Agent Based Model Framework for Quantifying the Propagation of Disinformation

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0.1 SEDPNR Model

By giving priority to the sentiment of rumours, we propose the SEDPNR (Susceptible-Exposed-Doubtful-Positively Infected-Negatively Infected-Restrained) model by splitting the conventional infected node based on the rumour sentiment along with adding a restrained state into the SEDIS model, taking into account the necessity of an "end state" for disease transmission while considering a single trending misinformation or fake news in online digital networks.

The Infected node is classified as Positively Infected and Negatively Infected based on the user's attitude whether the false information is viewed positively or negatively. The Restrained condition refers to people who have lost interest in the knowledge over time. Those who are no longer spreaders gradually fall into the restrained state. $S(t)$ denotes the individuals susceptible to the misinformation at time t, and $E(t)$ refers to exposed individuals who met the trending fake news/misinformation. A person exposed to the trending misinformation can become Infected or Doubter $D(t)$ depending upon his social and emotional intelligence at probability β_1 and β_2 respectively.

$$\beta_1 = \beta_1(\phi), \quad \beta_2 = \beta_2(\phi), \quad \beta_3 = \beta_3(\phi, \theta), \quad \beta_4 = \beta_4(\phi, \theta) \tag{1}$$

$$\lambda_1(t) = \lambda_0 + \rho\, e^{-\eta\, h}, \quad \lambda_2(t) = \lambda_0 + \rho\, e^{-\eta\, h} \tag{2}$$

1 depicts the SEDPNR model's discrete compartmental diagram.

This shows us that the model at figure 1 We can mathematically explain this model like:

$$\frac{ds}{dt} = \mu_1 e + \mu_2 d - \alpha s, \qquad \frac{de}{dt} = \alpha s - (\beta_1 + \beta_2 + \gamma + \mu_1)e,$$

$$\frac{dd}{dt} = \gamma e - (\beta_3 + \beta_4 + \mu_2)d, \qquad \frac{dp}{dt} = \beta_1 e + \beta_3 d - \lambda_1 p,$$

$$\frac{dn}{dt} = \beta_2 e + \beta_4 d - \lambda_2 n, \qquad \frac{dr}{dt} = \lambda_1 p + \lambda_2 n.$$

Regarding social network rumours and fake news, we ignore the word "Recovered" because it is essentially unattainable. An individual enters into the Restrained state only after becoming the spreader. This is because even though the Susceptible individual has returned from Doubter or Exposed state, he or she can become Infected (spreader) again if most of the community members share the misinformation and the topic attracts the community's interest positively. The principle of social reinforcement and membership closure plays a crucial role in driving a community to share fake news, even though the members within the community understand that the information they are about to share is false.

A distrustful individual may be more likely to dismiss rumours as false, even if they have no concrete evidence to support this belief. The distrustful members in the network do not spread rumours among themselves, but they are susceptible to infection if they encounter similar rumours. With newspapers, television channels, and fact-checking websites aggressively fighting against fake news, people are more likely to question the authenticity of the information. A person may choose to be in a doubter state and decide whether to accept or reject the rumour if the information loses clarity or authenticity. This aspect of human choice is unique to the Doubter condition. Considering α as the transition rate from the Susceptible to the Exposed state; β_1 and β_2 as the transition from the exposed state to the Doubter State and the Infected state, respectively; γ as the transition state from the Doubter state to the Infected State; λ as the transition state from the Infected state to the Restrained state; and μ_1 and μ_2 the transition rate to the Susceptible state from the Exposed and Doubter state; the model can be mathematically explained as;

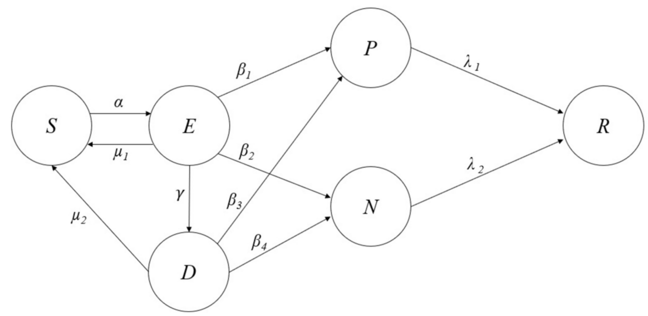

Figure 1: SEDPNR model's discrete compartmental diagram

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$$\frac{ds}{dt} = (\mu_1 e + \mu_2 d - \alpha s)(t), \qquad \frac{de}{dt} = \alpha s(t) - (\beta_1 + \beta_2 + \gamma + \mu_1)e(t),$$

$$\frac{dd}{dt} = \gamma e(t) - (\beta_3 + \beta_4 + \mu_2)d(t), \qquad \frac{dp}{dt} = (\beta_1 e + \beta_3 d - \lambda_1 p)(t),$$

$$\frac{dn}{dt} = (\beta_2 e + \beta_4 d - \lambda_2 n)(t), \qquad \frac{dr}{dt} = (\lambda_1 p + \lambda_2 n)(t).$$

In more simple terms, the transitions between states in our model closely mirror the behaviour observed in real social networks:

(1) 1.S to E (α): This represents users encountering misinformation for the first time, similar to seeing a shared post.

$$\alpha(t) = \alpha(\phi, \sigma)\, \theta\, (1 - e^{-\delta h}), \tag{3}$$

The 'Initial Appeal Score' of a claim C_i is a function of its emotional and cognitive appeals, weighted by their historical effectiveness. A higher $\Omega(C_i)$ score implies a higher probability of exposure. The initial psychographic appeal of a candidate claim C_i is scored as:

$$\Omega(C_i) = \sum_{j=1}^{k} w_j \cdot f_j(C_i) \tag{4}$$

- C_i: The candidate claim.
- $f_j(C_i)$: A function measuring the strength of cognitive or emotional appeal j in the claim.
- w_j: The learned weight representing the historical effectiveness of that appeal.
- k: The total number of appeals considered.

(2) E to D (γ): Some users, after initial exposure, become sceptical and start questioning the information, much like when social media users fact-check or ask for sources.The rate at which an exposed user transitions to a positively infected state, β_1, or a negatively infected state, β_2, can be modified by two key factors: semantic drift (ρ, measures how a claim's meaning changes across platforms, which can influence a user's decision to share it.) and social reinforcement (a user is more likely to adopt a belief state if their close connections already hold that belief).

$$\gamma(t) = \gamma + \kappa\, \frac{|\{\, j \in N(i) : D_j \}|}{|N(i)|} \tag{5}$$

The semantic drift ρ of a claim C_i between two platforms, p and q, is the cosine distance between the embeddings of its platform-specific versions, C_i^p and C_i^q:

$$\rho(C_i^p, C_i^q) = 1 - \frac{\vec{E}(C_i^p) \cdot \vec{E}(C_i^q)}{\|\vec{E}(C_i^p)\|_2 \|\vec{E}(C_i^q)\|_2} \tag{6}$$

- C_i^p, C_i^q: Versions of the same claim on platforms p and q, respectively.
- $\vec{E}(\cdot)$: The function that produces the embedding for a given text.

The transition rates can be enhanced as follows:

- $\beta_1(t) = \beta_1 \cdot \Pi(i, N_i) \cdot (1 - \rho), \quad \beta_2(t) = \beta_2 \cdot \Pi(i, N_i) \cdot (1 + \rho)$
- $\beta_3(t) = \beta_3 \cdot \Pi(i, N_i) \cdot (1 - \rho), \quad \beta_4(t) = \beta_4 \cdot \Pi(i, N_i) \cdot (1 + \rho)$

Where $\Pi(i, N_i)$ is a function of the user's neighbours N_i and their current states (P or N).
A user's individual γ is affected by how they perceive the information. Responses classified as 'Refute' or 'Attack' would increase the probability of a user transitioning to the Doubter state. The stance of a user response R_j is classified into a discrete set of categories:

$$\psi(R_j) \in \{\text{Support, Refute, Attack, Neutral}\} \tag{7}$$

The transition rate γ can be defined as a function of the stance classification:

$$\gamma(t) = \gamma \cdot \psi(R_j) \tag{8}$$

(3) E to P/N (β_1, β_2): This transition occurs when exposed users share the misinformation, either with a positive or negative sentiment, mirroring how people re- share posts with their commentary.

(4) D to P/N (β_3, β_4): Doubters may eventually choose a side and start spreading the information, similar to how users might initially be sceptical but later convince themselves and share. The intent ν behind an accusation of "fake news" A_k is determined by finding the most likely category l from a set of possible intents $\mathcal{L}$:

$$\nu(A_k) = \arg\max_{l \in \mathcal{L}} P(l|A_k) \tag{9}$$

- A_k: An instance of an accusation.
- $\mathcal{L}$: A set of intent categories, such as Authentic Disbelief, Delegitimization, and Trolling.
- $P(l|A_k)$: The probability of intent l given the accusation A_k.

(5) P/N to R (γ_1, γ_2): This represents users who stop spreading misinformation, perhaps due to fact-checking or platform interventions, or loss of interest analogous to users deleting posts or changing their stance. If a claim resurfaces, its score O would be high, suggesting that users are being reengaged with the content and thus delaying their transition to the restrained state.

The resurfacing score O for a claim C at time t is the maximum similarity between its current fingerprint and the fingerprint of any relevant prior claim seen at a time $\tau < t$:

$$O(C_t) = \max_{\tau < t} \{\varsigma(F(C_t), F(C_\tau))\} \tag{10}$$

- C_t: The current claim at time t.
- $\varsigma(\cdot,\cdot)$: A function that measures similarity between two claim fingerprints.
- $F(C_t)$: The fingerprint of claim C at time t.

The transition rates λ_1 and λ_2 can be modified as follows:

$$\lambda_1(t) = \lambda_1 \cdot (1 - \Upsilon), \quad \lambda_2(t) = \lambda_2 \cdot (1 - \Upsilon)$$

Spreaders of disinformation subtly manipulate people's cognitive biases to make the false information more emotional, and therefore easier to remember. A highly emotional or biased piece of content would manifest in the model as an increase in the transition rates $\beta_1$andβ_1 (the probability of moving from Exposed or Doubter to a Positive Spreader). Due to the fact that emotional things are 20% easier to remember, this content would also decrease the restrained rates λ_1 and λ_2 as it prolongs the loss of interest that will eventually happen.

For time t, the densities of all states can be represented as $S(t)$, $E(t)$, $D(t)$, $P(t)$, $N(t)$ and $R(t)$. We build upon the SENPDR Model by challenging the assumption that the probability that the chance of transferring from one state to another is equal.

$$\frac{ds}{dt} = \mu_1 e + \mu_2 d - \left[\alpha(\phi,\psi)\,\theta\,(1-e^{-\delta h})\right]s,$$

$$\frac{de}{dt} = \left[\alpha(\phi,\psi)\,\theta\,(1-e^{-\delta h})\right]s - \left[\beta_1(\phi)+\beta_2(\phi)+\gamma+\kappa\,\frac{|\{j\in N(i):D_j\}|}{|N(i)|}+\mu_1\right]e,$$

$$\frac{dd}{dt} = \left[\gamma+\kappa\,\frac{|\{j\in N(i):D_j\}|}{|N(i)|}\right]e - \left[\beta_3(\phi,\theta)+\beta_4(\phi,\theta)+\mu_2\right]d,$$

$$\frac{dp}{dt} = \beta_1(\phi)e + \beta_3(\phi,\theta)d - \left[\lambda_0+\rho\,e^{-\eta h}\right]p,$$

$$\frac{dn}{dt} = \beta_2(\phi)e + \beta_4(\phi,\theta)d - \left[\lambda_0+\rho\,e^{-\eta h}\right]n,$$

$$\frac{dr}{dt} = \left[\lambda_0+\rho\,e^{-\eta h}\right](p+n).$$

According to network theory, a network with all its nodes being the same kind and possessing the same properties is called homogenous. The number of nodes with which each node has direct interaction is about the same, and this number follows the Poisson distribution. Under this assumption, the homogeneous network discusses the condition that $S(t), E(t), D(t), P(t), N(t)$ and $R(t)$ signify the population densities at time t that are susceptible, exposed, doubters, and infected, respectively. They fulfil the requirement:

$$S(t)+E(t)+D(t)+P(t)+N(t)+R(t)=N_t(t) \tag{11}$$

where N_t is the total population within the network community at time t. Based on the equations to 5**??**to **??**; the algorithm for the model can be written as:

0.2 Equation 1.

The time complexity of the SEDPNR Model algorithm is $O(t \times n)$, where t is the number of time intervals and n is the number of nodes in the network. This complexity arises from the nested loop structure, where the outer loop iterates t times, and for each iteration, we process all n nodes in the network. The first stage in constructing a scale-free network and initializing states has a complexity of O(N + E), where N is the number of nodes and E is the number of edges. However, the main loop structure precedes this stage in most real-world settings. It's important to note that the algorithm's time complexity is predictably linear for both the number of time intervals and the number of nodes, ensuring a consistent performance. The actual runtime, however, can be significant for large networks or extended time periods. The space complexity is O(n), as we need to store the state for each node.

1. The rapid spread of misinformation through exposure and sharing (S to E to P/N).
2. The role of doubt and scepticism in potentially slowing spread (E to D).
3. The impact of emotional valence on sharing behaviour (separate P and N states).
4. The potential for users to eventually stop spreading misinformation (P/N to R).

These dynamics align with observed behaviours on platforms like Twitter and Facebook, where misinformation can quickly go viral, face pushback from sceptical users, and eventually be contained through user awareness and platform interventions. The breakdown of assumptions underpinning the SEDPNR model is as follows;

1. Limited Attention and Processing: When people come across false information, they may fail to critically assess it or pay careful attention to it, which puts them in the "Exposed" condition.
2. Cognitive Biases: The Doubter category illustrates how people may believe or mistrust the message due to pre-existing biases or incomplete knowledge.
3. Dynamic Belief States: The model acknowledges the potential of fact-checking or shifting opinions and permits people to go from accepting false information based on the sentiment or rejecting it.
4. Behavioural Change: The "Restrained" category suggests awareness-raising initiatives or interventions that reduce people's propensity to disseminate false information.

0.3 Properties of the Model

This section investigates the SEDPNR model's mathematical features, including existence, stability, and long-term behaviour. We will first establish the existence of solutions for the system, introduce the basic reproduction number R_0, prove positivity and validity of the model, and finally analyse the stability of the rumour-free equilibrium.

Basic Reproduction Number. In epidemic modelling, the basic reproduction number R_0 is a significant metric reflecting the average number of new infections induced by a single infectious individual in a fully susceptible network. In other words, it estimates a disease's potential spread within a population. In simple words, the average number of people infected from infection from a single infected person in a community where everyone is susceptible is referred to as the basic reproduction number. The user's behaviour can influence the value of R_0 in social networks, the network's structure, and the features of rumours.

R_0 is essential in determining an outbreak's severity and the interventions' efficacy. If R_0 is less than one, it indicates that each infectious person is likely to infect fewer than one person on average, implying that the outbreak will eventually die out. If $R_0$is more significant than one, the disease is more likely to spread quickly across the population and could produce an epidemic. To find the basic reproduction number R_0 for the given model, we first identify the transmission and recovery terms in the system of equations. Then, we construct the next-generation matrix F and the diagonal matrix $V$of recovery rates . Transmission terms:

$$\frac{ds}{dt} = \mu_1 e, \quad \frac{de}{dt} = \alpha s$$

$$\frac{dd}{dt} = \gamma e \quad \frac{dp}{dt} = \beta_1 e \quad \frac{dn}{dt} = \beta_2 e$$

Recovery terms:

$$\frac{dr}{dt} = \lambda_1 p + \lambda_2 n$$

Next-generation matrix F and the diagonal matrix of recovery rates V:

$$F = \begin{bmatrix} 0 & 0 & 0 & \beta_1 & \beta_2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad V = \mathrm{diag}(\lambda_1, \lambda_2, 0, 0, 0)$$

V is a diagonal matrix with the values λ_1 and λ_2 on the main diagonal and zeros everywhere else. The basic reproduction R_0 number is the spectral radius of FV^{-1} . After calculations, we find:

$$R_0 = \max\left\{\frac{\beta_1}{\lambda_1}, \frac{\beta_2}{\lambda_2}\right\}.$$

So, the basic reproduction number R_0 for this social network epidemiological model is the maximum of the ratios of the transmission rates to the recovery rates for the positively and negatively infected compartments. In simple terms, this means that this equation tells us "leverage points" for modelling a systems behaviour. The equation shows that the most effective countermeasures are those that either decrease the infection rate (β) or increase the rate of losing interest (λ).

Stability of the model and conditions under which the rumour dies or persists indefinitely

We can linearize the system of differential equations around the steady state when the number of exposed persons is zero in order to examine the stability of the rumour propagation model. The following matrix can be used to represent the linearized system:

$$\begin{bmatrix} -\alpha & \mu_1 & \mu_2 \\ \alpha & -\beta_1 - \beta_2 - \gamma - \mu_1 & 0 \\ 0 & \gamma & -\beta_3 - \beta_4 - \mu_2 \end{bmatrix}$$

An eigenvalue of a square matrix A is a unique scalar value that, when multiplied by a non-zero vector x, produces another vector that points in the same direction (or a scalar multiple of that direction), albeit possibly with a different magnitude. Eigenvalues of the square matrix A are denoted by the Greek letter lambda, λ. This may be stated mathematically as $Ax = \lambda x$. In other words, an eigenvalue is a vector that scales a non-zero vector without affecting its basic orientation. The eigenvalues of this matrix determine the system's stability. If all eigenvalues have negative real components, the system is stable, and the rumour will ultimately die. However, if at least one eigenvalue has a positive real part, the rumour will persist endlessly, indicating that the system is unstable.

We can use the following equation to examine the matrix's eigenvalues:

$$\det(\delta I - A) = 0$$

where I is the identity matrix, A denotes the matrix representing the linearized system, and δ is the eigenvalue. Solving this equation for δ will give us the eigenvalues of the matrix. For the system defined, the eigenvalues of the matrix can be represented as:

$$\delta_1 = \frac{-\alpha + \sqrt{\alpha^2 + 4\mu_1(\beta_1 + \beta_2 + \gamma + \mu_1)}}{2}$$

$$\delta_2 = \frac{-(\beta_3 + \beta_4 + \mu_2) + \sqrt{(\beta_3 + \beta_4 + \mu_2)^2 + 4\gamma(\beta_1 + \beta_2 + \gamma + \mu_1)}}{2}$$

$$\delta_3 = \frac{-(\beta_3 + \beta_4 + \mu_2) - \sqrt{(\beta_3 + \beta_4 + \mu_2)^2 + 4\gamma(\beta_1 + \beta_2 + \gamma + \mu_1)}}{2}$$

If and only if all three eigenvalues are negative, the rumour will ultimately die; which means that:

$$\alpha - \sqrt{\alpha^2 + 4\mu_1(\beta_1 + \beta_2 + \gamma + \mu_1)} > 0$$

$$\beta_3 + \beta_4 + \mu_2 - \sqrt{(\beta_3 + \beta_4 + \mu_2)^2 + 4\gamma(\beta_1 + \beta_2 + \gamma + \mu_1)} > 0$$

$$\beta_3 + \beta_4 + \mu_2 + \sqrt{(\beta_3 + \beta_4 + \mu_2)^2 + 4\gamma(\beta_1 + \beta_2 + \gamma + \mu_1)} > 0$$

These conditions stand in for the limits of the stable area in the proposed model's parameter space.

We can ascertain the effect of lower contact rates on rumour transmission by modelling the rumour's spread with various values of α. The social distancing strategies that result in the fewest number of people being exposed overall or the quickest time for rumours to circulate will be the most successful.

0.4 Impact of disinformation and distrust on rumour propagation

We have expanded on Golopan et al's work to include characteristics linked to information sharing and trust to examine the influence of disinformation and mistrust of public health authorities on rumour dissemination. For instance, we may change the transition probabilities to account for the impact of false information on rumour susceptibility or create a new state to represent people who do not trust public health authorities.

We may replicate the spread of the rumour under various circumstances and compare the outcomes once the model has been expanded to include disinformation and mistrust. This will enable us to ascertain how these variables impact the spread of rumours and pinpoint populations more or less vulnerable to false information.

An example of how to change the model to include mistrust of public health authority is explained as follows:

$$\frac{ds}{dt} = \mu_1 e + \mu_2 d - \alpha s, \qquad \frac{de}{dt} = \alpha s - (\beta_1 + \beta_2 + \gamma + \mu_1)e \tag{12}$$

(13)

$$\frac{dv}{dt} = \nu s - \tau v, \qquad \frac{dd}{dt} = \gamma e - (\beta_3 + \beta_4 + \mu_2)d, \tag{14}$$

(15)

$$\frac{dp}{dt} = \beta_1(1-\tau)ve + \beta_3 d - \rho_1 p, \qquad \frac{dn}{dt} = \beta_2(1-\tau)ve + \beta_4 d - \rho_2 n \tag{16}$$

where τ is the rate at which people grow distrustful, and v is the total number of distrustful people. In the Equation (14) and (16), the factor $(1-\tau)$ denotes the reduced sensitivity of distrustful people to disinformation.

We can ascertain the effect of mistrust on rumour propagation by modelling the spread of the rumour with various values of τ. This method can also be expanded to consider additional elements like the influence of social media influencers or the frequency of false information sources linked to trust and information sharing.

Finding equilibrium points

Equilibrium points in mathematical models, particularly differential equation models, are system states where some variables stay constant across time. An equilibrium point is a specific range of values for the model's variables at which their rates of change approach zero. It is sometimes referred to as a steady state or a critical point. Put another way, all of the system's variables cease to change, and the system stays in a steady state at the equilibrium point. To determine the equilibrium points, we must first set all of the rates of change in the system of differential equations of (5)–(10) to zero and then solve the resulting system of equations:

$$\mu_1 e + \mu_2 d - \alpha s = 0 \qquad \alpha s - (\beta_1 + \beta_2 + \gamma + \mu_1)e = 0$$

$$\gamma e - (\beta_3 + \beta_4 + \mu_2)d = 0 \qquad \beta_1 e + \beta_3 d - \rho_1 p = 0$$

$$\beta_2 e + \beta_4 d - \rho_2 n = 0 \qquad \rho_1 p + \rho_2 n = 0$$

We can identify the model's equilibrium points by solving this set of equations. From Eq **??**, we can solve for e as:

$$e = \frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}$$

Substituting this expression for e into the Eqs. (**??**), (**??**), and (**??**), we get:

$$\mu_1\left(\frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}\right) + \mu_2 d - \alpha s = 0$$

$$\gamma\left(\frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}\right) - (\beta_3 + \beta_4 + \mu_2)d = 0$$

$$\beta_1\left(\frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}\right) + \beta_3 d - \rho_1 p = 0$$

Now, from (**??**), we can solve for p as:

$$p = -\frac{\rho_2 n}{\rho_1}$$

Substituting this expression for p into the Eq. (**??**), we get:

$$\beta_2 e + \beta_4 d - \rho_2 n = 0$$

Adding these equations together:

$$\beta_2 e + \beta_4 d + \rho_1 n = 0$$

$$(\beta_2 + \beta_4)e + (\beta_4 + \mu_2)d = 0$$

Substituting the expression for e from the previous steps:

$$(\beta_2 + \beta_4)\left(\frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}\right) + (\beta_4 + \mu_2)d = 0$$

Solving for d, we get:

$$d = -\frac{(\beta_2 + \beta_4)(\alpha s)}{(\beta_4 + \mu_2)(\beta_1 + \beta_2 + \gamma + \mu_1)}$$

Substituting this expression for d into the expression for p from previous steps, we get:

$$p = \frac{\rho_2(\beta_2 + \beta_4)(\alpha s)}{(\beta_4 + \mu_2)(\beta_1 + \beta_2 + \gamma + \mu_1)\rho_1}$$

Substituting this expression for p into the Eq. (**??**), we get:

$$\beta_1 e + \beta_3 d - \rho_1 p = 0$$

$$\beta_1\left(\frac{\alpha s}{\beta_1 + \beta_2 + \gamma + \mu_1}\right) + \beta_3 d - \rho_1 p = 0$$

Finally, upon solving for s, we get:

$$s = \frac{\mu_1(\beta_3 + \beta_4 + \mu_2)(\rho_1 + \rho_2)}{\mu_1\beta_3 + \mu_2\rho_1} \tag{17}$$

Now; upon substituting the expressions for e, d, p and n from the previous steps into the original system of equations, we get:

$$e = \frac{\alpha(\beta_3 + \beta_4 + \mu_2)(\rho_1 + \rho_2)}{\alpha\beta_3 + (\beta_1 + \beta_2 + \gamma + \mu_1)\rho_1} \qquad d = \frac{\gamma(\rho_1 + \rho_2)(\beta_1 + \beta_2 + \gamma + \mu_1)}{\gamma\beta_1 + (\beta_3 + \beta_4 + \mu_2)\rho_2}$$

$$p = \frac{(\mu_1\beta_3 + \mu_2\rho_1)(\rho_2\, p + \rho_1\, n)}{\beta_1\,\rho_2} \qquad n = \frac{(\beta_1 + \beta_2 + \gamma + \mu_1)\,\rho_2\,(\rho_1\, p + \rho_2\, n)}{\beta_2\,\rho_1}$$

which are the equilibrium points of the system for a clustered network. This equilibrium point represents a state where the rumour is persistently circulating in the population. The values of the variables in this equilibrium point will depend on the values of the model parameters.

We may determine the effect of network clustering on rumour prevalence by comparing the values of p in the two equilibrium points. If the value of p is greater in the clustered network's equilibrium point, then clustering is thought to aid rumour spread. In contrast, if the value of p is smaller in the clustered network's equilibrium point, clustering is thought to impede rumour spread.

- If $\beta_1 > \beta_4, \beta_2$, then p will be higher in the equilibrium point for the clustered network, indicating that clustering facilitates rumour propagation among positively believing individuals.
- If $\beta_1 < \beta_4, \beta_2$, then p will be lower in the equilibrium point for the clustered network, indicating that clustering hinders rumour propagation among positively believing individuals.
- Similarly, if $\beta_2 > \beta_3, \beta_1$, then n will be higher in the equilibrium point for the clustered network, indicating that clustering facilitates rumour propagation among negatively believing individuals.

- If $\beta_2 < \beta_3, \beta_1$, then n will be lower in the equilibrium point for the clustered network, indicating that clustering hinders rumour propagation among negatively believing individuals.

This means that if the infection rate within a belief group is higher than the overall network rate, clustering can amplify the spread of disinformation within that specific group. This helps us understand social network structure's potential influence on spreading disinformation based on the underlying dynamics of belief-specific infection rates. This is because the parameters β_1 and β_2 represent the rates at which exposed individuals convert positively and negatively believing individuals, respectively. Similarly, β_3 and β_4 represent how distrustful individuals convert to positively and negatively believing individuals.

A system is considered stable when it maintains its proximity to the equilibrium point across time, having started from a condition that is near it. The system won't diverge greatly, even in response to minor perturbations. An equilibrium point is considered asymptotically stable if the system, from any nearby state, stays near and progressively moves closer to the equilibrium point over time. The system eventually approaches and stays arbitrarily near the equilibrium point.

0.5 Theorem 2

The SEDPNR model is globally asymptotically stable.

0.5.1 Proof. To establish the global asymptotic stability of the SEDPNR model, we introduce a Lyapunov functional and prove its monotonicity along the system's trajectories. A Lyapunov functional is an effective tool for evaluating the stability of a dynamic system. A Lyapunov function is defined on the state space of a dynamical system and is a scalar function, which means it produces a single numerical value. All different setups or states that the system may be in are represented by this state space. The designated Lyapunov functional for the SEDPNR model can be written as:

$$V\big(s, e, d, p, n, r\big) = s^2 + e^2 + d^2 + p^2 + n^2 + r^2. \tag{18}$$

By examining the derivative of the Lyapunov functional concerning time and replacing the equations for the rates of change in our SEDPNR model, we can demonstrate that this functional decreases along the model's trajectories.

$$\begin{aligned} V'\big(s, e, d, p, n, r\big) &= 2\Big[\, s\big(\mu_1 e + \mu_2 d - \alpha s\big) + e\big(\alpha s - (\beta_1 + \beta_2 + \gamma + \mu_1)e\big) \\ &\quad + d\big(\gamma e - (\beta_3 + \beta_4 + \mu_2)d\big) + p\big(\beta_1 e + \beta_3 d - \rho_1 p\big) \\ &\quad + n\big(\beta_2 e + \beta_4 d - \rho_2 n\big) + r\big(\rho_1 p + \rho_2 n\big) \,\Big] \\ &= 0 \end{aligned} \tag{19}$$

Since the derivative of the Lyapunov functional is zero, we can infer that the functional maintains a constant value along the trajectories of the SEDPNR model. This indicates that the SEDPNR model exhibits global asymptotic stability, signifying that the system will ultimately reach an equilibrium where the proportions of susceptible, exposed, doubtful, positively infected, negatively infected, and restrained individuals will remain unchanged.

Require: Probability of Susceptible to Exposed state α
Probability of Exposed to Positively Infected state β_1
Probability of Exposed to Negatively Infected state β_2
Probability of Exposed to Doubter state γ
Probability of Doubter to Positively Infected state β_3
Probability of Doubter to Negatively Infected state β_4
Probability of Exposed to Susceptible state μ_1
Probability of Doubter to Susceptible state μ_2
Probability of Positively Infected to Restrained state λ_1
Probability of Negatively Infected to Restrained state λ_2
State of nodes: state
Node of susceptible state: S
Node of exposed state: E
Node of doubter state: D
Node of positively infected state: P
Node of negatively infected state: N
Node of restrained state: R
Ensure: State of nodes after time interval t:state

Generating scale free network S=V,E, adjacent matrix A
Initialization:original state= [S,E,D,P,N,R]
while interval < t **do**
  for $i = 1 : n$ **do**
    Switch state[i]
      case S: transfer to E with α
      case E: transfer to P with β_1, to N with β_2, to D with γ and to S with μ_1
      case D: transfer to P with β_3, to N with β_4 and to S with μ_2
      case P: transfer to R with λ_1
      case N: transfer to R with λ_2
    end**Switch**
  end for
end while
Return state for each node