CECS 228
Practice 5a
Objectives:

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- Be able to apply Direct Proof
- Be able to apply Proof by contradiction
- Be able to apply Proof by contrapositive
- Be able to apply other proof methods

Exercise 1: Use direct proof to prove the following:
a. The product of two odd integers is an odd integer.

$$xy = (2t+1)(2a+1)$$
$$= 4at + 2t + 2a + 1$$
$$= 2(2at + t + a) + 1$$

$2at + t + a$ is an integer

$\therefore$ xy is odd

b. If r and s are rational numbers, then the product of r and s is a rational number.

$r = \frac{a}{b}$, $b \neq 0$ where a, b, c, d are integers

$s = \frac{c}{d}$ $d \neq 0$

$$rs = \frac{a}{b}\left(\frac{c}{d}\right) = \frac{ac}{bd}$$

$\therefore$ rs is rational because rs is the ratio of two integers

Exercise 2: Use proof by contrapositive to prove the following:
a. If x and y are real numbers and x + y is irrational, then x is irrational or y is irrational.

$x + y$ is irratdnal

Suppose, $x = \frac{a}{b}$, $b \neq 0$

$y = \frac{c}{d}$, $d \neq 0$

where a, b, c, d are integers

$$x + y = \frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$\therefore$ $x + y$ is rational

This statement is true by proof of contrapositive

b. If x and y are positive real numbers and xy > 400, then x > 20 or y > 20.

Suppose $x < 20$ and $y < 20$

$x = 19$

$y = 19$

$$xy = 19(19) = 361$$

$\therefore$ $xy < 400$

This statement is true by proof of contrapositive

Exercise 3: Use proof by contradiction to prove the following:
a. There is no smallest integer.

There is a smallest integer x

If x is integer, then x − 1 is integer

$x - 1 < x$

$\therefore$ x is not the smallest integer and there exist a smallest integer is false

b. $\sqrt[3]{2}$ is irrational. You can use the following fact in your proof: "If n is an integer and n3 is even, then n is even."

Suppose $\sqrt[3]{2}$ is rational

$\sqrt[3]{2} = \frac{a}{b}$ where a and b are integers

$2 = \frac{a^3}{b^3}$

If a is integer and a^3 is even then a is even

$2b^3 = a^3$

$a = 2d$

$2b^3 = (2d)^3 = 8d^3$

$b^3 = 4d^3$

b is even too

$\therefore$ there is no a and b such $\sqrt[3]{2} = \frac{a}{b}$ and the statement $\sqrt[3]{2}$ is rational is false

Exercise 4:

Case 1: $n=1$

$$1^2+1 \geq 2^1$$

$$2 \geq 2$$

Case 2: $n=2$

$$2^2+1 \geq 2^2$$

$$5 \geq 4$$

Case 3: $n=3$

$$3^2+1 \geq 2^3$$

$$10 \geq 8$$

Case 4: $n=4$

$$4^2+1 \geq 2^4$$

$$17 \geq 16$$

Exercise 5:

Case 1

$a = -1 \quad b = \frac{1}{2}$

$(-1)^{1/2} = \sqrt{-1}$

irrational number

Case 2

$a = 2 \quad b = \frac{1}{2}$

$(2)^{1/2} = \sqrt{2}$ irrational number

Exercise 6:

$(-a)^2 = a^2$

Case 1: $x = 1 \quad y = 1$

$2(1)^2 + 5(1)^2 = 7 \neq 14$

Case 2: $x = 1 \quad y = 2$

$2(1)^2 + 5(2)^2 = 22 \neq 14$

Case 3: $x = 1 \quad y = 3$

$2(1)^2 + 5(3)^2 = 47 \neq 14$

Case 4: $x = 2 \quad y = 1$

$2(2)^2 + 5(1)^2 = 13 \neq 14$

Case 5: $x = 2 \quad y = 2$

$2(2)^2 + 5(2)^2 = 28 \neq 14$

Case 6: $x = 2 \quad y = 3$

$2(2)^2 + 5(3)^2 = 53 \neq 14$

Case 7: $x=3$ $y=1$

$$2(3)^2+5(1)^2=23\neq 14$$

Case 8: $x=3$ $y=2$

$$2(3)^2+5(2)^2=38\neq 14$$

Case 9: $x=3$ $y=3$

$$2(3)^2+5(3)^2=63\neq 14$$