CIS 185
Practice 14
Objective:

- Be able to count permutations and combinations with repetition
- More counting with combination and permutation

Name: Timothy Bryant
ID: Date:

Exercise 1: A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

a) a dozen croissant?

6 types
a dozen
$n = 6$
$r = 12$
$C(6+12-1,12) = C(17,12) = \frac{17!}{12!(17-12)!} = 6188$

b) two dozen croissants with no more than two broccoli croissants?

No broc
1 broc
2 broc
$C(28,24) + C(27,23) + C(26,22) = \frac{28!}{24!(28-24)!} + \frac{27!}{23!(27-23)!} + \frac{26!}{22!(4)!}$
1 broc $n=5$, $r=23$
2 broc $n=5$, $r=22$
$= 52975$

Exercise 2:
How many ways are there to distribute five balls into seven boxes if each box must have at most one ball in it if

a) both the balls and boxes are labeled?

5 balls
7 boxes
$P(7,5) = \frac{7!}{(7-5)!} = 2520$

b) the balls are labeled, but the boxes are unlabeled?

5 balls = distinguishable
7 boxes = indistinguishable

There is only 1 way to distribute 5 balls into 7 boxes

Exercise 3: How many strings with five or more characters can be formed from the letters in SEERESS?

7 letters

7 characters $= \frac{7!}{3!3!1!} = 140$

3 E
3 S
1 R

6 characters
3E 3S $= \frac{6!}{3!3!} = 20$
3E 2S 1R $= \frac{6!}{3!2!1!} = 60$
2E 3S 1R $= \frac{6!}{3!2!1!} = 60$
$= 140$

3E 2S 2E 2S 1R
2E 3S
3E 1S 1R
1E 3S 1R

5 characters
$2\left(\frac{5!}{2!3!}\right) + 2\left(\frac{5!}{3!1!1!}\right) + \frac{5!}{2!2!1!} = 90$

$140 + 140 + 90 = 370$

6

Exercise 4: How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29$, where x_i, $i = 1, 2, 3, 4, 5, 6$ is a nonnegative integer such that

a) $x_1 \leq 5$?

$x_1 > 5$

$x_1 + 6 + x_2 \cdots + x_6 = 29$

$x_1 + x_2 \cdots + x_6 = 23$

$n = 6$

$r = 23$

None restricting

$n = 6$

$r = 29$

$C(28,23) = \frac{28!}{23!(28-23)!} = 98,280$

$C(34,29) = \frac{34!}{29!(34-29)!} = 278,256$

$278,256 - 98,280 = 179,976$

b) $x_1 < 8$ and $x_2 > 8$?

$x_2 > 8 \quad x_2 + 9$

$x_1 + x_2 + \cdots + x_6 = 20$

$n = 6$

$r = 20$

$C(25,20) = \frac{25!}{20!(5)!} = 53,130$

$x_1 \geq 8$ and $x_2 \geq 8$

$x_1 + 8 + x_2 + 9 = 29 - 17 = 12$

$n = 6 \quad r = 12$

$C(17,12) = \frac{17!}{12!(5)!} = 6188$

$53,130 - 6188 = 46,942$

Exercise 5: How many bit strings contain exactly five 0s and 14 1s if every 0 must be immediately followed by two 1s?

5 0's

14 1's

5 blocks for 011

$14 - 5 = 9$

$r = 4$ left

$n = 9$

$C(9,4) = \frac{9!}{4!(9-4)!} = 126$

Exercise 6: How many bit strings of length 10 have

a) exactly three 0s?

$n = 10$

$r = 3$

$C(10,3) = \frac{10!}{3!(10-3)!} = 120$

b) at least three 1s?

$n = 10$

$r \geq 3$

$C(10,3) = 120$

$C(10,4) = \frac{10!}{4!\,6!} = 210$

$C(10,5) = \frac{10!}{5!\,5!} = 252$

$C(10,6) = \frac{10!}{6!\,4!} = 210$

$C(10,7) = \frac{10!}{7!\,3!} = 120$

$C(10,8) = \frac{10!}{8!\,2!} = 45$

$C(10,9) \quad \frac{10!}{9!\,1!} = 10$

$C(10,10) = 1$

$2(120) + 2(210) + 45 + 10 + 1 = 968$