

## Left to Right Algorithm

numOfSwaps = 0

For  $i=0$  to  $n$  do (Where  $n = \text{half the list that is light}$ ) 1tu

For  $j=i$  to  $2n-1$  do (Where  $2n$  is the whole list)

If ( $\text{disk}[j] > \text{disk}[j+1]$ ) 1tu

Swap ( $\text{disk}[j], \text{disk}[j+1]$ ); 1tu

numOfSwaps++; 1tu

return ( $\text{disk}[], \text{numOfSwaps}$ ) 1tu

### Step Count

$$\left( \sum_{i=0}^n \sum_{j=i}^{2n-1} 1 + \max(2, 0) \right) + 1 + 1$$

$$\left( \sum_{i=0}^n \sum_{j=i}^{2n-1} 3 \right) + 2 = \left( \sum_{i=0}^n \left( \sum_{j=i}^{2n-1} 3 - \sum_{j=i}^{i-1} 3 \right) \right) = \sum_{i=0}^n 3(2n-1) - 3(i-1)$$

$$= \sum_{i=0}^n 6n - 3 + 3 - 3i = \sum_{i=0}^n 6n - \sum_{i=0}^n 3i$$

$$= 6n^2 - 3 \left( \frac{n(n+1)}{2} \right) = 6n^2 - 3 \left( \frac{n^2 + n}{2} \right) = 6n^2 - \frac{3}{2}n^2 + \frac{3}{2}n$$

$$= \boxed{\frac{9}{2}n^2 + \left(\frac{3}{2}n\right) + 2}$$

### Mathematical Analysis $O(n^2)$

By definition

$$\frac{9}{2}n^2 + \frac{3}{2}n + 2 \leq n^2 \quad \forall n \geq n_0$$

$c=8$

$n_0=1$

$$\frac{9}{2}n^2 + \frac{3}{2}n + 2 \leq 8n^2$$

$$5n^2 \leq 8n^2 \quad \checkmark$$

$\therefore$  the algorithm belongs to  $O(n^2)$

By limit theorem

$$9/2n^2 + (3/2n) + 2 \in O(n^2)$$

$$\lim_{n \rightarrow \infty} \frac{9/2n^2 + (3/2n) + 2}{n^2} = \lim_{n \rightarrow \infty} \frac{9n + (3/2)}{2n} = \lim_{n \rightarrow \infty} \frac{9}{2}$$

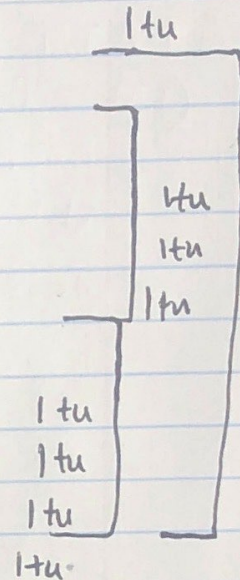
$\therefore$  By limit theorem, the algorithm belongs to  $O(n^2)$



## Lawnmower algorithm

```

numOfSwaps = 0;
For i=0 to n do
    For j=i do 2n-1 do
        If disk[j] > disk[j+1]
            swap(disk[j], disk[j+1])
            numOfSwaps ++;
    For j=2n-1 to j=0
        If disk[j] < disk[j-1]
            swap(disk[j], disk[j-1])
            numOfSwaps ++;
return(disk[], numOfSwaps)
    
```



### Step Count

$$\begin{aligned}
 & \left( \sum_{i=0}^n \left( \sum_{j=i}^{2n-1} 3 + \sum_{j=2n-1}^0 3 \right) \right) + 2 \\
 &= \left( \sum_{i=0}^n \left( 3(2n-i) - 3(i-1) + \sum_{j=2n-1}^0 3 \right) \right) + 2 \\
 &= \sum_{i=0}^n \left( 3(2n-i) - 3(i-1) + \left( \sum_{j=1}^0 3 - \sum_{j=1}^{2n-1} 3 \right) \right) \\
 &= \sum_{i=0}^n 6n - 3 + 3 - 3i + (3) + 6n - \left( \sum_{i=0}^n 12n^2 - 3i - 3 \right) + 2 \\
 &= \sum_{i=0}^n 12n^2 - \sum_{i=0}^n 3i - \sum_{i=0}^n 3 = 12n^2 - 3 \left( \frac{n(n+1)}{2} \right) - 3n + 2 \\
 &= 12n^2 - \frac{3}{2}n^2 - \frac{3}{2}n - 3n + 2 \\
 &= \boxed{\frac{21}{2}n^2 - \frac{9}{2}n + 2}
 \end{aligned}$$

## Mathematical Analysis $O(n^2)$

By definition

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \leq n^2 \quad \forall n \geq n_0$$

$$C=17 \quad n_0=1$$

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \leq 17n^2$$

$$8n^2 \leq 17n^2 \quad \checkmark$$

$\therefore$  the algorithm belongs to  $O(n^2)$

By limit theorem

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \in O(n^2)$$

$$\lim_{n \rightarrow \infty} \frac{2\frac{1}{2}n^2 - \frac{9}{2}n + 2}{n^2} = \lim_{n \rightarrow \infty} \frac{2\frac{1}{2}n - \frac{9}{2}}{2n} = \lim_{n \rightarrow \infty} \frac{2\frac{1}{2}}{2}$$

$\therefore$  By limit theorem, the algorithm belongs to  $O(n^2)$