Left to Right Algorithm

```
numOfSwaps = 0                                              1tu
For i=0 to n do (Where n = half the list that is light
    For j=i to 2n-1 do (Where 2n is the whole list)
        If (disk[j] > disk[j+1])                            1tu
            Swap (disk[j], disk[j+1]);                      1tu
            numOfSwaps++;                                   1tu
return (disk[], numOfSwaps)                                 1tu
```

Step Count

$$\left(\sum_{i=0}^{n}\sum_{j=i}^{2n-1} 1+\max(2,0)\right)+1+1$$

$$\left(\sum_{i=0}^{n}\sum_{j=i}^{2n-1} 3\right)+2 = \left(\sum_{i=0}^{n}\left(\sum_{j=1}^{2n-1} 3-\sum_{j=1}^{i-1} 3\right) = \sum_{i=0}^{n} 3(2n-1)-3(i-1)\right.$$

$$= \sum_{i=0}^{n} 6n-3+3-3i = \sum_{i=0}^{n} 6n - \sum_{i=0}^{n} 3i$$

$$= 6n^2 - 3\left(\frac{n(n+1)}{2}\right) = 6n^2 - 3\left(\frac{n^2+n}{2}\right) = 6n^2 - 3/2\,n^2 + -3/2\,n$$

$$= \boxed{\frac{9}{2}n^2 + (-3/2\,n) + 2}$$

Mathematical Analysis $O(n^2)$

By definition

$$9/2\,n^2 + -3/2\,n + 2 \le n^2 \quad \forall n \ge n_0$$

$c = 8 \quad n_0 = 1$

$$9/2\,n^2 + -3/2\,n + 2 \le 8n^2$$

$$5n^2 \le 8n^2 \ ✓$$

∴ the algorithm belongs to $O(n^2)$

By limit theorem

$9/2n^2 + (-3/2n) + 2 \in O(n^2)$

$$\lim_{n \to \infty} \frac{9/2n^2 + (-3/2n) + 2}{n^2} = \lim_{n \to \infty} \frac{9n + (-3/2)}{2n} = \lim_{n \to \infty} \frac{9}{2}$$

$\therefore$ By limit theorem, the algorithm belongs to $O(n^2)$

Lawnmower algorithm

```
numOfSwaps = 0 ;                                         1tu
  For i=0 to n do
  {   For j=i do 2n-1 do
        If disk[j] > disk[j+1]                           1tu
              swap(disk[j], disk[j+1])                   1tu
           numOfSwaps ++;                                1tu
      For j = 2n-1  to j = 0
         If disk[j] < disk[j-1]                          1tu
            Swap(disk[j], disk[j-1])                     1tu
            numOfSwaps ++;                               1tu
return(disk[], numOfSwaps)                               1tu
```

<u>Step Count</u>

$$\left(\sum_{i=0}^{n}\left(\sum_{j=i}^{2n-1} 3 + \sum_{j=2n-1}^{0} 3\right)\right) + 2$$

$$= \left(\sum_{i=0}^{n}\left(3(2n-1) - 3(i-1) + \sum_{j=2n-1}^{0} 3\right)\right) + 2$$

$$= \sum_{i=0}^{n}\left(3(2n-1) - 3(i-1) + \left(\sum_{j=1}^{0} 3 - \sum_{j=1}^{2n-1} 3\right)\right)$$

$$= \sum_{i=0}^{n} 6n^2 - 3 + 3 - 3i + (3) + 6n^2 = \sum_{i=0}^{n} 12n^2 - 3i - 3) + 2$$

$$= \sum_{i=0}^{n} 12n^2 - \sum_{i=0}^{n} 3i - \sum_{i=0}^{n} 3 = 12n^2 - 3\left(\frac{n(n+1)}{2}\right) - 3n + 2$$

$$= 12n^2 - \tfrac{3}{2}n^2 - \tfrac{3}{2}n - 3n + 2$$

$$= \boxed{\tfrac{21}{2}n^2 - \tfrac{9}{2}n + 2}$$

Mathematical Analysis $O(n^2)$

By definition

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \leq n^2 \quad \forall n \geq n_0$$

$c = 17 \quad n_0 = 1$

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \leq 17n^2$$

$$8n^2 \leq 17n^2 \checkmark$$

$\therefore$ the algorithm belongs to $O(n^2)$

By limit theorem

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \in O(n^2)$$

$$\lim_{n \to \infty} \frac{\frac{21}{2}n^2 - \frac{9}{2}n + 2}{n^2} = \lim_{n \to \infty} \frac{21n - \frac{9}{2}}{2n} = \lim_{n \to \infty} \frac{21}{2}$$

$\therefore$ By limit theorem, the algorithm belongs to $O(n^2)$