Name:
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Objective:

- Be able to understand definition of functions
- Be able to understand important functions such as ceiling, floor functions

Exercise 1: Why is f not a function from $\mathbb{R}$ to $\mathbb{R}$ if

a) $f(x) = 1/x$?

f is undefined at 0

b) $f(x) = \sqrt{x}$?

f is undefined for all negative numbers

c) $f(x) = \pm\sqrt{x^2+1}$?

f assigns more than one value to each value of x

Exercise 2: Determine whether f is a function from **Z** to **R.** If not, explain why.

a) $f(n) = \pm n$.

Not a function because f assigns more than one value to each value of X

b) $f(n) = \sqrt{n^2+1}$

Function

c) $f(n) = 1/(n^2-4)$.

Not a function because the function is undefined at $n=2$ and $n=-2$

Exercise :

Find these values.

a. $\lfloor 1/2 + \lfloor 1/2 \rfloor \rfloor = 0$

b. $\lceil \lfloor 1/2 + \lfloor 1/2 \rfloor \rfloor \rceil = 0$

$\lceil \lfloor 1/2 \rfloor \rceil$

c. $\lfloor 1/2 + \lceil 3/2 \rceil \rfloor$

$\lfloor 2\frac{1}{2} \rfloor = 2$

Exercise 5:

Prove or disprove each of these statements about the floor and ceiling functions

a. $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$ for all real numbers x and y.

$$\lceil \tfrac{1}{2}(2) \rceil = \lceil 1 \rceil = 1$$

$$\lceil 0.5 \rceil \lceil 2 \rceil = 0(2) = 0$$

$\therefore$ $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$ is false

b. $\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil = 0$ or 1 whenever x and y are real numbers.

1. Let x be an integer $\lceil x \rceil = x$

$$x + \lceil y \rceil - \lceil x + y \rceil = x + \lceil y \rceil - (x + \lceil y \rceil) = 0$$

$\therefore$ the statement is true for all integers x

2. Let y be an integer $\lceil y \rceil = y$

$$\lceil x \rceil + y - \lceil x + y \rceil = \lceil x \rceil + y - (\lceil x \rceil + y) = 0$$

$\therefore$ The statement is true for all integers y

3. Let x and y not be integers

n and m are integers $\therefore$ t and p are $0 < t < 1$ and $0 < p < 1$

$$\lceil x \rceil = n + 1 \quad \lceil y \rceil = m + 1$$

$\lceil x + y \rceil = n + m + 1$ or $n + m + 2$

$\lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil = 1$ when $0 < t + p < 1$

$\therefore$ This statement is true

$\lceil x \rceil + \lceil y \rceil + \lceil x + y \rceil = 0$ when $1 \leq t + p <$