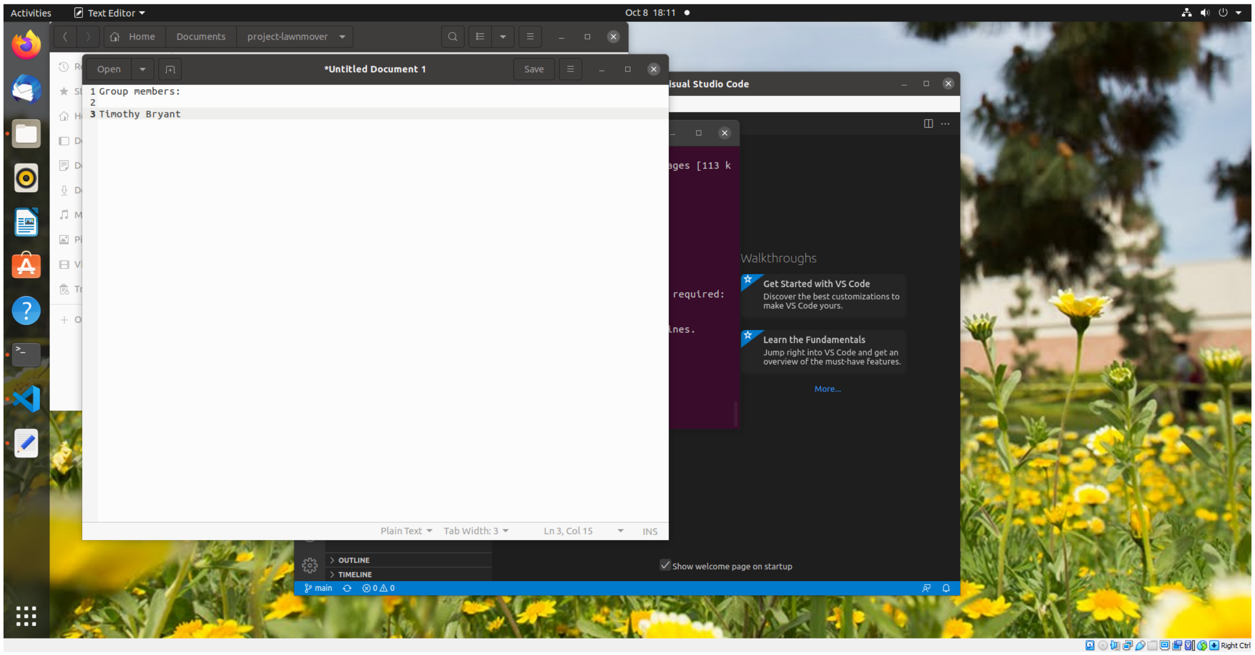
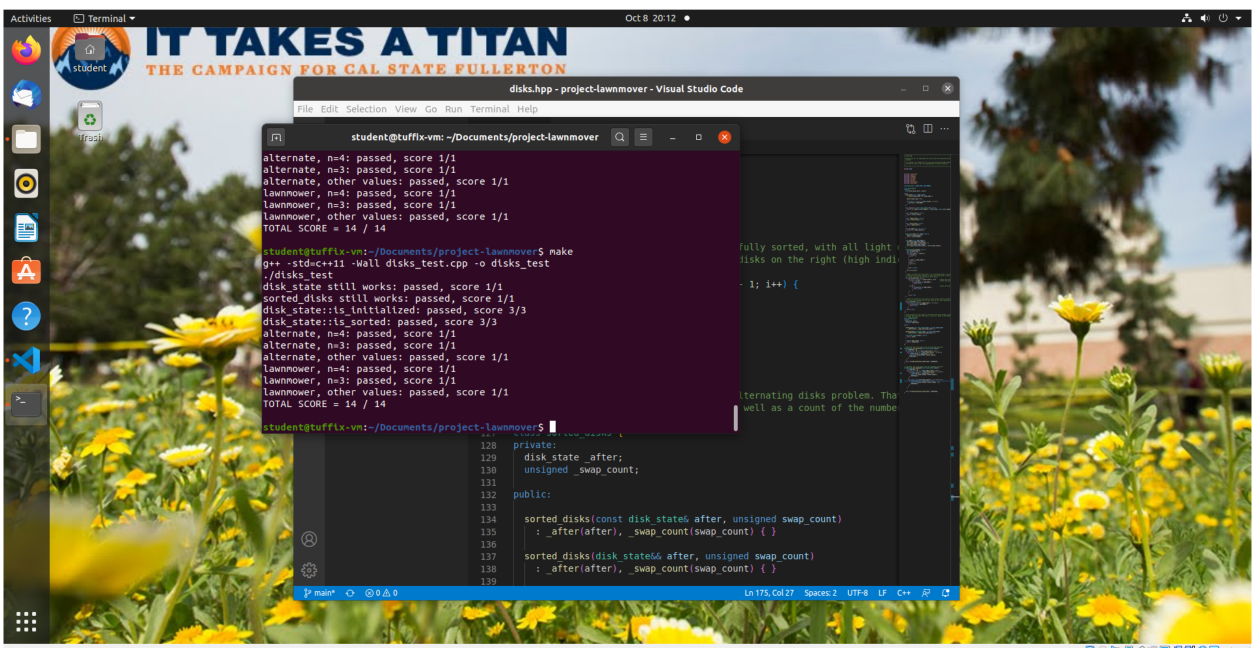
Team Members:
Timothy Bryant tbryant3614@csu.fullerton.edu
October 9, 2022

Project 1 Submission





Left to Right Algorithm

```
numOfSwaps = 0                                                          1tu
For i=0 to n do (Where n = half the list that is light
    For j=i to 2n-1 do (Where 2n is the whole list)
        If (disk[j] > disk[j+1])                                        1tu
                Swap (disk[j], disk[j+1]);                              1tu
                numOfSwaps++;                                           1tu
return (disk[], numOfSwaps)                                             1tu
```

Step Count

$$\left(\sum_{i=0}^{n}\sum_{j=i}^{2n-1} 1+\max(2,0)\right)+1+1$$

$$\left(\sum_{i=0}^{n}\sum_{j=i}^{2n-1} 3\right)+2 = \left(\sum_{i=0}^{n}\left(\sum_{j=1}^{2n-1} 3 - \sum_{j=1}^{i-1} 3\right)\right) = \sum_{i=0}^{n} 3(2n-1) - 3(i-1)$$

$$= \sum_{i=0}^{n} 6n - 3 + 3 - 3i = \sum_{i=0}^{n} 6n - \sum_{i=0}^{n} 3i$$

$$= 6n^2 - 3\left(\frac{n(n+1)}{2}\right) = 6n^2 - 3\left(\frac{n^2+n}{2}\right) = 6n^2 - 3/2n^2 + {-3/2}n$$

$$= \boxed{\frac{9}{2}n^2 + (-3/2n) + 2}$$

Mathematical Analysis $O(n^2)$

By definition

$$9/2n^2 + {-3/2}n + 2 \leq n^2 \quad \forall n \geq n_0$$

$c=8$ $\quad n_0 = 1$

$$9/2n^2 + {-3/2}n + 2 \leq 8n^2$$

$$5n^2 \leq 8n^2 \checkmark$$

$\therefore$ the algorithm belongs to $O(n^2)$

By limit theorem

$\frac{9}{2}n^2 + (\frac{3}{2}n) + 2 \in O(n^2)$

$$\lim_{n \to \infty} \frac{\frac{9}{2}n^2 + (\frac{3}{2}n) + 2}{n^2} = \lim_{n \to \infty} \frac{9n + (\frac{3}{2})}{2n} = \lim_{n \to \infty} \frac{9}{2}$$

$\therefore$ By limit theorem, the algorithm belongs to $O(n^2)$

Lawnmower algorithm

```
numOfSwaps = 0;                                        1tu
  For i=0 to n do
      For j=i do 2n-1 do
         If disk[j] > disk[j+1]                         1tu
               swap(disk[j], disk[j+1])                 1tu
            numOfSwaps ++;                              1tu
      For j = 2n-1 to j = 0
          If disk[j] < disk[j-1]                       1tu
              Swap(disk[j], disk[j-1])                 1tu
              numOfSwaps ++;                           1tu
return(disk[]; numOfSwaps)                             1tu
```

Step Count

$$\left(\sum_{i=0}^{n}\left(\sum_{j=i}^{2n-1} 3 + \sum_{j=2n-1}^{0} 3\right)\right) + 2$$

$$= \left(\sum_{i=0}^{n}\left(3(2n-1) - 3(i-1) + \sum_{j=2n-1}^{0} 3\right)\right) + 2$$

$$= \sum_{i=0}^{n}\left(3(2n-1) - 3(i-1) + \left(\sum_{j=1}^{0} 3 - \sum_{j=1}^{2n-1} 3\right)\right)$$

$$= \sum_{i=0}^{n} 6n - 3 + 3 - 3i + (3) + 6n = \left(\sum_{i=0}^{n} 12n - 3i - 3\right) + 2$$

$$= \sum_{i=0}^{n} 12n - \sum_{i=0}^{n} 3i - \sum_{i=0}^{n} 3 = 12n^2 - 3\left(\frac{n(n+1)}{2}\right) - 3n + 2$$

$$= 12n^2 - \frac{3}{2}n^2 - \frac{3}{2}n - 3n + 2$$

$$= \boxed{\frac{21}{2}n^2 - \frac{9}{2}n + 2}$$

Mathematical Analysis $O(n^2)$

By definition

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \le n^2 \quad \forall n \ge n_0$$

$c = 17 \quad n_0 = 1$

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \le 17n^2$$

$$8n^2 \le 17n^2 \checkmark$$

$\therefore$ the algorithm belongs to $O(n^2)$

By limit theorem

$$\frac{21}{2}n^2 - \frac{9}{2}n + 2 \in O(n^2)$$

$$\lim_{n \to \infty} \frac{\frac{21}{2}n^2 - \frac{9}{2}n + 2}{n^2} = \lim_{n \to \infty} \frac{21n - \frac{9}{2}}{2n} = \lim_{n \to \infty} \frac{21}{2}$$

$\therefore$ By limit theorem, the algorithm belongs to $O(n^2)$