# AG313 Treasury Management & Derivatives Coursework Summary

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# 1 Derivatives

# 1.1 Options

#### 1.1.1 Option vs. Forward Contracts

- Option: Right to buy/sell, in future, at rate (no future exchange rate saftey)
- Future: Obligation to buy/sell, in future, at rate (future exchange rate safety)

#### 1.1.2 Spot vs. Future/Forward Prices

- Spot Price: immediate delivery  $(S_0, S_T)$
- Future/Forward Price: future delivery price (locked-in today) (F<sub>0</sub>, F<sub>T</sub>)
  - $F_{\rm T} < S_{\rm T} :$  Forward = Spot grossed up @ r
  - Spot expected to be > r growth

#### 1.1.3 Short vs. Long Positions

- Short: Sell shares now  $(S_0 = Spot)$ , buy later  $(S_T = Delivery)$ 
  - Expect fall in share price, in future
  - Futures price  $(\uparrow)$ , loss
  - $Profit = S_0 K$
- Long: Buy shares now
  - Expect rise in share price, in future
  - Futures price (↑), gain
  - $Profit = K S_T$

#### 1.1.4 Call vs. Put Options

- "At The Money":  $S_T = K$
- Call: Agreement to buy at specified time and Strike Price
  - Profit ("In The Money"):  $S_T > K$
  - $\ Profit = N(S_T K) Cost; \ Cost = N(C_0)$
- Put: Agreement to sell at specified time and Strike Price

- Profit ("In The Money"):  $\mathrm{K}>\mathrm{S}_{\mathrm{T}}$
- $\ \mathrm{Profit} = \mathrm{Cost} \mathrm{N}(\mathrm{K} \mathrm{S_T}); \ \mathrm{Cost} = \mathrm{N}(\mathrm{P_0})$
- European Option: exercised only on expiration
- American Option: exercised any time up-to expiration and expiration

#### 1.1.5 Exchange vs. Over-the-Counter

- Exchange: \$60tn valuation; more standardized and regulated
  - Trades Futures contracts
- Over-the-Counter: \$600tn valuation; higher credit risk, higher prices
  - Trades Forward contracts

#### 1.2 Futures Markets

- Regulated by Commodities Futures Trading Commission (CFTC)
- Clearing House: always used in Futures Market to ensure payment method
- Central Clearing Parties: similar job to the above
- Haircut: difference between Market Value and Collateral usage of an asset
- Bilateral Clearing: group agree terms to trade w/ eash-other to minimise risk
- Limit Order: trader identifies worst at which trade can take place

#### 1.2.1 Forward vs. Future

- Futures based on a shorter period than Forwards
- Futures usually don't have final cash settlements

#### 1.2.2 Margin 'Curtain Call' Call

- Broker's demand that investor adds funds to retain minimum value of fund, daily
- Options up-to 9 months must be bought in full; post-9-months margin can be taken
- The seller posts the margin
- Margin acounts adjusted daily for gain/loss
- Reduce systematic risk  $\rightarrow$  ensure funds available  $\rightarrow$  reduce risk of back-out
- Margin Call when: Loss > (Initial Margin Maintenance Margin)
  - $\alpha$  If Short: ea. \$1 rise in price is a \$1 per-unit loss; find = to above
  - $\beta$  Add the per-unit rise to the per-unit price
  - $\gamma$  If Long: ea. \$1 rise in price is a \$1 per-unit gain; find = to above
  - $\delta$  Add the per-unit rise to the per-unit price

#### 1.2.3 Corn Futures Contract

- Initiated by party w/ Short Position; 'Notice of Intention' [to deliver]
- Exchange goes through procedure of choosing party to take Long Position

### 1.2.4 Hedging vs. Speculating

- *Hedging*: e.g. expect volatility, perhaps price rise to take Futures contract to lock in a better price now
- Speculating: e.g. act upon volatility expectation perhaps where there's expected fall in price, take a Short position and buy back for profit
- $\bullet$  Hedgers hold Long, Speculators hold Short: (F  $_{\rm T} > {\rm S}_{\rm T})$

## 1.3 Forward & Futures Prices

- Future Price quoted as no. of US\$ per-unit of foreign currency
- Lenders cannot issue instructions
- Investment Asset: traded but not usually physically usable or tangible
- Consumption Asset: traded and usable for consumption (e.g. Copper)
- Convenience Yield: 0/(+), measures benefit of owning rather than Forward/Future
  - Having real value vs. locked-in F value
  - Investment: 0
  - Consumption: (+)
  - Increase: F as % of S  $\downarrow$ ; more convenie int to own
  - Decrease: F as % of S  $\uparrow$ ; more convenient to F
- Dividend Yield: Div.'s as a % of stock price at t of Div. payment
- Contango:  $F_T > S_T$  abnormally

#### 1.3.1 Shorting With Dividends

- 1. Sell now  $(S_0)$ , buy later  $(S_T)$  (Gain-Per-Share  $= S_0 S_T$ )
- 2. Pay Dividend (Gain-Per-Share =  $S_0 S_T Div.$ )

#### 1.3.2 Spot-to-Forward Price

$$F_T = S_0 e^{rT}$$

$$F_T = (S_0 - Income)e^{rT}$$

$$\label{eq:Income} Income = Y_t e^{-rT} + ... + Y_{t+n} e^{-rT}$$

$$F_T = \mathrm{ER}_0 \mathrm{e}^{(r_1 - r_2)T}$$

# 1.4 Hedging Strategies With Futures

- Futures delivery month should be as close as possible to purchase of asset
- "Tailing the Hedge": corrects for daily settlement
- Hedging Futures leads to predictability

$$Basis = Spot_{At\ Close} - Futures_{At\ Close\ (For\ Maturity)}$$

Price Recieved = Basis + Futures<sub>At Purchase</sub> (ForMaturity)

Optimal Hedge Ratio = 
$$\rho_{A,B} \left( \frac{\sigma_A}{\sigma_B} \right)$$

"Movement in S price to movement in F price"

$$Optimal\ Folios = (\beta_{Current} - \beta_{Desired}) \left( \frac{V_{Folio}}{F_0 F_N} \right)$$

If (+): Short

If (-): Long

$$P_{Total} = w_{Hedged} P_{Hedged} + w_{Not-Hedged} P_{Not-Hedged}$$

Where:

Given 
$$S_0, F_0, S_T, F_T$$

$$P_{Hedged} = S_T - (F_0 F_T)$$

$$P_{\rm Not-Hedged} = S_{\rm T}$$

# 1.5 Option Market Mechanics

- Option Class: All Calls or Puts on a stock
- Option Series: All options on a certain stock type
- LEAPS: Long-Term Equity Anticipation Securities w/ long maturities
- Stock Split

$$- E.g.: N = 100, K = 20, 2-for-1 Split;$$

- Ans.: 
$$N = 2(100) = 200, K = \frac{1}{2}(20) = 10$$

• Stock Dividend

$$- \text{ E.g.: } N = 100, K = 20, \text{ Div. } = 25\%;$$

– Ans.: N = 1.25(100) = 125, K = 
$$\frac{4}{5}$$
(20) = 16

- Cash Dividend
  - No effect
- Option Value = Time Value + Intrinsic Value
  - At-the-Money Time Value = 0 so Option Value = Intrinsic Value
  - Call:  $(S_T K, 0)$
  - Put:  $(K S_T, 0)$

# 1.6 Option Pricing

#### 1.6.1 Binomial Option Tree: European Put

#### Step 1

$$u = e^{\sigma \sqrt{\Delta t}}$$
 
$$d = e^{-\sigma \sqrt{\Delta t}} = \frac{1}{u}$$

 $p=\frac{e^{r\Delta t}-d}{u-d}=Risk\ Neutral\ Probability\ of\ Up\ Movement}$   $(1-p)=Risk\ Neautral\ Probability\ of\ Down\ Movement}$ 

#### Step 2

 $S_{u/d} = Value of Stock Upon Increase/Decrease$ 

$$\mathrm{S}_{\mathrm{u}}=\mathrm{Pu}$$

$$S_d = Pd$$

$$S_{u,u} = Pu^2 \\$$

$$S_{u,d} = Pud$$

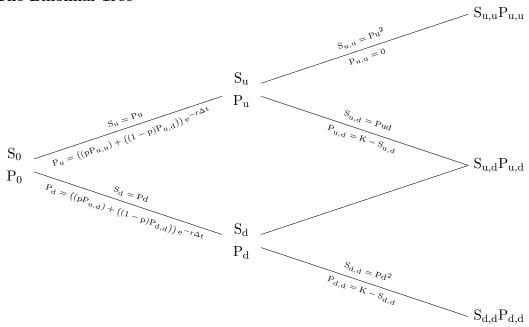
$$S_{d,d} = Pd^2$$

#### Step 3

 $P_{u/d} = Value of Option Upon Increase/Decrease$ 

$$\begin{split} P_{u,u} &= 0 \\ P_{u,d} &= K - S_{u,d} \\ P_{d,d} &= K - S_{d,d} \\ P_u &= \left( (pP_{u,u}) + ((1-p)P_{u,d}) \right) e^{-r\Delta t} \\ P_d &= \left( (pP_{u,d}) + ((1-p)P_{d,d}) \right) e^{-r\Delta t} \\ P_0 &= \left( (pP_u) + ((1-p)P_d) \right) e^{-r\Delta t} \end{split}$$

#### The Binomial Tree



# 1.6.2 Binomial Option Tree: Converting to American Put

$$P_{d_A} = \max\{K - S_d, P_d\}$$

 $P_{\rm d_A} = {\rm Larger~Outcome~of~Either~(K-S_d)~or~(P_d)}$ 

$$P_{u_A} = \operatorname{Remains} \, \operatorname{Same}$$

$$P_{0_A} = ((pP_{u_A}) + ((1-p)P_{d_A}))\,e^{-r\Delta t}$$

Reform Binomial Tree

# 1.7 Stock Options

- Stock Price ( $\uparrow$ ): Call ( $\uparrow$ ); Put ( $\downarrow$ )
- Strike Price (↑): Call (↓); Put (↑)
- Volatility (↑): Call Payoff (↑); Put Payoff (↑)
- Dividends ( $\uparrow$ ): Stock Price ( $\downarrow$ ); Call ( $\downarrow$ ); Put ( $\uparrow$ )
- Interest Rate (↑): Call (↑); Put (↓)
- Time-Maturity ( $\uparrow$ ): European Options ( $\uparrow / \downarrow$ )

#### 1.7.1 Call Lower-Bound

$$S_0 - \mathrm{Ke}^{-rT}$$

#### 1.7.2 Put Lower-Bound

$$\mathrm{Ke^{-rT}} - \mathrm{S}_0$$

#### 1.7.3 Put-Call Parity w/o Dividend (or 0 Interest)

$$C_0 + Ke^{-rT} = P_0 + S_0$$

$$C_0 + K = P_0 + S_0$$

#### 1.7.4 Put-Call Parity w/ Divided

$$C_0 + Ke^{-rT} = P_0 + (S_0 - Div.)$$

#### 1.7.5 Black & Scholes Models

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$
 
$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\begin{split} C_0 &= S(N(d_1)) - Ke^{-rT}(N(d_2)) \\ C_0 &= Se^{-yT}(N(d_1)) - Ke^{-rT}(N(d_2)) \\ P_0 &= K(1-N(d_1)) - Se^{-rT}(1-N(d_2)) \end{split}$$

# 2 Treasury Management

# 2.1 Foreign Exchange Market

Domestic in terms of foreign; foreign in terms of domestic

$$Spread = \frac{Ask - Bid}{Ask}$$

Direct Quotation = 
$$\mathcal{L}/\$ = \frac{1}{\$/\mathcal{L}}$$

Indirect Quotation = 
$$\$/\pounds = \frac{1}{\pounds/\$}$$

$$\text{Cross Rate} = \$/\pounds = \text{EUR}/\pounds\frac{1}{\text{EUR}/\$}$$

# 2.2 Interest Parity Relationships

#### 2.2.1 Interest Rate Arbitrage

$$A_n = \left(\frac{A_h}{S}\right)(1+i_f)(S(1+p))$$
 
$$S(1+p) = F$$

 $A_{h,n} = Home/New Home Currency$ 

 $i_{h,f} = \mathrm{Home}/\mathrm{Foreign}\ \mathrm{Currency}$ 

 $S = Spot Exchange Rate = N of \mathcal{L} Per Unit of $$ 

F = Forward (Locked) Exchange Rate = N of £ Per Unit of \$

 $p = Forward Premium = Amount By Which F is \uparrow / \downarrow Than S$ 

Convert To 
$$\$: \left(\frac{A_h}{S}\right)$$

End of Period  $\Priripal \ \& \ Interest: \ \left(\frac{A_h}{S}\right)(1+i_f)$ 

$$\$$
 Principal & Interest Back to  $\pounds:\ \left(\frac{A_h}{S}\right)(1+i_f)F$ 

#### 2.2.2 Interest Rate No-Arbitrage

$$A_h(1+i_h) = A_h(1+i_f)(1+p)$$

 $A_h(1+i_h) = Investing w / Home Interest = Investing w / Foreign Interest w / p$ 

$$\therefore p = \frac{(1+i_h)}{(1+i_f)} - 1 \therefore p \approx i_h - i_f$$

#### 2.2.3 Absolute PPP

$$S_f^d = \frac{P_s^d}{P_s^f}$$

$$As: P_s^d = S_f^d P_s^f$$

#### 2.2.4 Relative PPP w/ Inflation

$$P_{h}(1 + \pi_{h})$$

$$P_f(1+\pi_f)$$

If  $\pi_h > \pi_f$ : PP is greater when buying foreign goods  $\rightarrow$  foreign cheaper If  $\pi_h < \pi_f$ : PP is greater when buying domestic goods  $\rightarrow$  domestic cheaper

Adjust for Change in Currency:

$$P_f(1+\pi_f)(1+e_f)$$

 $e_f = \%$  Change Per Unit of Foreign Currency In Domestic Currency

Hence:

$$P_h(1+\pi_h) = P_f(1+\pi_f)(1+e_f)$$

$$e_f = \frac{P_h(1+\pi_h)}{P_f(1+\pi_f)} - 1 = \frac{(1+\pi_h)}{(1+\pi_f)}$$

Given 
$$P_h = P_f$$
:

If  $\pi_h > \pi_f$ :  $e_f$  (+): foreign should appreciate; domestic depreciate If  $\pi_h < \pi_f$ :  $e_f$  (-): foreign should depreciate; domestic appreciate

For Relatively Low Inflation:

$$e_f = \frac{(1 + \pi_h)}{(1 + \pi_f)} - 1 \approx (\pi_h - \pi_f)$$

# 2.3 Exchange Exposure

#### 2.3.1 Variance of Two-Asset Folio

$$\sigma_{x,y}^2 = \sigma_x^2 + \sigma_y^2 + 2(cov_{x,y})$$

Hence:

$$p = \{x, y\}$$

$$cov_{x,y} = \rho_{x,y}\sigma_x\sigma_y$$

$$\sigma_{\mathrm{p}}^2 = \sigma_{\mathrm{x}}^2 + \sigma_{\mathrm{y}}^2 + 2(\rho_{\mathrm{x,y}}\sigma_{\mathrm{x}}\sigma_{\mathrm{y}})$$

#### 2.3.2 Variance of Three-Asset Folio

$$\sigma_{p}^2 = \sigma_{x}^2 + \sigma_{y}^2 + \sigma_{z}^2 + 2(\rho_{x,y}\sigma_{x}\sigma_{y}) + 2(\rho_{x,z}\sigma_{x}\sigma_{z}) + 2(\rho_{y,z}\sigma_{y}\sigma_{z})$$

#### 2.3.3 Economic Exposure

$$V_{\rm MNC} = \sum \frac{\sum (E(CF_{\rm j,t})E(ER_{\rm j,t}))}{(1+k)^{\rm t}} \label{eq:VMNC}$$

Where:

 $E(\mathrm{CF}_{j,t}) = \mathrm{Expected}~\mathrm{CF}$  in Currency j Recieved At End of Period t

 $\mathrm{E}(\mathrm{ER})_{j,t} = \mathrm{Expeced} \ \mathrm{ER}$  of Currency j At End of Peiod t

k = Weighted Average Cost of Capital (WACC) of MNC

# 2.4 Value of A Multinational Corporation

- Value of Parent Company (p, perhaps in USD)
- Value of Subsidiary 1 (s1, perhasp in EUR)
- Value of Subsidiary 2 (s2, perhaps in GBP)

#### 2.4.1 Basic Values

$$V_t = \frac{E(C_{t+1})}{(1+r)^{t+1}} + \frac{E(C_{t+2})}{(1+r)^{t+2}} + \frac{E(C_{t+3})}{(1+r)^{t+3}}$$

Value of Cash Flows in USD Functional Currency:

$$V_{t,p} = \frac{E(C_{t+1,\$})}{(1+r_{\$})^{t+1}} + \frac{E(C_{t+2,\$})}{(1+r_{\$})^{t+2}} + ... + \frac{E(C_{t+n,\$})}{(1+r_{\$})^{t+n}}$$

Value of Cash Flows in EUR:

$$V_{t,s1} = \frac{E(C_{t+1,EUR})}{(1+r_{EUR})^{t+1}} + \frac{E(C_{t+2,EUR})}{(1+r_{EUR})^{t+2}} + ... + \frac{E(C_{t+n,EUR})}{(1+r_{EUR})^{t+n}}$$

Value of Cash Flows in GBP:

$$V_{t,s2} = \frac{E(C_{t+1,\pounds})}{(1+r_{\pounds})^{t+1}} + \frac{E(C_{t+2,\pounds})}{(1+r_{\pounds})^{t+2}} + ... + \frac{E(C_{t+n,\pounds})}{(1+r_{\pounds})^{t+n}}$$

#### 2.4.2 Value Exchnage Conversion

Value of Cash Flows in USD Functional Currency:

$$V_{t,p} = \frac{E\left(C_{t+1,\$}\left(\frac{\$}{\$}\right)_{t+1}\right)}{(1+r_{\$})^{t+1}} + \frac{E\left(C_{t+2,\$}\left(\frac{\$}{\$}\right)_{t+2}\right)}{(1+r_{\$})^{t+2}} + ... + \frac{E\left(C_{t+n,\$}\left(\frac{\$}{\$}\right)_{t+3}\right)}{(1+r_{\$})^{t+n}}$$

Value of Cash Flows in USD Converted from EUR:

$$\begin{split} V_{t,p} &= \frac{E\left(C_{t+1,EUR}\left(\frac{\$}{EUR}\right)_{t+1}\right)}{(1+r_\$)^{t+1}} + \frac{E\left(C_{t+2,EUR}\left(\frac{\$}{EUR}\right)_{t+2}\right)}{(1+r_\$)^{t+2}} + ... \\ &+ \frac{E\left(C_{t+n,EUR}\left(\frac{\$}{EUR}\right)_{t+3}\right)}{(1+r_\$)^{t+n}} \end{split}$$

Value of Cash Flows in USD Converted from GBP:

$$V_{t,p} = \frac{E\left(C_{t+1,\pounds}\left(\frac{\$}{\pounds}\right)_{t+1}\right)}{(1+r_{\$})^{t+1}} + \frac{E\left(C_{t+2,\pounds}\left(\frac{\$}{\pounds}\right)_{t+2}\right)}{(1+r_{\$})^{t+2}} + ... + \frac{E\left(C_{t+n,\pounds}\left(\frac{\$}{\pounds}\right)_{t+3}\right)}{(1+r_{\$})^{t+n}}$$

#### 2.4.3 Value of Each Domestic/Foreign Operation

Hence Total Value of Parent Corporation (p):

$$V_{t,p} = \sum_{i=1}^{n} \frac{E(C_{t+i,\$})}{(1+r_{\$})^{t+i}}$$

Hence Total Value of European Subsidiary (s1):

$$V_{t,s1} = \sum_{i=1}^{n} \frac{E\left(C_{t+i,EUR}\left(\frac{\$}{EUR}\right)_{t+i}\right)}{(1+r_{\$})^{t+i}}$$

Hence Total Value of British Subsidairy (s2):

$$V_{t,s2} = \sum_{i=1}^{n} \frac{E\left(C_{t+i,\pounds}\left(\frac{\$}{\pounds}\right)_{t+i}\right)}{(1+r_{\$})^{t+i}}$$

#### 2.4.4 Total Value of Multinational Corporation

Hence Total Value of Multinational Corporation:

$$V_p = V_{s1} + V_{s2}$$

Hence For 3 Currency Example Over n Periods (i):

$$V_{t,MNC} = \sum_{i=1}^{n} \frac{E(C_{t+i,\$})}{(1+r_{\$})^{t+i}} + \sum_{i=1}^{n} \frac{E\left(C_{t+i,EUR}\left(\frac{\$}{EUR}\right)_{t+i}\right)}{(1+r_{\$})^{t+i}} + \sum_{i=1}^{n} \frac{E\left(C_{t+i,\pounds}\left(\frac{\$}{\pounds}\right)_{t+i}\right)}{(1+r_{\$})^{t+i}}$$

Hence Generalised for 3 Unknown Currencies Over n Periods (i):

$$V_{t,MNC} = \sum_{i=1}^{n} \frac{E\left(C_{t+i,1}(ER_1)_{t+i}\right)}{(1+r_\$)^{t+i}} + \sum_{i=1}^{n} \frac{E\left(C_{t+i,2}(ER_2)_{t+i}\right)}{(1+r_\$)^{t+i}} + \sum_{i=1}^{n} \frac{E\left(C_{t+i,3}(ER_3)_{t+i}\right)}{(1+r_\$)^{t+i}}$$

Hence Generalised for n Unkown Currencies (j) Over n Periods (i):

$$V_{t,MNC} = \sum_{i=1}^n \left( \sum_{i=1}^n \frac{E\left(C_{t+i,j}(ER_j)_{t+i}\right)}{(1+r_\S)^{t+i}} \right)$$

## 2.4.5 How Can The Value Change?

If  $C_{t+i,j} < E(C_{t+i})_j$ :  $V_{t,MNC}$  lower than expected country business risk

If  $r_\$ > Er_\$ \colon \, V_{t,MNC}$  lower than expected country policy risk

If  $(ER_{\$})_{t+i} < (ER_{\pounds})_{t+i}$ :  $V_{t,MNC}$  lower than expected foreign exchange risk (where \$ is domestic)

## 2.5 Interest Rate Risk

- $\frac{1}{100}$  of a %pt. is a 'Basis Point'
- Must convert period to days

$$R = Simple Interest Rate$$

$$r = \frac{R}{m} = \text{Periodic Interest Rate}$$

"m periods per n"

$$(1+r)^{mn} - 1 = Compound Interest Rate$$

$$EAR = (1+r)^{\frac{year}{days}} - 1$$

#### 2.5.1 Duration

$$\Delta B = -DB\Delta y$$

$$B = \sum \frac{CF_t}{(1+v)^t}$$

$$D = \sum t \left( \frac{\frac{CF_t}{(1+y)^t}}{P} \right) = \sum tw_t$$

$$y = Yield on Bond$$

$$P = Bond Price$$

$$D_{Zero-Coupon} = Maturity = T$$

Constant Maturity : 
$$D(\uparrow)$$
,  $CF(\downarrow)$ 

Constant Coupon : 
$$D(\uparrow)$$
,  $T(\uparrow)$ 

Constant All Other: 
$$D(\uparrow)$$
,  $y(\downarrow)$ 

#### 2.5.2 Forward Rate Agreements

Payoff = (Notional Amount) (LIBOR – Agreed Upon Rate) 
$$\left(\frac{m}{360}\right)$$

Payoff = (Notional Amount) 
$$\left( (\text{LIBOR}) - \text{Agreed Upon Rate} \right) \frac{\left(\frac{\text{m}}{360}\right)}{\left(1 + \text{LIBOR}\right)\left(\frac{\text{m}}{360}\right)} \right)$$

#### 2.5.3 Interest Rate Option

$$\begin{split} \operatorname{Payoff}_{\operatorname{Call}} &= \left( \operatorname{Notional\ Amount} \right) \left( \operatorname{Max} \left( 0, \operatorname{LIBOR} - X \right) \left( \frac{m}{360} \right) \right) \\ & \quad \operatorname{If\ LIBOR} > X : \ \operatorname{Exercise} \\ & \quad \operatorname{Payoff}(\uparrow), \ \operatorname{LIBOR}(\uparrow) \end{split}$$

Protection Against Rising i (e.g. future borrowing)

$$\begin{split} \operatorname{Payoff}_{\operatorname{Put}} &= (\operatorname{Notional\ Amount}) \left( \operatorname{Max} \left( 0, X - \operatorname{LIBOR} \right) \left( \frac{m}{360} \right) \right) \\ & \quad \operatorname{If\ LIBOR} < X : \ \operatorname{Exercise} \\ & \quad \operatorname{Payoff}(\uparrow), \ \operatorname{LIBOR}(\downarrow) \end{split}$$

Protection Against Falling i (e.g. future investing)