University of Strathclyde, Department of Mathematics and Statistics

MM102 Applications of Calculus
Exercises for Week 4

1. Determine the equations for the tangent and the normal to the graph of the function

$$f(x) = \sin x$$

at the point $x = \dfrac{\pi}{4}$.

2. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x and y given that

   (a) $x^3 + y^3 = 1$

   (b) $2x^3 \sin y + y^2 - xy^3 = 1$

   (c) $\sqrt{xy} + \sin x + \cos y = 0$

   (d) $\sin(xy) = \cos x \cdot \cos y$

   (e) $\sin(x + y^2) = y$

   (f) $\sin x + \cos y = 1$

   (g) $e^y - xy^2 = 3$

3. Show that the given point lies on the curve. Moreover, find the tangent to the curve at that point.

   (a) $y^2 = 2x^3, \qquad (2, -4)$

   (b) $(x + y)^3 = 2x + y + 3, \qquad (3, -1)$

   (c) $xy^3 - x^3y = 30, \qquad (2, 3)$

   (d) $x = y - \cos y, \qquad \left(\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

4. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ as functions of x and y given that

   (a) $xy^2 + y = 1$

   (b) $y^4 + y = x^3$

5. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of the parameter t when x and y are given by

   (a) $x = 4t^2 - 1, \quad y = 2t + 1$

   (b) $x = 2\sec t, \quad y = \tan t$

   (c) $x = \dfrac{1-t^2}{1+t^2}, \quad y = \dfrac{2t}{1+t^2}$

   (d) $x = t + \dfrac{1}{t}, \quad y = t - \dfrac{1}{t}$

6. Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ as functions of the parameter t when x and y are given by

   (a) $x = \ln t + 2, \quad y = t^3 + 2t$

   (b) $x = \cos t + t, \quad y = \sin t + t^2$

   (c) $x = t^2, \quad y = t^3$

   (d) $x = t^2 + t, \quad y = 2t^3 + t^2 + 1$

7. Find the equations for the tangent and the normal to the curve given parametrically by

$$x = t^2 + \frac{1}{t}, \qquad y = t^2 - t + 1$$

   at the point where $t = 1$.

8. Find the length of the given curve:

   (a) $x = t - \dfrac{t^2}{2}, \quad y = \dfrac{4}{3}t^{3/2}, \quad t \in [0, 1]$

   (b) $x = \ln t, \quad y = \dfrac{1}{2}\left(t + \dfrac{1}{t}\right), \quad t \in [1, 2]$

   (c) $x = 3t^2, \quad y = 3t^3 - t, \quad t \in [0, 1]$

   (d) $x = 2t^{3/2} + 1, \quad y = 4t - 2, \quad t \in [0, 1]$

9. Find the surface area when the following parametric curve is rotated about the x-axis by 360°:

$$\begin{aligned} x &= t - \frac{t^2}{2} \\ y &= \frac{4}{3}t^{3/2} \end{aligned} \qquad t \in [0, 1]$$