Algoritmi e Strutture Dati

Alberi

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Sommario

Albero radicato – Definizione 1

Albero radicato (Rooted tree)

Un albero consiste di un insieme di nodi e un insieme di archi orientati che connettono coppie di nodi, con le seguenti proprietà:

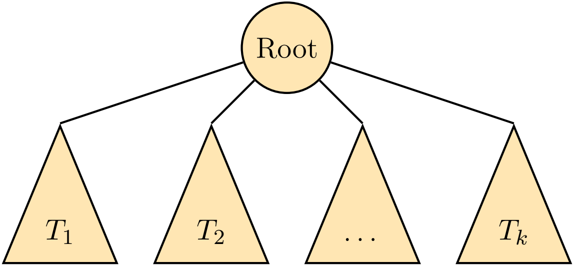
- Un nodo dell'albero è designato come nodo radice;
- Ogni nodo n, a parte la radice, ha esattamente un arco entrante;
- Esiste un cammino unico dalla radice ad ogni nodo;
- L'albero è connesso.

Albero radicato – Definizione 2 (Ricorsiva)

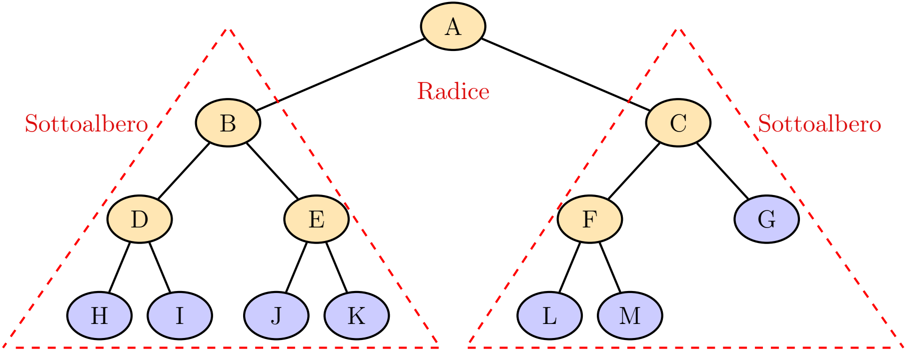
Albero radicato (Rooted tree)

Un albero è dato da:

- un insieme vuoto, oppure
- un nodo radice e zero o più sottoalberi, ognuno dei quali è un albero; la radice è connessa alla radice di ogni sottoalbero con un arco orientato.

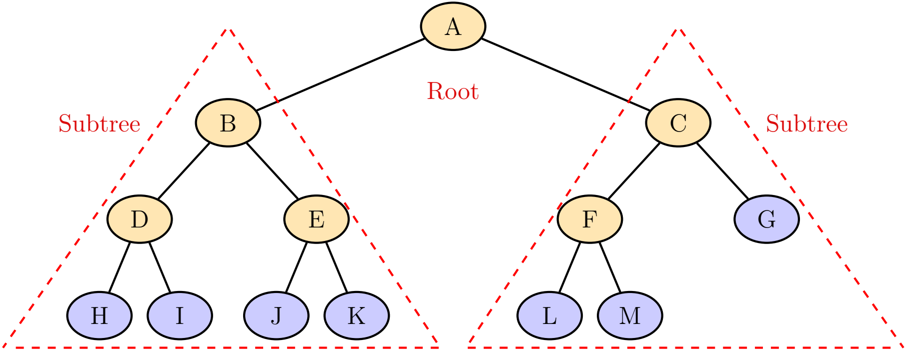


Terminologia



- A è la radice
- B, C sono radici dei sottoalberi
- D, E sono fratelli
- D, E sono figli di B
- B è il padre di D, E
- I nodi viola sono foglie
- Gli altri nodi sono nodi interni

Terminology (English)



- A is the tree root
- B, C are roots of their subtrees
- D, E are siblings

- D, E are children of B
- B is the parent of D, E

- Purple nodes are leaves
- The other nodes are internal nodes

Terminologia

Profondità nodi (Depth)

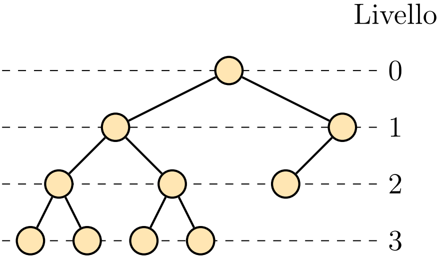
La lunghezza del cammino semplice dalla radice al nodo (misurato in numero di archi)

Livello (Level)

L'insieme di nodi alla stessa profondità

Altezza albero (Height)

La profondità massima della sue foglie



Altezza di questo albero = 3

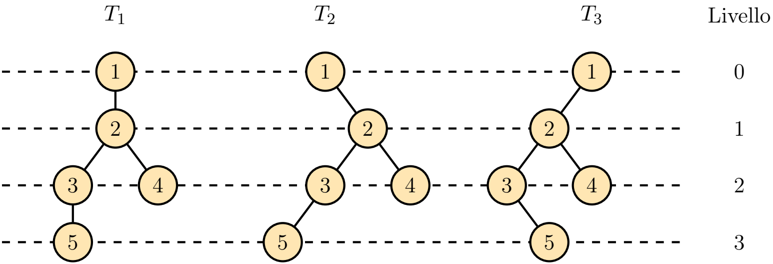
Sommario

Albero binario

Albero binario

Un albero binario è un albero radicato in cui ogni nodo ha al massimo due figli, identificati come figlio sinistro e figlio destro.

Nota: *Due alberi T e U che hanno gli stessi nodi, gli stessi figli per ogni nodo e la stessa radice, sono distinti qualora un nodo u sia designato come figlio sinistro di v in T e come figlio destro di v in U.*



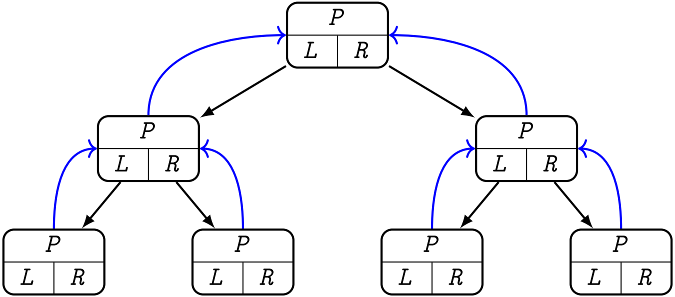
Specifica (Albero binario)

```
TREE
% Costruisce un nuovo nodo, contenente $v$, senza figli o genitori
Tree(ITEM $v$)

% Legge il valore memorizzato nel nodo
ITEM read()

% Modifica il valore memorizzato nel nodo
write(ITEM $v$)

% Restituisce il padre, oppure nil se questo nodo è radice
TREE parent()
```

Specifica (Albero binario)

```
TREE
% Restituisce il figlio sinistro (destro) di questo nodo; restituisce nil
  se assente
TREE left()
TREE right()

% Inserisce il sottoalbero radicato in t come figlio sinistro (destro)
  di questo nodo
insertLeft(TREE t)
insertRight(TREE t)

% Distrugge (ricorsivamente) il figlio sinistro (destro) di questo
  nodo
deleteLeft()
deleteRight()
```

Memorizzare un albero binario

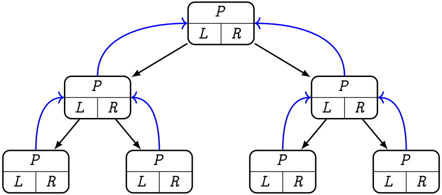


Campi memorizzati nei nodi

- *parent*: reference al nodo padre
- *left*: reference al figlio sinistro
- *right*: reference al figlio destro

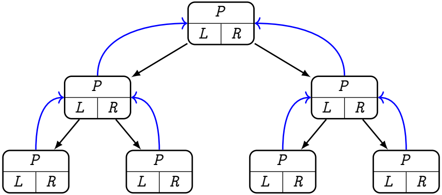
Implementazione

```
TREE
Tree(ITEM v)
    TREE t = new TREE
    t.parent = nil
    t.left = t.right = nil
    t.value = v
    return t

insertLeft(TREE T)
    if left == nil then
        T.parent = this
        left = T

insertRight(TREE T)
    if right == nil then
        T.parent = this
        right = T
```



Implementazione

```
TREE
deleteLeft()
    if $left \neq$ nil then
        $left$.deleteLeft()
        $left$.deleteRight()
        delete $left$
        $left$ = nil

deleteRight()
    if $right \neq$ nil then
        $right$.deleteLeft()
        $right$.deleteRight()
        delete $right$
        $right$ = nil
```



Visite di alberi

Visita di un albero / ricerca

Una strategia per analizzare (visitare) tutti i nodi di un albero.

Visità in profondità
Depth-First Search (DFS)

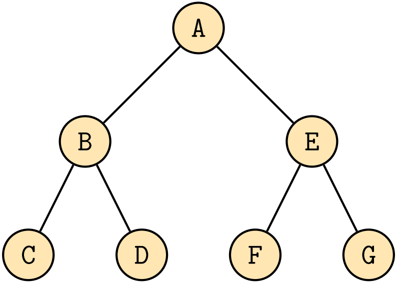
- Per visitare un albero, si visita ricorsivamente ognuno dei suoi sottoalberi
- Tre varianti: pre/in/post visita (pre/in/post order)
- Richiede uno stack

Visita in ampiezza
Breadth First Search (BFS)

- Ogni livello dell'albero viene visitato, uno dopo l'altro
- Si parte dalla radice
- Richiede una queue

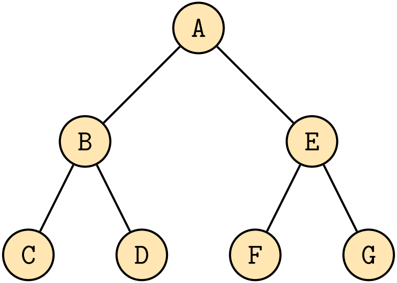
Depth-First Search

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```



Depth-First Search - Pre-Order

```
dfs(TREE t)
if t ≠ nil then
    % pre-order visit of t
    print t
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

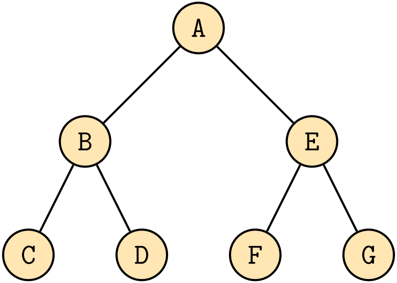


Sequence: **A**

Stack: **A**

Depth-First Search - Pre-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

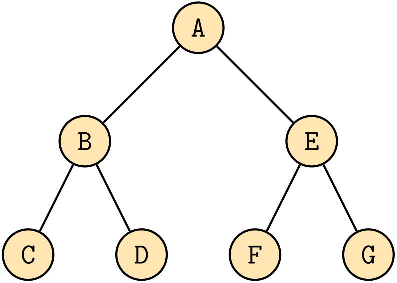


Sequence: A B

Stack: A B

Depth-First Search - Pre-Order

```
dfs(TREE t)
if $t \neq$ nil then
    % pre-order visit of t
    print t
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

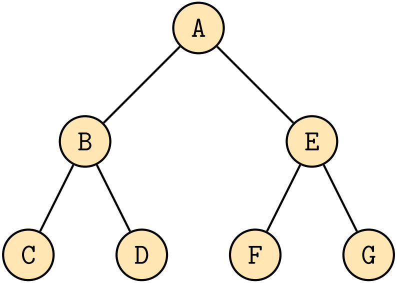


Sequence: A B C

Stack: A B C

Depth-First Search - Pre-Order

```
dfs(TREE t)
if $t \neq$ nil then
    % pre-order visit of t
    print t
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

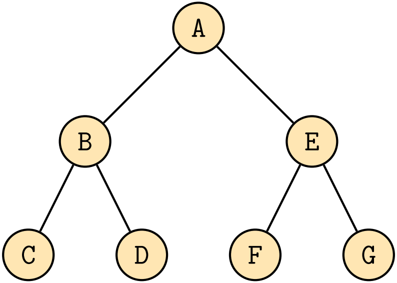


Sequence: A B C

Stack: A B

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

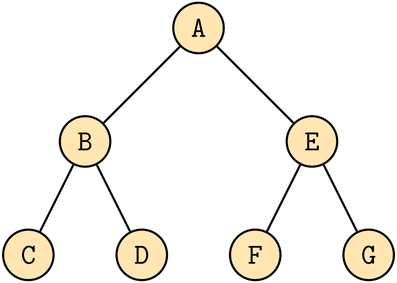


Sequence: A B C D

Stack: A B D

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

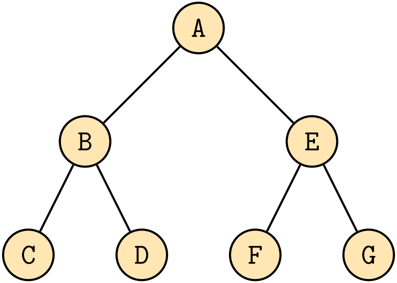


Sequence: A B C D

Stack: A B

Depth-First Search - Pre-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

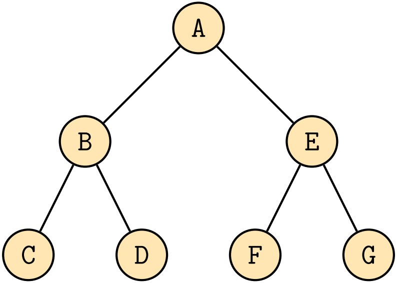


Sequence: A B C D

Stack: A

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

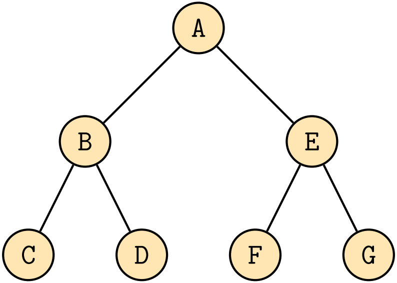


Sequence: `A B C D E`

Stack: `A E`

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

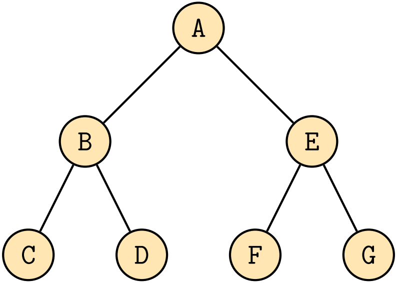


Sequence: A B C D E F

Stack: A E F

Depth-First Search - Pre-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

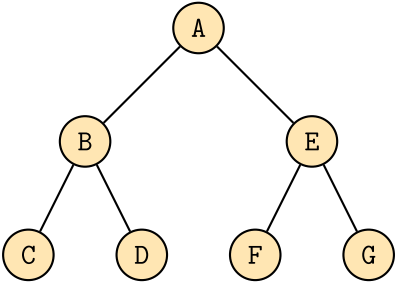


Sequence: A B C D E F

Stack: A E

Depth-First Search - Pre-Order

```
dfs(Tree t)
if t ≠ nil then
    % pre-order visit of t
    print t
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

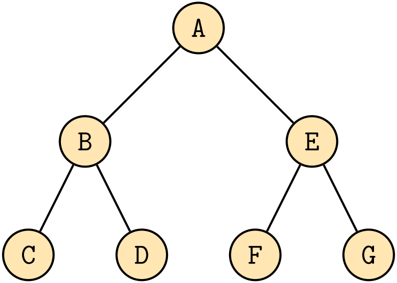


Sequence: A B C D E F G

Stack: A E G

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

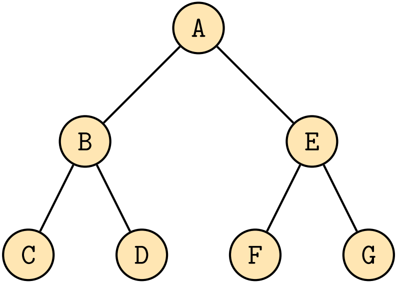


Sequence: A B C D E F G

Stack: A E

Depth-First Search - Pre-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of $t$
    print $t$
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

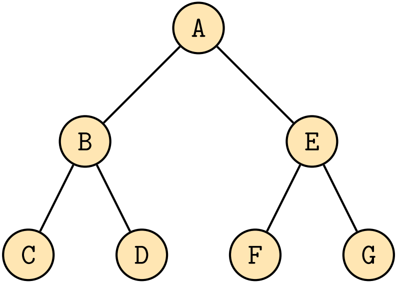


Sequence: A B C D E F G

Stack: A

Depth-First Search - Pre-Order

```
dfs(TREE t)
if $t \neq$ nil then
    % pre-order visit of t
    print t
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

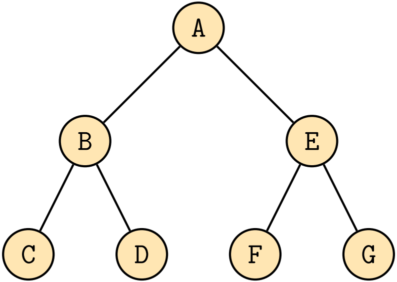


Sequence: A B C D E F G

Stack:

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

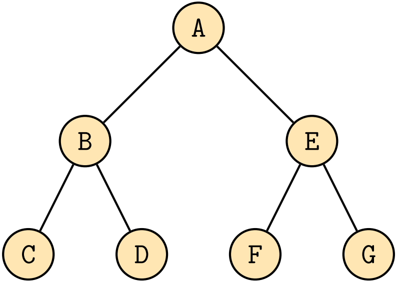


Sequence:

Stack: A

Depth-First Search - In-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

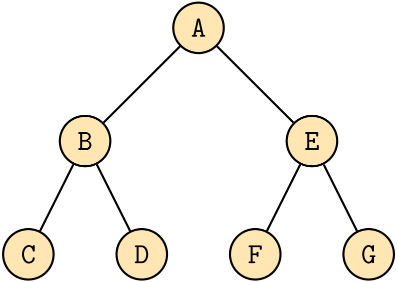


Sequence:

Stack: `A B`

Depth-First Search - In-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

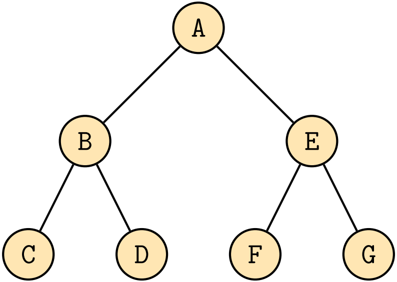


Sequence: C

Stack: A B C

Depth-First Search - In-Order

```
dfs(TREE t)
if $t \neq$ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    % in-order visit of t
    print t
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

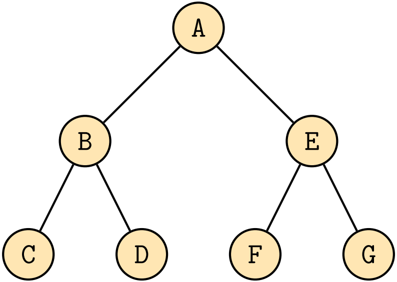


Sequence: C B

Stack: A B

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

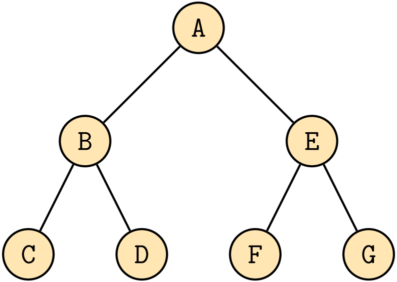


Sequence: C B D

Stack: A B D

Depth-First Search - In-Order

```
dfs(TREE t)
if t ≠ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    % in-order visit of t
    print t
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

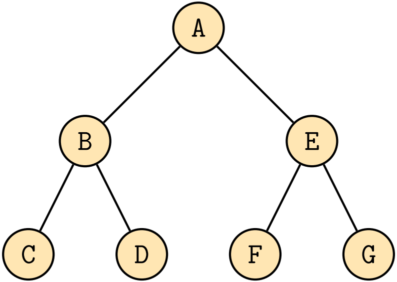


Sequence: C B D

Stack: A B

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

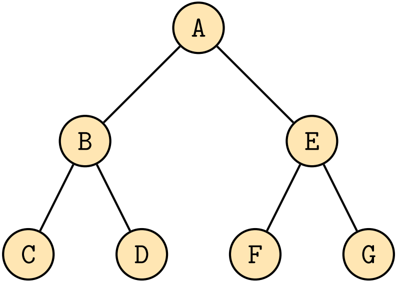


Sequence: C B D A

Stack: A

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

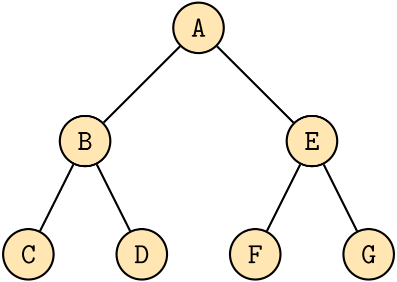


Sequence: C B D A

Stack: A E

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

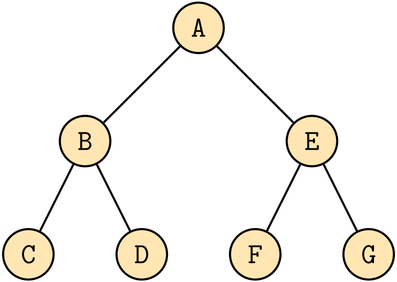


Sequence: C B D A F

Stack: A E F

Depth-First Search - In-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

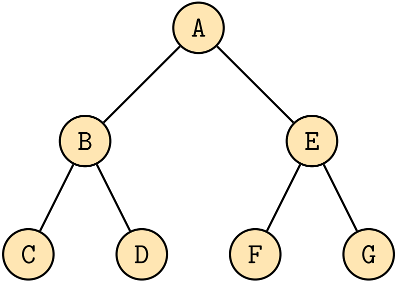


Sequence: C B D A F E

Stack: A E

Depth-First Search - In-Order

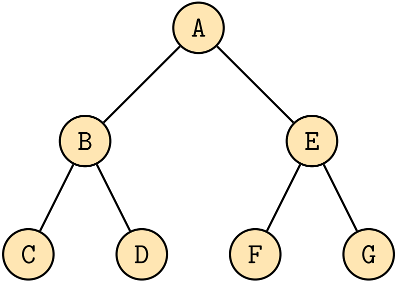
```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```



Sequence: C B D A F E G
Stack: A E G

Depth-First Search - In-Order

```
dfs(TREE t)
if $t \neq$ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    % in-order visit of t
    print t
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```

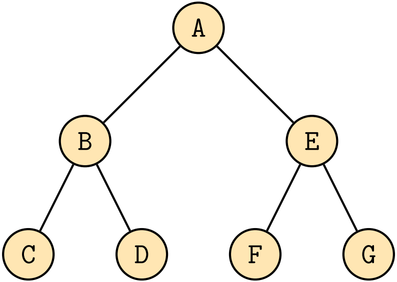


Sequence: C B D A F E G

Stack: A E

Depth-First Search - In-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    % in-order visit of $t$
    print $t$
    dfs($t$.right())
    ~~% post-order visit of $t$~~
    ~~print $t$~~
```

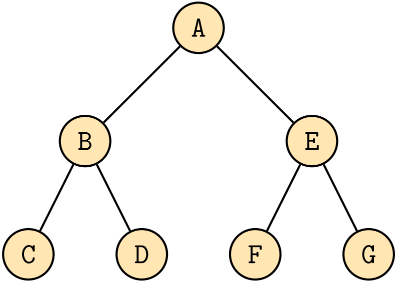


Sequence: C B D A F E G

Stack: A

Depth-First Search - In-Order

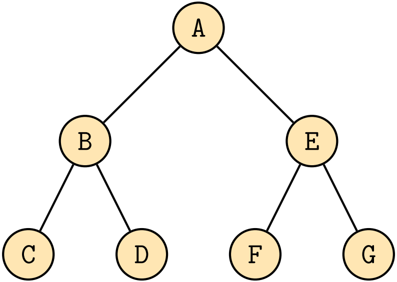
```
dfs(TREE t)
if $t \neq$ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    % in-order visit of t
    print t
    dfs(t.right())
    ~~% post-order visit of t~~
    ~~print t~~
```



Sequence: C B D A F E G
Stack:

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

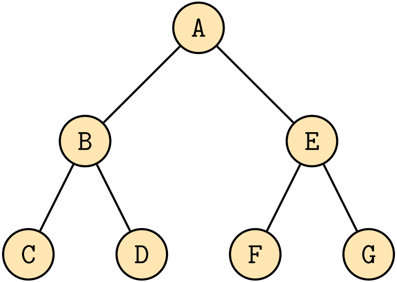


Sequence:

Stack: A

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

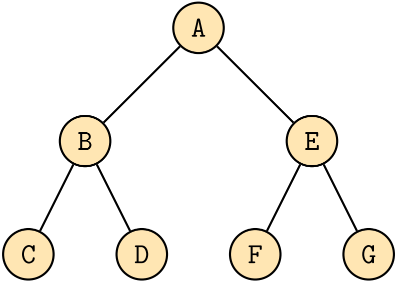


Sequence:

Stack: A B

Depth-First Search - Post-Order

```
dfs(TREE t)
if $t \neq$ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    % post-order visit of t
    print t
```

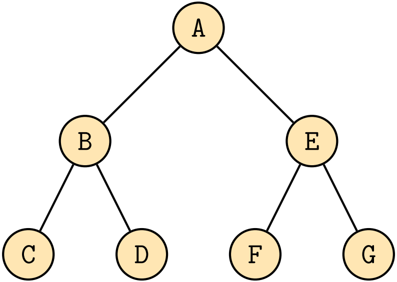


Sequence: C

Stack: A B C

Depth-First Search - Post-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

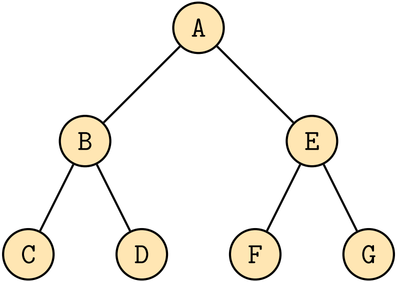


Sequence: C

Stack: A B

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

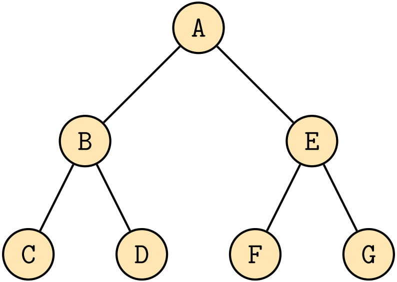


Sequence: C D

Stack: A B D

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

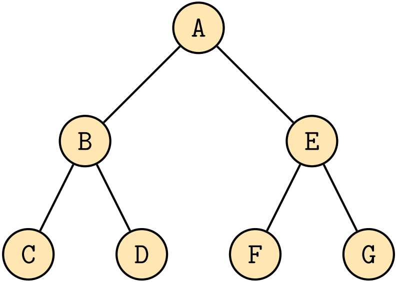


Sequence: `C D B`

Stack: `A B`

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

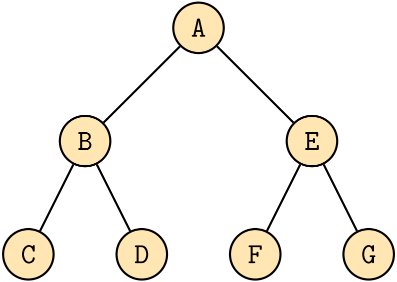


Sequence: `C D B`

Stack: `A`

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

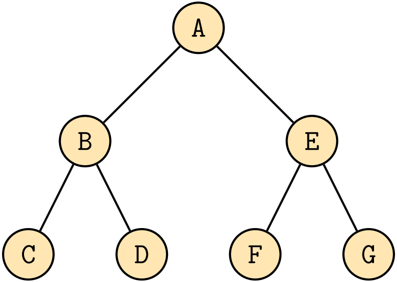


Sequence: C D B

Stack: A E

Depth-First Search - Post-Order

```
dfs(Tree $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

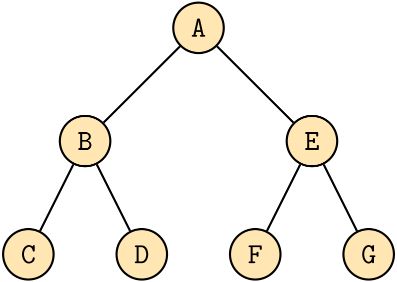


Sequence: C D B F

Stack: A E F

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

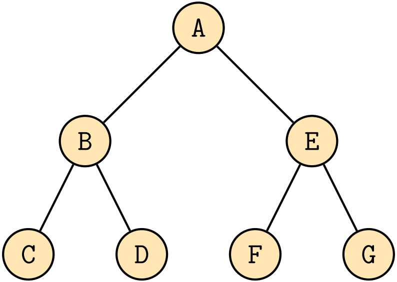


Sequence: C D B F

Stack: A E

Depth-First Search - Post-Order

```
dfs(TREE t)
if $t \neq$ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    % post-order visit of t
    print t
```

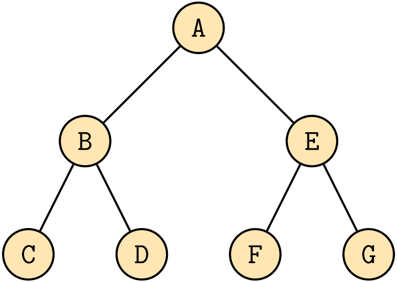


Sequence: C D B F G

Stack: A E G

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

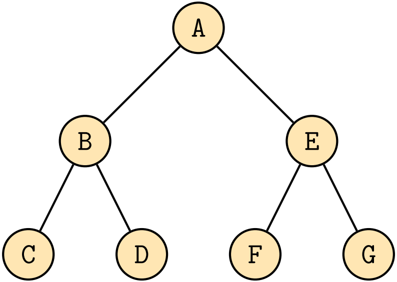


Sequence: C D B F G E

Stack: A E

Depth-First Search - Post-Order

```
dfs(TREE $t$)
if $t \neq$ nil then
    ~~% pre-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.left())
    ~~% in-order visit of $t$~~
    ~~print $t$~~
    dfs($t$.right())
    % post-order visit of $t$
    print $t$
```

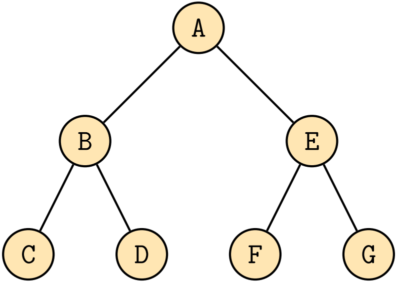


Sequence: C D B F G E A

Stack: A

Depth-First Search - Post-Order

```
dfs(TREE t)
if t ≠ nil then
    ~~% pre-order visit of t~~
    ~~print t~~
    dfs(t.left())
    ~~% in-order visit of t~~
    ~~print t~~
    dfs(t.right())
    % post-order visit of t
    print t
```



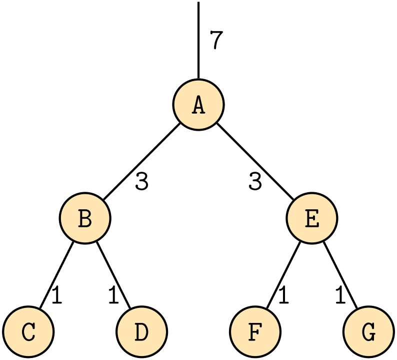
Sequence: C D B F G E A

Stack:

Esempi di applicazione

Contare nodi – Post-visita

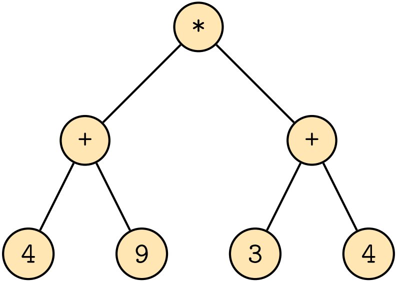
```
int count(Tree T)
if $T == \mathbf{nil}$ then
    return 0
else
    $C_\ell = \mathsf{count}(T.\mathsf{left}())$
    $C_r = \mathsf{count}(T.\mathsf{right}())$
    return $C_\ell + C_r + 1$
```



Esempi di applicazione

Stampare espressioni – In-visita

```
int printExp(Tree $T$)
if $T$.left() == nil and
  $T$.right == nil then
    print $T$.read()
else
    print "("
    printExp($T$.left())
    print $T$.read()
    printExp($T$.right())
    print ")"
```



((4+9) * (3+4))

Costo computazionale

Il costo di una visita di un albero contenente n nodi è $\Theta(n)$, in quanto ogni nodo viene visitato al massimo una volta.

Sommario

Specifica (Albero generico)

```
TREE
% Costruisce un nuovo nodo, contenente $v$, senza figli o genitori
Tree(ITEM $v$)
% Legge il valore memorizzato nel nodo
ITEM read()
% Modifica il valore memorizzato nel nodo
write(ITEM $v$)
% Restituisce il padre, oppure nil se questo nodo è radice
TREE parent()
```

Specifica (Albero generico)

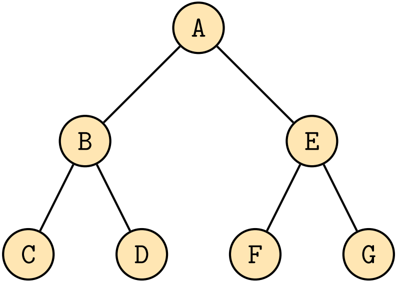
```
TREE
% Restituisce il primo figlio, oppure nil se questo nodo è una foglia
TREE leftmostChild()
% Restituisce il prossimo fratello, oppure nil se assente
TREE rightSibling()

% Inserisce il sottoalbero t come primo figlio di questo nodo
insertChild(TREE t)

% Inserisce il sottoalbero t come prossimo fratello di questo nodo
insertSibling(TREE t)

% Distruggi l’albero radicato identificato dal primo figlio
deleteChild()

% Distruggi l’albero radicato identificato dal prossimo fratello
deleteSibling()
```

Esempio: Class Node (Java 8)

```
package org.w3c.dom;
public interface Node {

  /** The parent of this node. */
  public Node   getParentNode();

  /** The first child of this node. */
  public Node   getFirstChild()

  /** The node immediately following this node. */
  public Node   getNextSibling()

  /** Inserts the node newChild before the existing child node refChild. */
  public Node   insertBefore(Node newChild, Node refChild)

  /** Adds the node newChild to the end of the list of children of this node. */
  public Node   appendChild(Node newChild)

  /** Removes the child node indicated by oldChild from the list of children. */
  public Node   removeChild(Node oldChild)

  [...]
}
```

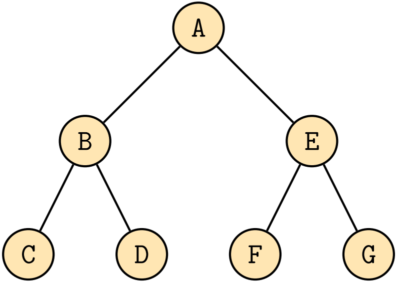
Depth-First Search

```
dfs(TREE $t$)
if $t \neq$ nil then
    % pre-order visit of node $t$
    print $t$
    TREE $u = t$.leftmostChild()
    while $u \neq$ nil do
        dfs($u$)
        $u = u$.rightSibling()
    % post-order visit of node $t$
    print $t$
```



Breadth-First Search

```
bfs(TREE $t$)
QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```

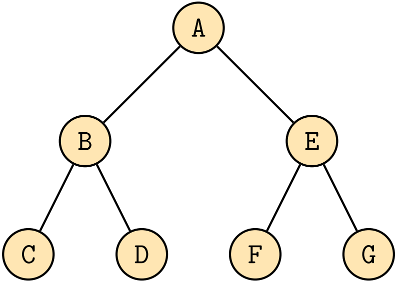


Sequence:

Queue: A

Breadth-First Search

```
bfs(Tree $t$)
Queue $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    Tree $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```

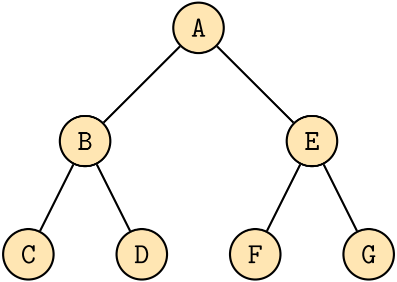


Sequence: A

Queue: B E

Breadth-First Search

```
bfs(TREE $t$)

QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u$ = $Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u$ = $u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u$ = $u$.rightSibling()
```

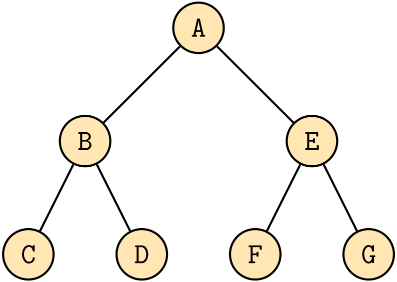


Sequence: A B

Queue: E C D

Breadth-First Search

```
bfs(TREE $t$)
QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```

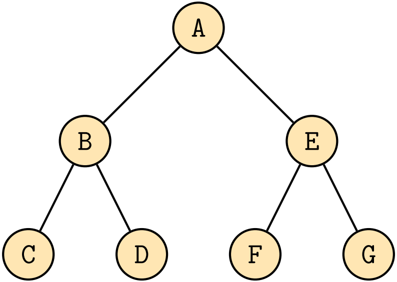


Sequence: A B E

Queue: C D F G

Breadth-First Search

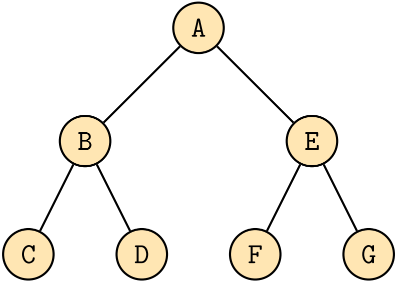
```
bfs(TREE $t$)

QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```



Sequence: A B E C
Queue: D F G

Breadth-First Search

```
bfs(TREE $t$)
QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```

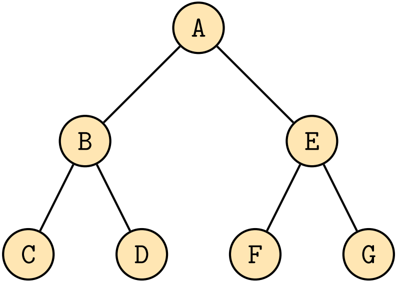


Sequence: A B E C D

Queue: F G

Breadth-First Search

```
bfs(TREE $t$)
QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u$ = $Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```

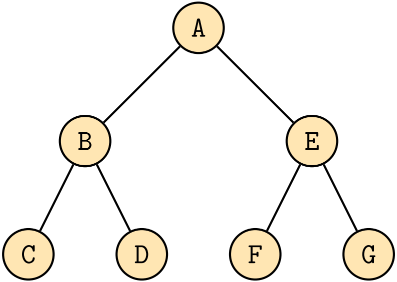


Sequence: A B E C D F

Queue: G

Breadth-First Search

```
bfs(TREE $t$)

QUEUE $Q$ = Queue()
$Q$.enqueue($t$)
while not $Q$.isEmpty() do
    TREE $u = Q$.dequeue()
    % visita per livelli nodo $u$
    print $u$
    $u = u$.leftmostChild()
    while $u \neq$ nil do
        $Q$.enqueue($u$)
        $u = u$.rightSibling()
```



Sequence: A B E C D F G

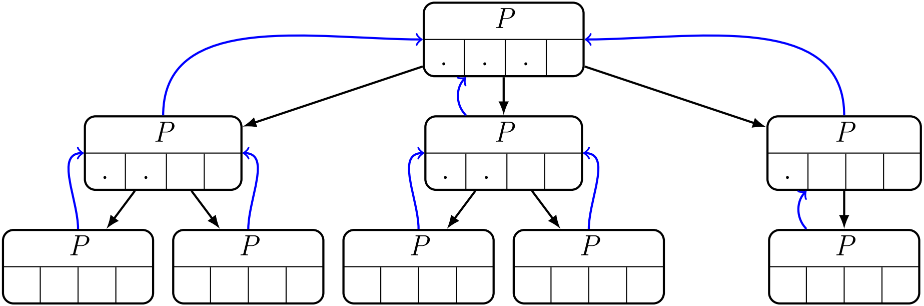
Queue:

Memorizzazione

Esistono diversi modi per memorizzare un albero, più o meno indicati a seconda del numero massimo e medio di figli presenti.

- Realizzazione con vettore dei figli
- Realizzazione primo figlio, prossimo fratello
- Realizzazione con vettore dei padri

Realizzazione con vettore dei figli

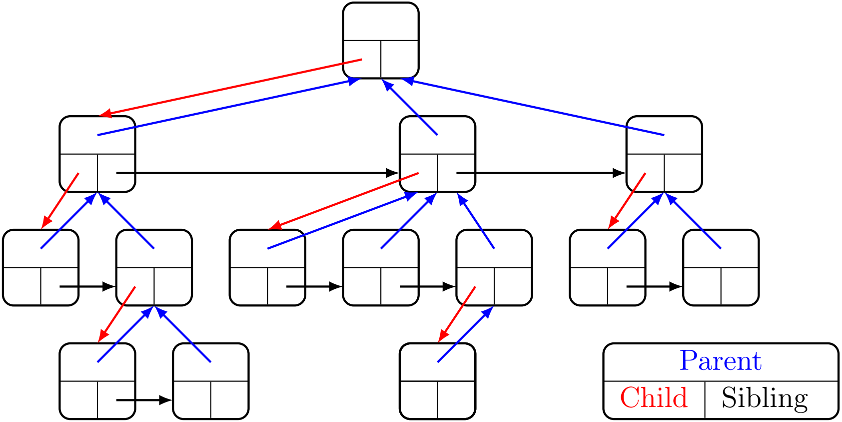


Campi memorizzati nei nodi

- *parent*: reference al nodo padre
- Vettore dei figli: a seconda del numero di figli, può comportare una discreta quantità di spazio sprecato

Realizzazione basata su Primo figlio, prossimo fratello

Implementato come una lista di fratelli



Implementazione

```
TREE
TREE parent                                   % Reference al padre
TREE child                                    % Reference al primo figlio
TREE sibling                                  % Reference al prossimo fratello
ITEM value                                    % Valore memorizzato nel nodo

Tree(ITEM v)                                  % Crea un nuovo nodo
    TREE t = new TREE
    t.value = v
    t.parent = t.child = t.sibling = nil
    return t

insertChild(TREE t)
    t.parent = self
    t.sibling = child                         % Inserisce t prima dell'attuale primo figlio
    child = t

insertSibling(TREE t)
    t.parent = parent
    t.sibling = sibling                       % Inserisce t prima dell'attuale prossimo fratello
    sibling = t
```

Implementazione

```
TREE
deleteChild()
    TREE $newChild$ = $child$.rightSibling()
    delete($child$)
    $child = newChild$

deleteSibling()
    TREE $newSibling$ = $sibling$.rightSibling()
    delete($sibling$)
    $sibling = newSibling$

delete(TREE $t$)
    TREE $u$ = $t$.leftmostChild()
    while $u \neq$ nil do
        TREE $next$ = $u$.rightSibling()
        delete($u$)
        $u = next$
    delete $t$
```

Realizzazione con vettore dei padri

L'albero è rappresentato da un vettore i cui elementi contengono il valore associato al nodo e l'indice della posizione del padre nel vettore.

1	A	0
2	B	1
3	E	1
4	C	2
5	D	2
6	F	3
7	G	3