# Problem Set 1

1a. True

b. False

c. True

d. False

2. a  Sam had pizza last night or Chris finished her homework

b  Chris finished her homework and Pat watched the news this morning

c. Sam had pizza last night and Chris finished her homework, or Pat watched the news this morning

d. Sam did not have pizza last night or Chris finished her homework

3. a. $p \to q$

b. $p \to r$

c. $(p \vee q) \to q$

d. $r \to q$

e. $p \to r$

4 a.    $p: xy = 0$    $q: x = 0$    $r: y = 0$

  $p \leftrightarrow (q \lor r)$

  b.  $p: n = 4$        $q: 7n - 5 = 23$

  $p \leftrightarrow q$

  c.  $p: x = 2$    $q: x^4 - x^2 - 12 = 0$

  $p \rightarrow q$

  d.  $p: x = 2$        $q: x^4 - x^2 - 12 = 0$

  $q \rightarrow p$

  e.  $p: x = 2$        $q: x^4 - x^2 - 12 = 0$

  $p \leftrightarrow q$

  f.  $p: x^2 + y^2 > 1$    $q: x > 1$    $r: y > 1$

  $p \leftrightarrow (q \land r)$

5 a. $(p \rightarrow q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$

$\neg(\neg p \lor q) \lor r \qquad \equiv \neg p \lor (\neg q \lor r)$ (False)

$p \land \neg q \lor r \qquad \not\equiv \quad \neg p \lor q \lor r$

b. $p \rightarrow (q \lor r) \equiv (p \rightarrow q) \lor (p \rightarrow r)$

$\equiv (\neg p \lor q) \lor (\neg p \lor r)$ (True)

$\equiv \neg p \lor (q \lor r)$

$\equiv p \rightarrow (q \lor r)$

c. $p \rightarrow (q \land r) \equiv (p \rightarrow q) \land (p \rightarrow r)$

$\equiv (\neg p \lor q) \land (\neg p \lor r)$

$\equiv \neg p \lor (q \land r)$ (True)

$\equiv p \rightarrow (q \land r)$

6 a. True

b. True

c. True

d. True

7. a. There exists an integer $n$ such that $n$ is prime and $n$ is even

   b. For all integers $n$, $n > 2$ only if $n$ is prime or $n$ is even

   c. There exists an integer $n$ such that $n$ is prime, if $n$ is even or $n > 2$

   d. For all integers $n$, $n$ is prime and $n > 2$ only if $n$ is not even