# A Cognitively Plausible, Game-Theoretic Framework for Simulating Disinformation Propagation with Fractional-Order Dynamics

Feargal Browne Ashkan Samali St. Vincent's Castleknock College Dublin, Ireland

#### **ACM Reference Format:**

gent-Based Modeling, Disinformation, Fractional Calculus, Evolutionary Game Theory, Quantum Cognition, Complex Systems

#### 0.1 Introduction

The digital information ecosystem is a complex adaptive system where narrative and belief co-evolve. Traditional epidemiological models, while useful, often oversimplify agent behaviour, while purely rational choice models may neglect the stochastic and path-dependent nature of large-scale social propagation. To address these limitations, we propose a unified framework that bridges these scales. Our model integrates a sophisticated cognitive architecture for individual agents within a game-theoretic strategic environment, whose collective actions drive a system-level propagation model endowed with long-memory properties via fractional calculus. This provides a more holistic and mechanistically detailed simulation of disinformation dynamics.

#### 0.2 Integrated Model Framework

The model is a hybrid architecture composed of four interacting layers: the agent's cognitive-strategic core, the evolutionary dynamics of the population, the multiplex network environment, and the system-level propagation physics.

- 0.2.1 1. Agent Cognitive and Strategic Architecture. Each agent i is an autonomous entity endowed with a complex internal state, including quantum-like beliefs and platform-specific reputations.
- 0.2.2 2. Quantum Cognition for Belief States. An agent's belief regarding a piece of information is not a classical probability but a quantum state vector in a 2D Hilbert space, representing a superposition of 'Skeptic' and 'Believer' states.

$$|\psi_i(t)\rangle = \alpha_i(t)|S\rangle + \beta_i(t)|B\rangle \tag{1}$$

- $|\psi_i(t)\rangle$ : The belief state of agent *i* at time *t*.
- |S>, |B>: The basis states representing pure 'Skeptic' and 'Believer'.

•  $\alpha_i(t)$ ,  $\beta_i(t)$ : Complex probability amplitudes, where  $|\alpha_i(t)|^2 + |\beta_i(t)|^2 = 1$ .

Exposure to information acts as a unitary operator (a rotation matrix) on this state.

$$|\psi_i(t+1)\rangle = \frac{U_{\text{info}}|\psi_i(t)\rangle}{\|U_{\text{info}}|\psi_i(t)\rangle\|}$$
 (2)

- *U*<sub>info</sub>: A 2 × 2 unitary matrix representing the influence of an information piece (e.g., a fact-check rotates the vector towards |S⟩).
- 0.2.3 3. Reputation and Utility-Based Decisions. Agents maintain a reputation on each platform within the multiplex network and make sharing decisions by calculating expected utility.

$$R_{i,p}(t+1) = \lambda R_{i,p}(t) + (1-\lambda) \cdot \mathbb{I}(\operatorname{succ}_{i,p})$$
(3)

- $R_{i,p}(t)$ : Reputation of agent i on platform p at time t.
- $\lambda$ : A persistence parameter for reputation decay.
- I(succ<sub>i,p</sub>): An indicator function, equal to 1 if agent i's action on platform p was successful (e.g., high engagement, not debunked), and 0 otherwise.

The decision to share is governed by a utility function that balances potential gains and losses, modulated by the platform's moderation policy.

$$U_{i,p}(a_s) = \mathcal{P}(R_{i,p}, S_k) - C(R_{i,p}, M_p)$$
 (4)

- *U<sub>i,p</sub>(a<sub>s</sub>)*: The expected utility for agent *i* on platform *p* of taking sharing action *a<sub>s</sub>*.
- $\mathcal{P}$ : The potential payoff function, dependent on reputation  $R_{i,p}$  and agent strategy  $S_k$ .
- C: The potential cost function, dependent on reputation and the platform's moderation level  $M_p$ .
- 0.2.4 Evolutionary Game Dynamics. Agent strategies are not fixed but co-evolve through social learning. We model this using replicator dynamics, a cornerstone of Evolutionary Game Theory.

$$x_k(t+1) = x_k(t) \frac{\pi_k(t)}{\bar{\pi}(t)}$$
(5)

- $x_k(t)$ : The proportion of the agent population using strategy k at time t.
- • π<sub>k</sub>(t): The average payoff (fitness) for strategy k at time t, derived from outcomes like reputation change.
- $\bar{\pi}(t)$ : The average payoff of the entire population,  $\sum_{i} x_{i}(t)\pi_{i}(t)$ .

0.2.5 System-Level Propagation Dynamics. The aggregate effect of agent decisions is modelled using a fractional-order SEDPNR (Susceptible, Exposed, Doubtful, Positively-Infected, Negatively-Infected, Recovered) system.

0.2.6 Fractional Predictor-Corrector Method. To capture the long-memory effects inherent in social systems, we solve the system of FDEs using the high-accuracy Adams-Bashforth-Moulton Predictor-Corrector (PECE) scheme. The state of each agent evolves according to:

$$Ct_0 D_t^{\alpha} y(t) = f(t, y(t)) \tag{6}$$

- $Ct_0D_t^{\alpha}$ : The Caputo fractional derivative of order  $\alpha$ .
- y(t): The vector of an agent's compartmental probabilities (S, E, D, P, N, R).
- f(t, y(t)): The function governing the system's dynamics, including the force of infection.

The predictor step  $(y^P)$  estimates the next state:

$$y^{P}(t_{k+1}) = y(t_0) + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{k} a_{j,k} f(t_j, y(t_j))$$
 (7)

- $y^P(t_{k+1})$ : The predicted state vector at the next time step.
- $\Gamma(\alpha)$ : The Gamma function.
- a<sub>j,k</sub>: The Adams-Bashforth convolution weights, which depend on the history of the system.

The corrector step refines the prediction:

$$y(t_{k+1}) = y(t_0) + \frac{h^{\alpha}}{\Gamma(\alpha+1)} \Big( f(t_{k+1}, y^P(t_{k+1})) + \sum_{i=0}^k b_{j,k} f(t_j, y(t_j)) \Big)$$
(8)

- $y(t_{k+1})$ : The final, corrected state vector.
- *h*: The time step size.
- $b_{j,k}$ : The Adams-Moulton convolution weights.

0.2.7 Meta-Adaptation of System Memory. The fractional order  $\alpha$  is not static. It dynamically adapts based on the global state of the system, representing an evolution of the system's own memory properties.

$$\alpha(t) = \alpha_{\text{max}} - (\alpha_{\text{max}} - \alpha_{\text{min}}) \cdot \tanh(c \cdot V(t))$$
 (9)

- $\alpha(t)$ : The effective fractional order of the system at time t.
- a<sub>min</sub>, a<sub>max</sub>: The minimum and maximum possible values for the fractional order.
- V(t): A measure of system polarization or variance in agent beliefs at time t.
- c: A sensitivity parameter.

#### 0.3 Model Framework

Our framework synthesizes system dynamics with agent-level heterogeneity. The core of the model is a system of fractional-order differential equations governing the state transitions of each agent, situated within a complex network environment. The parameters

for these equations are not assumed but are explicitly derived from empirical data.

#### 0.4 The Fractional-Tsallis SEDPNR Model

We extend classical compartmental models by introducing fractionalorder derivatives to capture long-range temporal memory and a Tsallis q-exponent to model non-extensive statistical effects in social interactions. The state of each agent i is a vector of probabilities across six compartments: Susceptible (S), Exposed (E), Doubtful (D), Procrastinating-Infected (P), Non-Adopting-Infected (N), and Recovered (R).

$${}^{C}D_{t}^{\alpha_{S}}S_{i}(t) = -\sum_{j \in \mathcal{N}(i)} \beta_{ij} \left[ S_{i}(t) \right]^{q} \left[ F_{ij}(t) \right]^{q}$$
 (10)

- $S_i(t)$ : The probability that agent i is in the Susceptible state at time t.
- ${}^CD_t^{\alpha_S}$ : The Caputo fractional derivative of order  $\alpha_S$ , representing the memory of the susceptible state.
- $\mathcal{N}(i)$ : The set of neighbours of agent i in the social network.
- $\beta_{ij}$ : The transmission intensity from agent j to agent i.
- $F_{ij}(t)$ : The effective infectiousness exerted by neighbour j upon agent i.
- q: The Tsallis exponent, modelling non-extensive (fractal) coupling in social interactions.

$${}^{C}D_{t}^{\alpha_{E}}E_{i}(t) = \sum_{i \in N(i)} \beta_{ij} [S_{i}(t)]^{q} [F_{ij}(t)]^{q} - \sigma_{i} E_{i}(t) \quad (11)$$

- $E_i(t)$ : The probability that agent i is in the Exposed state.
- α<sub>E</sub>: The fractional order governing the dynamics of the exposed state.

$${}^{C}D_{t}^{\alpha_{D}}D_{i}(t) = \sigma_{i}E_{i}(t) - \rho_{i}D_{i}(t)$$
(12)

- $D_i(t)$ : The probability that agent i is in the Doubtful or deciding state.
- $\rho_i$ : The agent-specific rate of decision resolution from the Doubtful state.

$${}^{C}D_{t}^{\alpha_{P}}P_{i}(t) = p_{i}^{(D \to P)} \rho_{i} D_{i}(t) - \gamma_{i}^{(P \to R)} P_{i}(t) \qquad (13)$$

- $P_i(t)$ : The probability that agent i is in the Procrastinating-Infected state (believes but does not act).
- $p_i^{(D\to P)}$ : The probability that agent *i* transitions to the Procrastinating state.
- γ<sub>i</sub><sup>(P→R)</sup>: The agent-specific rate of transition from Procrastinating to Recovered.

$${}^{C}D_{t}^{\alpha_{N}}N_{i}(t) = (1 - p_{i}^{(D \to P)}) \rho_{i} D_{i}(t) - \eta_{i} N_{i}(t)$$
 (14)

- $N_i(t)$ : The probability that agent i is in the Non-Adopting-Infected state (actively disbelieves/resists).
- $\eta_i$ : The rate at which a non-adopter transitions to the Recovered state.

$${}^{C}D_{t}^{\alpha_{R}}R_{i}(t) = \gamma_{i}^{(P \to R)} P_{i}(t) + \eta_{i} N_{i}(t)$$

$$(15)$$

- $R_i(t)$ : The probability that agent i is in the Recovered or stifler state
- $\alpha_R$ : The fractional order for the recovery dynamics.

The effective infectiousness term  $F_{ij}(t)$  is a weighted sum of the neighbour's infectious states.

$$F_{ij}(t) = w_E E_j(t) + w_D D_j(t) + w_P P_j(t)$$
 (16)

w<sub>E</sub>, w<sub>D</sub>, w<sub>P</sub>: Calibrated global hyper parameters for the relative infectiousness of the E, D, and P states.

#### 0.5 Agent Heterogeneity via Psychometric Mapping

The agent-specific parameters ( $\beta_i$ ,  $\sigma_i$ ,  $\rho_i$ , etc.) are not arbitrary but are derived from empirical data through a multi-stage pipeline.

- Survey Data Collection: A survey instrument (e.g., IPIP-50) is administered to a human subject pool to collect data on Big Five personality traits, network size proxies, and behavioural indicators.
- (2) **Item Response Theory (IRT):** A Graded Response Model is used to convert ordinal survey responses into continuous latent trait scores for each participant.
- (3) **Individual Parameter Contribution (IPC) Regression:** A regression model (e.g., using a log-link function) is fitted to map the latent trait scores  $\mathbf{x}_i$  to the behavioural parameters for the ABM.

$$\beta_i = \exp(\mathbf{w} \cdot \mathbf{x}_i + c) \tag{17}$$

- $\beta_i$ : The transmission propensity for agent i.
- $\mathbf{x}_i$ : The vector of latent trait scores (e.g., Big Five) for agent i.
- w: A vector of regression weights indicating the influence of each trait.
- *c*: The intercept term.

## 0.6 Network Architecture: Fractal Multiplex Environment

Agents interact on a multiplex network designed to reflect realworld information ecosystems.

- Layer 1 (Social Graph): A scale-free network with fractal community structure, generated using a hierarchical Stochastic Block Model (SBM) with power-law distributed community sizes. This captures the self-similar social structures observed in society.
- Layer 2 (Authoritative Graph): A sparse, centralized network representing connections to authoritative sources (e.g., fact-checkers, official news).

The transmission intensity between agents is a function of the agent's intrinsic propensity and the weight of their connection.

$$\beta_{ij} = \beta_i \cdot w_{ij} \tag{18}$$

 w<sub>ij</sub>: The edge weight between agents i and j, determined by factors like tie strength and platform context.

#### 0.7 Calibration and Uncertainty Quantification

The model's global hyper parameters  $(\{q, \alpha_*, w_*\})$  are calibrated against real-world time-series data using a two-phase process.

- (1) Global Search with Genetic Algorithm (GA): A GA is employed to find optimal parameter sets by minimizing the Dynamic Time Warping (DTW) distance between simulated and empirical data curves.
- (2) Bayesian Refinement: Following the GA search, Bayesian methods (e.g., MCMC) are used to estimate the posterior distributions of the parameters, providing full credible intervals and capturing parameter uncertainty. Priors are informed by related literature.

To assess model robustness, we perform Uncertainty Quantification (UQ) and Sensitivity Analysis (SA) by running large ensembles of the simulation. We use Sobol or Morris methods to identify the parameters that contribute most to the variance in the model's output.

#### 0.8 Policy and Intervention Experiments

The calibrated framework serves as a digital laboratory for testing counter-disinformation strategies.

- Targeted Hub Suppression: We assess the efficacy of interventions that target high-centrality nodes (hubs) versus random suppression. Fractal networks are known to be resilient to random attacks but vulnerable to targeted ones.
- Early Refutation Impulse: We model fact-checking as a time-dependent forcing term that multiplicatively reduces  $\beta_{ij}$  and test the impact of varying the intervention's timing and reach across the multiplex layers.
- Personality-Targeted Messaging: Leveraging the IPC regression results, we simulate interventions tailored to specific personality profiles (e.g., high Neuroticism, low Conscientiousness) and measure their effect on population-level outcomes.

#### 1 FORMALIZED GAME-THEORETIC SEDPNR MODEL

# **1.1** Repeated Game: Reputational Dynamics Players:

- Dis-informant (D)
- Fact-Checker (F)
- Consumer (C)
- Platform (P)

#### State Variables:

- Dis-informant's Reputation:  $R_D(t) \in [0, 1]$
- Fact-Checker's Reputation:  $R_F(t) \in [0, 1]$
- Consumer's Belief:  $\pi_C(t) = P(T_S|O(t))$

**Discount Factor:**  $\delta \in (0, 1)$ 

1.1.1 Dis-informant's Utility. The dis-informant's utility is the discounted sum of per-period payoffs.

$$U_D = \sum_{t=0}^{\infty} \delta^t \left( S_t(a_D, a_F, R_F) - \alpha \cdot \mathbb{I}(d_t) \cdot g(R_D(t)) \right)$$

Where:

- $S_t$ : Immediate spread of a claim.  $S_t = s(a_D, a_F, R_F)$ .
- α: Penalty coefficient for a debunk.
- $\mathbb{I}(d_t)$ : Indicator function, 1 if debunked, 0 otherwise.
- $g(R_D)$ : Penalty function for low reputation,  $g(R_D) = \frac{1}{1+R_D}$ .

Reputation Dynamics: Reputation evolves as a stochastic process.

$$R_i(t+1) = \lambda R_i(t) + (1-\lambda) \cdot \mathbb{I}(\operatorname{succ}_i)$$

Where  $\lambda \in [0, 1]$  is the persistence parameter and  $\mathbb{I}(\operatorname{succ}_i)$  is an indicator for a successful outcome for player i.

### 1.2 Incomplete Information: Bayesian Signaling Game

This is a game between D and C. C has incomplete information about D's type. **Players**: D (Sender), C (Receiver) **Types**: D's type  $T_D \in \{T_r, D_c\}$ , where  $T_r$  is Truthful and  $D_c$  is Deceptive. **Prior Beliefs**:  $P(T_D = T_r) = \pi_0$ . **Actions**:

- $a_D \in \{sh, \neg sh\}$  (share, not share)
- $a_C \in \{b, \neg b\}$  (believe, don't believe)

**Belief Update (Bayes' Rule):** C's posterior belief after observing D's action  $a_D = sh$ .

$$P(T_D = T_r | a_D = sh) = \frac{P(sh|T_D = T_r) \cdot \pi_0}{P(sh|T_D = T_r) \cdot \pi_0 + P(sh|T_D = D_c) \cdot (1 - \pi_0)}$$

**Consumer Payoffs (Conditional):** Let the value of a true claim be  $V_T$  and a false one be  $V_F < 0$ . The cost of effort is c.

$$U_C(a_C, T_{claim}) = \begin{cases} V_T - c & \text{if } a_C = b, T_{claim} = \text{True} \\ V_F - c & \text{if } a_C = b, T_{claim} = \text{False} \\ 0 & \text{otherwise} \end{cases}$$

Equilibrium Concept: Perfect Bayesian Equilibrium (PBE).

#### 1.3 Platform Dynamics: Stackelberg Game

The Platform (P) is the leader, setting its policy  $M \in [0, 1]$ . D and F are followers, reacting optimally. **Players:** P (Leader), D (Follower), F (Follower) **Platform Policy:** M, strength of moderation. **Followers' Strategies:**  $a_D(M)$ ,  $a_F(M)$ . **Platform Payoff:** 

$$U_P(M, a_D(M), a_F(M)) = E(a_D, a_F) - C_M(M) - C_R(a_D)$$

- *E*: Engagement,  $E(a_D, a_F) = f(a_D, a_F)$ .
- $C_M(M)$ : Moderation Cost,  $C_M(M) = k_M M^{\gamma}$ .
- $C_R(a_D)$ : Reputational damage,  $C_R(a_D) = k_R \cdot S(a_D)$ .

#### Followers' Best Response:

$$a_D^*(M) = \arg\max_{a_D} U_D(a_D, a_F(M), M)$$

$$a_F^*(M) = \arg\max_{a_F} U_F(a_D(M), a_F, M)$$

**Platform's Optimization Problem:** The platform solves for the optimal policy  $M^*$  by anticipating the followers' best responses.

$$M^* = \arg\max_{M} U_P(M, a_D^*(M), a_F^*(M))$$