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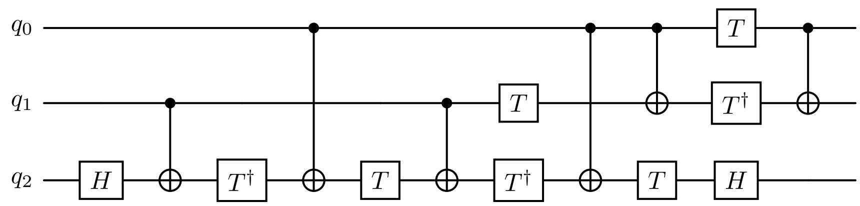
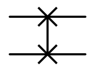
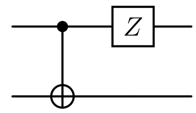
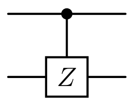
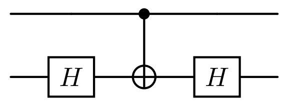
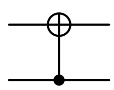
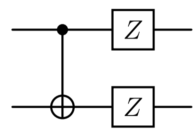
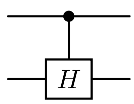
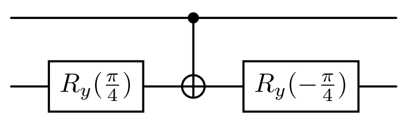
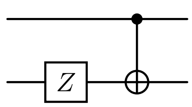
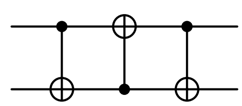
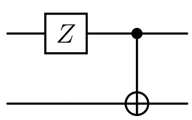
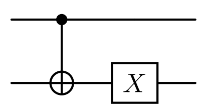
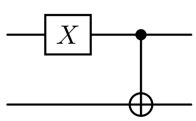
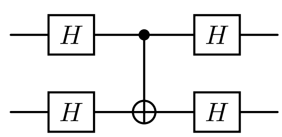
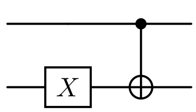
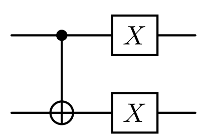
Quantum Basics

1. Construct the following circuit. Then, using the provided Qiskit tools, characterize the behaviour of the system by filling the truth table reported below.



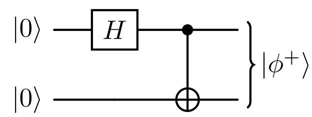
Input $\|q_2q_1q_0\rangle$			Output State
q_2	q_1	q_0	$\|\psi\rangle$
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

2. Check which of the following equivalences are true using the unitary representation of a circuit. Connect the equivalent circuits.

Swap		
$\mathbf{CZ}_{0,1}$		
$\mathbf{CX}_{1,0}$		
$\mathbf{CH}_{0,1}$		
Error propagation through CNOT		
		
		
		

3. Solve the exercises reported below, related to the entanglement generation.

(a) Consider the following quantum circuit which generates an entangled pair



where $|\phi^+\rangle = \frac{|00\rangle+|11\rangle}{\sqrt{2}}$. Check the behaviour of the quantum circuit using the state vector evolution and the unitary gate representation.

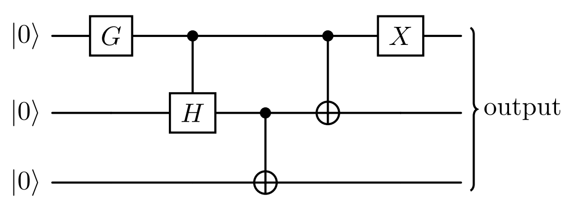
(b) Construct three new schemes able to generate

$$|\phi^-\rangle = \frac{|00\rangle - |11\rangle}{\sqrt{2}}, \quad |\psi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}}, \quad |\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}.$$

(c) Construct a scheme able to generate

$$|GHZ\rangle = \frac{|000\rangle + |111\rangle}{\sqrt{2}}.$$

(d) What is the output of the following circuit?



where $G = \begin{pmatrix} 1/\sqrt{3} & -\sqrt{2/3} \\ \sqrt{2/3} & 1/\sqrt{3} \end{pmatrix}$. Verify it using the evolution of a state vector.

Hint: Note that you can write G using a rotation gate.

4. Implement a quantum circuit able to generate the general quantum state

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$$

starting from the initial state $|0\rangle$. To this aim, find θ and ϕ such that

$$\mathbf{R}_{\mathrm{z}}(\phi)\mathbf{R}_{\mathrm{x}}(\theta)\,|0\rangle = |\psi\rangle$$

where

$$\mathbf{R}_{\mathrm{z}}(\phi) = \begin{pmatrix} e^{-j\phi/2} & 0 \\ 0 & e^{j\phi/2} \end{pmatrix}$$

and

$$\mathbf{R}_{\mathrm{x}}(\theta) = \begin{pmatrix} \cos(\theta/2) & -j\,\sin(\theta/2) \\ -j\,\sin(\theta/2) & \cos(\theta/2) \end{pmatrix}$$

Note: Since global phase can be neglected, defining $\alpha = |\alpha|\,e^{j\varphi_\alpha}$*, we have that an equivalent state is* $|\psi\rangle = |\alpha|\,|0\rangle + \beta\,e^{-j\varphi_\alpha}\,|1\rangle$*.*

Hint 1: Calculate the vector $\mathbf{R}_{\mathrm{z}}(\phi)\mathbf{R}_{\mathrm{x}}(\theta)\,|0\rangle$ *using the matrix/vector representation.*

(a) Implement the scheme generating

$$|\psi\rangle = 0.8\,|0\rangle + 0.6e^{j\pi/3}\,|1\rangle\ .$$

Verify, using the evolution of a state vector, the correct generation.

(b) Implement the scheme generating

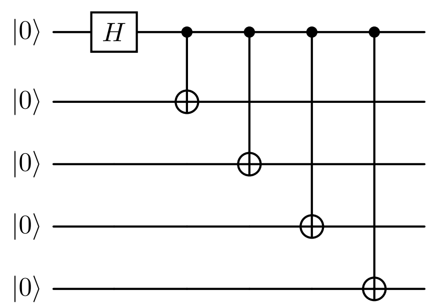
$$|\psi\rangle = \frac{j}{2}\,|0\rangle + \left(\frac{\sqrt{3}}{2\sqrt{2}} + \frac{\sqrt{3}}{2\sqrt{2}}\,j\right)|1\rangle\ .$$

Verify, using the evolution of a state vector, the correct generation.

5. Verify the transpiled quantum circuit of the schematic reported below using: IBM Vigo 5, IBM Athens 5, and IBM Yorktown 5 (see figure below the table for the architectures). Assume that the quantum computers could use the following gates: [id, rz, sx, x, cx, h].

   (a) Fill the table reporting the number of double qubit gates in the transpiled circuits using both the level 0 optimization and the full optimization.

   (b) Count the double qubit gate for the optimized circuit on Vigo 5 running 10 different simulations. What happens to the optimized quantum circuit?



Quantum Computer:	Vigo 5	Athens 5	Yorktown 5
Opt. Lvl 0			
Optimized			

DG: double gates.

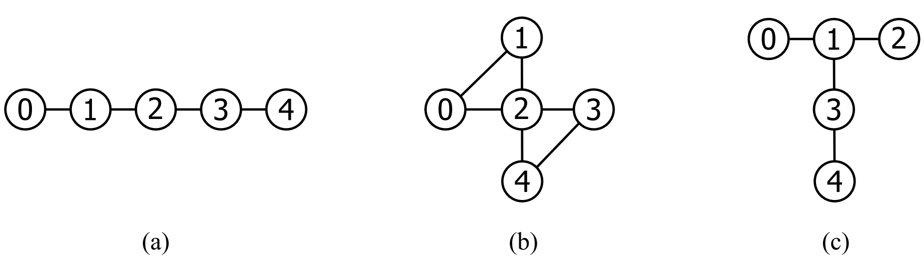


Figura 1: 5 Qubit quantum computer architectures. (a) Athens 5 qubit with linear connectivity. (b) Yorktown 5 qubit with bow-tie connectivity. (c) Vigo 5 qubit with "T" connectivity.