

- Be able to determine whether a function is bijective, surjective or injective.
- Be able to find inverse function and compositional function

Exercise 1: Give an example of a function from \mathbb{N} to \mathbb{N} that is

a) one-to-one but not onto.

$$f(x) = x^2$$

b) onto but not one-to-one.

$$f(x) = \frac{x}{2}$$

c) both onto and one-to-one (but different from the identity function).

$$f(x) = \begin{cases} 2n-1 & \text{if } x \text{ is odd} \\ 2n+1 & \text{if } x \text{ is even} \end{cases}$$

d) neither one-to-one nor onto.

$$f(x) = 0$$

Exercise 2:

Sets A and X are defined as:

$$A = \{a, b, c, d\}$$

$$X = \{1, 2, 3, 4\}$$

A function $f: A \rightarrow X$ is defined to be

$$f = \{(a, 3), (b, 1), (c, 4), (d, 1)\}$$

a. What is the target (or co-domain) of function f ?

$$f(x)$$

b. What is the range of function f ?

$$[1, 4]$$

c. What is $f(c)$?

$$4$$

d. What is the domain of function f ?

$$[a, d]$$

Exercise 3: Consider three functions f , g , and h , whose domain and target are \mathbb{Z} . Let

$$f(x) = x^2$$

$$g(x) = 2^x$$

$$h(x) = \lceil x^5 \rceil$$

a. Evaluate $g \circ h \circ f(4)$

$$g(h(f(4))) = 2^{1048576}$$

b. Give a mathematical expression for $f \circ g$.

$$f(g(x)) = (2^x)^2$$

Exercise 4: For each of the following functions, indicate whether the function has a well-defined inverse. If the inverse is well-defined, give the input/output relationship of f^{-1} .

a. $f: \mathbb{Z} \rightarrow \mathbb{Z}$. $f(x) = 2x + 3$

The function is not onto, so f^{-1} is not well defined

b. $f: \mathbb{R} \rightarrow \mathbb{R}$. $f(x) = 2x + 3$

$$f^{-1}(x) = \frac{x-3}{2}$$

$$(x-3) = 2y$$