MM104/ MM106/ BM110

Topic 3: Rules for Probability

The Essence of Probability

Ainsley Miller
ainsley.miller@strath.ac.uk

Complement

We will again introduce this lecture by introducing more definitions.

The **complement** of an event is all outcomes that are not the event.

For example:

- If the event is heads, the complement is tails
- If the event is {Monday and Thursday}, the complement is {Tuesday, Wednesday, Friday, Saturday, Sunday }

So the Complement of an event is all the other outcomes (not the ones we want).

Complement

If we are interested in the event A, then the probability of observing the complement of A is $\mathrm{P}(A^C)$.

Together the Event and its Complement make all possible outcomes.

$$\boxed{\mathrm{P}(A^C) = 1\text{- } \mathrm{P}(A)}$$

Combining Probabilities

Sometimes we may want to combine probabilities

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B)$$

The symbol $\cap$ is the intersection and we read this as “and”.

The above formula is only valid if the two events are independent from each other. Two events are independent if the occurrence of one does not affect the probability of occurrence of the other.

Combining Events - Example

In many statistical investigations the probability of two or more combined events is required. It is often easier to visualise these using a Venn diagram.

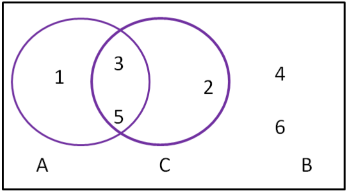
For example let's consider the events which may happen if a dice is thrown.

- A={odd numbers}=1,3,5
- B={even numbers}={2,4,6}
- C={prime numbers}= {2,3,5}

Find P(A and C)

Combining Events - Example

Find P(A and C)



A={odd numbers}=1,3,5 and C={prime numbers}= {2,3,5}
From the diagram there and the sample spaces the only numbers that are odd and prime are 3 and 5. Since the dice has 6 numbers in total.

P(A and C) $= \frac{2}{6} = \frac{1}{3}$.

But what about the formula ?

We cannot use the formula $\mathrm{P}(A \cap C) = \mathrm{P}(A)\mathrm{P}(C)$ because these events are **not independent**.

Mutually Exclusive

Mutually exclusive is a statistical term describing two or more events that cannot happen simultaneously.

In the previous example the roll of a dice cannot be both odd and even. Therefore, we can say that A and B are mutually exclusive.

Additive Law of Probability.

If A and B are not mutually exclusive

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$\cup$ is the union, and we read this as "or". We subtract off $P(A \cap B)$ so that we do not count the same element twice i.e. remove double counting

If A and B are mutually exclusive

$$P(A \cup B) = P(A) + P(B)$$

Additive Law - Example

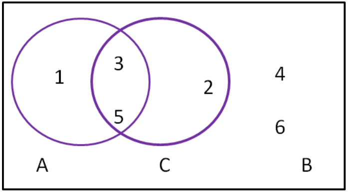
For example let's consider the events which may happen if a dice is thrown.

- A={odd numbers}=1,3,5
- B={even numbers}={2,4,6}
- C={prime numbers}= {2,3,5}

Find P(A or C)

Additive Law- Example

Find P(A or C)



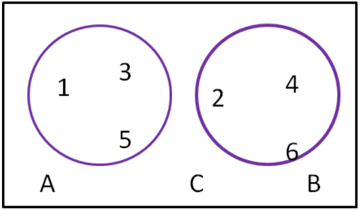
A={odd numbers}=$\{1, 3, 5\}$ and C={prime numbers}= $\{2, 3, 5\}$
These events are not mutually exclusive as they overlap in the Venn diagram above.

P($A \cup C$)= P(A)+P(C)-P($A \cap C$)= $\frac{3}{6} + \frac{3}{6} - \frac{2}{6} = \frac{4}{6} = \frac{2}{3}$.

Additive Law - Example 2

Find P (A or B)

A={odd numbers}=$\{1, 3, 5\}$ and B={even numbers}= $\{2, 4, 6\}$



These events are mutually exclusive as they do not overlap in the Venn Diagram.

P$(A \cup B)$= P(A)+P(B) $= \frac{3}{6} + \frac{3}{6} = \frac{6}{6} = 1$

Conditional Probability

Conditional probability is a measure of the probability of an event occurring given that another event has occurred

$$P(A|B) = \frac{P(A \cap B)}{P(B)},$$

we read the $|$ as the word "given", so $P(A|B)$ reads as the probability of A given that B has occurred

Conditional Probability Example

Example 1

Find the probability that a number on the die is prime given that it is odd.

$$\begin{aligned} P(\text{Prime} \mid \text{Odd}) &= \frac{P(\text{Prime and Odd})}{P(\text{Odd})} \\ &= \frac{2/6}{3/6} \\ &= \frac{2}{3} \end{aligned}$$

The probability that a number on the die is prime given that it is odd is 2/3.

Conditional Probability- Example 2

The following table gives the joint probabilities of having high blood pressure and of being a smoker:

	High blood pressure	Low blood pressure	Total
Smoker	0.26	0.29	0.55
Non Smoker	0.10	0.35	0.45
Total	0.36	0.64	1

a. Estimate the probability that a randomly selected person from the population is a smoker given that they have high blood pressure.

b. Estimate the probability that a randomly selected person from the population does not have high blood pressure, given that they are a smoker.

Estimate the probability that a randomly selected person from the population is a smoker given that they have high blood pressure.

Solution - A

$$
\begin{aligned}
\mathrm{P}(\text{Smoker} \mid \text{High b.p}) &= \frac{\mathrm{P}(\text{Smoker and High bp})}{\mathrm{P}(\text{High bp})} \\
&= \frac{0.26}{0.36} \\
&= 0.722
\end{aligned}
$$

In any probability question try to give your answer **to at least 3 decimal places**.

Estimate the probability that a randomly selected person from the population does not have high blood pressure, given that they are a smoker.

Solution - B

$$\begin{aligned}
\mathrm{P}(\text{Low b.p} \mid \text{Smoker}) &= \frac{\mathrm{P}(\text{Low b.p and Smoker})}{\mathrm{P}(\text{Smoker})} \\
&= \frac{0.29}{0.55} \\
&= 0.558
\end{aligned}$$

In any probability question try to give your answer **to at least 3 decimal places**.