

CIS 185

Practice 9

Objective:

- Be able to identify a relation
- Be able to identify properties of a relation
- Be able to represent relation using digraphs and matrices.

Name: Timothy Bryant
ID: _____ Date: _____

Exercise 1: List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

a) $a | b$

$$R = \{(1,0), (2,0), (3,0), (4,0), (1,1), (1,2), (2,2), (1,3), (3,3)\}$$

b) $\gcd(a, b) = 1$.

$$R = \{(1,0), (0,1), (1,1), (1,2), (2,1), (3,1), (4,1), (2,3), (3,2), (4,3)\}$$

Exercise 2: Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, asymmetric, and/or transitive, where $(x, y) \in R$ if and only if (explain your reasoning)

$x - y = \text{rational number}$

Reflexive: Let $x \in \mathbb{R}$. Then $x - x = 0 \in \mathbb{Q}$

Transitive: Let $x, y, z \in \mathbb{R}$ with $x - y \in \mathbb{Q}$ and $y - z \in \mathbb{Q}$. Then $x - z = (x - y) + (y - z) \in \mathbb{Q}$

Symmetric: Let $x, y \in \mathbb{R}$ with $x - y \in \mathbb{Q}$. Then $y - x = -(x - y) \in \mathbb{Q}$

Exercise 3: For each of these relations on the set $\{1, 2, 3, 4\}$, decide whether it is reflexive, whether it is symmetric, whether it is antisymmetric, whether it is asymmetric, whether it is irreflexive and whether it is transitive.

$\{(1, 3), (1, 4), (2, 3), (2, 4), (3, 1), (3, 4)\}$

Non-reflexive as $(1,1), (2,2), (3,3), (4,4)$ missing

Asymmetric as $(3,2), (4,1), (4,2)$ missing

Not antisymmetric because $(1,3)$ and $(3,1)$ present

Not transitive because we don't have $(2,1)$ for $(2,3), (3,1)$

Exercise 4:

$R_1 = \{(a, b) \in \mathbb{R}^2 \mid a > b\}$, the "greater than" relation,

$R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}$, the "greater than or equal to" relation,

$R_3 = \{(a, b) \in \mathbb{R}^2 \mid a < b\}$, the "less than" relation,

$R_4 = \{(a, b) \in \mathbb{R}^2 \mid a \leq b\}$, the "less than or equal to" relation,

$R_5 = \{(a, b) \in \mathbb{R}^2 \mid a = b\}$, the "equal to" relation,

$R_6 = \{(a, b) \in \mathbb{R}^2 \mid a \neq b\}$, the "unequal to" relation.

a) $R_1 \cup R_3$.

$$R_1 \cup R_3 = \text{Region above and below } y = x = R_6$$

b) $R_1 - R_2$.

$$\text{Region which is excess above } y = x = R_5$$

Exercise 5:

A. Represent each of these relations on $\{1, 2, 3\}$ with a matrix (with the elements of this set listed in increasing order).

B. Draw the digraphs representing each of the relations below.

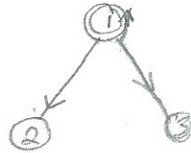
C. Determine whether the relations represented by the directed graphs in part B are reflexive, symmetric, antisymmetric, asymmetric, irreflexive and/or transitive.

a) $\{(1, 1), (1, 2), (1, 3)\}$

A.

		1	2	3
1		1	1	1
2		0	0	0
3		0	0	0

B



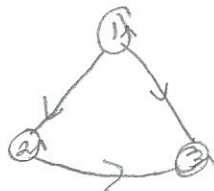
C antisymmetric

b) $\{(1, 2), (2, 1), (2, 2), (3, 3)\}$

A.

		1	2	3
1		0	1	0
2		1	1	0
3		0	0	1

B



C

Symmetric