UNIVERSITY OF STRATHCLYDE
DEPARTMENT OF MATHEMATICS AND STATISTICS

MM101 Introduction to Calculus

Examples 2A

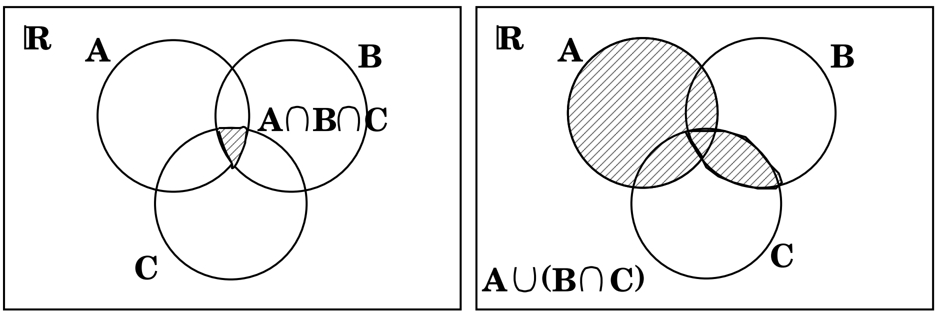
1 List the members of the following sets of real numbers.

$$\text{(i) } \{x \mid x^2 = 1\}, \quad \text{(ii) } \{y \mid y^2 = -5\},$$

$$\text{(iii) } \{z \mid z \text{ is the square of an integer and } z < 100\}.$$

(i) $\{x \mid x^2 = 1\} = \{-1, 1\}$.
(ii) $\{y \mid y^2 = -5\} = \emptyset$.
(iii) $\{z \mid z$ is the square of an integer and $z < 100\} = \{0, 1, 4, 9, 16, 25, 36, 49, 64, 81\}$.

2 Given three sets A, B and C of real numbers, draw Venn diagrams to illustrate the combinations $A \cap B \cap C$ and $A \cup (B \cap C)$.



3 Suppose that $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 5\}$, $C = \{3, 5\}$, and $D = \{3, 5, 6\}$. Determine which of these sets are subsets of the other sets.

$B \subset A$, $C \subset A$, $C \subset D$.

Examples 2B

1 Use (P1) to (P9) to show that $a \cdot 0 = 0$.

We have	$a \cdot 0 + a \cdot 0 = a \cdot (0+0)$	using (P9)
hence	$a \cdot 0 + a \cdot 0 = a \cdot 0$	using (P2)
hence	$(a \cdot 0 + a \cdot 0) - a \cdot 0 = a \cdot 0 - a \cdot 0$	adding $-a \cdot 0$ to both sides
hence	$a \cdot 0 = 0$	using (P1) and (P3).

Examples 2C

1 List five members of the following sets.

$$\text{(i) } \{x \mid x = 2k,\ k \in \mathbb{Z}\}, \qquad \text{(ii) } \{y^2 \mid y \in \mathbb{Z} \text{ and } -4 \le y \le 4\},$$

$$\text{(iii) } \{4 - 3z \mid z \in \mathbb{Q} \text{ and } 0 \le z < 3\}.$$

(i) $2, 4, 6, 162, -10$ (for $k = 1, 2, 3, 81, -5$).
(ii) $16, 9, 4, 1, 0$ (for $y = -4, -3, -2, -1, 0$).
(iii) $4, 1, -2, \frac{5}{2}, 3$ (for $z = 0, 1, 2, \frac{1}{2}, \frac{1}{3}$.)

2 Show that if $a < b$ and $b < c$, then $a < c$.

If $a < b$ and $b < c$ then $b - a$ and $c - b$ are positive. So $(c-b) + (b-a) = c - a$ is positive by (P11) and $a < c$.

3 Show that if $a < b$ then $-b < -a$.

If $a < b$ then $b - a$ is positive so $(-a) - (-b)$ is positive so $-b < -a$.

Examples 2D

1 Write the following sets as intervals and draw them on the real line:

$$\text{(a)}\ \{x : -2 < x \leq 1\} \qquad \text{(b)}\ \{\alpha : \alpha \geq 7\}.$$

(a) $\{x : -2 < x \leq 1\}$ is equivalent to $(-2, 1]$.
(b) $\{\alpha : \alpha \geq 7\}$ is equivalent to $[7, \infty)$.

2 Write the following intervals as sets and draw them on the real line:

$$\text{(a)}\ (-1, 6) \quad \text{(b)}\ (-\infty, \pi]$$

(a) $(-1, 6)$ is $\{x : -1 < x < 6\}$.
(b) $(-\infty, \pi]$ is $\{x : x \leq \pi\}$.

Examples 2E

1 Write the following sets as intervals and draw them on the real line:

$$\text{(a)}\ \{x : |x| \leq 1\} \qquad \text{(b)}\ \{\alpha : |\alpha| > 5\}.$$

(a) $|x| \leq 1$ means $-1 \leq x \leq 1$ so $[-1, 1]$.

(b) $|\alpha| > 5$ means $\alpha > 5$ or $\alpha < -5$ so $(-\infty, -5) \cup (5, \infty)$.

2 Evaluate (i) $|7|$, (ii) $|-3|$, (iii) $|1 + \sqrt{2} - \sqrt{3}|$.

(i) $|7| = 7$.
(ii) $|-3| = 3$.
(iii) We have

$$(1 + \sqrt{2})^2 = 1 + 2\sqrt{2} + 2 = 3 + 2\sqrt{2}$$

so

$$1 + \sqrt{2} = \sqrt{3 + 2\sqrt{2}} > \sqrt{3}.$$

Hence $|1 + \sqrt{2} - \sqrt{3}| = 1 + \sqrt{2} - \sqrt{3}$.