

- Be able to determine whether two graphs are isomorphic
- Be able to determine connectivity of a graph.

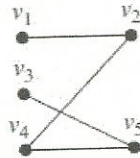
Exercise 1:

Determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists.

a.



isomorphic



$$\begin{aligned} f(u_1) &= v_1 \\ f(u_2) &= v_2 \\ f(u_3) &= v_4 \\ f(u_4) &= v_5 \\ f(u_5) &= v_3 \end{aligned}$$

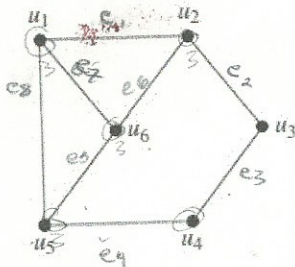
$$12 = 12$$

$$23 = 24$$

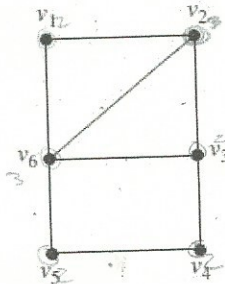
$$34 = 45$$

$$45 = 53$$

b.



not isomorphic



$$f(u_1) =$$

Exercise 2:

Are the simple graphs with the following adjacency matrices isomorphic?

a.

$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

isomorphic

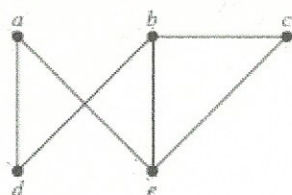
b.

$$\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

not isomorphic

Exercise 3:

Does each of these lists of vertices form a path in the following graph? Which paths are simple? Which are circuits? What are the lengths of those that are paths?



a) a, e, b, c, b

It is a path of length 4 but not simple path

b) a, e, a, d, b, c, a

Not a path because c is not connected to a

c) e, b, a, d, b, e

Not a path because b is not connected to a

d) c, b, d, a, e, c

It is a circuit of length 5

Exercise 4:

Determine whether the given graph is connected.

a.



disconnected

b.

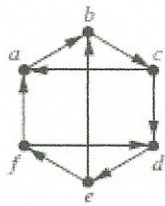


connected

Exercise 5:

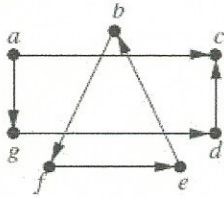
Determine whether this graph is strongly connected and if not, whether it is weakly connected.

a.



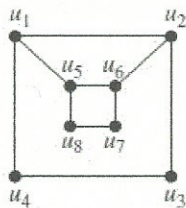
strongly connected

b.

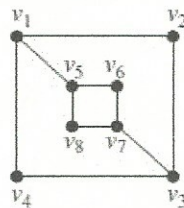


not strongly nor weakly connected

Exercise 6: Use paths either to show that these graphs are not isomorphic or to find an isomorphism between these graphs.



G



H

Both graphs have 8 vertices and 10 edges

Both graphs have 4 vertices w/ degree 3

Both graphs have 4 vertices w/ degree 2

$v_3 \text{ deg}(3)$

No such circuit in G

$\therefore$  not isomorphic

$v_7 \text{ deg}(3)$

$v_8 \text{ deg}(2)$

$v_5 \text{ deg}(3)$

$v_1 \text{ deg}(3)$

$v_4 \text{ deg}(2)$

$v_3 \text{ deg}(3)$