CIS 185

Practice 10

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Objective:

- Be able to identify equivalence relation
- Be able to compute composition of relation
- Be able to identify partial ordering relation

Exercise 1: Which of these relations on the set of all people are equivalence relations? Determine the properties of an equivalence relation that the others lack.

a) {(a, b) | a and b are the same age}

Equivalence relation

b) {(a, b) | a and b have met}

It is not transitive. If a met b and b met c it doesn't mean a met c

Exercise 2:

Let R be the relation {(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)}, and let S be the relation {(2, 1), (3, 1), (3, 2), (4, 2)}. Find S ∘ R.

$$S \circ R = \{(1,1), (1,2), (2,1), (2,2)\}$$

Exercise 3:

R1 = {(a, b) ∈ $\mathbb{R}^2$ | a > b}, the "greater than" relation,

R3 = {(a, b) ∈ $\mathbb{R}^2$ | a < b}, the "less than" relation,

R4 = {(a, b) ∈ $\mathbb{R}^2$ | a ≤ b}, the "less than or equal to" relation,

a) R1 ∘ R4.

$(a, c) \in R : a \leq b$ and $b > c = R_2$

b) R3 ∘ R3.

$R_3 \circ R_3 = R_3$

Exercise 4: Let R₁ and R₂ be relations on a set A represented by the matrices

$$\mathbf{M}_{R_1} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \quad \text{and} \quad \mathbf{M}_{R_2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find the matrices that represent

a) R1 ∪ R2.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

b) R1 ∘ R1.

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \circ \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Exercise 5: Answer these questions for the poset ({{1}, {2}, {4}, {1, 2}, {1, 4}, {2, 4}, {3, 4}, {1, 3, 4}, {2, 3, 4}}, ⊆).

a) Find the maximal elements.

$\{1,2\}, \{2,3,4\}, \{1,3,4\}$

b) Find the minimal elements.

$\{1\}, \{2\}, \{4\}$

c) Is there a greatest element?

Does Not Exist

d) Is there a least element?

Does not Exist

e) Find all upper bounds of {{2}, {4}}.

$\{2,4\}, \{2,3,4\}$

f) Find the least upper bound of {{2}, {4}}, if it exists.

$\{2,4\}$

g) Find all lower bounds of {{1, 3, 4}, {2, 3, 4}}.

$\{3,4\}, \{4\}$

h) Find the greatest lower bound of {{1, 3, 4}, {2, 3, 4}}, if it exists.

$\{3,4\}$