Home Assignment 2
Thermal-Hydraulics in Nuclear Energy Engineering

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1 Introduction

In this home assignment will be analyzed some crucial aspects regarding the hydraulics of a nuclear reactor. The analyzed BWR, is assumed to have a uniform surface heat flux, and the following data:

Table 1: Parameters of the nuclear reactor fuel assembly

Parameter	**Value**
Lattice type	5x5 Square
Pressure	70 bar
Inlet subcooling	10.7 °C
Fuel assembly height	3.65 m
2 short-rod height	1.64 m
2 intermediate-rod height	2.7 m
Fuel rod diameter	10.7 mm
Channel box inner width	67 mm
Exit qualiti	0.34
Water flow mass flux	1 200 kg/m^2.s
Spacers	7 every 0.5m from inlet
Wall roughness	0.003 mm

In particular, the principal topic of discussion will be the evaluation of the total pressure drop from inlet to outlet of the fuel assembly.

2 Pressure Drop

The evaluation of pressure drop across the fuel assembly is conducted by dividing it into distinct regions, each subjected to separate calculations. This subdivision is essential because, in the initial region where the fluid remains subcooled and the equilibrium quality is negative, the pressure drop is determined using single-phase flow equations. Beyond this point, a two-phase flow approach becomes necessary to account for the increasing presence of steam in the channel. Furthermore, the analysis incorporates additional regions to account for geometric variations caused by the presence of shorter rods, which influence the flow area and the resultant calculations.

To evaluate the geometric details, it has been adopted the Whole-Assembly Model for square lattice:

- The hydraulic diameter

$$D_h = \frac{4w^2 - N\pi d_r^2}{4w + N\pi d_r} \tag{1}$$

- The hydraulic perimeter

$$P_h = 4w + N\pi d_r \tag{2}$$

- The subchannel flow area:

$$A_{flow} = w^2 - \frac{N\pi d_r^2}{4} \tag{3}$$

To summarize, 4 regions will be considered:

1. Single-phase region with 25 rods until $x_e = 0$;
2. Two-phase region with 25 rods until the end of the end of the two short rods;
3. Two-phase with 23 rods between the end of the short rods and the end of the intermediate rods;
4. Two phase with 21 rods between the end of the intermediate rods and the exit of the fuel assembly.

Having defined the geometric properties, is possible to evaluate the heat flux thanks to an energy balance between inlet and outlet, which are well defined by the known data.

```
WW = GG1*Aflow1;
Aheated = (Nfr-4)*LL*pi*dCo + 2*shortrod*pi*dCo + 2*midrod*pi
   *dCo;
qflux =(ii_out-ii_in)*WW/Aheated;
```

As already stated, the heat flux is assumed to be uniform. However, by multiplying the heat flux for each heated perimeter of every region, different linear powers are calculated. All these preliminary results are reported in the following table.

Table 2: Geometric Information of the 4 Regions

Property	Region 1	Region 2	Region 3	Region 4
Hydraulic Diameter (m)	8.087×10^{-3}	8.087×10^{-3}	9.301×10^{-3}	1.068×10^{-2}
Wetted Perimeter (m)	1.108	1.108	1.041	0.974
Flow Area (m^2)	2.241×10^{-3}	2.241×10^{-3}	2.421×10^{-3}	2.601×10^{-3}
Mass Flux ($kg/m^2/s$)	1.200×10^{3}	1.200×10^{3}	1.111×10^{3}	1.034×10^{3}
Linear Power (W/m)	4.477×10^{5}	4.477×10^{5}	4.119×10^{5}	3.761×10^{5}

2.1 Region 1 - Liquid Phase

The initial step involves evaluating the length over which the fluid remains subcooled, corresponding to the region where the equilibrium enthalpy is negative. This marks the transition point where the liquid reaches its saturation state.

This can be easily done with the classic enthalpy balance:

$$L_{sp} = \frac{(i_L - i_{in})\, W}{q_{lin,1}} = 0.3395m \tag{4}$$

Now, to evaluate the pressure drop for single phase fluid, the equation is :

$$-\Delta p_{\text{tot,sp}} = -\Delta p_{\text{fric}} - \Delta p_{\text{loc}} - \Delta p_{\text{elev}} = \left(\frac{4C_f L}{D_h} + \sum_i \xi_i \right) \frac{G^2}{2\rho} + L\rho g \tag{5}$$

To solve that, each pressure loss will be evaluated individually:

- **Friction** pressure loss: here to evaluate the Fanning friction coefficient it has been used the Aljoshin *et al.* correlation, valid for rod assemblies :

$$C_f = 0.38 \frac{P_{w,\text{ch}}}{P_{w,\text{r}}} \left(\frac{A_{\text{ch}}}{A_{\text{r}}} \right)^{0.45} \text{Re}^{-0.25} = 0.0279 \tag{6}$$

  where $P_{w,ch}$ and $P_{w,r}$ are the wetted perimetes of the channel and of all the rods, respectively. Similarly, A_{ch} and A_r are the cross-section of the channel and of all rods.

  Thus:

$$-\Delta p_{\text{fric}} = \frac{4C_f L_{sp}}{D_{h,1}} \frac{G_1^2}{2\rho} = 4.494 \text{ kPa} \tag{7}$$

  This value is strongly affected by the method chosen for the evaluation of the friction factor. Aljoshin's correlation doesn't include values for roughness, however it has been chosen since is the only one available in the textbook developed for rod assemblies. Other correlation in literature, such us Colebrook's correlation, are developed for fluid in pipes.

```
fColebrook = @(fric) -2*log10(rough/Dhy/3.7 + 2.51/(ReL *
   sqrt(fric))) - 1/sqrt(fric);
Cf = fsolve(fColebrook,0.002)/4;
```

  Computing in this way the value of friction factor, using FSOLVE in MATLAB, the value obtianed for the Fanning friction coefficient is: 0.0047. This value is six times lower than the previous one, thus giving an equally lower friction pressure drop.

  In the other regions, friction factor will be evaluated using 6. However, this large difference should be taken into account when consider the final validity of the result.

- **Elevation** pressure drop:

$$-\Delta p_{\text{elev}} = L_{sp}\, \rho\, g = 7.358 \text{ kPa} \tag{8}$$

- **Local** losses at the inlet due to the contraction of the flow area with $\xi_{in} = 0.5$:

$$-\Delta p_{\text{loc}} = \frac{1}{2}\frac{G_1^2}{2\rho} = 0.480 \text{ kPa} \tag{9}$$

The total pressure drop, along the subcooled region with single phase fluid is:

$$-\Delta p_{\text{tot,sp}} = 12.332 \text{ kPa} \tag{10}$$

To be complete, it has to be mentioned that all the fluid properties, such as density and viscosity, necessary for the evaluation of the Reynolds number, are evaluated by making the average between the inlet and the saturation length.

The results of this region are summarized in the following table:

Table 3: Results of Region 1

Pressure Drop	**Value [kPa]**
Friction	4.494
Local	0.480
Elevation	7.358
Total	12.332

2.2 Region 2 - Two-Phase 25 rods

The second region extends from the onset of two-phase flow to the termination of the shorter rods. As the equations used for single-phase liquid water are no longer applicable, new formulations are required.

The prediction of the two-phase pressure drop is inherently more complex due to the presence of void fractions within the flow.

The total two-phase flow pressure drop in the fuel assembly can be calculated as:

$$\begin{aligned} -\Delta p_{\text{tot,tp}} &= -\Delta p_{\text{acc}} - \Delta p_{\text{fric}} - \Delta p_{\text{loc}} - \Delta p_{\text{elev}} = \\ &r_2\frac{G^2}{\rho_l} + r_3 C_{f,lo}\frac{4L}{D_h}\frac{G^2}{2\rho_l} + r_4 L\rho_l g + \left(\sum_{i=1}^{N}\phi_{lo,i}^2\xi_i\right)\frac{G^2}{2\rho_l} \end{aligned} \tag{11}$$

Where r_2, r_3 and r_4 are integral two-phase pressure drop multipliers, while $\phi_{lo,i}^2$ and ξ_i are local loss multiplier and local pressure loss coefficient respectively. Moreover, the Fanning friction factor $C_{f,lo}$ is found as described in the previous section, for single-phase liquid in Eq. 6.

The integral and local two-phase multiplier are evaluated applying the Homogeneous Equilibrium Model. Thus, first are evaluated the main parameters useful for the calculations:

- Mass flux is the same as the previous region, since there are still 25 rods;

- Quality at the exit of the region after an energy balance;

- Void Fraction evaluated using Homogeneous Equilibrium Model in order to remain coherent with multiplier:

$$\alpha_{HEM} = \frac{1}{1 + \frac{\rho_g}{\rho_f}\left(\frac{1-x_e(z)}{x_e(z)}\right)} \tag{12}$$

```
GG2 = WW/Aflow2;
xx_2 = qlin2*(shortrod-Lsp)/((iiV-iiL)*WW);
alpha2 =  1/(1+rhoV/rhoL*((1-xx_2)/xx_2));
```

- **Friction** pressure loss: the integral friction multiplier is found generally as follow:

$$r_3 = \frac{1}{z_{out} - z_{in}} \int_{z_{in}}^{z_{out}} \left[1 + \left(\frac{\mu_l}{\mu_v} - 1\right) x\right]^{-0.25} \left[1 + \left(\frac{\rho_l}{\rho_v} - 1\right) x\right] dz \tag{13}$$

Where, here $z_{in} = L_{sp}$ and $z_{out} = L_{shortrod}$.

The integral is solved on MATLAB using the following line of code:

```
xx = @(z) xx_in + qlin2*(z)/((iiV-iiL)*WW);
r3_integral = @(zeta) (1 + xx(zeta) * ((myL / myV) - 1))
   .^-0.25 .* ((1 + xx(zeta)* ((rhoL / rhoV) - 1)));
r3 = 1/(shortrod-Lsp)*integral(r3_integral, Lsp, shortrod);
```

Thus, the friction pressure drop in the second region is:

$$-\Delta p_{fric} = r_3 C_{f,lo} \frac{4(L_{shortrod} - L_{sp})}{D_h} \frac{G^2}{2\rho_l} = 38.868 \text{ kPa} \tag{14}$$

- **Elevation** pressure drop: the gravity integral multiplier is defined as follow:

$$r_4 = 1 - \frac{\rho_l - \rho_v}{\rho_l} \frac{1}{L_{shortrod} - Lsp} \int_{z_{in}}^{z_{out}} \alpha \, dz \tag{15}$$

Also this integral is solved on MATLAB using *integral* function:

```
r4_integral = @(zz) 1./(1+rhoV/rhoL.*(((1-xx(zz))./xx(zz))));
r4 = 1 - (rhoL-rhoV)/rhoL * 1/(shortrod-Lsp).*integral(
   r4_integral,Lsp,shortrod);
```

So, the gravity pressure drop:

$$-\Delta p_{\text{elev}} = r_4 \left(L_{shortrod} - Lsp\right) \rho_l \, g = 4.522 \text{ kPa} \tag{16}$$

- **Acceleration** pressure drop: this is an additional component due to the presence of two-phase flow. This loss is caused by changes in the flow velocity due to variations in phase composition and density as the phases accelerate or decelerate to conserve momentum. Its integral two-phase pressure drop is obtained as:

$$r_2 = \left[\frac{x^2\rho_l}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)}\right]_{ex} - \left[\frac{x^2\rho_l}{\alpha\rho_v} + \frac{(1-x)^2}{(1-\alpha)}\right]_{in} \tag{17}$$

Where the properties in the parenthesis are calculated at the region exit and inlet, respectively. Since for the inlet of this region the quality is zero, then the second bracket is just 1.

$$-\Delta p_{acc} = r_2 \frac{G^2}{\rho_l} = 5.392 \text{ kPa} \tag{18}$$

- **Local** pressure drop: in this region are evaluated the pressure drop due to first three grid spacer and the pressure drop for the area change due to part-length rods. This kind of loss uses a local pressure drop rather than an integral one. In this case is used a correlation developed by *Reddy et al.* :

$$\phi_{lo}^2 = 1 + x \left(\frac{\rho_l}{\rho_v} - 1 \right) C \tag{19}$$

Where for $p > 41.4$ bar:

$$C = 1.02 x^{-0.175} \left(\frac{G}{1356.2} \right)^{-0.45} \tag{20}$$

Since this losses are local, variables such as quality and Reynolds number are evaluated at the exact location of interest. Lastly, the local pressure loss coefficient for spacer loss and enlargement from region 2 to region 3 are:

$$\xi_{spacer} = 1.95 \; Re^{-0.08}; \qquad \xi_{enl} = \left(1 - \frac{A_2}{A_3} \right)^2 \tag{21}$$

Then, final value of pressure drop is evaluated summing each component:

$$-\Delta p_{loc} = \left(\sum_{i=1}^{N} \phi_{lo,i}^2 \xi_i \right) \frac{G^2}{2\rho_l} = 4.522 \text{ kPa} \tag{22}$$

The value of each pressure drop at each location are reported in Table 7 ant 8 in the next section.

Results of this region are summarized in the following table:

Table 4: Results in Region 2

Pressure Drop	**Value [kPa]**
Friction	38.868
Local	7.470
Elevation	4.522
Acceleration	5.392
Total	56.252

2.3 Region 3 - Two-Phase 23 rods

In this third region, is used exactly the same approach of the previous region. However, in this region calculation are carried out considering 23 fuel rods instead of 25. Then, each step of the previous section is repeated.

Final results in this region are:

Table 5: Region 3

Pressure Drop	**Value [kPa]**
Friction	47.506
Local	8.802
Elevation	1.625
Acceleration	3.465
Total	61.398

Where local pressure drop includes the losses due to 4^{th} and 5^{th} spacers and also the loss for change of area due to part-length rods, from region 3 to region 4.

2.4 Region 4 - Two-Phase 21 rods

Also in this region are repeated the same step of the others two, considering only 21 rods and all the parameters related to this region. Local losses here consider 6^{th} and 7^{th} spacers. Also, in this part is computed the local pressure drop due to exit from fuel assembly, where $\xi_{ex} = 1.0$. Result are summarized:

Table 6: Results in Region 4

Pressure Drop	**Value [kPa]**
Friction	46.252
Local	17.658
Elevation	1.035
Acceleration	2.457
Total	67.402

3 Final Consideration

In this section are collected final data and consideration. First of all, are reported in detail local pressure losses:

Spacer Number	Pressure Drop [kPa]
1	1,311
2	2,542
3	3,590
4	3,990
5	4,780
6	4,831
7	5,519

Table 7: Pressure drop for each spacer grid.

Area Change	Pressure Drop [kPa]
Inlet	4,800
From 25 to 23 rods	0,028
From 23 to 21 rods	0,031
Outlet	7,308

Table 8: Values for different Areachange numbers.

For semplicity each pressure drop is reported in Table 9, by summing every losses is possible to obtained the total pressure drop along the fuel assembly:

$$-\Delta p_{tot} = 197.384 \text{ kPa} \tag{23}$$

Category	Region 1 [kPa]	Region 2 [kPa]	Region 3 [kPa]	Region 4[kPa]
Friction	4,494	38,868	47,506	46,252
Elevation	7,358	4,522	1,625	1,035
Acceleration	0	5,392	3,465	2,457

Table 9: Values for different categories across regions.

Graphic evaluation of this pressure drop are reported in Figure 1 and 2.

In Figure 1 are reported the different causes of pressure drop, each with different colors. Similarly, in figure 2 this values are weighted on the length in order to obtain comparable values. Here we can notice, as expected that friction losses increases for increasing quality. On the other hand, gravity pressure drops are reduced significantly with increasing pressure drop.

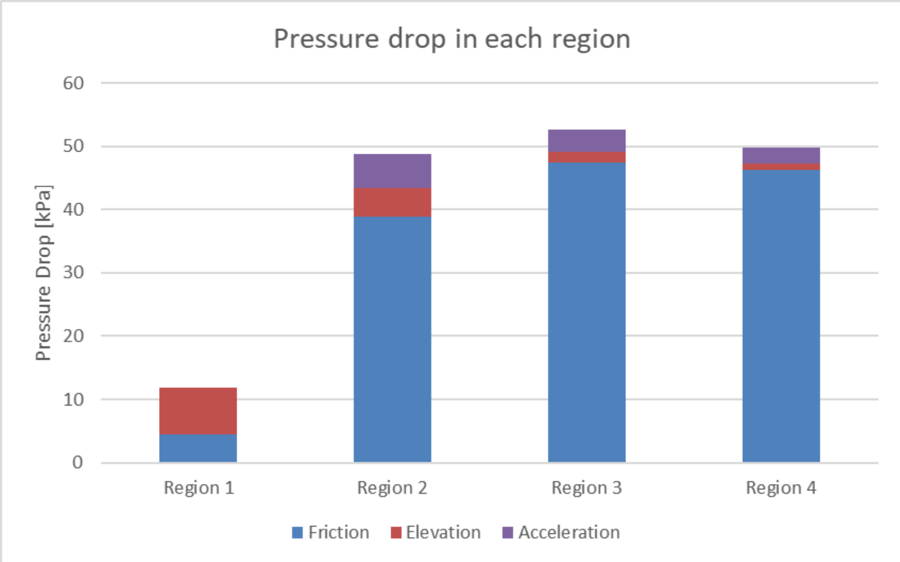

Figure 1: Contribution of different pressure drop in each region

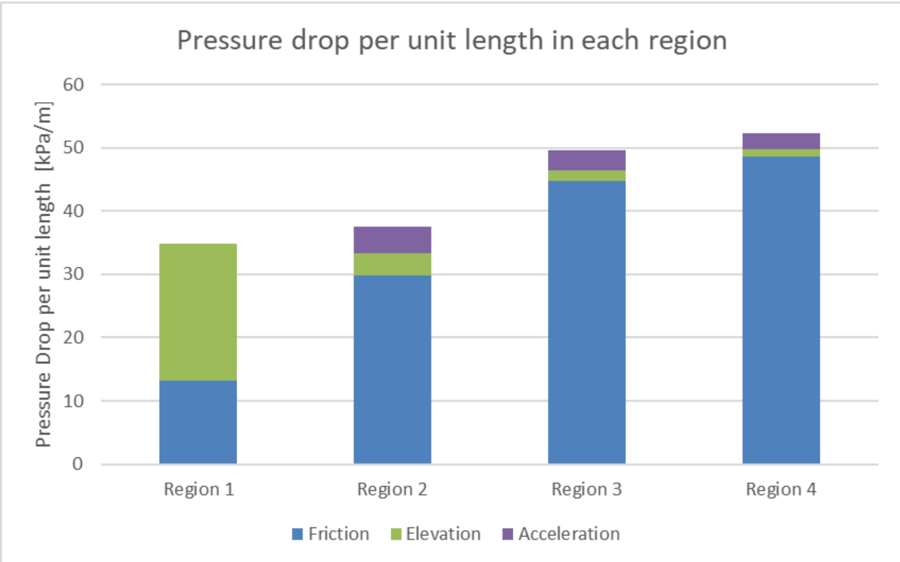

Figure 2: Contribution of different pressure drop per unit length.

3.1 Friction Factor

To conclude, as already anticipated, can be quickly analyzed the different results obtained using Colebrook or Aljoshin correlation to evaluate friction factor. From Figure 1 can be quickly understood that pressure drop to friction are the biggest losses, so a different correlation can cause significant variation in the final result.

Friction Correlation	Region 1	Region 2	Region 3	Region 4
Aljoshin	4,494 kPa	38,868 kPa	47,506 kPa	46,252 kPa
Colebrook	0.793 kPa	6,834 kPa	7,621 kPa	6,707 kPa

Table 10: Pressure drop according to friction correlation.

As can be clearly seen from the table, in each region there is a large difference, leading to a final pressure drop using Colebrook's correlation of $\Delta p_{tot,Colebrook} = 84,488 kPa$.

I personally believe that both this correlation are not fully correct, for this reason I thought it necessary to underline this difference.