Home Assignment 1
Thermal-Hydraulics in Nuclear Energy Engineering

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1 Introduction

In this home assignment will be analyzed some crucial aspects regarding the thermal-hydraulics of a nuclear reactor. This reactor, a BWR, has the following data:

Table 1: Parameters of the nuclear reactor fuel assembly

Parameter	**Value**
Lattice type	Triangular
Pressure	77 bar
Inlet subcooling	11 °C
Fuel assembly height	3.65 m
Fuel rod diameter	9.4 mm
Fuel lattice pitch	12.4 mm
Uniform heat flux	0.34 MW/m^2
Water flow mass flux	1,980 kg/m^2/s

In particular, the principal topic of discussion will be:

- The determination of the location of the major points of interest:
    1. Onset of Nucleate Boiling;
    2. Onset of Significant Void;
    3. Bulk Saturation Point .
- The axial Void Fraction distribution using numerical tool.

2 Major Location

First of all, it has been adopted the Isolated Sub-Channel Model. For a triangular lattice, the principal geometric characteristics are;

- The hydraulic diameter evaluated as follow:

$$D_h = d_r \left[\frac{2\sqrt{3}}{\pi} \left(\frac{p}{d_r} \right)^2 - 1 \right] = 7.857 \times 10^{-3}\,\mathrm{m} \tag{1}$$

- The hydraulic perimeter

$$P_h = \frac{1}{2}\,\pi\, d_r = 1.477 \times 10^{-2}\,\mathrm{m} \tag{2}$$

- The subchannel flow area:

$$A_{flow} = \frac{\sqrt{3}}{4}p^2 - \frac{\pi\, d_r^2}{8} = 3.188 \times 10^{-5}\, m^2 \tag{3}$$

2.1 Onset of Nucleate Boiling

ONB (Onset of Nucleate Boiling) refers to the point at which bubble formation begins on a heated surface in a liquid. Before this point, heat is transferred primarily through single-phase convection. Once ONB occurs, nucleate boiling starts, characterized by bubble nucleation at specific sites on the surface and the heat transfer mode is subcooled boiling.

In order to expect boiling to occur, the wall temperature should be higher than the saturation temperature of the liquid. Since there isn't a direct criterion which define such wall superheat at which subcooled boiling will start, it has been used the **Bowring correlation**. Bowring suggested that at the ONB point the wall superheat is equal to which results from a subcooled boiling correlation.

The ONB location is determined as follow:

$$z_{\mathrm{ONB}} = \frac{c_p G A}{P_H}\left[\frac{\psi(q'')^n + \Delta T_{\mathrm{subi}}}{q''} - \frac{1}{h}\right] = 0.2652\; m \tag{4}$$

Where $\psi(q'')^n = \Delta T_{sup}$ has to be found with suitable correlation for subcooled boiling heat transfer. The choosen correlation is proposed by **Thom et al.** :

$$\Delta T_{sup} = T_w - T_s + 22.65\left(\frac{q''}{10^6}\right)^{0.5} e^{-P/87} \;= 5.450\,\mathrm{K} \tag{5}$$

At this point, we only need to find the convective heat transfer coefficient. Here, the choosed correlation, commonly used for turblent fluid in heated pipes is the following **Dittus-Boelter** correlation:

$$Nu_{D-B} = 0.023\, Re^{0.8}\, Pr^{0.4} = 330.831 \tag{6}$$

However, to adapt this correlation to a bundle-wide approach the previous equation is corrected by the **Markoczy correlation**:

$$Nu_{bundle} = Nu_{D-B}\left[1 + 0.91\, Re^{-0.1}\, Pr^{0.4}\,(1 - 2e^{-B})\right] = 375.094 \tag{7}$$

Where B fot triangular lattice is:

$$B = \frac{2\sqrt{3}}{\pi}\left(\frac{p}{d_r}\right)^2 - 1 \tag{8}$$

In this way, according to the Nusselt number defintion, the heat transfer coefficient is determined, hence, also the ONB location.

It is important to clarify that all thermophysical properties used in the analysis are evaluated using the XSteam implementation for MATLAB.

In particular, since the correlation for the heat transfer coefficient are dependent on the fluid properties, which in turn vary due to the increasing enthalpy, a discretized and iterative approach are implemented:

- First, the location of ONB is evaluated considering the properties of the fluid at the inlet.
- Afterwards, the length from the inlet to the ONB point is discretized. For each segment dz, new properties are evaluated making the average between z and $z+dz$. Then for each segment, the heat transfer coefficient is calculated and added to the final value weighting on the length dz.
- Finally, having a new convective coefficient, a new value of z_{ONB} can be calculated and the previous point is repeated iteratively until convergence.

Final result of the last iteration and the fluid properties at the z_{ONB} point are reported in the following tables:

Table 2: Thermal Properties of the Fluid

Parameter	**Value**
Specific Isobaric Heat Capacity	$c_p = 5.36 \times 10^3 \, \frac{J}{kg\, K}$
Thermal Conductivity	$k = 5.710 \times 10^{-1} \, \frac{W}{m\ K}$
Viscosity	$\mu = 9.171 \times 10^{-5}\, \text{Pa s}$

Table 3: Summary of the Results

Parameter	**Values**
Prandtl	$\text{Pr} = 0.862$
Reynolds	$\text{Re} = 1.696 \times 10^5$
Heat Transfer Coefficient	$\text{h} = 2.725 \times 10^4 \, \frac{W}{m^2\ K}$
ONB location	$z_{ONB} = 0.268\,\text{m}$

It can finally be verified that all the range of validity of the previous correlation are respected. In other correlation, such as the Subbotin correlation for Nusselt value, Prandtl number wouldn't respect the range.

2.2 Onset of Significant Void

While ONB is the initial formation of vapor bubbles at the heated surface, Onset of Significant Void occurs further downstream when bubble generation becomes volumetrically significant, affecting the bulk fluid.

OSV marks the transition from isolated bubble formation to a regime where bubble coalescence and depart from heated wall to the coolant.

To determine this location it has been used the correlation proposed by **Saha and Zuber**, which states that OSV is located at a certain position in the channel where the local equilibrium quality is determined as follow:

$$x_{OSV} = -154 \frac{q''}{G\ i_{fg}} = -0.0181 \tag{9}$$

This correlation is valid for our case, where the Peclet number is greater than 70 000:

$$Pe = Re\ Pr = 1.462 \times 10^5 \tag{10}$$

Thus, the corresponding location can be evaluated thanks to an energy balance:

$$z_{OSV} = (x_{OSV} - x_{in}) \frac{G\ A_{flow}\ i_{lg}}{q'' P_h} = 0.415 m \tag{11}$$

2.3 Bulk Saturation Point

The Bulk Saturation Point represents the location where the equilibrium quality is equal to zero and is no more negative.

To evaluate it, it's sufficient to make an energy balance:

$$z_{BSP} = (i_l - i_{in}) \frac{G\ A_{flow}}{q'' P_h} = 0.747\ m \tag{12}$$

This point has a great interest in the Thermal-Hydraulics of the reactor. From this location, the fluid is not subcooled, hence for the heat trasfer, correlation such as Jens-Lottes or Thom are no longer valid. Rather, for saturated convective boiling other correlations, such as Chen correlation, will be necessary.

Moreover, starting from this point a different approach is required when calculating the pressure drop. The presence of steam in the liquid makes the single-phase equation not valid anymore and a two-phase pressure drop should be evaluated.

The resulting location are summarized in the following Table:

Table 4: Summary of the Results

Major Location	**Values**
Onset of Nulceate Boiling	$z_{ONB} = 0.268\ m$
Onset of Significant Void	$z_{OSV} = 0.415\ m$
Saturation Bulk Point	$z_{BSP} = 0.747\ m$

3 Void Fraction

The second part of this analysis regards the evaluation of the void fraction along the fuel assembly.

In order to do so, a discretization of the axial coordinate over the whole length is made. In this way, using the following function on MATLAB, is possible to evaluate the equilibrium quality profile.

```
xx_fun = @(z) xx_in + pH*z*qflux/GG/Aflow/(iiV-iiL)
```

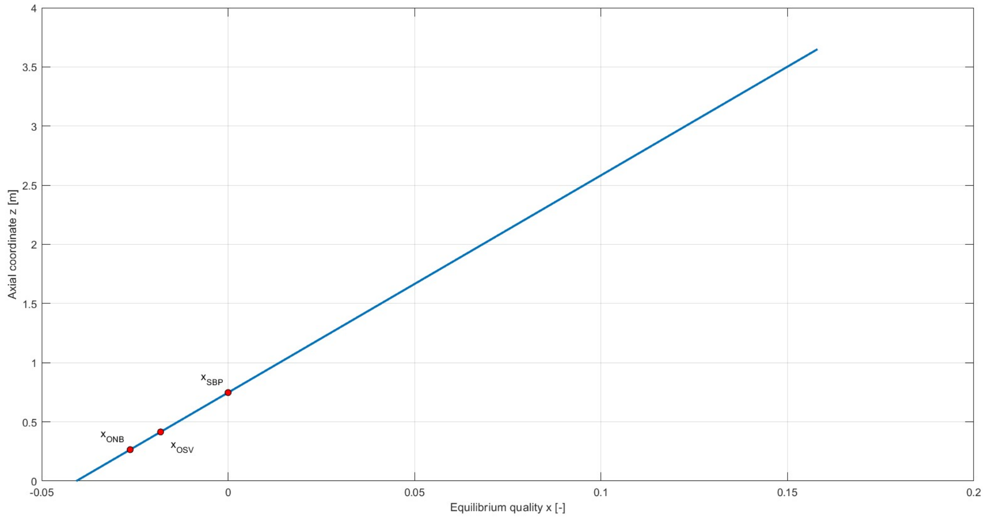

Figure 1: Equilibrium Quality Profile

Then, in order to evaluate the void fraction, a first analysis using the Homogeneous Equilibrium Model has been achieved.

Using the result, a more detailed analysis using the Drift Flux Model will be conducted.

The Drift Flux Model (DFM) and the Homogeneous Equilibrium Model (HEM) differ primarily in their treatment of phase velocities and equilibrium. The HEM assumes no relative motion and full thermodynamic and mechanical equilibrium between phases, treating the two phases as a single effective fluid. In contrast, the DFM allows relative motion between the liquid and vapor phases. It requires additional correlations for drift velocity and void fraction, depending on the flow pattern, hence the void fraction itself. For this reason an iterative process is required using the calculation with HEM as a first guess.

Thus, void fraction using HEM is evaluated as follow:

$$\alpha_{HEM} = \begin{cases} 0 & x_e \leq 0 \\ \dfrac{1}{1 + \frac{\rho_g}{\rho_f}\left(\frac{1-x_e(z)}{x_e(z)}\right)} & 0 < x_e < 1 \\ 1 & x_e \geq 1 \end{cases} \tag{13}$$

Then, starting from this values to firstly discern the flow pattern, the following calculation are executed:

$$\alpha_{DFM} = \frac{J_v}{C_0 J + U_{vj}} \tag{14}$$

Where the superficial velocities are evaluated as:

$$J_v = \frac{G_v}{\rho_v} = \frac{xG}{\rho_v}, \quad J_v = \frac{G_l}{\rho_l} = \frac{(1-x)G}{\rho_l}, \quad J = J_v + J_l \tag{15}$$

While C_0 and U_{vj} are the distribution parameter and the drift velocity respectively. They are flow-regime dependent:

Table 5: Drift Flux Parameters for Different Flow Patterns

Flow pattern	**Distribution parameter**	**Drift velocity**
Bubbly $0 < \alpha \leq 0.25$	$C_0 = 1.2 \quad p/p_\sigma < 0.5$	$U_{\mathrm{vj}} = 1.41 \left(\frac{\sigma g(\rho_\ell - \rho_{\mathrm{v}})}{\rho_\ell^2} \right)^{0.25}$
Slug/churn $0.25 < \alpha \leq 0.75$	$C_0 = 1.15$	$U_{\mathrm{vj}} = 0.35 \left(\frac{gD(\rho_\ell - \rho_{\mathrm{v}})}{\rho_\ell} \right)^{0.5}$
Annular $0.75 < \alpha \leq 0.95$	$C_0 = 1.05$	$U_{\mathrm{vj}} = 23 \left(\frac{\mu_\ell j_\ell}{\rho_{\mathrm{v}} D_{\mathrm{h}}} \right)^{0.5} \cdot \frac{(\rho_\ell - \rho_{\mathrm{v}})}{\rho_\ell^2}$
Mist $0.95 < \alpha \leq 1$	$C_0 = 1.0$	$U_{\mathrm{vj}} = 1.53 \left(\frac{\sigma t(\rho_\ell - \rho_{\mathrm{v}})}{\rho_{\mathrm{v}}^2} \right)^{0.25}$

The iterative process used on MATLAB is reported below:

```
alpha_old = alphaHEM;
alpha = zeros(length(zz),1);
for iteration = 1:100
    for index = 1:length(zz)
        if alpha_old(index)<=0.25 && alpha_old(index)>0
            C0 = 1.2;
            %C0 = 1.13;
            Uvj = 1.41*(sigma*9.81*(rhoL-rhoV)/rhoL^2)^0.25;
            Jv = xx(index)*GG/rhoV;
            Jl = (1-xx(index))*GG/rhoL;
            alpha(index) = Jv/(C0*(Jv+Jl)+Uvj);
        elseif alpha_old(index) > 0.25 && alpha_old(index) <=
           0.75
            C0 = 1.15;
            Uvj = 0.35*(9.81*DH*(rhoL-rhoV)/rhoL)^0.5;
            Jv = xx(index)*GG/rhoV;
            Jl = (1-xx(index))*GG/rhoL;
            alpha(index) = Jv/(C0*(Jv+Jl)+Uvj);
        elseif alpha_old(index) > 0.75
            C0 = 1.05;
            Jv = xx(index)*GG/rhoV;
            Jl = (1-xx(index))*GG/rhoL;
            Uvj = 23*(myL*Jl/rhoV/DH)^0.5 * (rhoL-rhoV)/rhoL;
            alpha(index) = Jv/(C0*(Jv+Jl)+Uvj);
        end
```

```
    end
    alpha_old = alpha;
end
```

One hundred iteration are repeated, however, even after few iteration the result is converged.

One last step is to consider the subcooled void where the equilibrium quality is negative and the void fraction is not considered included in the Drift Flux Model.

To do so, it has been used the **Levy's model**:

1. First is evaluated the actual quality, defined as:

$$x_a = x_e(z) - x_e(z_{OSV})\, e^{\frac{x_e(z)}{x_e(z_{OSV})}-1} \tag{16}$$

2. While the void fraction for subcooled boiling region is found with the same equation of the previous models:

$$\alpha_{sub} = \frac{J_v}{C_0 J + U_{vj}} \tag{17}$$

However, here C_0 and U_{vj} are:

$$C_0 = \beta\left[1 + \left(\frac{1}{\beta}\right)^b\right], \quad U_{vj} = 2.9\left(\frac{\sigma\, g\, (\rho_f - \rho_g)}{\rho_f^2}\right)^{0.25} \tag{18}$$

Where:

$$\beta = \frac{1}{1 + \frac{\rho_g}{\rho_f}\,\frac{1-x_a(z)}{x_a(z)}}, \quad b = \left(\frac{\rho_g}{\rho_f}\right)^{0.1} \tag{19}$$

The final result is represented in the figure below:

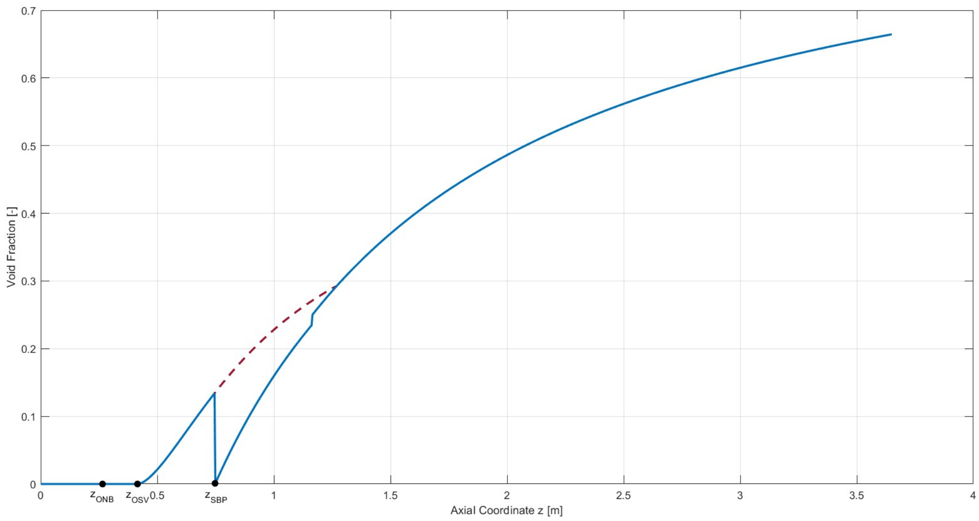

Figure 2: Void Fraction profile

From Figure 2 can be distinguished different parts:

- First, until Onset of Significant Void Fraction, α is negligible.
- Then, starting from z_{OSV} to z_{BSP}, the correlation for subcooled boiling is applied. Since DFM doesn't include this region,is present a discontinuity in the curve. For this reason, a dotted line deriving from subcooled correlation can be use as a bridge to DFM model.
- Finally, the curve using the Drift Flux Model. Here, can be also noticed a small discontinuity of the curve at $\alpha = 0.25$, probably due to the formulation of the correlation.