MM104/ MM106/ BM110

Topic 4: Normal Distribution

Introduction to the Normal Distribution

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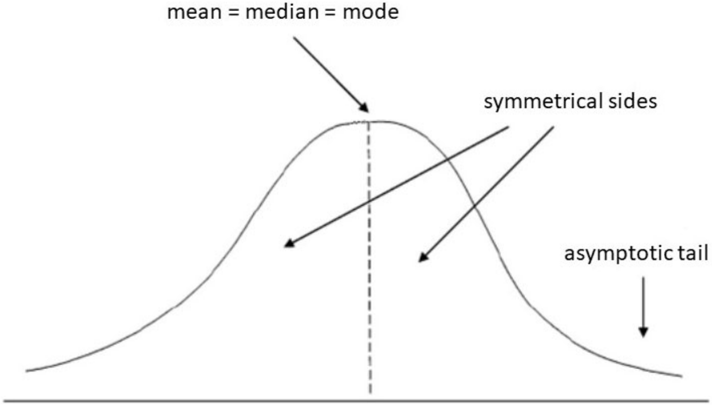
Normal Distribution

In today's class we are going to learn about arguably the most important concept in statistics. **The Normal Distribution**

The normal distribution is the most important probability distribution in statistics because many continuous data in nature and psychology displays this bell-shaped curve when compiled and graphed.

What does it look like ?

The normal distribution is a continuous probability distribution that is symmetrical on both sides of the mean, so the right side of the center is a mirror image of the left side.



Parameters of the Normal Distribution

There are two parameters in the normal distribution.

1. Mean
    - The mean is the central tendency of the distribution. It defines the location of the peak for normal distributions and most data values cluster around the mean.
2. Standard Deviation (also called numerical data)
    - The standard deviation defines the width of the normal distribution. The standard deviation determines how far away from the mean the values tend to fall. It represents the typical distance between the observations and the average.
      On a graph, changing the standard deviation either tightens or spreads out the width of the distribution along the X-axis. Larger standard deviations produce distributions that are more spread out

Common Properties for All Forms of the Normal Distribution

- Theyre all symmetric. The normal distribution cannot model skewed distributions.
- The mean, median, and mode are all equal.
- Half of the population is less than the mean and half is greater than the mean.
- The tails of the normal distribution are asymptotic so this means that they will never touch the x axis and go on to ∞ and $-\infty$.

The Standard Normal Distribution

The standard normal distribution is a normal distribution with a mean of zero and standard deviation of 1.

Mathematically we write this as: $Z \sim N(\mu, \sigma) \Rightarrow Z \sim N(0, 1)$,

where the μ (pronounced mu) is the population mean and σ (pronounced sigma) is the population standard deviation.

The Z random variable is called a **standard normal variable**.

Normal Distribution and Probability

From the previous topic we know that **the sum of total probability is equal to one.**

> The area under the normal distribution curve represents probability and the total area under the curve sums to one.

We can use this fact to carry out calculations involving the normal distribution.

Normal Distribution Calculations

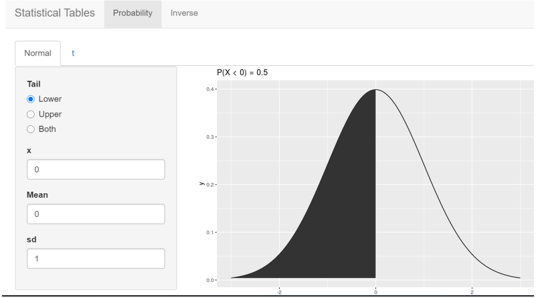
When working with this distribution there are two possible calculations

1. Find $P(X < x)$:
    - What is the probability that the random variable X takes the values less than x?
    - What is the area under the curve to the LEFT (as the $<$ sign points to the left) of x?
2. Find x when $P(X < x) = 0.225$
    - What is the value of x that gives a probability of 0.225?
    - What would x need to be is the area under the curve to the LEFT of it is 0.225?

Normal Distribution Calculations

To answer any questions in this topic we need to make use of a **Normal Distribution table**.

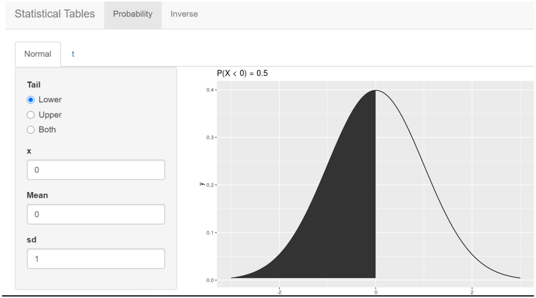
You will find this table on the Myplace page, under the link Statistical Tables. This will take you to a page which looks like this...



Normal Distribution Calculations

Simply select if you are doing Probability (Question Type 1 on Slide 8) or Inverse (Question Type 2 on Slide 8 Calculations), and fill in the relevant information. This set of slides will only consider Probability Questions.

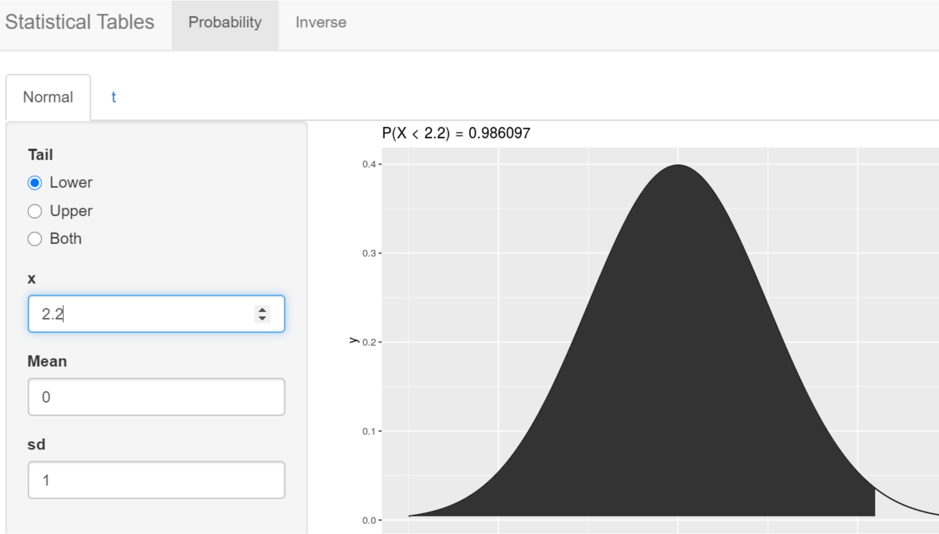
Lower, upper and both refer to the mathematical sign in the question, which we will discuss in the coming slides.



Normal Distribution Calculations - Example 1

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X < 2.22)$.

We are being asked for a **Probability** so our answer must be between 0 and 1.
Putting the values from the question into the Statistical tables we get the following and selecting "lower tail" as we have a single less than sign.


Statistical Tables
Probability
Inverse
Normal
t
Tail
Lower
Upper
Both
x
2.2
Mean
0
sd
1
P(X < 2.2) = 0.986097
y
0.4
0.3
0.2
0.1
0.0

Normal Distribution Calculations - Example 1

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X < 2.22)$.

Therefore, $\mathrm{P}(X < 2.22) = \underline{\underline{0.986097}}$.

Make sure you write all of the decimal places as your answer may be marked incorrectly if you are not accurate enough.

Normal Distribution Calculations - Example 2

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X > 2.22)$.

This is the same as the previous question only the sign is now $>$. Since we know the sum of total probability is 1, we can make use of the following result.

$$\mathrm{P}(X > a) = 1 - \mathrm{P}(X < a)$$

Normal Distribution Calculations - Example 2

Therefore,

$$\begin{aligned} \mathrm{P}(X > 2.22) &= 1 - \mathrm{P}(X > 2.22) \\ &= 1 - 0.986097 \\ &= \underline{\underline{0.013903}} \end{aligned}$$

Or you could have used the tables as in Example 1 and selected upper tail instead of lower tail.

Normal Distribution Calculations - Example 3

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(0.9 < X < 1.02)$.

$$\mathrm{P}(a < X < b) = \mathrm{P}(X < b) - \mathrm{P}(X < a)$$

$$\begin{aligned}\mathrm{P}(0.9 < X < 1.02) &= \mathrm{P}(X < 1.02) - \mathrm{P}(X < 0.9)\\ &\quad \text{Using the Statistical tables}\\ &= 0.846136 - 0.81594\\ &= \underline{\underline{0.030196}}\end{aligned}$$

Normal Distribution Calculations - Example 4

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X < 0.7 \text{ or } X > 1)$.

Or in maths means addition

$\mathrm{P}(X < 0.7 \text{ or } X > 1) = \mathrm{P}(X < 0.7) + P(X > 1)$

Using the Statistical tables.

$\mathrm{P}(X < 0.7) = 0.758036$ (Use setting lower tail as we have a $<$)

$\mathrm{P}(X > 1) = 0.158655$ (Use setting upper tail as we have a $>$)

$= 0.758036 + 0.158655$

$= \underline{\underline{0.916691}}$