MM104/MM106/BM110
Statistics and Data Presentation

Lecture 6-4:
Confidence Intervals
t distribution

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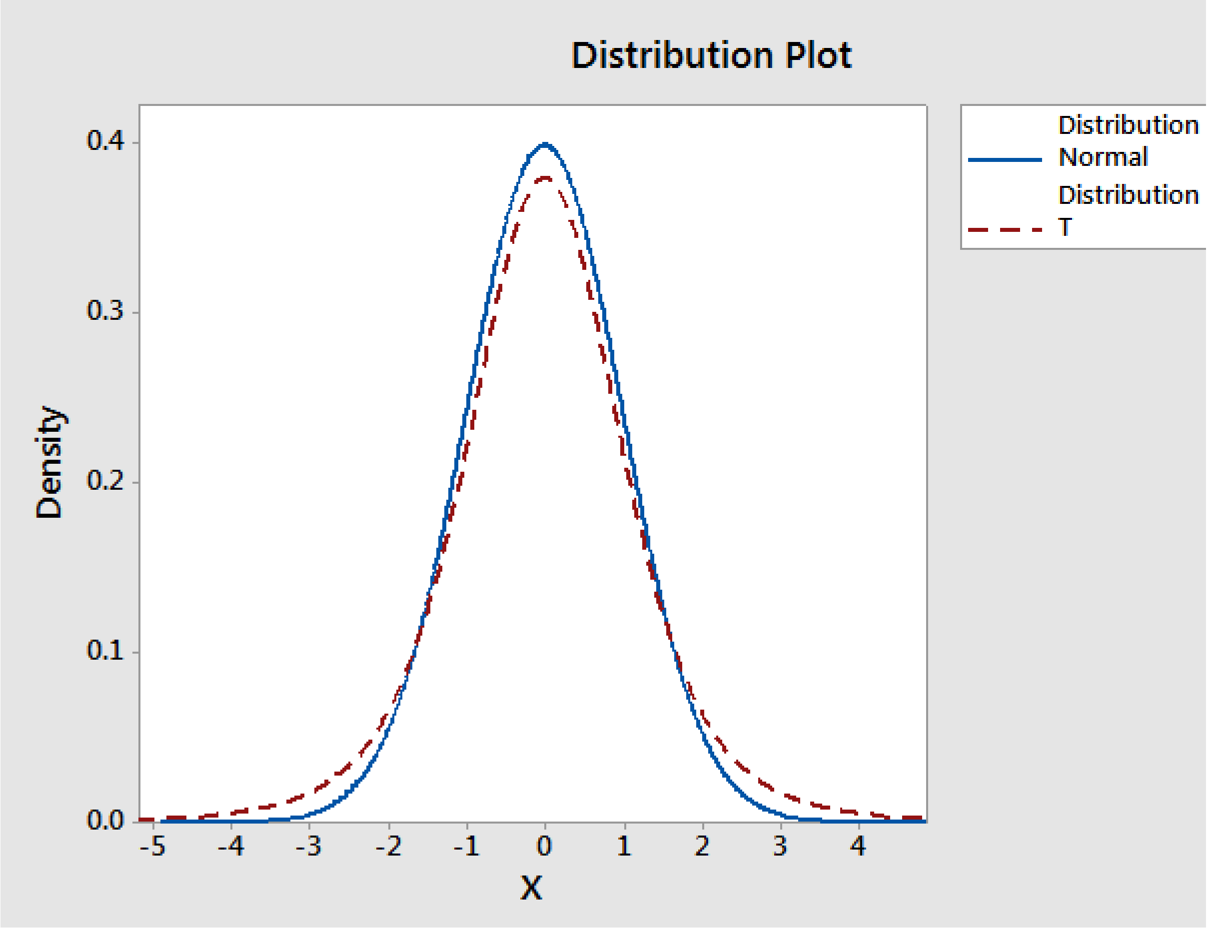
T distribution

- The standard normal distribution has a mean of zero and a standard deviation of 1.
- It is a very useful distribution and has many uses
- With the standard normal distribution
- 95% of the population lies within the range -2 to 2 and
- 99.7% of the population lies in the range -3 to 3
- Sometimes this spread is not enough and you need a symmetric distribution which has a bigger range of values
- The t distribution is the simplest extension of the normal distribution which as a bigger standard deviation and is also symmetric

T-distribution

- Student’s t-distribution was derived in 1908

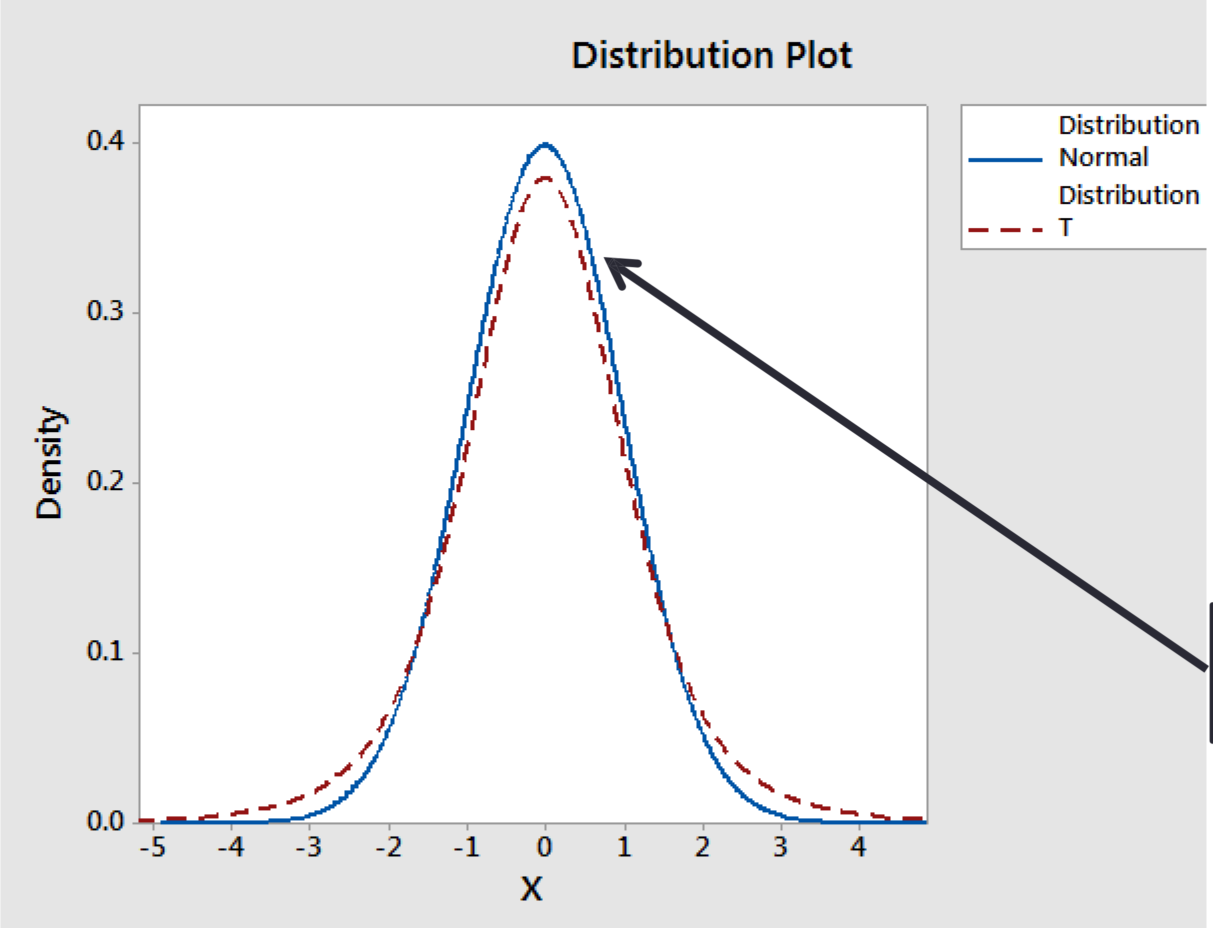


Student

W S Gossett
Worked for
Guinness

T-distribution

- Student's t-distribution was derived in 1908



Student

Both distributions are symmetric about 0

T-distribution

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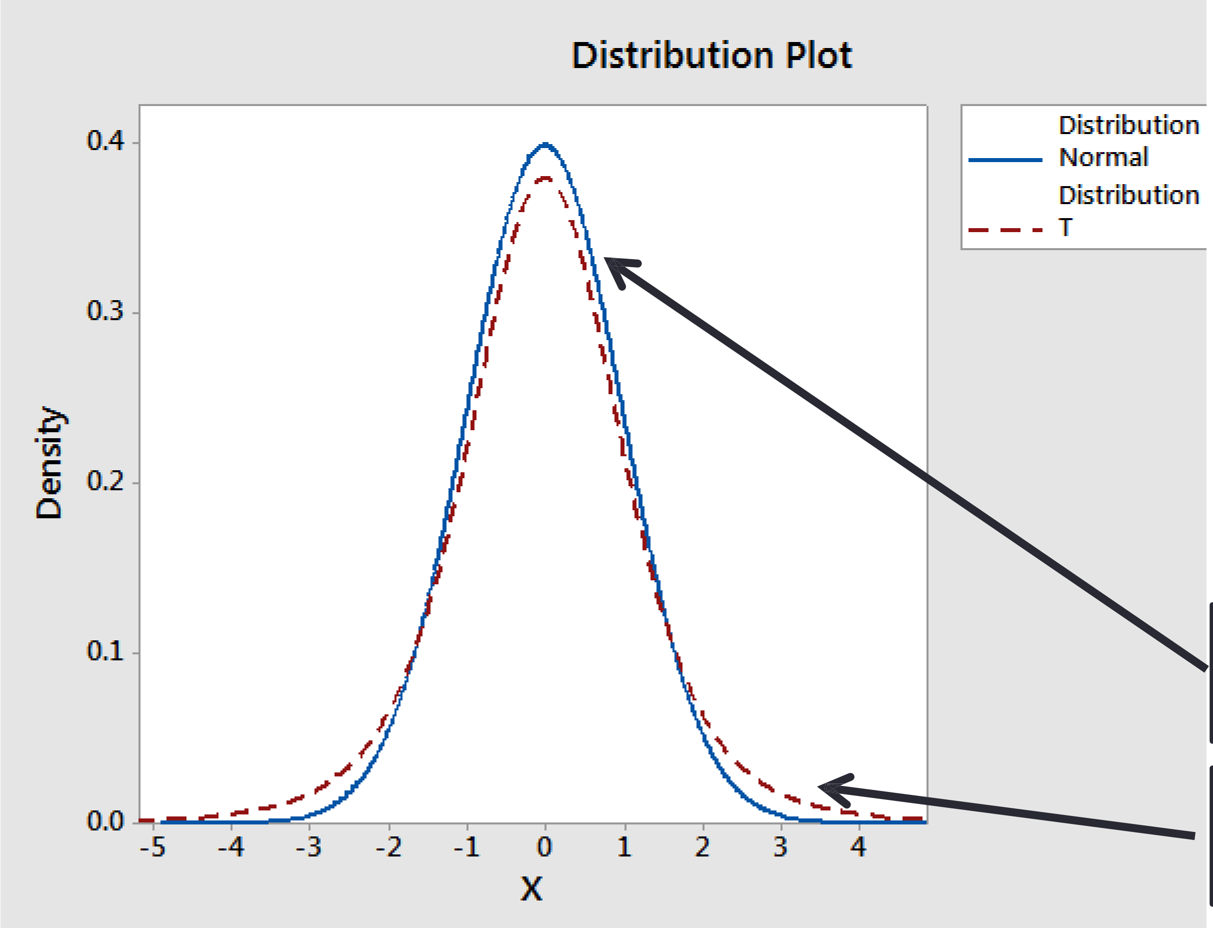


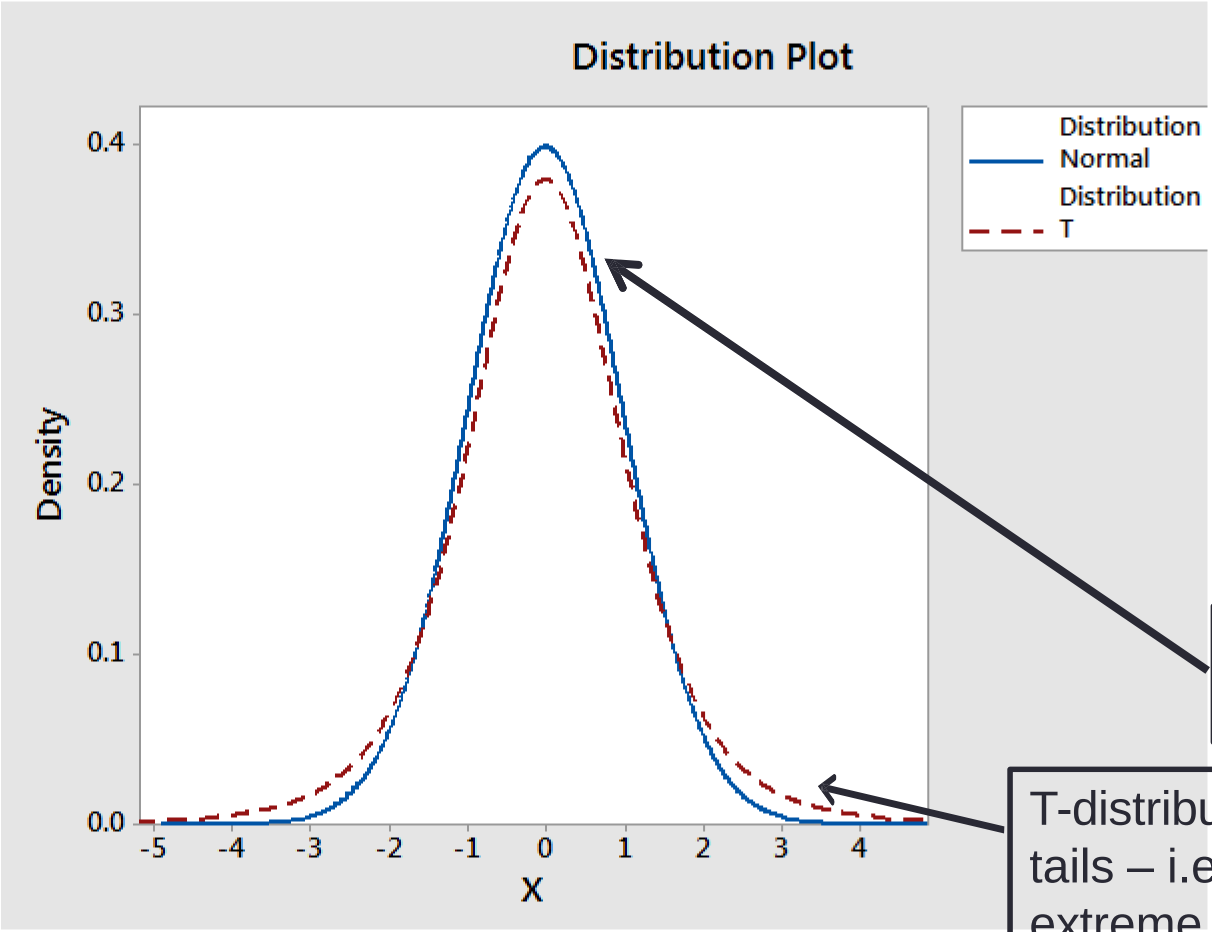
Student

Both distributions are symmetric about 0

T-distribution is heavier in the tails

T-distribution

- Student’s t-distribution was derived in 1908



With a t distribution you can have values further away from zero than with the standard normal distribution

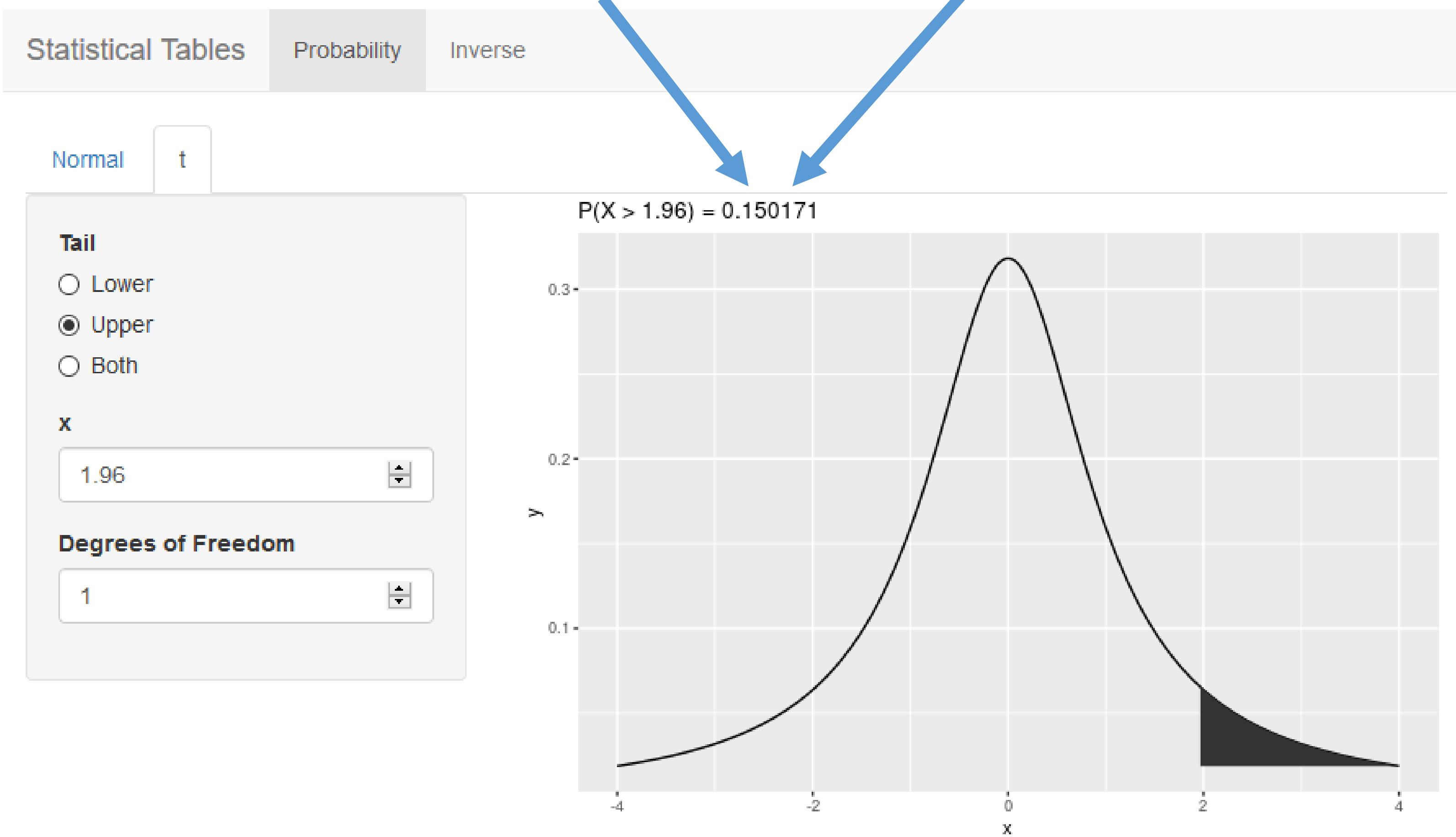
Both distributions are symmetric about 0

T-distribution is heavier in the tails – i.e more probability for extreme values

t-distribution tables

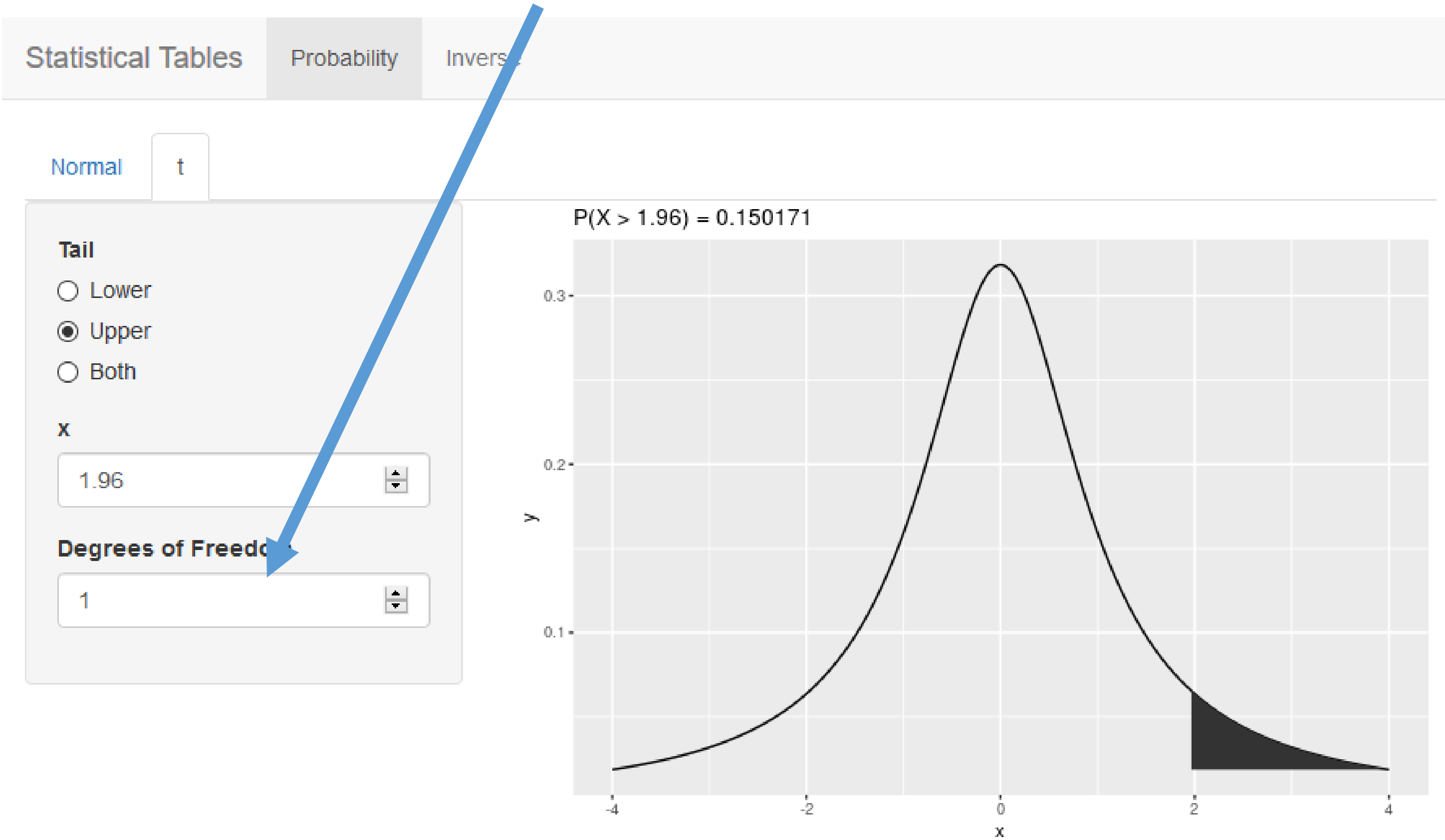
T distribution has more probability in the tails

For a standard normal distribution the probability of being above 1.96 is 0.025



t-distribution - degrees of freedom

The t distribution has an extra parameter compared to the standard normal distribution. This is known as the degrees of freedom (ν) which, in our examples, is an integer greater than or equal to 1. As the degrees of freedom increase the t distribution becomes more like the standard normal distribution



T-distribution: Examples

- **Inverse Calculations:**
  - Find with the value t for the t-distribution with ν degrees of freedom that has the tails described by the following probabilities $P(T < t) = 0.39$, with $\nu = 41$
    - Use Inverse T table

- **Solution:**
  - We are interested in finding t such that $P(T < t) = 0.39$
  - Since we have $<$ in the bracket we will use the low option

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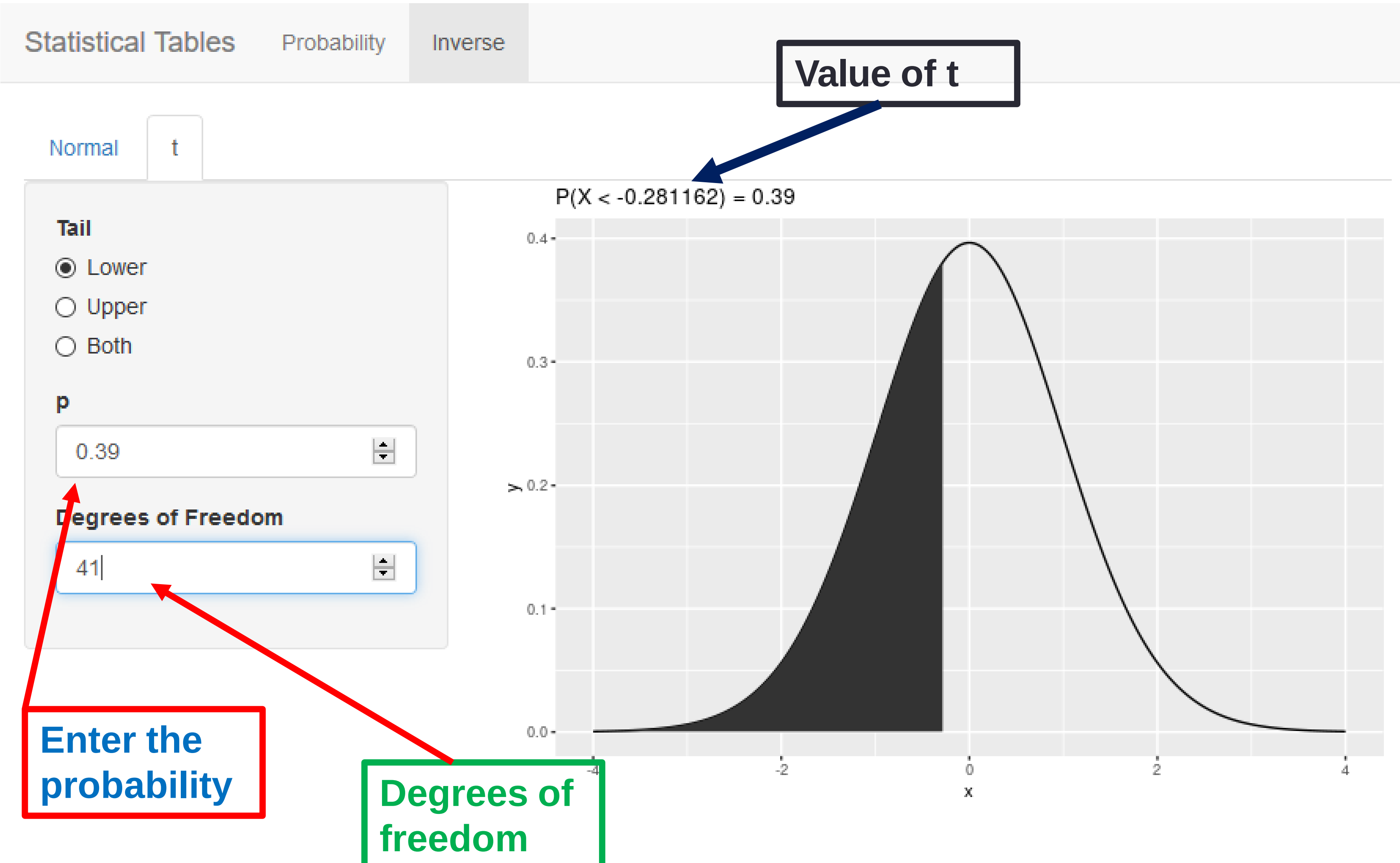
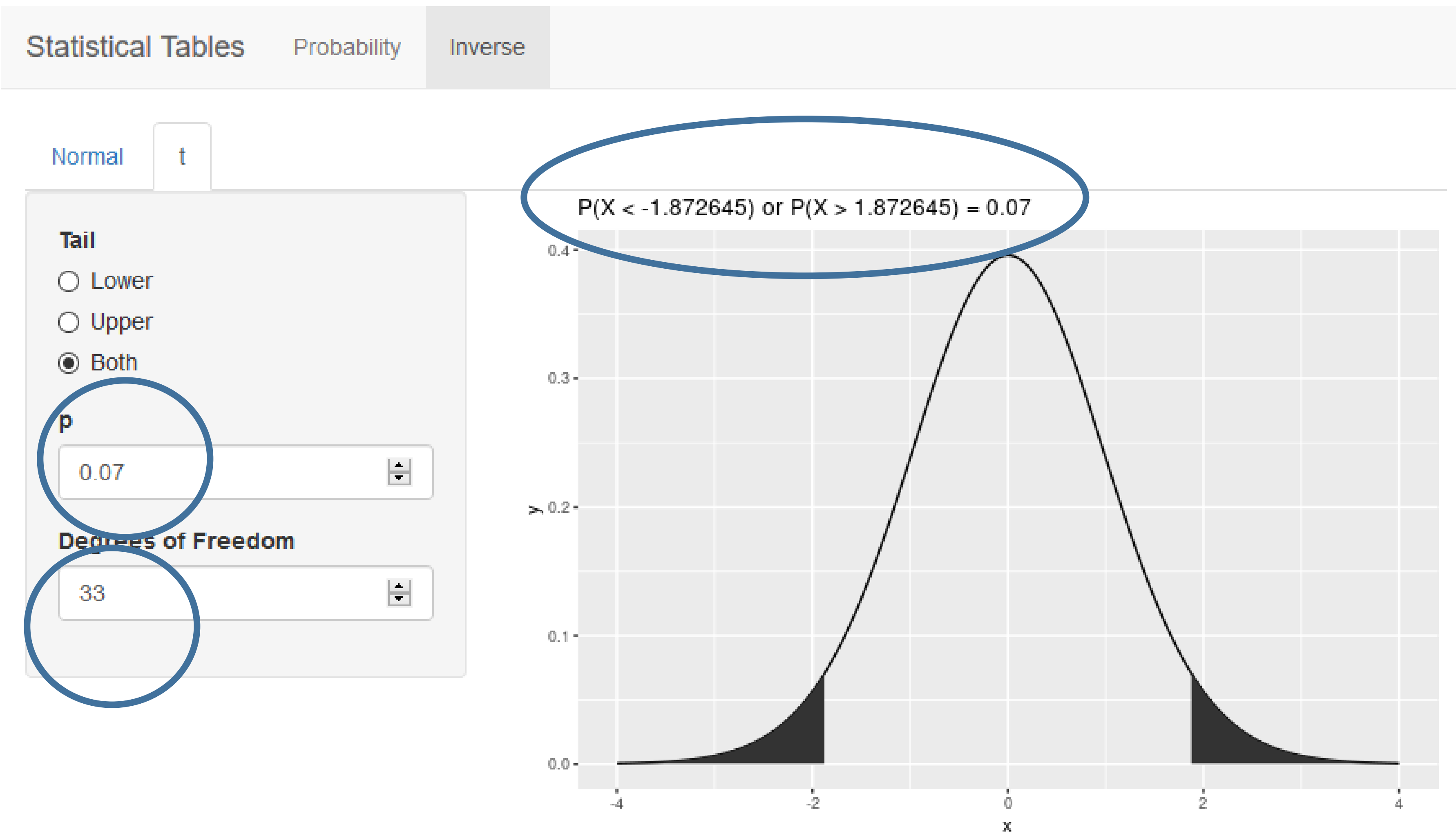
T-distribution: Examples



T-distribution: Examples

- **Inverse Calculations:**
  - Find the value t for the t-distribution with ν degrees of freedom that has the tails described by the following probabilities $P(T < -t \text{ or } T > t) = 0.07,$ with $\nu = 33$

- **Solution:**
  - Since we have $<$ and $>$ in the bracket we will use the both option



Statistical Tables
Probability
Inverse
Normal
t
Tail
Lower
Upper
Both
p
0.07
Degrees of Freedom
33
P(X < -1.872645) or P(X > 1.872645) = 0.07
0.4
0.3
0.2
0.1
0.0
y
-4
-2
0
2
4
x

t-distribution

- Symmetric distribution like the normal
- Mean of zero
- T-distribution has more probability in the tails than the standard normal
- Parameter known as the degrees of freedom
- Probabilities and percentiles calculated like the standard normal distribution – from tables