Numerical Solution of PDEs Using the Finite Element Method

Exercise 1: Creating and manipulating Triangulations

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Some useful resources

https://www.dealii.org/8.5.1/doxygen/deal.II/step_1.html
https://www.dealii.org/8.5.1/doxygen/deal.II/step_49.html

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1. Using `step-1` as a base:
   (a) Compile and run this tutorial on the command line or inside a suitable IDE, and inspect at the output.
   (b) Create a helper function that takes a reference to a Triangulation and prints the following information:
   - number of levels
   - number of cells
   - number of active cells

   Test this with all of your meshes.

2. Modifying an existing meshing function
   (a) Comment out the `.set_manifold(0, ...)` line in `second_grid()`. What happens now?
   (b) Output mesh two as an svg file instead of eps. Open it in a browser to display it (`Firefox`, for example).
   (c) Go into `second_grid()` and remove the last line (`.set_manifold(0);`). The program will crash when you run it. Try to find out what is going on by debugging the program (e.g. For `Qt Creator`: "Debug" → "Start debugging") and stepping through the function `second_grid()`. You can fix this problem in a more elegant way than putting the line you removed back in. How? See the tutorial description for more info.

3. Creating a mesh from scratch
   (a) Generate a circle using `GridGenerator::hyper_ball()` in 2d (add a function `third_grid()` to `step-1`).
      i. Use a `SphericalManifold` everywhere, only on the boundary, or on all cells except the center cell and refine the mesh globally twice.
      ii. Set the output format of the previous example to `vtk` and inspect the mesh in `Paraview`.
   (b) Create an image of an L-shape domain with one global refinement.
      i. Inspect the mesh in `Paraview`.
      ii. Refine the L-shaped mesh adaptively:
         α) Refine all cells with the distance between the center of the cell and re-entrant corner is smaller than $\frac{1}{3}$.
         β) Refine exactly at the re-entrant corner (i.e. those with the corner as a vertex) several times.

4. Reading in a mesh
   (a) Take a look at `step-49` and read the included `.msh` file in your modified `step-1` program.

(b) Add two levels of refinement to the cells at the boundary of the cut-outs.

5. Additional tasks

(a) Create a mesh that represents the surface of a torus and refine it 2 times globally. Output to `vtk` format and check the output. Note that your `Triangulation` needs to be of type `Triangulation<2,3>` (not explicitly discussed in this course).