CIS 185
Practice 3
Objectives:

Name: Timothy Bryant
ID:
Date:

- Be able to understand and apply nested predicate
- Be able to negate predicate
- Be able to translate from English into logical expressions

Exercise 1: Let P(x) be the statement "x spends more than five hours every weekday in class," where the domain for x consists of all students. Express each of these quantifications in English.

a. $\exists x \neg P(x)$

There exists a student that does not spend more than five hours every weekday in class.

b. $\forall x P(x)$

All students spend more than five hours every weekday in class.

Exercise 2: Translate in each of these statements into logical expressions using predicates, quantifiers, and logical connectives. Also, provide a negation in English. First, let the domain consist of the students in your class.

x = consists of all students in your class

a. There is a person in your class who cannot swim.

$\exists x \neg R(x)$

R(x) = "x can swim"

All students in your class can swim.

b. All students in your class can solve quadratic equations.

$\forall x\, Q(x)$

Q(x) = "x can solve quadratic equations"

There exists a student that cannot solve quadratic equations

Exercise 3: Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that."). Assume that the domain is all animals.

x = koalas

a) Every koala can climb.

B(x) = "x can climb"

$\forall x\, B(x)$

There exists a koala that cannot climb

b) No monkey can speak French.

x = monkeys

D(x) = "x can speak French"

$\forall x \neg D(x)$

There exists a monkey that can speak French

Exercise 4: Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y" and where the domain for x and y consists of all systems and all possible states, respectively.

a) $\exists x S(x, \text{open})$

There exists a system that is in the state open

b) $\forall x \neg S(x, \text{working})$

All systems are not in the state working

Exercise 5: Let $F(x, y)$ be the statement "x can fool y," where the domain consists of all people in the world.

i. Use quantifiers to express each of these statements.

ii. Negate these statements using quantifiers and write them in simple English

a) Everybody can fool Fred.

i. $\forall x\, F(x, \text{Fred})$

ii. $\exists x\, \neg F(x, \text{Fred})$

There exist a person that cannot fool Fred

b) Evelyn can fool everybody.

i. $\forall y\, F(\text{Evelyn}, y)$

ii. $\exists y\, \neg F(\text{Evelyn}, y)$

Evelyn cannot fool one person

c) There is exactly one person whom everybody can fool.

i. $\exists y\, (\forall x F(x,y)) \wedge (\forall z\, F(x,z) \Rightarrow (z = y))$

ii. $(\forall y\, \exists x\, \neg F(x,y)) \vee (\exists z\, F(x,z) \Rightarrow (z = y))$

There is exactly one person that cannot fool everyone

d) No one can fool himself or herself.

i. $\neg \exists x\, F(x,x)$

ii. $\neg \forall\, \neg F(x,x)$

Some people cannot fool themselves

Exercise 6: Let Q(x,y) denote the statement "x is the capital of y." What are these truth values?
a. Q(Denver, Colorado)

The statement is True. Denver is the capital of Colorado

b. Q(Massachusetts, Boston)

The statement is false because Boston is the capital of Massachusetts

Exercise 7: Determine the truth value of each of these statements if the domain consists of all real numbers. If the universal statement is false, provide a counterexample. If the existential statement is true, provide an example.
a. $\exists x(x^3 = -1)$

True $x = -1$

b. $\forall x((-x)^2 = x^2$

True the square of any number is positive

therefore $(-1)^2 = (1)^2$

$(-2)^2 = (2)^2$

and so on