

- Be able to prove set identities
- Be able to understand Cartesian products

Exercise 1: Let A, B, and C be sets. Show that  $A \cup (B \cap C) = (C \cup B) \cap A$ .

$$\begin{aligned} A \cup (B \cap C) &= (A \cup B) \cap (A \cup C) \\ &= (C \cup B) \cap A \end{aligned}$$

Exercise 2: Let A, B, and C be sets. Show that  $\bar{A} \cup (A \cap B) = \bar{A} \cup B$ .

$$\begin{aligned} \bar{A} \cup (A \cap B) &= (\bar{A} \cup B) \cap (\bar{A} \cup A) \\ &= \bar{A} \cup B \cap U = \bar{A} \cup B \end{aligned}$$

Exercise 3: Use set builder notation and logical equivalences to establish the first De Morgan law  $\overline{A \cap B \cap C} = \bar{A} \cup \bar{B} \cup \bar{C}$ .

$$\begin{aligned} \overline{A \cap B \cap C} &= \{x \mid x \notin A \cap B \cap C\} \text{ complement} \\ &= \{x \mid \neg(x \in (A \cap B \cap C))\} \text{ does not belong symbol} \\ &= \{x \mid \neg(x \in A \wedge x \in B \wedge x \in C)\} \text{ intersection} \\ &= \{x \mid \neg((x \in A) \wedge (x \in B) \wedge (x \in C))\} \text{ Demorgans} \\ &= \{x \mid x \notin A \vee x \notin B \vee x \notin C\} \text{ does not belong symbol} \\ &= \{x \mid x \in \bar{A} \vee x \in \bar{B} \vee x \in \bar{C}\} \text{ complement} \\ &= \{x \mid x \in \bar{A} \cup \bar{B} \cup \bar{C}\} \text{ union} = \boxed{\bar{A} \cup \bar{B} \cup \bar{C}} \end{aligned}$$

Exercise 4:

Let  $A = \{a, b, c\}$ ,  $B = \{x, y\}$ , and  $C = \{0, 1\}$ . Find

a)  $A \times B$ .

$$\{(a, x), (a, y), (b, x), (b, y), (c, x), (c, y)\}$$

b)  $C \times B \times A$ .

$$\begin{aligned} &\{(0, x, a), (0, x, b), (0, x, c), (0, y, a), (0, y, b), (0, y, c), \\ &(1, x, a), (1, x, b), (1, x, c), (1, y, a), (1, y, b), (1, y, c)\} \end{aligned}$$