POLITECNICO DI TORINO

Corso di Laurea Magistrale
in Sustainable Nuclear Energy

Nuclear Fission Plant

Exercise 3:Thermo-mechanical Verification of a Fuel Pin and Sizing of the Gas Plenum

Professor
Alessandro MORANDI
Cristina BERTANI

Student
Paolo GUIDI s331322
Lara MASSA s331285
Federico PATI s328987

2023/2024

Chapter 1

Introduction

The AP1000 studied in this work is the same as the one studied in the Exercise 2.

During operation, gaseous fission products are released inside the pin. These fission gases dissolve poorly in the UO_2 matrix and are ultimately release in the gap and plenum, reducing the thermal properties of the rod and increasing its inner pressure. Due to these reasons in this exercise, it is done the thermo-mechanical verification of the fuel pin cladding and the sizing of the gas plenum. This is typical of the “Core design” and its process is controlled by the fuel element behaviour and it is composed by different analysis correlated between them. These analyses are the following [9]:

1. Thermal analysis: there the cladding thickness and material are tentatively chosen and they must be verified later with the fuel-element structural analysis. This analysis is done in exercise 2.
2. Fuel element composition and diameter: here the diameter is assumed and the heat flux is compared with the one used in the point 1.
3. Core sizing: there is necessary to evaluate the number of fuel elements and the geometrical measures. Once defined the core size the neutronic and thermal analysis are recomputed.
4. Fuel cycle economic analysis
5. Structural analysis of the fuel element: it is important to evaluate not only the geometry of the fuel but also the pressure inside it. It is analysed in order to choose the fuel rod plenum length: the length between the higher part of the core and the rods where there is the high plenum. This choice is necessary because there the fission gases, reported in the initial part of the discussion, are collected.
6. Hydraulic analysis: the pressure loss across the fuel element bundle is calculated and compared with an allowable loss determined from pumping power. If the loss is excessive is needed to re-iterate the core sizing.
7. Safety analysis: reactivity changes are compared with the acceptable ones.
8. Fuel reliability analysis: how many damages there are in a fuel element.
9. Post irradiation fuel handling: possible damages to the cladding are analysed to determine if it is possible to manage the irradiated fuel that has to be water washed.

Chapter 2

Case study

Firstly, is done the thermo-mechanical verification of the fuel pin cladding studying two of the three main stresses, the pressure and thermal stresses. So, it is followed the ASME code [3], specifically the Section III: "Rules for Construction of Nuclear Facility Components" guidelines.

Then is possible to pass to the second part of the work, where the gas plenum of the AP1000 fuel pin at EOL is sized.

2.1 Data

The data are taken by the «AP1000 Design Control Document» number ML11171A443 [1] which describes the AP1000 design, so the same as the Exercise 2:

Thermal and Hydraulic Design Parameters	
Reactor core heat output (MWt)	3400
Heat generated in fuel (%)	97.4
System pressure, nominal (MPa)	1.55
Rod pitch (m)	1.26×10^{-2}
Heat flux hot channel factor (FQ)	2.60
Number of fuel assemblies	157
Uranium dioxide rods per assembly	264
Fuel Rods	
Number	41,448
Outside diameter (m)	9.5×10^{-3}
Clad thickness (m)	5.715×10^{-4}
Fuel Pellets	
Density (% of theoretical)	95.5
Diameter (m)	8.2×10^{-3}
Structure Characteristics	
Core height, cold, active fuel (m)	4.267

Concerning the Zircaloy-4 choice:

- The yield strength is $\sigma_y = 241\ MPa$ and it limits the stress until go to deformation;
- The ultimate strength is $\sigma_u = 413\ MPa$ and it limits the stress before arrive to the failure of the material.

About the fuel:

- Impurities:
  - N_2: 25 ppm;
  - H_2O: 75 ppm;
- Fission yield: $Y = 0.28$;
- Burnup: 6000 MWD/BTU [1];
- UO_2 density: 10970 kg/m^3 [2];
- Energy released per fission: 200 MeV [4];
- Molecular weights:
  - N_2: 14.01 kg/mol;
  - H_2O: 18.02 kg/mol;
  - U: 238.03 kg/mol;
  - UO_2: 270.03 kg/mol [8].

2.2 Assumptions

For the thermo-mechanic verification the assumptions are the following:

- Reactor operating at steady-state in nominal conditions;
- 20% overpower when computing the thermal stresses;
- Only static loads are considered;
- Bending stresses, peak stresses and local stresses are all neglected;
- Cladding material is Zircaloy-4;
- The maximum internal pressure is computed assuming reactor depressurization ($p_o = 1bar$).

For the gas plenum sizing:

- The gaseous fission products in the plenum can be modelled as ideal gases;
- The volume available in the gap is neglected because of the swelling and buckling of the fuel pellet;

- The fuel releases about 40% of the gas produced ($R_r = 0.4$);
- All the gases and the vapours can be treated as ideal gases (initially);
- The gas plenum has the same temperature of the cladding inner surface at the same height.

2.2.1 Radii consideration

In some cases, the fuel radii are evaluated as the modified ones resulting by the analysis of the exercise 2 and, in other cases, as the results of an overpower of the 20%.

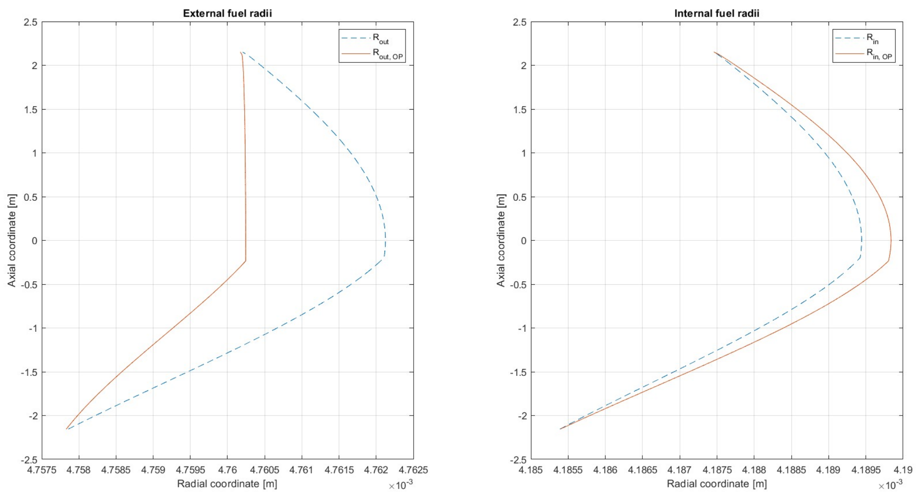

Figure 2.1: Fuel radii deformations.

As it is possible to understand from the Figure 2.1 above, the results do not change very much, but where is it possible in the analysis are used the ones more coherent.

2.2.2 Temperature considerations

The temperatures with overpower change very much and this has to be considered in the evaluations.

2.3 Objectives

The objectives for this analysis are to verify the design using the limits provided by ASME and size the gas plenum checking the assumption done.

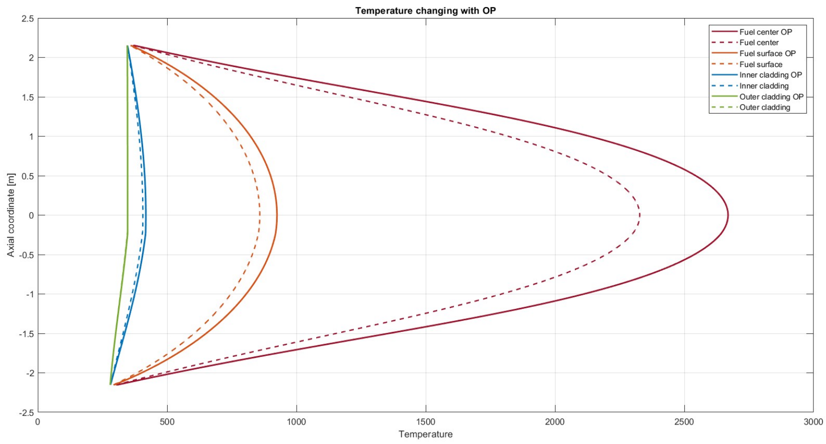

Figure 2.2: Temperature with and without overpower.

Chapter 3

3. Simulation setup

3.1 Correlations and definitions

Main mechanical types of limiting stresses (Figure 3.1):

- Linear or allowable: it is the maximum one allowed to be applied to material, set by the design code, to remain in the elastic region without any problem [7];
- Yield: it is the stress point where the material starts to have a plastic behaviour: from that point the deformations are irreversible;
- ideal or ultimate: it is the maximum stress before the reducing of area of the material.

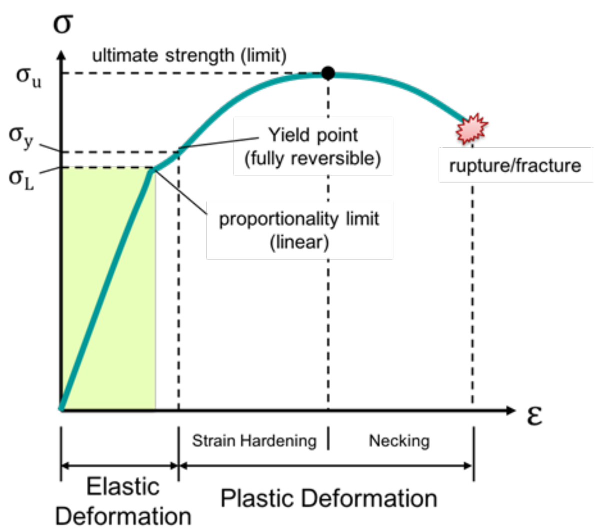

Figure 3.1: Main stress and strain concepts in mechanics [6].

3.2 Thermo-mechanical verification

3.2.1 Preliminary verification against buckling

Buckling means “collapsing” and is necessary to evaluate the critical pressure at which buckling starts to develop and it is possible to do with the following expression:

$$p_{cr} = \frac{E}{4 * (1 - \nu^2)} * \left(\frac{t}{r_{avg}} \right)^3 \tag{3.1}$$

Where from the “Effect of High Temperature Steam Oxidation on Yielding of Zircaloy-4 PWR Fuel Cladding”:

- E is the Young’s modulus:

$$E = \left[9.9 * 10^3 - 5.669 * (T - 273) \right] * 9.81$$

  With T in K and E in MPa. Young’s modulus is a property of the material that describe the tensile/compressive stiffness when a force is applied, also defined in the elastic region as the ratio between the applied stress and the resulting axial strain.

- ν is the Poisson’s ratio:

$$\nu = 0.3303 + 8.376 * 10^{-5} * (T - 273)$$

  Again, T as to be in K. This parameter is the ratio between the lateral and longitudinal strain and represents the deformation of a material in directions perpendicular to the specific direction of loading.

Both E and ν are evaluated with an average cladding temperature calculated as:

$$T_{c_{avg}} = \frac{T_{ci} * r_i^2 + T_{co} * r_o^2}{r_i^2 + r_o^2}$$

So, T_{ci} and T_{co} are weighted on the cladding area that they affect; T_{ci} and T_{co} are taken by the result of Exercise 2.

Talking about the geometrical part t is the cladding thickness and r_{avg} is the average distance of the cladding from the fuel pellet centerline and is measured with the outer radius without overpower.

Than to verify that the fuel structure does not collapse on itself it is necessary to confront the critical pressure with the maximum external one that is present in the worst scenario: if the first one is higher than the last one the verification is successful. So, as to be verify the following expression:

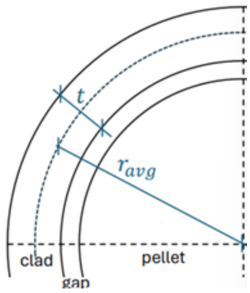

Figure 3.2: Geometrical distances of the fuel structure.

$$p_{cr} > p_{out_{\max}} \tag{3.2}$$

$p_{out_{\max}}$ as said in the assumption if is taken as almost 1 bar.

3.2.2 Identification of the type of stresses

- **Maximum internal pressure in case of depressurized reactor**

  It is necessary to depressurize a reactor when the release of fission gases is performed or in case of incidents. So, it is needed to calculate the maximum internal pressure and to do that is used the Mariotte's formula that is studied for thin cylinder:

$$\sigma_{p,\ t_{\max}} = \Delta p * \frac{r_{avg}}{t} \tag{3.3}$$

  Where in $\sigma_{p,\ t_{\max}}$ t stays for tangential (or hoop) and p stays for "due to pressure". Then:

$$\Delta p = p_{in} - p_{out} \tag{3.4}$$

  So:

$$p_{in} = p_{out} + \frac{\sigma_{p,t_{\max}} * t}{r_{avg}} \tag{3.5}$$

  Since we are in condition of depressurization, we can assume ($p_{out} = 1$ bar. While for the thickness and average radius, the new values after deformation are assumed. Finally, $\sigma_{p,t_{\max}}$ is assumed equal to σ_{yield}. So, maximum internal pressure could be valuated

- **Thermal stresses on the cladding inner and outer surfaces**

  The considered thermal stresses are only in tangential and longitudinal direction and are evaluated as:

$$\sigma_{tht} = \sigma_{th_l} \tag{3.6}$$

So, it is possible to refer to the thermal stresses only as σ_{th}. It is necessary to evaluate both internal and external thermal stresses refer to the surface and it is possible to do that with the following expression:

$$\sigma_{th_x} = E * \alpha * \frac{T_i - T_o}{2 * (1 - \nu)} * \left(1 - \frac{\frac{2 * r_y^2}{r_o^2 - r_i^2} * \ln\left(\frac{r_o}{r_i}\right)}{\ln\left(\frac{r_o}{r_i}\right)} \right) \tag{3.7}$$

Where E is the one in (3. 2) and ν is the Poisson ratio in (3. 3) but evaluated with the peak temperatures with overpower; x stays for the part of surface that is wanted to calculate the stress and y for the other one. Regarding the thermal expansion coefficient, it is possible to use this correlation:

$$\alpha = \frac{6.72 * 10^{-6} * T - 2.07 * 10^{-3}}{T - 308} \tag{3.8}$$

Again, evaluated at the same temperature of E and ν.

- **Stresses due to pressure**

  To obtain the stress components due to pressure in the three principal directions and so have 6 different stresses the following calculations are needed:

  - Inner surface:

$$\sigma_{p_t} = \frac{p_i * \left(r_e^2 + r_i^2\right) - 2 * p_e * r_e^2}{r_e^2 - r_i^2} \tag{3.9}$$

$$\sigma_{p_r} = -p_i \tag{3.10}$$

$$\sigma_{p_l} = \frac{r_i^2 * (p_i - p_e)}{r_e^2 - r_i^2} \tag{3.11}$$

  - Outer surface:

$$\sigma_{p_t} = \frac{2 * p_i * r_i^2 - p_e * \left(r_e^2 + r_i^2\right)}{r_e^2 - r_i^2} \tag{3.12}$$

$$\sigma_{p_r} = -p_e \tag{3.13}$$

$$\sigma_{p_l} = \frac{r_i^2 * (p_i - p_e)}{r_e^2 - r_i^2} \tag{3.14}$$

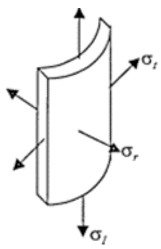

Figure 3.3: Directions of computed stresses on the cladding surface.

There are used radii without overpower because there what is important is not the thermal load but the pressure one.

- **Total stresses in the three** directions

  The total stress is calculated as the algebraic sum of pressure and thermal stresses.

- **Ideal and Tresca stresses**

  The ideal and Tresca stresses are uniaxial representations of the stresses:

$$\sigma_{ideal} = \frac{\sqrt{(\sigma_t - \sigma_r)^2 + (\sigma_r - \sigma_l)^2 + (\sigma_l - \sigma_t)^2}}{\sqrt{2}} \tag{3.15}$$

  Ideal (or ultimate) stress is evaluate using the technique of the Mohr circle and the maximum shear stress corresponds to the radius of this circle: $\sigma_{ideal} = R_{Mohr}$.

$$\sigma_{Tresca} = \max\ \{|\sigma_t - \sigma_r|\,, |\sigma_r - \sigma_l|\,,\ |\sigma_l - \sigma_t|\} \tag{3.16}$$

  Tresca stress is proposed for ductile materials, and it is a method to represents the maximum tangential stress at which the yield point of the material is reached.

  These stress limits should be computed for both the inner and outer surfaces, for principal and total stresses.

3.2.3 Verify the stresses on the inner and outer surface, finding the safety factors

In mechanics the safety factor is a critical design parameter that ensures structures can perform safely under anticipated conditions and it provide a margin of error due to uncertainties on material properties and load estimations.

The traditional method to calculate a safety factor is the Yield Safety factor compares the yield stress with the working stress, here the ideal one:

$$\frac{\sigma_{yield}}{\sigma_{ideal}} > 1 \tag{3.17}$$

So, (3.17) indicate how much times the material can handle the maximum expected stress before reaching its yield point.

The other method, used in the ASME code Sec. III for dealing with membrane stress for nuclear applications, is the maximum tangential stress method, also known as Tresca criterion:

$$\frac{\sigma_{allowed}}{\sigma_{Tresca}} > 1 \tag{3.18}$$

Also, for ASME safety factor the translation is almost the same: it indicates how much times the material can handle the maximum stress, calculated by Tresca method, before reaching the allowed stress.

But the allowed stress depends on the type of stress:

- For primary stresses:

$$\sigma_{allowed} = S_m \tag{3.19}$$

- For primary + secondary stresses:

$$\sigma_{allowed} = 3 * S_m \tag{3.20}$$

Where:

$$S_m = \min\left(\frac{2}{3} * \sigma_{yield}, \frac{1}{3} * \sigma_{ultimate}\right) \tag{3.21}$$

σ_{yield} and $\sigma_{ultimate}$ are given in the data.

3.2.4 Identify the maximum internal pressure allowed

If one or more safety factors is below 1, it is needed to identify the maximum internal pressure allowed to satisfy the safety factors.

3.3 Sizing of the gas plenum

The sizing of the gas plenum is done following the assumptions of ideal gases and so their general equation of state:

$$V_{\min} = \frac{n * R * T}{p_{\max}} \tag{3.22}$$

3.3.1 Total mass of uranium dioxide

In order to calculate the gases produces during burnup, it is needed the total mass of UO_2 in the rod:

$$M_{UO_2} = \rho_{UO_2} * \pi * \frac{d_p^2}{4} * H \tag{3.23}$$

Where ρ_{UO_2} is the effective density of UO_2, so must be corrected by the correction density factor equal to 0.995.

3.3.2 Computation of the moles

- **Moles of gas impurities**

  The gas impurities are released during normal operation and here are evaluated two different impurities:

  - Nitrogen (N_2)
  - Water (H_2O)

    For both of them the moles of gas are calculated as:

  $$n_g = \frac{c_g * M_{UO_2}}{u_g} \tag{3.24}$$

  Where:

  - c_g is the concentration of "g" gas impurity: given in ppm so have to be divided by 10^6;
  - u_g is the molecular weight of "g" gas impurity: given in the data.

- **Moles of fission gases**

  The number of moles of the fission gases depends on the number of fissions:

  $$N_{fissions} = \frac{BU * M_U}{E_f} \tag{3.25}$$

  Where:

  $$M_U = M_{UO_2} * \frac{u_U}{u_{UO_2}} \tag{3.26}$$

  Finally:

$$n_{Xe+Kr} = N_f * Y * \frac{R_r}{N_A} \tag{3.27}$$

Where:

- R_r from the assumption i;
- Y and N_f given by the data;
- N_A: Avogadro number equal to $6.0221 * 10^{23}\ mol^{-1}$.

3.3.3 Size the gas plenum

In order to size the gas plenum it is possible to use, as said before, the ideal gas law to find the minimum volume (3. 22) where T is the cladding temperature, R is the universal gas constant (8.3144 J/mol K) and $p_{\max}$ the allowed maximum pressure in the initial part of the work.

It is also possible to find the corresponding minimum height:

$$H_{\min} = \frac{V_{\min}}{\pi * r_i^2}$$

The used radius here is the one resulting from the exercise 2 without overpower.

Then accounting for the spring that push down the pellets:

$$H = H_{\min} + 15\ cm$$

3.3.4 Check the ideal gas assumption

It is needed to check the assumption done before on the water vapor treated as an ideal gas. To consider it conservative the partial pressure of the vapour has to be lower than the ideal gas case:

$$p_{H_2O_{vap}} < p_{H_2O_{ideal}} \tag{3.28}$$

This correlation is conservative because if the partial pressure is higher the contribution of water is overestimate on the total pressure and it is designed a gas plenum that can sustain higher pressure than necessary.

For the evaluation of this pressure:

$$p_{H_2O_{ideal}} = \frac{n_{H_2O}RT}{V_{min}}; \tag{3.29}$$

While, $p_{H_2O_{vap}}$ is evaluated with online tool [10], inserting in input the specific volume and the temperature. This temperature for both the pressure is the ending temperature of the fuel rods, near where the gas plenum is located.

Chapter 4

Results

4.1 Thermomechanical verification of the fuel pin

- **Verification-against-the-buckling**

  The verification against the buckling is confirmed:

$$\min\left(p_{cr}\right) = 46.01\ MPa > p_{e_{\max}} = 15.51\ MPa$$

This means that the fuel pellet does a good work also as structural elements.

The following table reports all the evaluated stresses in the three directions and on inner or outer surface:

	Pressure [MPa]	**Thermal [MPa]**	**Total [MPa]**
Inner surface $\boldsymbol{\sigma_t}$	105.60	-3.27	98.39
Inner surface $\boldsymbol{\sigma_r}$	-30.94	0	-30.94
Inner surface $\boldsymbol{\sigma_l}$	49.23	-3.60	45.63
Outer surface $\boldsymbol{\sigma_t}$	87.47	-2.69	84.79
Outer surface $\boldsymbol{\sigma_r}$	-15.51	0	-15.51
Outer surface $\boldsymbol{\sigma_l}$	50.15	-2.69	47.46

Table 4.1: Maximum stresses in the three directions.

The resulting radial stresses are all negative that means are compressive due to the pressure on external and internal surfaces (are equal to them). It is also possible to comment that are the lowest ones and this confirm the theory of the thin cylinder mechanics: the radial components are little and sometimes also negligible. The longitudinal and tangential stresses, instead, are all tensile and the higher contribution is given by the pressure and not by the thermal one.

- **Traditional and ASME verifications**

  The traditional verification gives 2 safety factors and the ASME one gives 4 safety factors and each one has to be higher than 1:

Safety factors	Results
Traditional method: INNER	2.08
Traditional method: OUTER	2.67
ASME method: primary INNER	1.04
ASME method: total INNER	3.11
ASME method: primary OUTER	1.34
ASME method: total OUTER	4.01

  As it is possible to understand from the table above all the safety factors are higher than one and so the verifications are all confirmed: the fuel structure can work in elastic deformation regime without loosing performance.

  Due to all the factors up to 1 the maximum internal pressure satisfies the safety factors and has not to be computed again.

4.2 Sizing of the gas plenum

Individuated the maximum pressure allowable it was possible to compute the minimum volume for the released gases and it is equal to 12.73 cm^3 with a height of 38.22 cm.

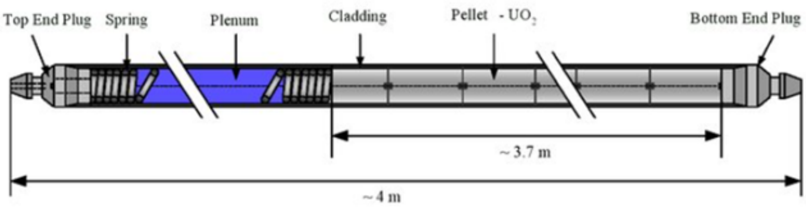

Figure 4.1: Schematic of a fuel rod of a PWR [5]

.

A volume like that roughly speaking 1% of the total volume of the rod, so not so much and the height has almost the same order of magnitude than the one considered by Blair in the figure above: its gas plenum has a length of almost 0.3 m on 4 m of total length, in the case study of this work the total length is 4.267m.

Finally, the last point to be discussed is the pressure of the superheated steam. Since we treat also steam as an ideal gas, it has be verified the realism of this assumption.

- Pressure of the ideal gas: $p_{H_2O_{id}} = 39.66bar$
- Pressure of the superheated vapor $p_{H_2O_{vap}} = 36.82bar$

Even if the difference in not so big, we can assume the assumption valid. Temperature and specific vapour used for the evaluation of the pressure are respectively: $T_{cl,i_{end}} = 619.26K$ and $v = 0.0721m^3/kg$

Chapter 5

Conclusions

In this analysis we verified the integral structure of a fuel cladding in a AP1000. We go through all the comparison needed and trying to assume always the worst condition of operation. As it has been demonstrated all the verifications done are positive and this means that the fuel structure remain in elastic region and whatever deformation from the stresses studied can be reversible. It is also proved that the main character on the analysis is the pressure that give high contributes to the stresses, instead the thermal ones are very lower also taking in account the possible overpower and the radii's deformation. So, it is possible to say that fission released gases are important to study and the critical contribute that they give is mostly on the increasing of inner pressure in the fuel. Also a gas plenum is designed. The sizing of this latter part of the cladding, allows to store all the gases produced during the reactor life to avoid extra pressure inside.

Bibliography

[1] Westinghouse AP1000 Design Control Document Rev. 19. *NRC: Package ML11171A500 -Westinghouse AP1000 Design Control Document Rev. 19 — nrc.gov*. https://www.nrc.gov/docs/ML1117/ML11171A500.html.

[2] INTERNATIONAL ATOMIC ENERGY AGENCY. «Thermal Conductivity of Uranium Dioxide». In: *TECHNICAL REPORTS SERIES No. 59* (1966).

[3] ASME. *BPVC Section III Rules for Construction of Nuclear Facility Components Division 1 Subsection NCD Class 2 and Class 3 Components - ASME — asme.org*. https://www.asme.org/codes-standards/find-codes-standards/bpvc-iii-ncd-bpvc-section-iii-rules-construction-nuclear-facility-components-division-1-subsection-ncd-class-2-class-3-components/2023/print-book. 2023.

[4] World Nuclear Association. *Physics of Uranium and Nuclear Energy - World Nuclear Association — world-nuclear.org*. https://world-nuclear.org/information-library/nuclear-fuel-cycle/introduction/physics-of-nuclear-energy.

[5] Blair, Paul. «Modelling of fission gas behaviour in high burnup nuclear fuel». en. In: (2008). DOI: 10.5075/EPFL-THESIS-4084. URL: http://infoscience.epfl.ch/record/118662.

[6] TU Delft. *1.1.6 Main Takeaways on Stress and Strain - TU Delft OCW — ocw.tudelft.nl*. https://ocw.tudelft.nl/course-readings/1-1-6-main-takeaways-on-stress-and-strain/. 2024.

[7] Mervin Fingas, ed. *Above ground storage tank oil spills*. en. Austin, TX: Gulf Publishing Company, Sept. 2022.

[8] PubChem. *Uranium oxide (UO2) — pubchem.ncbi.nlm.nih.gov*. https://pubchem.ncbi.nlm.nih.gov/compound/Uranium-oxide-_UO2.

[9] Alexander Sesonske. «NUCLEAR POWER PLANT DESIGN ANALYSIS». In: *Technical Information Center, Office of Information Services UNITED STATES ATOMIC ENERGY COMMISSION* (1973).

[10] SPIRAX SPARCO. *Steam Table Calculator | Superheated Steam Region | Spirax Sarco — spiraxsarco.com*. https://www.spiraxsarco.com/resources-and-design-tools/steam-tables/superheated-steam-region.

Appendix A

A.1 Exercise 2

```
clc;
close all;
clear all;

%% FROM EX2
TT_NP = load('T_NP.mat');
TT_co =TT_NP.TT_co;
TT_ci = TT_NP.TT_ci;
TT_gap = TT_NP.TT_gap_ave;
TT_fuel_surf=TT_NP.TT_fuel_new;
TT_fuel_center=TT_NP.TT_fuel_center_new;

TT_OP = load('T_OP.mat');
TT_co_OP = (TT_OP.TT_co) ;
TT_ci_OP = (TT_OP.TT_ci);
TT_gap_OP = TT_OP.TT_gap_ave;
TT_fuel_surf_OP=TT_OP.TT_fuel_new;
TT_fuel_center_OP=TT_OP.TT_fuel_center_new;

%% DATA
sigma_yield = 241e6; %Pa
sigma_uts = 413e6; %Pa

power=3400e6; %W
overpower = power*1.2;
heat_fuel=0.974; % percentuale
psia_to_Pa=6894.76;
pp_nom=psia_to_Pa*2250;
lb_h_ft2_to_kg_s_m2=0.001356;
ft_to_m=0.3048;
pa_to_psi=0.000145038;
number_assembly=157;
fahrenheit_to_celsius=@(T) (T-32)*5/9;
```

```
T_in_coolant_core=fahrenheit_to_celsius(535);
T_out_coolant_core=fahrenheit_to_celsius(535+81);
inches=0.0254;
lbm_h_to_kg_s=0.000125998;
ft_2_to_m_2=0.092903;
dd_out_rods=0.374*inches;
rr_cl_out = dd_out_rods/2;
thickness_cladding_rods=0.0225*inches;
dd_in_rods=dd_out_rods-2*thickness_cladding_rods;
rr_cl_in = dd_in_rods/2;
cross_section_rod=dd_out_rods^2*pi/4;
dd_pellet=0.3225*inches;
cross_section_fuel_pellet=pi*dd_pellet^2/4;
HH=168*inches;
transport_length=0.29e-2;
diffusion_coefficient_core=transport_length/3;
diffusion_coefficient_reflector=0.16;
diffusion_length_reflector=2.85;
reflector_saving=diffusion_coefficient_core*
    diffusion_length_reflector/diffusion_coefficient_reflector;
HH_e=HH+1.42*transport_length+2*reflector_saving;
total_number_rods=41448;
V_fuel=total_number_rods*cross_section_fuel_pellet*HH;
T_amb = 20; p_amb = 101325;

%% normal power Vs overpower
zz=linspace(-HH_e/2,HH_e/2,1000);
figure(1)
plot(TT_fuel_center_OP,zz,'Color',"#A2142F",'LineWidth',1.5,'
    DisplayName','Fuel center OP')
hold on
plot(TT_fuel_center,zz,'Color',"#A2142F",'LineStyle','--','
    LineWidth',1.5,'DisplayName','Fuel center')
hold on

plot(TT_fuel_surf_OP,zz,'Color',"#D95319",'LineWidth',1.5,'
    DisplayName','Fuel surface OP')
hold on
plot(TT_fuel_surf,zz,'Color',"#D95319",'LineStyle','--','LineWidth
    ',1.5,'DisplayName','Fuel surface')
hold on

plot(TT_ci_OP,zz,'Color',"#0072BD",'LineWidth',1.5,'DisplayName','
    Inner cladding OP')
hold on
plot(TT_ci,zz,'Color',"#0072BD",'LineStyle','--','LineWidth',1.5,'
    DisplayName','Inner cladding')
hold on

```

```
plot(TT_co_OP,zz,'Color',"#77AC30",'LineWidth',1.5,'DisplayName','Outer cladding OP')
hold on
plot(TT_co,zz,'Color',"#77AC30",'LineStyle','--','LineWidth',1.5,'DisplayName','Outer cladding')
hold on
grid on

ylabel('Axial coordinate [m]');
xlabel('Temperature');
legend('-dynamiclegend')
title('Temperature changing with OP')

%% Modified radius
alpha_cl=@(T)5.62e-6+3.162e-9.*T;


rr_out_rods=dd_out_rods*0.5*(1+alpha_cl(TT_co).*(TT_ci-20));
rr_in_rods=dd_in_rods*0.5*(1+alpha_cl(TT_ci).*(TT_ci-20));

rr_out_rods_OP=dd_out_rods*0.5*(1+alpha_cl(TT_co_OP).*(TT_co_OP-20));
rr_in_rods_OP=dd_in_rods*0.5*(1+alpha_cl(TT_ci_OP).*(TT_ci_OP-20));

figure(2)
subplot(1,2,1)
plot(rr_out_rods, zz, LineStyle="--")
hold on
plot(rr_out_rods_OP, zz, LineStyle="-")
hold on
grid on
xlabel('Radial coordinate [m]')
ylabel('Axial coordinate [m]')
legend('R_{out}', 'R_{out, OP}')
title('External fuel radii')
subplot(1,2,2)
plot(rr_in_rods, zz, LineStyle="--")
hold on
plot(rr_in_rods_OP, zz, LineStyle="-")
hold on
xlabel('Radial coordinate [m]')
ylabel('Axial coordinate [m]')
grid on
legend('R_{in}', 'R_{in, OP}')
title('Internal fuel radii')
%% pressure at which buckling starts developing and confront with max ext press
```

```
TT_clad_ave = (TT_ci.*rr_in_rods.^2+TT_co.*rr_out_rods.^2)./(
    rr_out_rods.^2+rr_in_rods.^2);
TT_clad_ave_OP = (TT_ci_OP.*rr_in_rods_OP.^2+TT_co_OP.*
    rr_out_rods_OP.^2)./(rr_out_rods_OP.^2+rr_in_rods_OP.^2);
E_zr_funz = @(T) (9.9e3-5.669.*T)*9.81*1e6;
E_zr = E_zr_funz(TT_clad_ave);
ni_funz = @(T) (0.3303+8.376e-5.*T);
ni = ni_funz(TT_clad_ave);
tt = (rr_out_rods-rr_in_rods);
rr_avg = (rr_out_rods-tt./2);
p_cr = E_zr./(4-4*ni.^2).*(tt./rr_avg).^3;
if all(p_cr > pp_nom)
    fprintf('prima verifica (BUKLING) TRUE\n');
else
    fprintf('prima verifica (BUKLING) FALSE\n');
end

%% compute Max int pressure in case of depressurized
sigma_pt_max = sigma_yield;
delta_p = sigma_pt_max.*tt./rr_avg;
p_in = (delta_p+p_amb);

%% compute thermal stresses
alpha_fun = @(T)(6.72e-6.*(T+273.15)-2.07e-3)./((T+273.15)-308);
alpha_in = alpha_fun(TT_ci_OP); %OP
alpha_out = alpha_fun(TT_co_OP); %OP
E_zr_OP = E_zr_funz(TT_clad_ave_OP);

sigma_th_in = E_zr.*alpha_in.*(TT_ci_OP-TT_co_OP)./2./(1-ni)
    .*(1-((2.*rr_out_rods_OP.^2)./(rr_out_rods_OP.^2-rr_in_rods_OP
    .^2).*log(rr_out_rods_OP./rr_in_rods_OP))./log(rr_out_rods_OP
    ./rr_in_rods_OP));
sigma_th_out = E_zr.*alpha_out.*(TT_ci_OP-TT_co_OP)./2./(1-ni)
    .*(1-((2.*rr_in_rods_OP.^2)./(rr_out_rods_OP.^2-rr_in_rods_OP
    .^2).*log(rr_out_rods_OP./rr_in_rods_OP))./log(rr_out_rods_OP
    ./rr_in_rods_OP));

sigma_th_in = E_zr.*alpha_in.*(TT_ci_OP-TT_co_OP)./2./(1-ni)
    .*(1-((2.*rr_out_rods.^2)./(rr_out_rods.^2-rr_in_rods.^2).*log
    (rr_out_rods./rr_in_rods))./log(rr_out_rods./rr_in_rods));
sigma_th_out = E_zr.*alpha_out.*(TT_ci_OP-TT_co_OP)./2./(1-ni)
    .*(1-((2.*rr_in_rods.^2)./(rr_out_rods.^2-rr_in_rods.^2).*log(
    rr_out_rods./rr_in_rods))./log(rr_out_rods./rr_in_rods));



%% compute stresses due to pressure
sigma_pt_in = (p_in.*(rr_out_rods.^2+rr_in_rods.^2)-2*pp_nom.*
    rr_out_rods.^2)./(rr_out_rods.^2-rr_in_rods.^2);
```

```
sigma_pr_in = -p_in;
sigma_pl_in = (rr_in_rods.^2.*(p_in-pp_nom))./(rr_out_rods.^2-rr_in_rods.^2);

sigma_pt_out = (-pp_nom.*(rr_out_rods.^2+rr_in_rods.^2)+2*p_in.*rr_in_rods.^2)./(rr_out_rods.^2-rr_in_rods.^2);
sigma_pr_out = -pp_nom;
sigma_pl_out = (rr_in_rods.^2.*(p_in-pp_nom))./(rr_out_rods.^2-rr_in_rods.^2);


%% total stresses
sigma_tt_in = sigma_th_in+sigma_pt_in;
sigma_rr_in = sigma_pr_in;
sigma_ll_in = sigma_th_in+sigma_pl_in;

sigma_tt_out = sigma_th_out+sigma_pt_out;
sigma_rr_out = sigma_pr_out;
sigma_ll_out = sigma_th_out+sigma_pl_out;

%% ideal and Tresca stresses

sigma_id_in = sqrt((sigma_tt_in-sigma_rr_in).^2+(sigma_rr_in-sigma_ll_in).^2+(sigma_ll_in-sigma_tt_in).^2)./sqrt(2);
sigma_id_out = sqrt((sigma_tt_out-sigma_rr_out).^2+(sigma_rr_out-sigma_ll_out).^2+(sigma_ll_out-sigma_tt_out).^2)/sqrt(2);

sigma_tresca_in = max([max(abs((sigma_tt_in-sigma_rr_in))),max(abs((sigma_rr_in-sigma_ll_in))),max(abs((sigma_ll_in-sigma_tt_in)))]);
sigma_tresca_out = max([max(abs((sigma_tt_out-sigma_rr_out))),max(abs((sigma_rr_out-sigma_ll_out))),max(abs((sigma_ll_out-sigma_tt_out)))]);

%% Verification
%verification
if sigma_yield>max(sigma_id_in)
    fprintf('seconda verifica (traditional sup interna) TRUE\n');
else
    fprintf('seconda verifica (traditional sup interna) FALSE\n');
end
SF_tr_in=sigma_yield/max(sigma_id_in);

if sigma_yield>max(sigma_id_out)
    fprintf('terza verifica (traditional sup esterna) TRUE\n');
else
    fprintf('terza verifica (traditional sup esterna) FALSE\n');
end
SF_tr_out=sigma_yield/max(sigma_id_out);
```

```
Sm = min([2/3*sigma_yield,1/3*sigma_uts]);
sigma_allowded_primary = Sm;
sigma_allowded_total = 3*Sm;

if sigma_allowded_primary>max(sigma_tresca_in)
    fprintf('quarta verifica (ASME primary in) TRUE\n');
else
    fprintf('quarta verifica (ASME primary in) FALSE\n');
end
SF_ASME_pr_in=sigma_allowded_primary/max(sigma_tresca_in);

if sigma_allowded_primary>max(sigma_tresca_out)
    fprintf('quinta verifica (ASME primary out) TRUE\n');
else
    fprintf('quinta verifica (ASME primary out) FALSE\n');
end
SF_ASME_pr_out=sigma_allowded_primary/max(sigma_tresca_out);

if sigma_allowded_total>max(sigma_tresca_in)
    fprintf('sesta verifica (ASME total in) TRUE\n');
else
    fprintf('sesta verifica (ASME total in) FALSE\n');
end
SF_ASME_tot_in=sigma_allowded_total/max(sigma_tresca_in);

if sigma_allowded_total>max(sigma_tresca_out)
    fprintf('settima verifica (ASME total out) TRUE\n');
else
    fprintf('settima verifica (ASME total out) FALSE\n');
end
SF_ASME_tot_out=sigma_allowded_total/max(sigma_tresca_out);

%% 3.2

% https://inis.iaea.org/collection/NCLCollectionStore/_Public
    /34/065/34065217.pdf
%pg 11 del pdfm 3 del libro
density_correction_factor=0.955;
rho_uranium=10970*density_correction_factor; %kg/m^3
m_UO2=rho_uranium*pi*dd_pellet^2*HH/4 %kg

% total uranium mass(knonw data)/total number fuel rods
%m_UO2=211588/2.20462/(264*157)  %QUALE PRENDERE? l'esercizio ne
    dice un altra, ma hh_e? e la molla?

N2_impurity=25/1e6;
H2O_impurity=75/1e6;

```

```
%g/mol
molecular_N2_weight=14.0067;
molecular_H2O_weight=18.01528;
%molecular_U_weight=238.02891;
molecular_U_weight=238.03*0.96+235*0.04;
molecular_UO2_weight=270.03;


moles_N2=N2_impurity*m_UO2*1000/molecular_N2_weight;
moles_H2O=H2O_impurity*m_UO2*1000/molecular_H2O_weight;

Rr=0.4;
fission_yield=0.28;
Na=6.0221408e23;

m_uranium=m_UO2*molecular_U_weight/molecular_UO2_weight; %kg

% Questi e il fission yeld dove sono stati presi?
burnup=60000/1e3;
energy_fission= 3.2e-11/8.64/1e10;%8.9009851666667e-21/24; %200MeV %MWd

fission_number=burnup*m_uranium/energy_fission;

moles_XeKr=fission_number*fission_yield*Rr/Na;

moles_total=moles_XeKr+moles_H2O+moles_N2;
RR=8.31446;

TT=TT_ci(end)+273.15;

V_min=moles_total*RR*TT/max(p_in);

H_min=V_min/(pi*(dd_in_rods/2)^2);
H_final=0.15+H_min;

%% check ideal gas assumpition


p_h2o=moles_H2O*RR*TT/V_min

M_vapour=moles_H2O*molecular_H2O_weight/1000;

v_vapour=V_min/M_vapour;

%FROM TABLE: T = 346.11; v = 0.0721.

p_sat = 36.83;

```

```
if p_sat>p_h2o
    fprintf('ideal gas assumption VERIFIED')
else
    fprintf('ideal gas assumption NOT VERIFIED')
end
```