ORIE 5132: Pricing Analytics and Revenue Management
Assignment 1
Due on February 10th, 11:59pm (EST)

Notes

- Each assignment must be submitted on Gradescope by the due date.
- All questions in this homework must be individual work. You are allowed to discuss the assignment with others but please mention in your write-up if you have discussed the solution with someone.
- You should also submit your programming code (Python, R, Matlab, etc) along with the answers.

Problem 1 (Single Fare Class Overbooking). In class, we have seen a problem of finding optimal overbooking amount for single fare class. Suppose the capacity is C seats, the price per seat is fixed at p, the number of no-show customers is a random variable $Y \sim$ cdf $F(\cdot)$ and the number of overbooked tickets is q. Assume we are able to sell all $(C+q)$ seats for any q. The profit or the revenue function is as follows.

$$\Pi(q) = p * (C+q) - \gamma * E_Y[(q-Y)_+],$$

where $\gamma > p$, is the penalty per customer who can't get a seat. Suppose Y is a Poisson random variable with mean 5, $p = 100, \gamma = 300, C = 100$.

1. Compute the optimal overbooking limit, q^* that maximizes the expected profit.
2. For q^* computed in part (a), find the expected penalty paid due to overbooking.

Problem 2 (Callable Tickets). US Tennis Association (USTA) is trying to devise a strategy to sell tickets for the Men's Quarterfinals for US Open 2016 in New York. The stadium has a seating capacity of 20000. Since the quarterfinals are held on a weekday, the demand is quite uncertain (assume all quarter finals are held on a single day and there is a single ticket that grants admission for all four quarter finals). USTA plans to offer a low-price ticket of \$50 with a restriction that USTA can buy it back from the customer anytime at the price of \$75. USTA estimates that there is enough demand for the low-price tickets and any number will be sold. USTA also plans to offer a high-price ticket of \$100 that does not have any restrictions. However, demand for the high-price tickets is uncertain and USTA estimates that is uniformly distributed between 8000 and 17000. Assume that all the low-price demand arrives before the first high-price demand arrives and there is enough demand for low price tickets in the initial period that any number of low-price tickets can be sold. However, no low-price tickets can be sold after high price demand arrives.

1. What is the optimal number of tickets that USTA should offer at high-price initially to maximize the expected revenue from sales?
2. What is the expected revenue if USTA offers the high-price tickets as computed in part (1)?

Problem 3 (2-Fare Model). A hotel with 400 rooms is trying to optimize the protection levels for business travelers. Leisure travelers typically book in advance and we will assume that there are enough of them to take the room at the "leisure" price of \$160 per night. (To make things simple, we assume that everyone stays in this hotel just for one night.) The hotel can block some rooms (not make them available to leisure travelers who book in advance) and make them available only closer to the actual date at the business-traveler price of $200. We have one year of business demand data in the file *HotelProtectionLevelData.xls*.

1. Let us first treat all data the same (not adjusting the booking limit by days etc.). Suppose that the underlying distribution is normal with the empirical mean and empirical standard deviation. What is the protection level that you recommend for business travelers ?

2. What is the protection level for business travelers (still without distinguishing between days) but using the empirical distribution instead of assuming normality?

3. Next, consider making different decisions for different days. What is your recommended protection level for business travelers for a non-holiday Monday? Use the empirical distribution for this question.