CIS 185
Practice 13
Objective:

- Be able to apply the Binomial Theorem
- Be able to use Pigeonhole Principle
- Be able to use Inclusion-Exclusion Principle

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Exercise 1:

a. Find the coefficient of x^6 in $(2x^2 + 3)^6$.

$$(2x^2+3)^6 = \sum_{n=0}^{6} \binom{6}{k}(2x^6)^k \cdot 3^{6-k} = \binom{6}{6} 2^6 \cdot 3^0 = \binom{6}{6} 2^6$$

b. Find the expansion of $(x + y)^4$

$$(x+y)^4 = \binom{4}{0} x^4 - \binom{4}{1} x^3 y + \binom{4}{2} x^2 y^2 - \binom{4}{3} x y^3 + \binom{4}{4} y^4$$

c. Prove the following identity Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

$$\binom{2n}{2} = \frac{2n(2n-1)}{2} = 2n^2 - n = n^2 + (n^2 - n)$$

$$= n^2 + 2\binom{n}{2}$$

Exercise 2: What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x - 3y)^{200}$?

$$(2x-3y)^{200} = \sum_{n=99}^{200} \binom{200}{99} 2x^{200-99} (-3y)^{99}$$

$$= \binom{200}{99} 2x^{101} - 3y^{99}$$

$$\boxed{= -\binom{200}{99} 2^{101} \cdot 3^{99}}$$