A.

Step 1

Prove $n=1$

$$1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \dots + n\cdot 3^n = \frac{3}{4}\left[(2n-1)\cdot 3^n + 1\right]$$

$$1\cdot 3 = \frac{3}{4}\left[(2(1)-1)\cdot 3^1 + 1\right]$$

$$3 = \frac{3}{4}(3+1)$$

$$3 = 3 \checkmark$$

Step 2

Asume $n \leq k$ $\quad 1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \dots + k\cdot 3^k = \frac{3}{4}\left[(2k-1)\cdot 3^k + 1\right]$

Find $n = k+1$

$$1\cdot 3 + 2\cdot 3^2 + 3\cdot 3^3 + \dots + k\cdot 3^k + (k+1)\cdot 3^{k+1} = \frac{3}{4}\left[(2(k+1)-1)\cdot 3^{k+1} + 1\right]$$

$$\frac{3}{4}\left[(2k-1)\cdot 3^k + 1\right] + k+1\cdot 3^{k+1} = \frac{3}{4}\left[(2(k+1)-1)\cdot 3^{k+1} + 1\right]$$

$$\frac{3}{4}\left[6k^k - 3^k + 1\right] + k+1 + 3^{k+1} = \frac{3}{4}\left[(2k+2-1)\cdot 3^{k+1} + 1\right]$$

$$\frac{9}{2}k^k - \frac{9}{4}^k + \frac{3}{4} + k+1 + 3^{k+1} = \frac{3}{4}\left[6k^{k+1} + 6^{k+1} - 3^{k+1} + 1\right]$$

$$\frac{9}{2}k^k - \frac{9}{4}^k + 3^k\cdot 3 + k + \frac{7}{4} = \frac{9}{2}k^{k+1} + \frac{9}{2}^{k+1} - \frac{9}{4}^{k+1} + \frac{3}{4}$$

B.

a. Reflexive For (f, g) fRf

$f(1) = g(1)$ ✓

$\therefore (f, f)$ is true. It is reflexive

Symmetric If (f, g) then (g, f)

$f(1) = g(1)$ ✓

$g(1) = f(1)$ ✓

$(f(1), g(1))$ then $(g(1), f(1))$

$\therefore$ It is symmetric

Transitive: If (f, g) and (g, k) then (f, k)

$f(1) = g(1)$ $\therefore f(1) = g(1) = k(1)$

fRg gRk and fRk

$\therefore$ It is transitive

$\therefore$ It is equivalence relation

b. Reflexive fRf

$f(0) = g(0)$
$f(1) = g(1)$

If $f(1) = g(1)$ then $(f,g) = (f,f)$
or $g(0) = f(0)$
$\therefore$ It is reflexive

Symmetric If fRg then gRf

If $f(1) = g(1)$ or $f(0) = g(0)$ then (f,g)
is symmetric since $f = g$

Transitive: If fRg and gRK then fRK

If $f(1) = g(1)$ or $f(0) = g(0)$ is transitive

since g is always related to f

$\therefore$ It is not an equivalence relation