LECTURE 8: FURTHER HYPOTHESIS TESTING
t test

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Re-cap: t-distribution

- In Section 7.4 (Confidence Intervals) we introduced the t-distribution and how we use that distribution for a confidence interval for the mean in small samples when the population standard deviation is unknown

- The same is true when we are performing hypothesis tests on the mean

- In this study session you will have to first get used to calculating p-values using the t-distribution before you perform any hypothesis tests

Re-cap – hypothesis tests

- One sample Z-test which compares the sample mean to a hypothesised mean – $H_0: \mu = \mu_0$; $H_1: \mu \neq \mu_0$

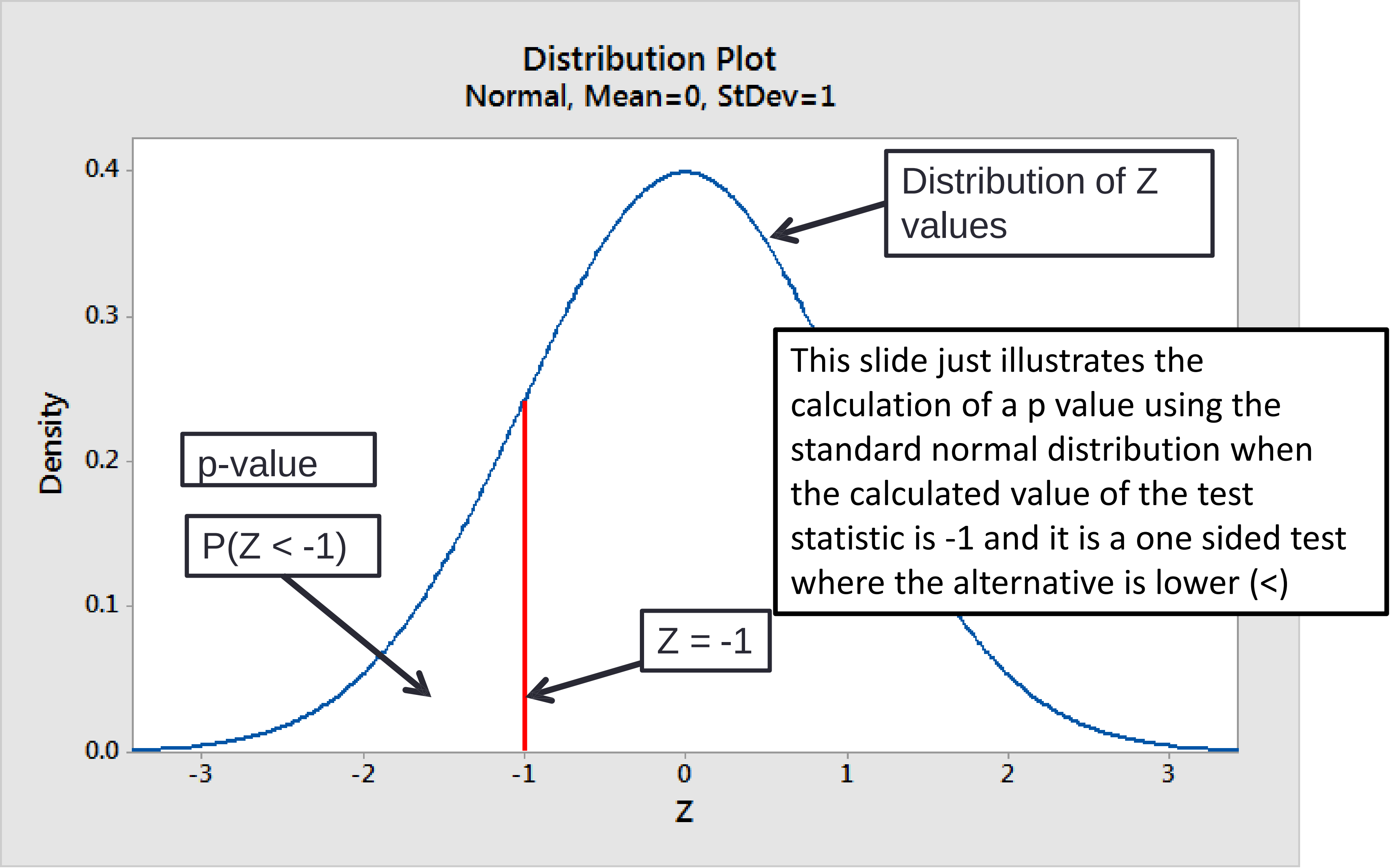
- You calculated a standardised test statistic (Z statistic) using

$$Z = \frac{(\bar{x} - \mu_0)}{\sigma/\sqrt{n}}$$

- If the null hypothesis is true and there is no difference between your sample mean and your hypothesised mean, this value of Z will come from the z-distribution (standard normal distribution)

- You then use normal distribution tables to find where that value lies and calculate the p-value associated with it [P(Z<-1) say]

Z-distribution



Re-cap: Decisions and assumptions

- You then used the p-value to make decisions
  - If $p \leq$ significance level: Reject the null hypothesis
  - If $p >$ significance level: Do not reject the null hypothesis

- This test assumes that we either
  - Know the standard deviation for the population
    - **For example: it is known that the standard deviation is 0.06 inch**
    - **Not very common**
  - The sample standard deviation is sufficiently close to the true standard deviation (large samples of 100 or more)
    - **Assume that the standard deviation estimated from the data is sufficiently close to the true standard deviation**

t-distributions

- It is not always possible/reasonable to make these assumptions
  - **Particularly when we have a small sample size (how small is too small, certainly less than 30 observations)**
  - **and**
  - **the population standard deviation is unknown (i.e. no info. given on this)**

- Therefore, we must use a t-distribution (Student’s t-distribution), instead of the standard normal distribution, to perform our hypothesis test
  - One sample t-test (one sample student t-test)

T-distribution

- The shape of a t-distribution depends on
  - The degrees of freedom (df) associated with the sample standard deviation: $\nu = n - 1$ (Greek letter pronounced nu)

- Test statistic:
  - The t test statistic formula is very similar to the formula for the z test statistic

$$t = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}}$$

  - As before $\bar{x}$ denotes the sample mean, μ denotes the hypothesised mean and n denotes the sample size
  - Note that we now use s for standard deviation rather than σ
    - This is to differentiate between a sample estimate (s) and a population parameter (σ)

T-distribution

- The rest of the calculations are similar to those used in Study Session 7

- Use a calculator to calculate t and then the distribution tables to calculate the probability
  - You may have to use Minitab to calculate the sample mean and sample standard deviation **(Store Descriptive Statistics)**

- You will need to decide on the use of the Lower, Upper or Both options for the t-distribution
  - Follow the same rules that you used for the z-distribution in Study Session 7 based on the alternative hypothesis – one or two tailed tests.

T-test of the mean

- The main use of Student’s t-test is to test the mean value of a sample

- This is very similar to Study Session 8.2 (testing the mean)
  - Read the question to identify the alternative hypothesis
  - Calculate the test statistic
  - Calculate the p-value for the test statistic using t distribution tables
  - Based on the p-value make a decision about the null hypothesis

- The only difference here is you will have to calculate/use the mean and standard deviation of the sample

T-test of the mean - Example

- A machine in a food production plant fills packets of corn flakes. The usual fill follows a normal distribution with a mean 430g. A new machine is introduced and a sample of 26 packets gives a mean contents of 412g with a standard deviation 37g. On the assumption that the amount put into the packets still follows a normal distribution, does this sample provide evidence at the 5% significance level to suggest that the new machine is under filling the packets?

- What is α?

T-test of the mean – Significance level

- A machine in a food production plant fills packets of corn flakes. The usual fill follows a normal distribution with a mean 430g. A new machine is introduced and a sample of 26 packets gives a mean contents of 412g with a standard deviation 37g. On the assumption that the amount put into the packets still follows a normal distribution, does this sample provide evidence at the 5% significance level to suggest that the new machine is under filling the packets?

- What is α? $\alpha = 0.05$

T-test of the mean - Hypotheses

- A machine in a food production plant fills packets of corn flakes. **The usual fill follows a normal distribution with a mean 430g.** A new machine is introduced and a sample of 26 packets gives a mean contents of 412g with a standard deviation 37g. On the assumption that the amount put into the packets still follows a normal distribution, does this sample provide evidence at the 5% significance level to **suggest that the new machine is under filling the packets**?

H_0: $\mu = 430$

H_0: $\mu < 430$

T-test of the mean – Sample values

- A machine in a food production plant fills packets of corn flakes. The usual fill follows a normal distribution with a mean 430g. **A new machine is introduced and a sample of 26 packets gives a mean contents of 412g with a standard deviation 37g.** On the assumption that the amount put into the packets still follows a normal distribution, does this sample provide evidence at the 5% significance level to suggest that the new machine is under filling the packets?

$$\boxed{\begin{aligned} x &= 412g \\ s &= 37g \\ n &= 26 \end{aligned}}$$

T-test of the mean – test statistic

- Null Hypothesis: $\mu = 430$

$x = 412g$
$s = 37g$
$n = 26$
$\mu_0 = 430$

- Alternative hypothesis?
  1. $\boldsymbol{\mu < 430}$

- Calculate the standardised t-statistic needed for the test

$$t = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}} = \frac{412 - 430}{37/\sqrt{26}} = -2.481$$

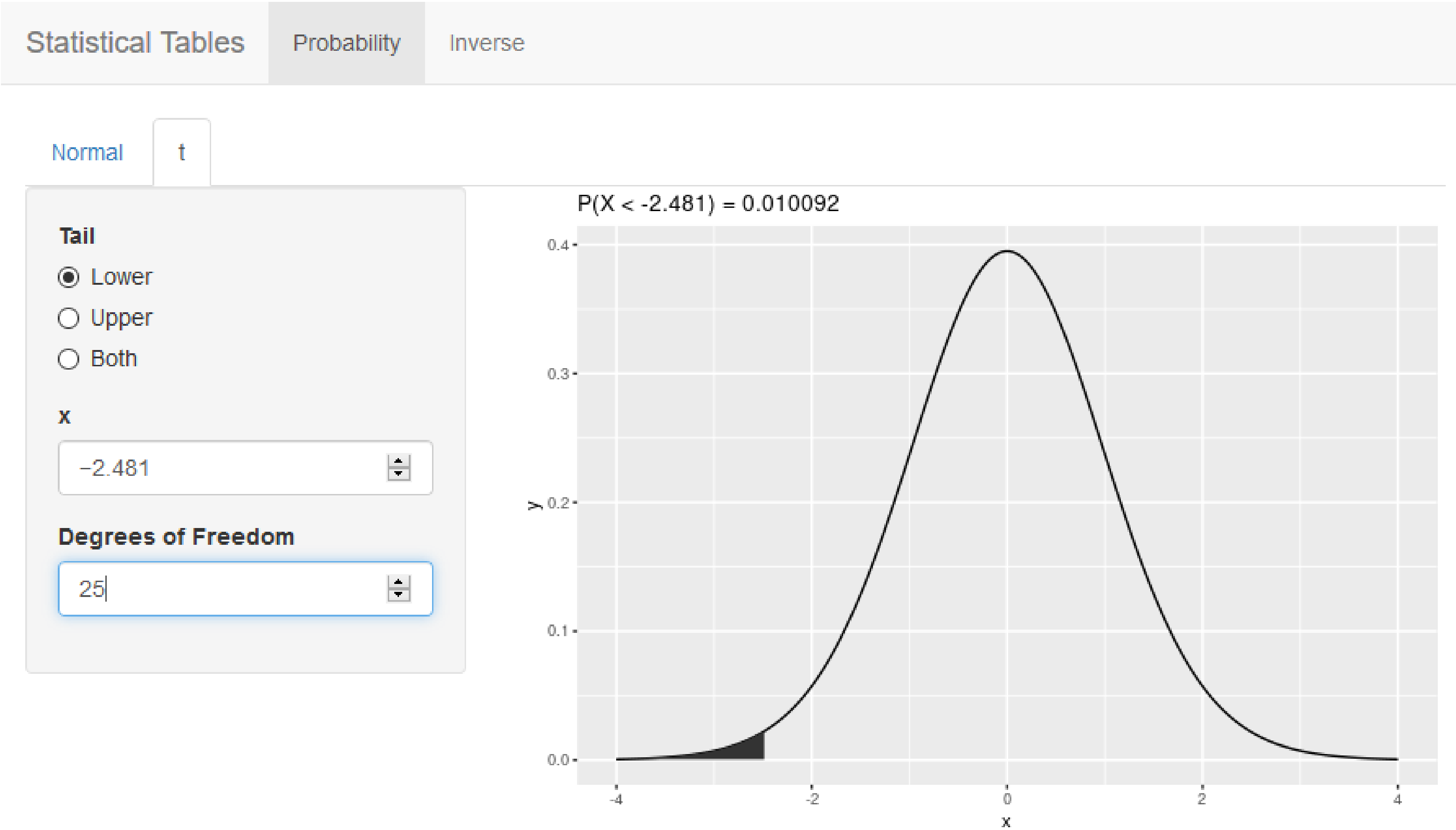
T-test of the mean - Example

- In relation to this t-statistic, state the p-value for the test
  - What is $P(T < -2.481)$?

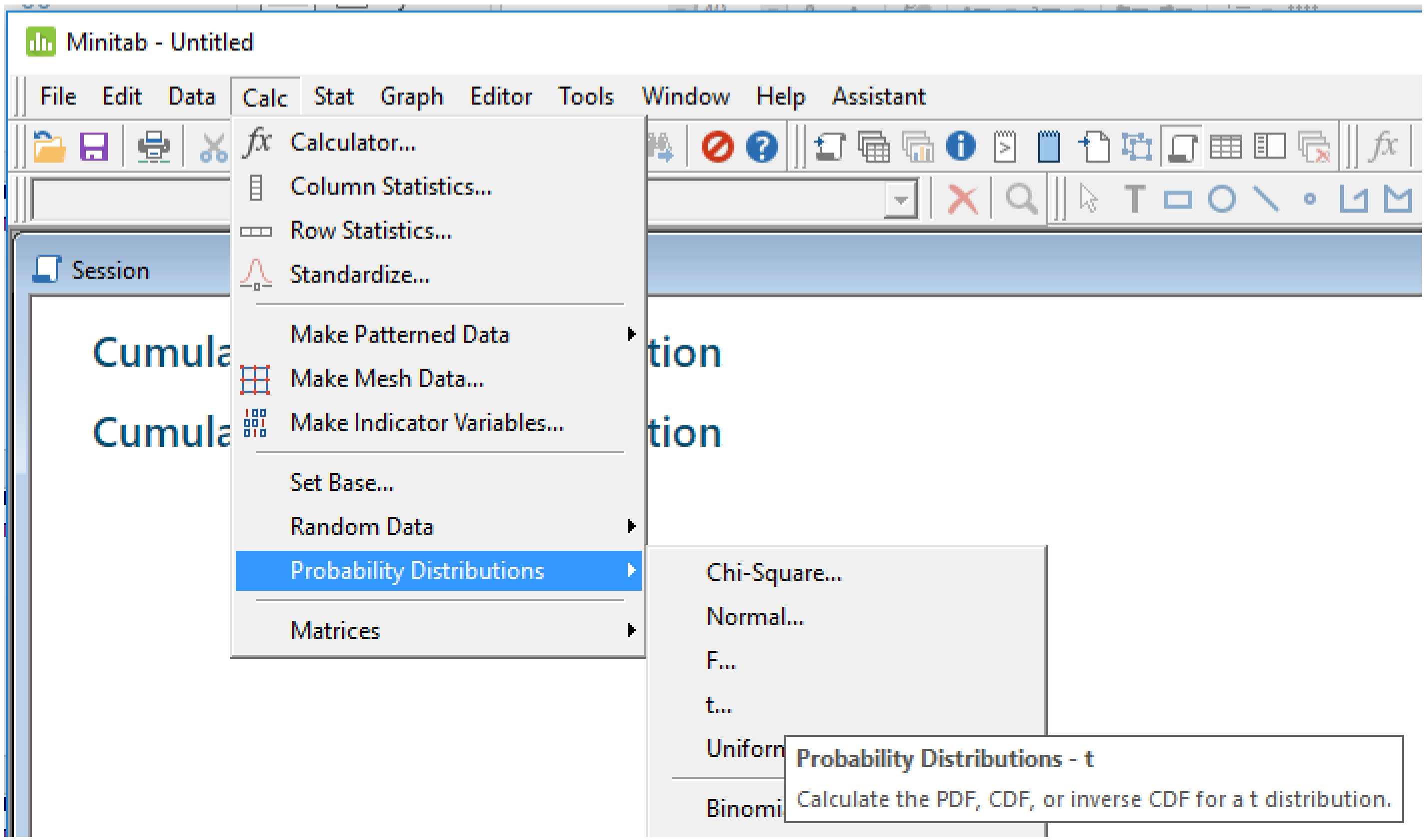
  - Go to tables select the Lower t and enter the values for t and υ
    - $t = -2.481,$
    - $\nu = n - 1 = 26 - 1 = 25$

  - P-value for the test is 0.0100924

P Value for the t test from tables



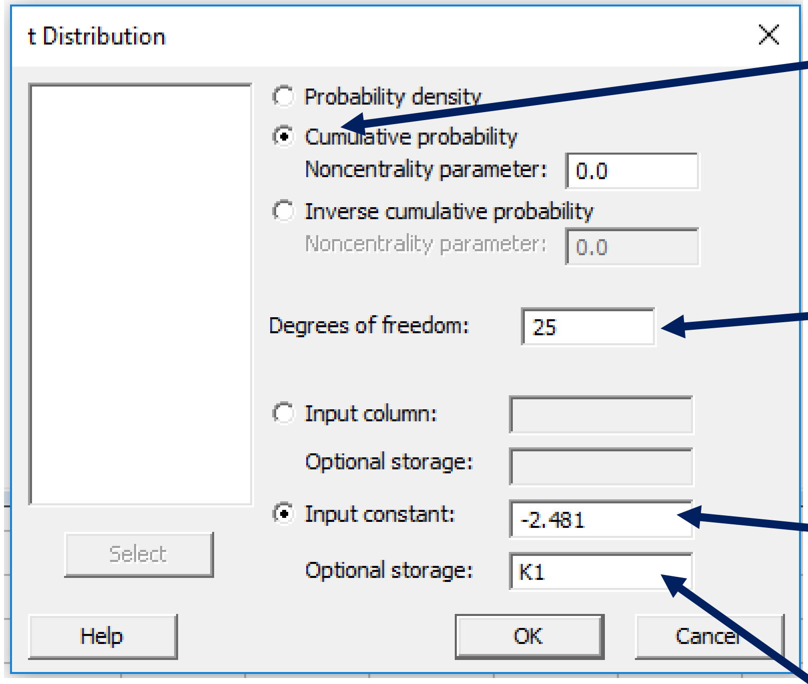
P Value for the t test - Minitab



Select Calc, then Probability Distributions, then t…

P Value for the t test - Minitab

Fill in the Dialogue Box



Check Cumulative Probability

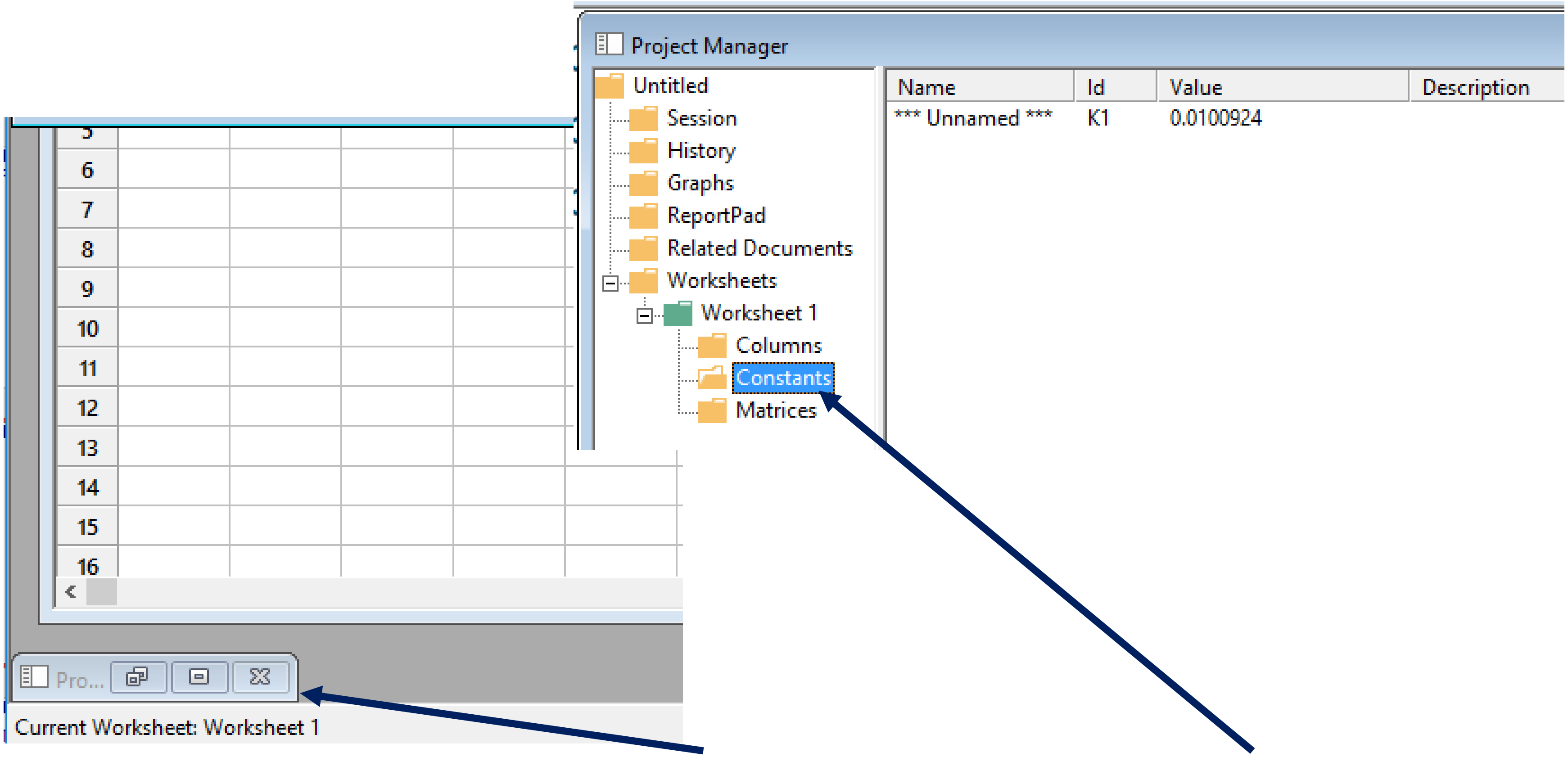
Enter Degrees of Freedom

Enter t statistic as a constant

Enter a constant K1, K2 etc to store the result to get sufficient decimal places

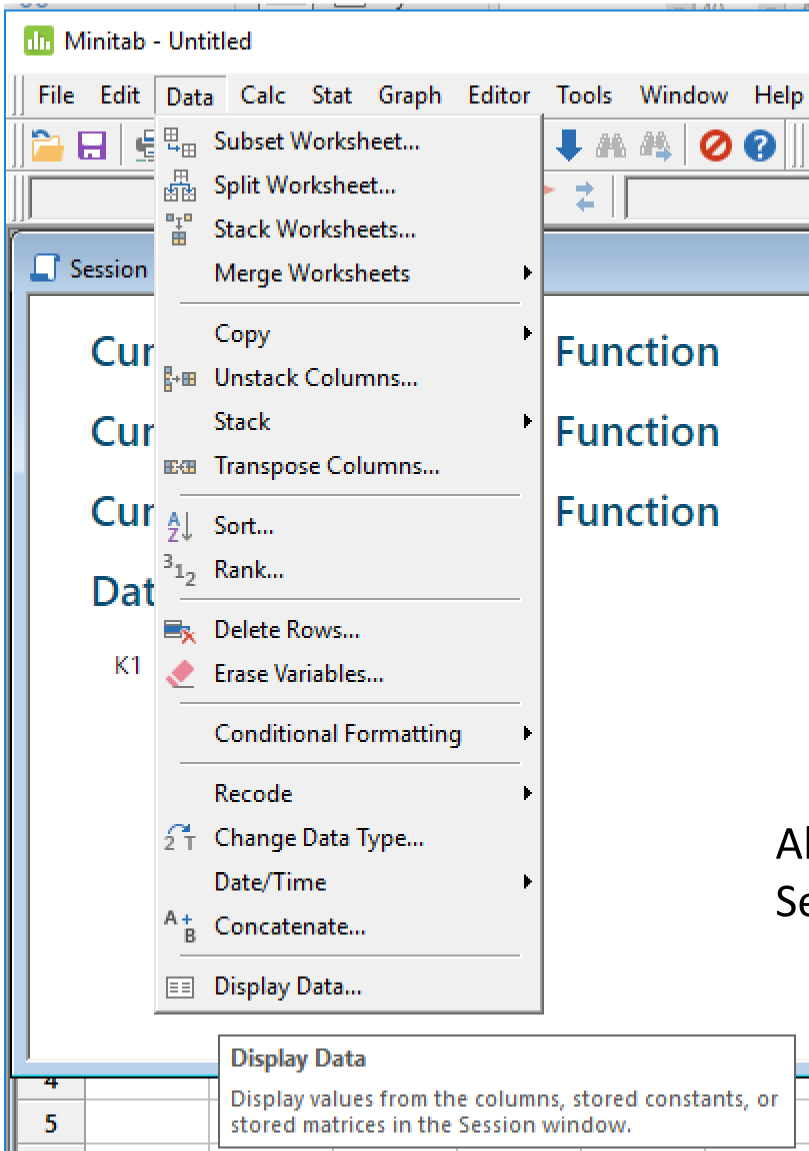
Press OK

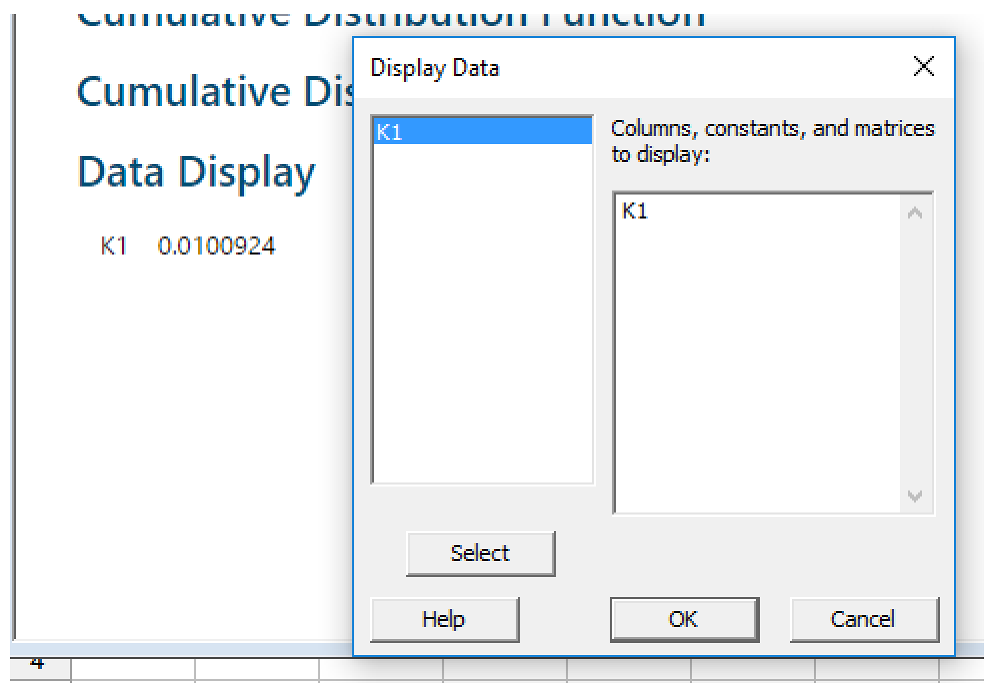
P Value for the t test - Minitab



To see the constant open the project window and choose constants

P Value for the t test - Minitab





Alternatively
Select Data > Display Data

Select K1 and click OK

K1 is printed in the session window

T-test of the mean - Example

- Based on this p-value do we
  1. Reject H_0
  2. Do not reject H_0

  - Since 0.01009 < 0.05 we **Reject $\boldsymbol{H_0}$**

- What do you conclude?
  - Rejection of the null hypothesis means that we must fall in line with the alternative hypothesis
  - There is clear evidence at the 5% significance level that the mean quantity being packed by the new machine is less that 430g, or
  - There is clear evidence at the 5% significance level to suggest that the new machine is under filling the packets

T-test of the mean – Example 2

- A dietary expert claims that, on average UK men are more than 7.5kg overweight. To test her assertion a random sample of 18 men is assembled and their excess weight is calculated (the difference between their weight and their ideal weight)

7.3	13	9.8	10.9	7.4	10.7
10.6	7	7	8.3	10.3	12.4
11.2	11	6	7	5.4	7

- On the assumption that these excess weights are normally distributed, test whether the dietician's claim is supported by the evidence. Use a 1% significance level

T-test of the mean - Example

- The test has to be applied with level of significance 1% and null hypothesis $\mu = 7.5$. What is the alternative hypothesis?

- Alternative hypothesis:
  - Person thinks that men are **MORE** than 7.5kg overweight on average.
  - $H_1 \colon \mu > 7.5$

- Calculate the standardised t-statistic needed for the test
  - We need to copy and paste the data into Minitab
  - Minitab needs the data in a single column format so we need to stack columns
    - Data -> Stack -> Columns

T-test of the mean - Example

Worksheet 2 ***

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
1	7.3	13.0	9.8	10.9	7.4	10.7	10.6	7		7.3	
2	7.0	8.3	10.3	12.4	11.2	11.0	6.0	7		7.0	
3	5.4	7.0								5.4	
4										13.0	
5										8.3	
6										7.0	
7										9.8	
8										10.3	
9										10.9	

- To calculate the test statistic we need to know the sample mean and sample standard deviation
  - Stat -> Basic Statistics -> **Store Descriptive Statistics**
  - $x = 9.01667, s = 2.32233$

T-test of the mean - Example

- Using these values we find

$$t = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}} = \frac{9.01667 - 7.5}{2.32233/\sqrt{18}} = 2.771$$

- In relation to this t-statistic, calculate the p-value for the test
  - Since the alternative hypothesis contained > we use the Upper option in the t-tables on myplace with $t = 2.771$ and $\upsilon = 17$

T-test of the mean - Example

- The p-value for the test was 0.00654

- Since 0.0065394 < 0.01 (significance level in question) we conclude
  - The test is significant at the 1% significance level
  - Reject the null hypothesis
  - There is clear evidence at the 1% significance level to suggest that UK men are, on average, more than 7.5kg overweight

Key Points

One sample t-test for a mean follows the same principles as the one sample z test for a mean

The t-test is more realistic than the z test as it is unlikely that the population standard deviation will be known if the mean is unknown.

The degrees of freedom associated with the t test are $\upsilon = n - 1$, where n is the sample size

To use a t-test the variable must follow a normal distribution over the population.

The test statistic is

$$t = \frac{(\bar{x} - \mu_0)}{s/\sqrt{n}}$$