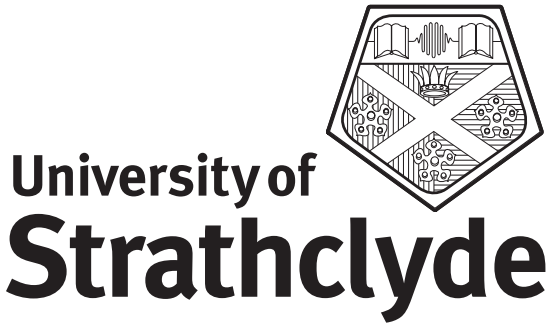

MM104/ MM106/ BM110

Topic 3: Probability

The Essence of Probability

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Probability Definitions

Before we begin the topic of probability we need to start of by defining some terms which will be used throughout this topic.

Experiment or **Trial**: any process which yields an observable outcome (observation or measurement).

Outcome: a possible value that may result from a trial
Example: tossing a coin is an experiment which will yield one of two outcomes – a head or a tail.

Random: an unpredictable event, where we cannot predict the outcome of an experiment.

Probability Definitions continued

Sample Space (SS): the set of all possible outcomes from an experiment.
Example: tossing a coin twice will result in the following sample space SS={HH, TT, TH, HT}, where H and T are heads and tails respectively. We always write our sample space using curly brackets { }.

Probability Definitions continued

Event (E) a subset of the sample space SS, a collection of all possible outcomes in that subset.
Example: Rolling a die and observing an odd number

P(E) the probability that the event E occurs in a single performance of an experiment.

What is Probability

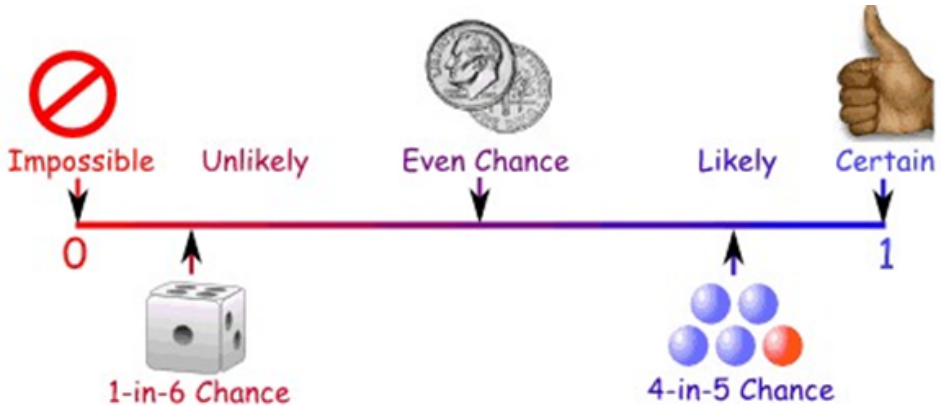
- Many outcomes/ events cannot be predicted with absolute certainty.
- As statisticians, we are often interested in predicting the outcome from a series of trials.
- To do this we need to be able to quantify how likely an outcome is
- We do this with **probability**

Probability is the chance of observing one outcome as a proportion of all possible outcomes.

Probability

A probability is a number between 0 and 1.

If P=0, the event is impossible. If P=1, the event is certain.



The higher an outcome's probability, the more likely it is to be observed.

Total Probability

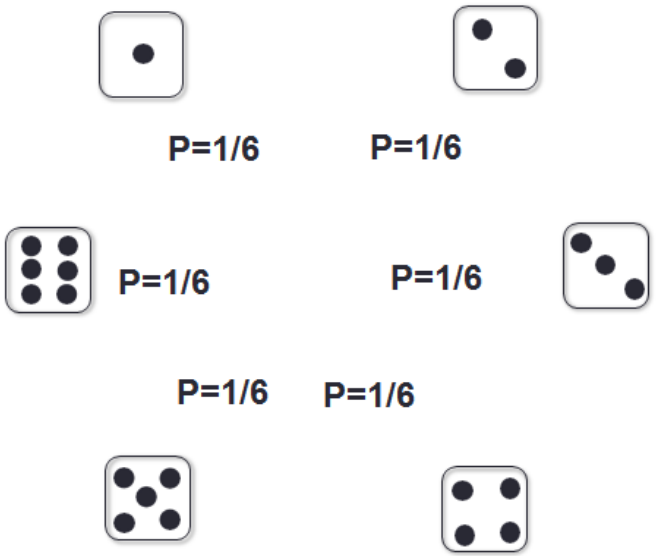
The sum of **all** possible outcomes' probabilities is 1, that is, we are certain that one of our outcomes will be observed.

This is an extremely important result and will be used throughout this topic.

When all outcomes are equally likely.

There are some case where the probability of observing each outcome is the same.

- The roll of a dice
- A fair coin toss
- The lottery



There are a total of six possible outcomes when rolling a normal die (N=6). Providing the dice is fair, all probabilities are equally likely so the probability of getting any one of the numbers is 1/6. $\Rightarrow$ P= 1/6

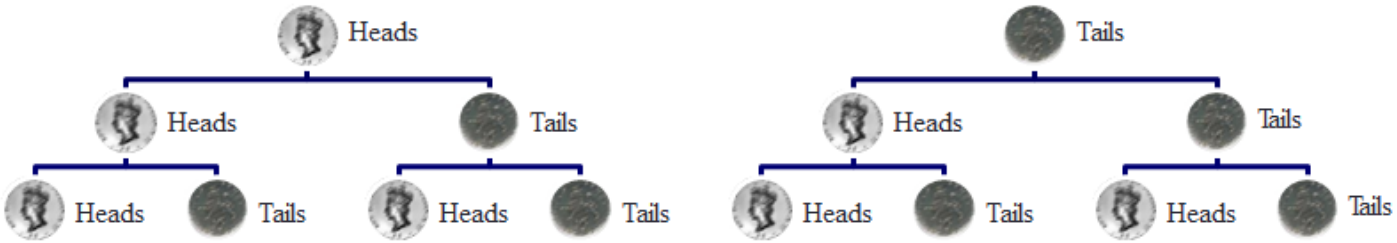
When all outcomes are equally likely.

When all outcomes are equally likely the probability of observing one of the outcomes is P$= \frac{1}{N}$, where N is the total number of possible outcomes.

If all outcomes are equally likely then the experiment follows a **uniform distribution**.

Calculating the Probability of an Event

To be able to calculate the probability of an event occuring we firstly need to calculate the total number of possible events.
For examples, the sequence of three coin tosses:



At each of our three outcomes making up the event there are always two possibilities {H,T}. By counting the number of elements in the bottom row there are 8.
Therefore the number of outcomes is 8.

Probability of Events

The number of possible events denoted by E is given by the number of outcomes N to the power of the number of trials T, that is,

$$E = N^T$$

When Outcomes are not equally likely

The probability of E is P(E) and

$$\mathrm{P(E)} = \frac{\text{number of different possible outcomes in E}}{\text{total number of different possible outcomes in SS}}$$

Probability Example

Example 1

A building firm has 10 lorries available. Of these 5 are 1 year old, 3 are 3 years old, and 2 are 5 or more years old.

a. Define the sample space SS.

b. What is the probability that a randomly selected lorry is 5 or more years old?

c. What is the probability that a randomly selected lorry is 1 year old?

Solution - A

The sample space, SS, contains all possible outcomes

- 5 lorries are 1 year old: SS $= \{1, 1, 1, 1, 1\}$.
- 3 lorries are 3 years old: SS $= \{3, 3, 3\}$.
- 2 lorries are 5+ years old: $SS = \{5, 5\}$.

Hence the sample space is : $\{1, 1, 1, 1, 1, 3, 3, 3, 5, 5\}$.

Solution - B

Let A denote the event that our lorry is 5+ years old
From part (a.) our sample space contains 10 elements. From the question we know that two out of the ten lorries are 5+ years old.
Hence $\mathrm{P}(A) = \frac{2}{10} = 0.2$.

Solution - C

Let B denote the event that our lorry is 1 years old
From the question we know that five out of the ten lorries are 1 years old.
Hence $\mathrm{P}(B) = \frac{5}{10} = 0.5$.