Numerical Solution of PDEs Using the Finite Element Method

Exercise 3: Solving the Poisson problem

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Some useful resources

https://www.dealii.org/8.5.1/doxygen/deal.II/step_3.html
https://www.dealii.org/8.5.1/doxygen/deal.II/step_4.html

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1. Using `step-3` as a base:
   (a) Compile and run this tutorial, and inspect at the output.
   (b) Modify the code so that the problem is dimension-independent.
   (c) Switch to `vtk` output and visualize in `Paraview`. Figure out how to warp the solution by the solution variable.
   (d) Add a zero Neumann boundary condition to one edge of the domain. Assign this Neumann boundary the indicator 1.
   Tip: Look at the instructions in "Modify the type of boundary condition" in the "Possibilities for extensions" section of the tutorial.
   (e) Add a non-zero Dirichlet boundary condition to one edge of the domain.
      i. Set the value to -0.5 for the boundary with indicator 1.
      Tip: Look at the instructions in "A slight variation of the last point" in the "Possibilities for extensions" section of the tutorial.
      ii. Change the setup to have $f = 0$. Compare this result to that where f is non-zero.
2. Additional tasks
   (a) Do "Convergence of the mean". Can you see the order h^2?
   (b) Increase the polynomial order (you need to increase all orders of the quadratures in the program!) and check the convergence of the mean now.
   (c) Switch to an L-shaped domain and experiment with a combination of Dirichlet and Neumann boundary conditions. By experimentation, identify the faces adjacent to the re-entrant corner and apply Dirichlet conditions only there.
   Tip: There is more than one way to generate such a grid using the built-in functions.