

- Be able to use Basic Counting Principle

Exercise 1: Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

6

7

$$6 \cdot 7 = 42 \text{ trips}$$

Exercise 2: How many strings of eight English letters are there
a) that contain no vowels, if letters can be repeated?

26 letters

5 vowels

1 letter: 21

6 letter: 21

2 letter: 21

7 letter: 21

3 letter: 21

8 letter: 21

4 letter: 21

5 letter: 21

$$21^8 = 37,822,859,361$$

b) that start with a vowel, if letters can be repeated?

1 letter: 5

5 letter: 26

2 letter: 26

6 letter: 26

3 letter: 26

7 letter: 26

4 letter: 26

8 letter: 26

$$5 \cdot 26^7 = 40,159,050,880$$

Exercise 3: How many positive integers between 100 and 999 inclusive
a) are not divisible by 4?

900 integers

$$\frac{900}{4} = 225 \text{ integer divisible by 4}$$

$$900 - 225 = 675 \text{ integers not divisible by 4}$$

b) are divisible by 3 or 4?

$$A \cup B = A + B - A \cap B$$

$$\frac{900}{3} = 300$$

$$\frac{900}{4 \cdot 3} = \frac{900}{12} = 75$$

$$300 + 225 - 75 = 450 \text{ integers divisible by 3 or 4}$$

Exercise 4: Use the principle of inclusion-exclusion to find the number of positive integers less than 1,000,000 that are not divisible by either 4 or by 6.

$$\frac{999,999}{4} = 249,999$$

$$999,999 - 249,999 = 750,000$$

$$\frac{999,999}{6} = 166,666$$

$$999,999 - 166,666 = 833,333$$

$$\frac{999,999}{12} = 83,333$$

$$999,999 - 83,333 = 916,666$$

$$750,000 + 833,333 - 916,666 = 666,667$$

Exercise 5: In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if

a) the bride must be next to the groom?



$$1 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 24$$

$$5 + 5 = 10$$

$$24 \cdot 10 = 240$$

b) the bride is not next to the groom?

$$6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 6! = 720$$

$$720 - 240 = 480$$