow \sum^{m} \bar{y}_i - \sum^{m} (a\varphi_i + c) = 0$$

$$\Rightarrow \underbrace{\sum^{m} \bar{y}_i}_{b} = \underbrace{\sum^{m} (a\varphi_i + c)}_{A}$$

für $A\underset{\sim}{x} = b$

$x = \begin{pmatrix} a \\ c \end{pmatrix}$

$\to$ Matrix Form

$$\underbrace{\begin{bmatrix} \sum^{m} \varphi_i\varphi_i & \sum^{m} \varphi_i \\ \sum^{m} \varphi_i & \sum^{m} 1 \end{bmatrix}}_{M \to \text{Code}} \cdot \underbrace{\begin{bmatrix} a \\ c \end{bmatrix}}_{\text{KOEFF}} = \underbrace{\begin{bmatrix} \sum^{m} \bar{y}_i\varphi_i \\ \sum^{m} \bar{y}_i \cdot 1 \end{bmatrix}}_{B}$$

$\to$ solve via Jacobi Method

~~d) F(b) =~~

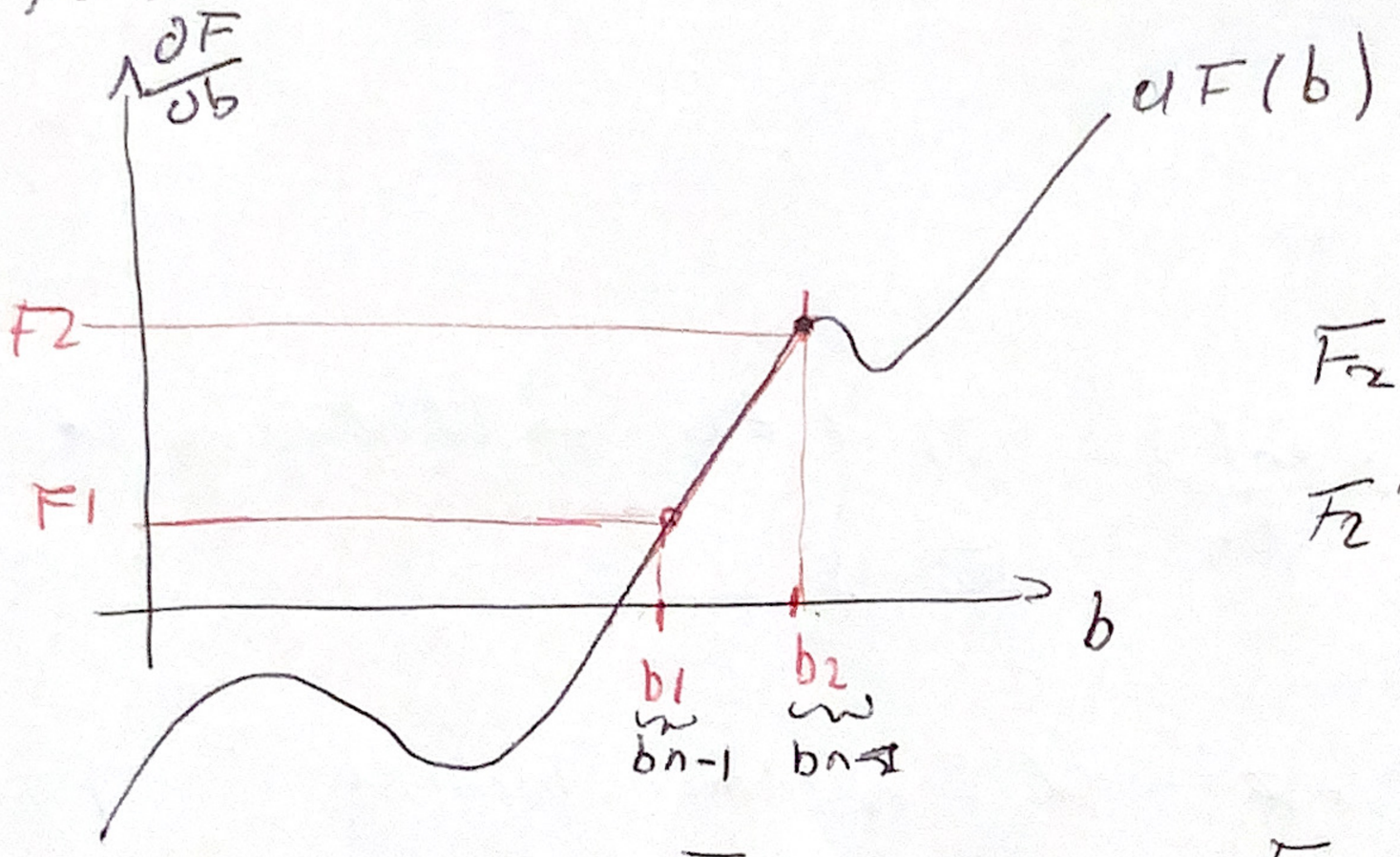
$$\frac{\partial F}{\partial b} = 0$$

$$\frac{\partial F}{\partial b} = \overbrace{\sum^{m} (\bar{y}_i - (a x_i^b + c)) \cdot \ln(x_i) \cdot x_i^b}^{dF(b)} \overset{!}{=} 0$$

a, c gegeben durch vorherige Bestimmung

Schätzung von b_1 und b_2 (Initiale Schätzung

Newton



F_2' über Differenzquotient

$$F_2' = \frac{F_2 - F_1}{b_2 - b_1}$$

$$b_{n+1} = b_2 - \frac{F_2}{F_2'} = b_2 - \frac{F_1}{F_2 - F_1}(b_2 - b_1).$$

↳ so lange bis $\|F_2\| <$ toleranz → sollte 0 werden