

Team Members:

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Project 1 Submission

The screenshot displays a Linux desktop environment with a yellow flower field background. Two windows are open:

- Terminal Window:** Shows the command-line output of a C++ program named `disks.cpp`. The program tests various disk state configurations and swaps them to check if they are sorted. The output includes scores for each test case and a final total score.
- Visual Studio Code Window:** Shows the source code for the `disks.cpp` file. The code defines a class `disk_state` with methods for swapping disks and checking if a sequence of disks is sorted. It also includes a `main` function that runs the test cases.

```
student@tuffix-vm:~/Documents/project-lawnmover$ ./disks_test
alternate, n=4: passed, score 1/1
alternate, other values: passed, score 1/1
lawnmower, n=4: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, other values: passed, score 1/1
TOTAL SCORE = 14 / 14

student@tuffix-vm:~/Documents/project-lawnmover$ g++ -std=c++11 -Wall disks.cpp -o disks_test
./disks test
disk_state still works: passed, score 1/1
sorted_disks still works: passed, score 1/1
disk_state: sorted: passed, score 3/3
disk_state::is_sorted: passed, score 3/3
alternate, n=4: passed, score 1/1
alternate, n=3: passed, score 1/1
alternate, other values: passed, score 1/1
lawnmower, n=4: passed, score 1/1
lawnmower, n=3: passed, score 1/1
lawnmower, other values: passed, score 1/1
TOTAL SCORE = 14 / 14

student@tuffix-vm:~/Documents/project-lawnmover$ cat disks.cpp
128     private:
129         disk_state _after;
130         unsigned _swap_count;
131
132     public:
133
134     sorted_disks(const disk_state& after, unsigned swap_count)
135     : _after(after), _swap_count(swap_count) { }
136
137     sorted_disks(disk_state&& after, unsigned swap_count)
138     : _after(after), _swap_count(swap_count) { }
```

Left to Right Algorithm

```

numOfSwaps = 0           1 tu
For i=0 to n do (Where n = half the list that is left)
    For j=i to 2n-1 do (Where 2n is the whole list)
        If (disk[i] > disk[j+1])
            Swap(disk[i], disk[j+1]);      1 tu
            numOfSwaps++;                 1 tu
    return (disk[], numOfSwaps)          1 tu

```

Step Count

$$\begin{aligned}
& \left(\sum_{i=0}^n \sum_{j=i}^{2n-1} 1 + \max(2, 0) \right) + 1 + 1 \\
& \left(\sum_{i=0}^n \sum_{j=i}^{2n-1} 3 \right) + 2 = \left(\sum_{i=0}^n \left(\sum_{j=1}^{2n-1} 3 - \sum_{j=1}^{i-1} 3 \right) \right) = 3 \sum_{i=0}^n (2n-1) - 3(i-1) \\
& = \sum_{i=0}^n 6n - 3 + 3 - 3i = \sum_{i=0}^n 6n - \sum_{i=0}^n 3i \\
& = 6n^2 - 3 \left(\frac{n(n+1)}{2} \right) = 6n^2 - 3 \left(\frac{n^2+n}{2} \right) = 6n^2 - \frac{3}{2}n^2 + \frac{3}{2}n \\
& = \boxed{\frac{9}{2}n^2 + \frac{3}{2}n + 2}
\end{aligned}$$

Mathematical Analysis $O(n^2)$

By definition

$$\frac{9}{2}n^2 + \frac{3}{2}n + 2 \leq n^2 + n^2 n.$$

$$c=8 \quad n_0=1$$

$$\frac{9}{2}n^2 + \frac{3}{2}n + 2 \leq 8n^2$$

$$5n^2 \cancel{5n^2} \leq 8n^2 \checkmark$$

\therefore the algorithm belongs to $O(n^2)$

By limit theorem

$$\frac{9}{2}n^2 + f(3/2)n + 2 \in O(n^2)$$

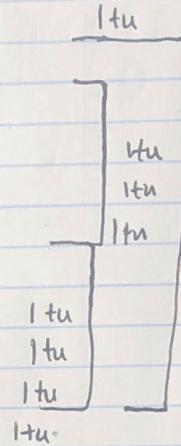
$$\lim_{n \rightarrow \infty} \frac{\frac{9}{2}n^2 + f(3/2)n + 2}{n^2} = \lim_{n \rightarrow \infty} \frac{9n + f(3/2)}{2n} = \lim_{n \rightarrow \infty} \frac{9}{2}$$

∴ By limit theorem, the algorithm belongs to $O(n^2)$

Lawnmower algorithm

```

numOfSwaps = 0;
For i=0 to n do
    For j=i to 2n-1 do
        If disk[j] > disk[j+1]
            swap(disk[j], disk[j+1])
            numOfSwaps++;
    For j = 2n-1 to j=0
        If disk[j] < disk[j-1]
            Swap(disk[j], disk[j-1])
            numOfSwaps++;
return(disk[], numOfSwaps)
    
```



Step Count

$$\begin{aligned}
& \left(\sum_{i=0}^n \left(\sum_{j=i}^{2n-1} 3 + \sum_{j=2n-1}^0 3 \right) \right) + 2 \\
& = \left(\sum_{i=0}^n \left(3(2n-1) - 3(i-1) + \sum_{j=2n-1}^0 3 \right) \right) + 2 \\
& = \sum_{i=0}^n \left(3(2n-1) - 3(i-1) + \left(\sum_{j=1}^0 3 - \sum_{j=1}^{2n-1} 3 \right) \right) \\
& = \sum_{i=0}^n 6n - 3 + 3 - 3i + (3) + 6n - \left(\sum_{i=0}^n 2n - 3i - 3 \right) + 2 \\
& = \sum_{i=0}^n 12n - \sum_{i=0}^n 3i - \sum_{i=0}^n 3 = 12n^2 - 3 \left(\frac{n(n+1)}{2} \right) - 3n + 2 \\
& = 12n^2 - \frac{3}{2}n^2 - \frac{3}{2}n - 3n + 2 \\
& = \boxed{\frac{21}{2}n^2 - \frac{9}{2}n + 2}
\end{aligned}$$

Mathematical Analysis $O(n^2)$

By definition

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \leq n^2 \quad \forall n \geq n_0$$

$$c = 17 \quad n_0 = 1$$

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \leq 17n^2$$

$$8n^2 \leq 17n^2 \checkmark$$

\therefore the algorithm belongs to $O(n^2)$

By limit theorem

$$2\frac{1}{2}n^2 - \frac{9}{2}n + 2 \in O(n^2)$$

$$\lim_{n \rightarrow \infty} \frac{2\frac{1}{2}n^2 - \frac{9}{2}n + 2}{n^2} = \lim_{n \rightarrow \infty} \frac{2\frac{1}{2}n - \frac{9}{2}}{2n} = \lim_{n \rightarrow \infty} \frac{21}{2}$$

\therefore By limit theorem, the algorithm belongs to $O(n^2)$