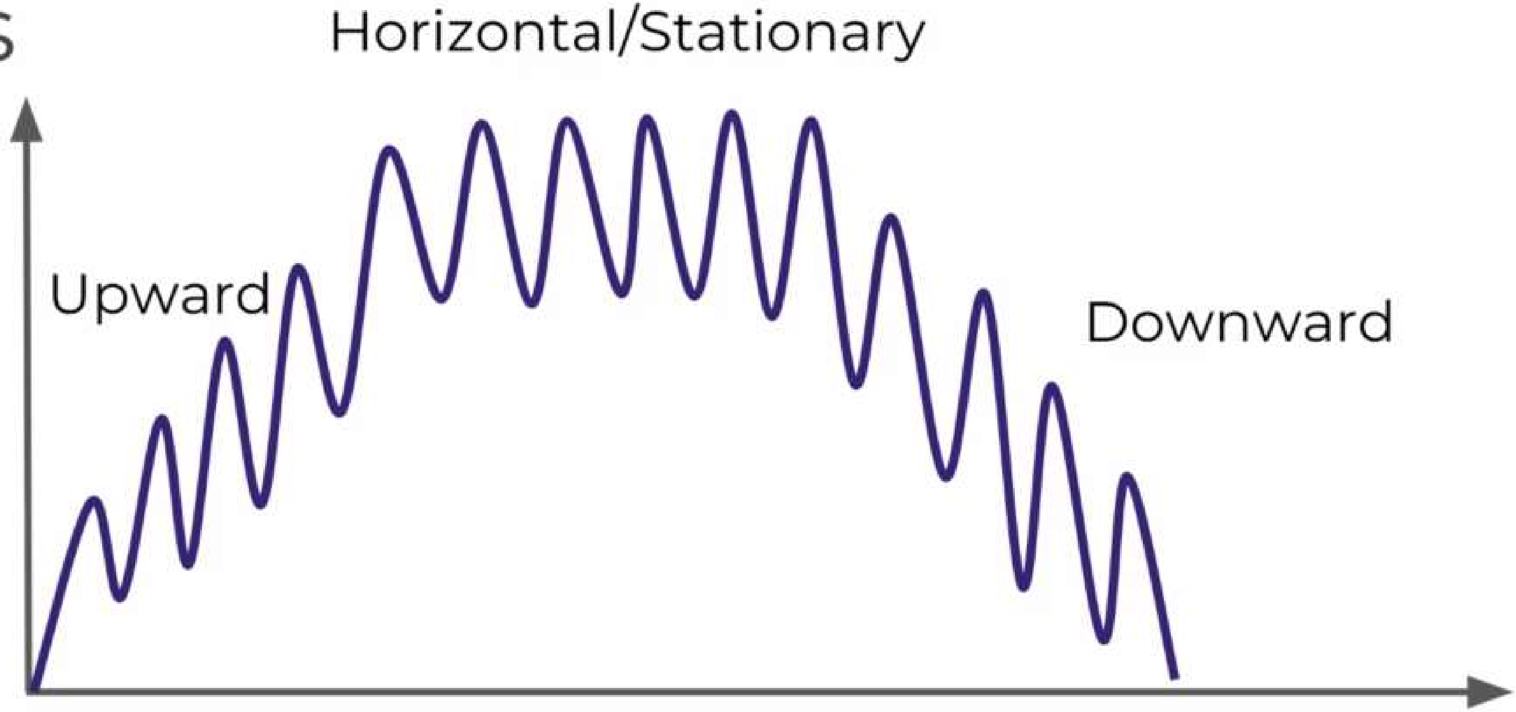
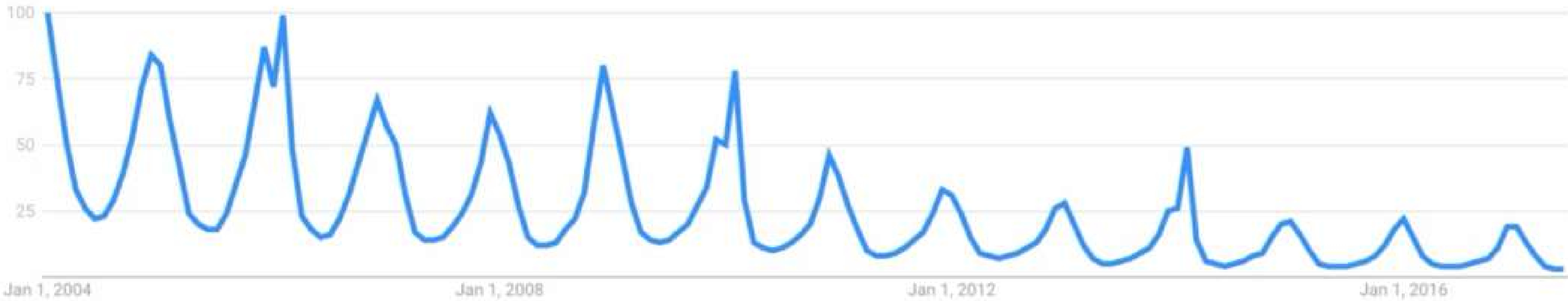
Time Series

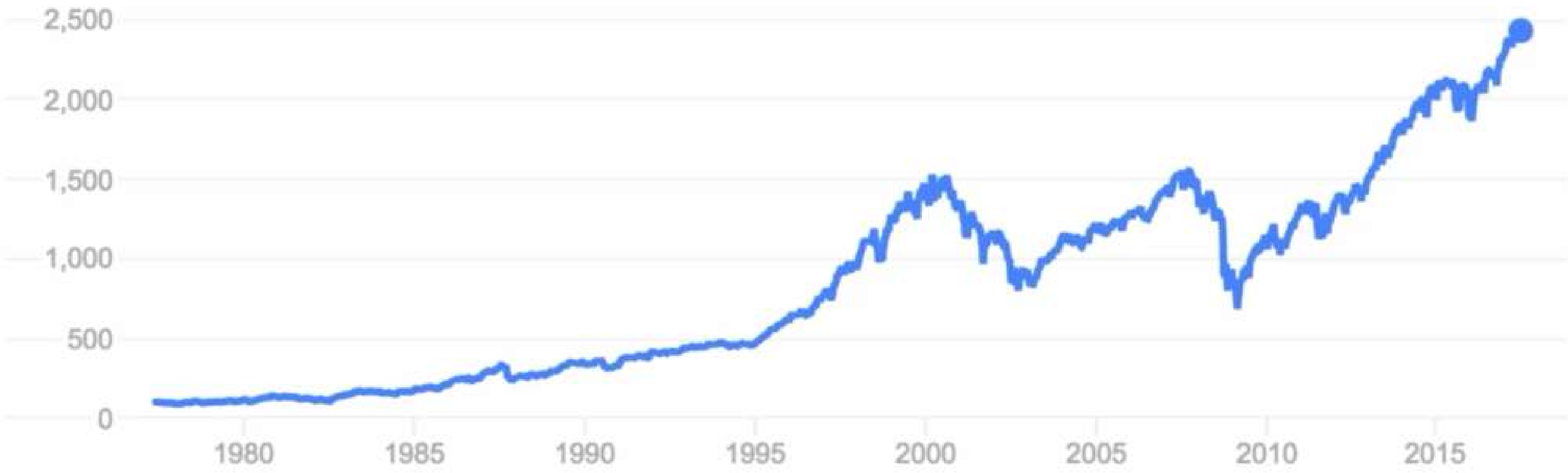
- Trends



Seasonality - Repeating trends



Cyclical - Trends with no set repetition.



SP500

The Hodrick-Prescott filter separates a time-series y_t into a trend component τ_t and a cyclical component c_t

$$y_t = \tau_t + c_t$$

- The λ value above handles variations in the growth rate of the trend component.

$$\min_{\tau_t} \sum_{t=1}^{T} c_t^2 + \lambda \sum_{t=1}^{T} [(\tau_t - \tau_{t-1}) - (\tau_{t-1} - \tau_{t-2})]^2$$

- Statsmodels provides a seasonal decomposition tool we can use to separate out the different components.
- We've already seen a simplistic example of this in the Introduction to Statsmodels section with the Hodrick-Prescott filter.

- Time Series Decomposition with ETS (Error-Trend-Seasonality).
- Visualizing the data based off its ETS is a good way to build an understanding of its behaviour.

- We could expand off the idea of an SMA (simple moving average) by utilizing a EWMA (Exponentially weighted moving average).
- As issue with SMA is that the entire model will be constrained to the same window size.

- It would be nice if we could have more recent data be **weighted** more than older data.
- We do this by implementing a EWMA instead of SMA.

- Basic SMA has some "weaknesses".
    - Smaller windows will lead to more noise, rather than signal
    - It will always lag by the size of the window
    - It will never reach to full peak or valley of the data due to the averaging.

- Does not really inform you about possible future behaviour, all it really does is describe trends in your data.
- Extreme historical values can skew your SMA significantly
- To help fix some of these issues, we can use an EWMA (Exponentially-weighted moving average).

The amount of weight applied to the most recent values will depend on the actual parameters used in the EWMA and the number of periods given a window size.

Let's explore the Holt-Winters methods , which allow us to add on double and triple exponential smoothing.

Single Exponential Smoothing

$$
\begin{aligned}
y_0 &= x_0 \\
y_t &= (1-\alpha)y_{t-1} + \alpha x_t
\end{aligned}
$$

In Double Exponential Smoothing (aka Holt's Method) we introduce a new smoothing factor **β** (beta) that addresses trend:

Double Exponential Smoothing

$$
\begin{aligned}
l_t &= (1-\alpha)l_{t-1} + \alpha x_t, && \text{level} \\
b_t &= (1-\beta)b_{t-1} + \beta(l_t - l_{t-1}) && \text{trend} \\
y_t &= l_t + b_t && \text{fitted model} \\
\hat{y}_{t+h} &= l_t + hb_t
\end{aligned}
$$

With Triple Exponential Smoothing (aka the Holt-Winters Method) we introduce a smoothing factor (gamma) that addresses seasonality:

$$
\begin{aligned}
l_t &= (1-\alpha)l_{t-1} + \alpha x_t, && \text{level}\\
b_t &= (1-\beta)b_{t-1} + \beta(l_t - l_{t-1}) && \text{trend}\\
c_t &= (1-\gamma)c_{t-L} + \gamma(x_t - l_{t-1} - b_{t-1}) && \text{seasonal}\\
y_t &= (l_t + b_t)c_t && \text{fitted model}\\
\hat{y}_{t+m} &= (l_t + mb_t)c_{t-L+1+(m-1)modL}
\end{aligned}
$$