

ctice 4

$P(x)$  = "I take the day  $x$  off"

$R(x)$  = "It rains on  $x$  day"

$Q(x)$  = "It snows on  $x$  day"

Reason:

Steps

1.  $\forall x (P(x) \rightarrow (R(x) \vee Q(x)))$

Premise

2.  $P(\text{Tuesday}) \vee P(\text{Thursday})$

Premise

3.  $\neg (R(\text{Tuesday}) \vee Q(\text{Tuesday}))$

Premise

4.  $\neg Q(\text{Thursday})$

Premise

5.  $P(\text{Tuesday}) \rightarrow (R(\text{Tuesday}) \vee Q(\text{Tuesday}))$

Universal instantiation from Step 1

6.  $P(\text{Thursday}) \rightarrow (R(\text{Thursday}) \vee Q(\text{Thursday}))$

Universal instantiation from Step 1

7.  $\neg P(\text{Tuesday})$

modus tollens from step 3 and 5

8.  $P(\text{Thursday})$

Disjunctive syllogism from step 2 and 7

9.  $R(\text{Thursday}) \vee Q(\text{Thursday})$

Modus ponens from step 6 and 8

10. R(Thursday)

Disjunctive syllogism from step  
4 and 6

Conclusions: "I did not take Tuesday off": Step 7

"I took Thursday off": Step 8

"It rained Thursday": Step 10

b)  $P(x) =$  "x is a computer science major"

$Q(x) =$  "x has a personal computer"

Step

Reason

1.  $\forall x (P(x) \rightarrow Q(x))$

Premise

2.  $\neg Q(\text{Ralph})$

Premise

3.  $Q(\text{Ann})$

Premise

4.  $P(\text{Ralph}) \rightarrow Q(\text{Ralph})$

Universal instantiation from step 1

5.  $\neg P(\text{Ralph})$

Modus tollens from step 2 and 4

Conclusions: "Ralph is not a computer science major" from step 5

Exercise 2: Identify the error or errors in this argument that supposedly shows that if  $\forall x(P(x) \vee Q(x))$  is true then  $\forall xP(x) \vee \forall xQ(x)$  is true. Explain your reasoning

- |   |                                   |
|---|-----------------------------------|
| 1. $\forall x(P(x) \vee Q(x))$          | Premise                           |
| 2. $P(c) \vee Q(c)$                     | Universal instantiation from (1)  |
| 3. $P(c)$                               | Simplification from (2)           |
| 4. $\forall xP(x)$                      | Universal generalization from (3) |
| 5. $Q(c)$                               | Simplification from (2)           |
| 6. $\forall xQ(x)$                      | Universal generalization from (5) |
| 7. $\forall x(P(x) \vee \forall xQ(x))$ | Conjunction from (4) and (6)      |

Error in steps 3 and 5 for Simplification

Simplification cannot be used, because it needs a conjunction, not a disjunction.