MM104 / MM107
Statistics and Data Presentation Semester 2

Project 5: Statistical Process Control

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Lecture Overview

In this lecture we will

- introduce statistical process control (SPC).
- introduce control charts and how to generate them in Minitab.
- learn about common causes and special causes.
- discuss statistical processes being in control and being out of control.
- learn how to calculate lower and upper control limits, and lower and upper warning limits.

Statistical Process Control

Statistical Process Control (SPC) is an area of statistics to do with the quality control of processes.

SPC uses data analysis to monitor and manage quality in an ongoing production process and managing variability (e.g. drug potency in pharmaceutical products, measuring blood sugar levels if you have diabetes, etc.)

Statistical Process Control

Statistics
Collection, presentation and interpretation of data (e.g. random sampling of outputs from process over time, estimating mean of outputs at each time point, visualising and analysing data).

Processes
Converting inputs to outputs (e.g. manufacturing of products).

Control
Management (e.g. organisation and coordination of activities in order to achieve defined objectives, control and management of variability of outputs, etc).

Control Charts

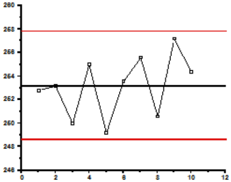
Control charts are the basis for monitoring the performance of many ongoing processes, we will refer to them as x**-bar charts**.

Control charts

- are based on very simple statistical principles and designed for easy implementation.
- were developed in late 1920s and Walter Shewhart first published them in 1931.
- have undergone 80 years of evolution and are probably the most widely used SPC tool.

x-bar charts

- are a type of line graph.
- track trends.
- observe fluctuations between statistically derived limits.
- provide a continuous picture of process.



x-bar charts- Where are they used ?

Every manufactured process has features used to rate its quality, e.g. a 25g packet of crisps should weight at least 25g.

X-bar charts are used

- to identify any variation on the production line.
- to provide early warning signs of bad production.
- to determine when to leave the process alone.
- to assess any changes that should be made to the process.

Causes of Variation - Common Causes

The factors that affect the average quality of the items being produced, or the variance in their quality, are often divided into two main categories.

Common Causes

- affect all items produced.
- have a small cumulative effect.
- are hard to remove/reduce.

Examples of Common Causes

Common causes are mostly caused by poor quality machinery and atmospheric conditions e.g.

- humidity.
- temperature.
- machine wear and tear.
- compression force.

Causes of Variation - Special Causes

Special Causes

- affect the process at some times but not other times.
- are rare.
- have a large impact.
- are easy to correct for.

Examples of Special Causes

- change of supplier.
- machine break-down.
- wrong compression force.
- incorrect assembly.

Common and Special Causes

Process	Common Causes	Special Causes
Baking a cake	The oven's thermostat allows the temperature to drift up and down slightly.	Changing the oven's temperature or opening the oven door during baking can cause the temperature to fluctuate needlessly.
Recording customer contact information	An experienced operator makes an occasional error.	An untrained operator new to the job makes numerous data-entry errors.
Injection moulding of plastic toys	Slight variations in the plastic from a supplier result in minor variations in product strength from batch to batch.	Changing to a less reliable plastic supplier leads to an immediate shift in the strength and consistency of your final product.

Being in Control

A production process that is influenced only by common causes is said to be **in control**. It is stable and predictable, which is an advantage to the producer.

A production process that is influenced by special causes is said to be **out of control**. It is unstable and unpredictable, and it is not possible to predict the mean or variance in the quality of items being produced.

Improvement

Improvement of production quality requires the detection and elimination of special causes

Control Limits

Lower Control Limits (LCL) and **Upper Control Limits (UCL)**.

It is very unlikely that the value of the sample mean ($\bar{x}$) will fall below the LCL or the above the UCL.

$$LCL = \mu - \left(\frac{3\sigma}{\sqrt{n}}\right), \qquad UCL = \mu + \left(\frac{3\sigma}{\sqrt{n}}\right),$$

where μ is the population mean (usually historically known), σ is the population standard deviation and n is the number of observations per time point.

Warning Limits (2σ limits)

Lower Warning Limits (LWL) and **Upper Warning Limits (UWL)**.

$$LWL = \mu - \left(\frac{2\sigma}{\sqrt{n}}\right), \qquad UWL = \mu + \left(\frac{2\sigma}{\sqrt{n}}\right),$$

where μ is the population mean (usually historically known), σ is the population standard deviation and n is the number of observations per time point.

Consider an Example

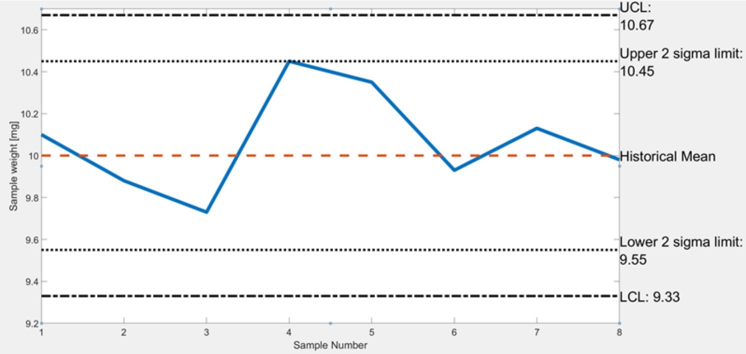
One strategy adopted by many manufacturers to detect special causes involved sampling a few items off the production line at regular intervals. Often, the quantity of these items is measured by a quality variable (this is a continuous measurement, such as weight).

Example

An important component in the manufacture of laptop screens is liquid crystals. Historical records show that the liquid crystal content follows a normal distribution with mean 10 mg, and variance 0.2 mg, when the process is in control. For a period of 8 hours, four laptops screens were sampled from the production line each hour and the liquid crystal content was determined.

The means were: 10.10, 9.88, 9.73, 10.45, 10.35, 9.93, 10.13 and 9.98.
Plot an x-bar chart and determine if the process is in control.

Consider an Example cont...



The process is in control, however does reach an upper warning limit at the 4th sample.

How did we get the results ?

From the question, $\mu = 10$, $\sigma = \sqrt{0.2}$ (we are told in the question the variance is 0.2) and n is 4.

$$\text{UCL} = \mu + \left(\frac{3\sigma}{\sqrt{n}}\right) = 10 + \left(\frac{3\sqrt{0.2}}{\sqrt{4}}\right) = 10.67$$

$$\text{UWL} = \mu + \left(\frac{2\sigma}{\sqrt{n}}\right) = 10 + \left(\frac{2\sqrt{0.2}}{\sqrt{4}}\right) = 10.45$$

$$\text{LCL} = \mu - \left(\frac{3\sigma}{\sqrt{n}}\right) = 10 - \left(\frac{3\sqrt{0.2}}{\sqrt{4}}\right) = 9.33$$

$$\text{LWL} = \mu - \left(\frac{2\sigma}{\sqrt{n}}\right) = 10 - \left(\frac{2\sqrt{0.2}}{\sqrt{4}}\right) = 9.55$$

Sigma Relationships

Remember, standard deviation is a measure of spread. As the standard deviation increases the data gets more spread out and has greater variability.

If a process is in controls you expect

1. 33% of observations outside the 1σ limits – 16.5% each side.
2. 5% of observations outside the 2σ limits – 2.5% each side.
3. 0.27% of observations outside the 3σ limits – 0.13% each side.

Assumptions of an SPC

SPC is similar to other statistical techniques, in that it has assumptions. The assumptions of an SPC are

1. The data are independent of each other.
2. Real valued data are approximately normally distributed and counting data may be approximated by the normal distribution.
3. There is no trend in the historical data used to estimate the mean and variance.

Recall that you can use the Anderson Darling test to test for normality. The Anderson Darling has the following hypotheses

- H_0: Data are normally distributed.
- H_1: Data are not normally distributed.

SPC in Minitab

Firstly, let's make the control chart.

Stat > Control Chart > Variable charts for subgroups > X-bar S. The subgroup size is the number of data observations that you have at each observation i.e. 4 in our example.

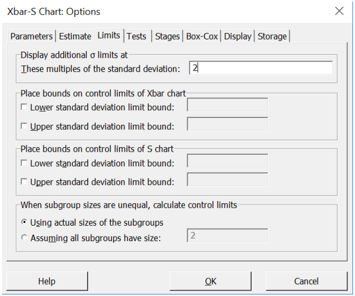
Historical data ?

Click *Xbar-S Options* and the *Parameters* tab and type in the mean i.e. 10 and standard deviation $\sqrt{0.2} = 0.4472$

SPC in Minitab ...

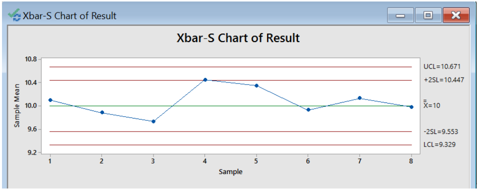
Minitab will not give you the warning limits unless you ask for them - it will only give the upper and lower control limits. To add these in

Click *Xbar-S Options* and the *Limits* and type a 2 in the top box.



What do the plots mean ?

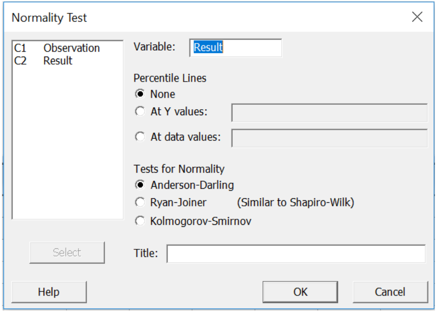
You will get a plot of the mean and the standard deviation. The standard deviation plot has been omitted from the slides (due to the way the data was generated). You should comment on both the mean and standard deviation control plots.



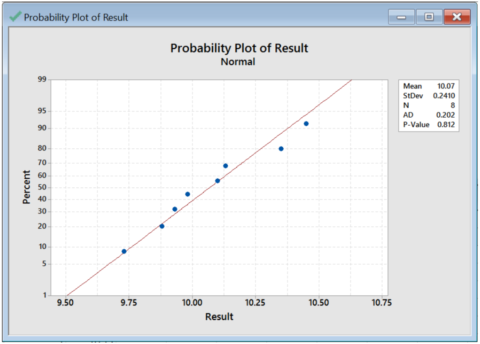
SPC in Minitab...

We will now check the assumption of SPC, starting with normality.

Stat > Basic Statistics > Normality Test



SPC in Minitab cont...



Since p is greater than 0.05, we fail to reject our null hypothesis and conclude that the observations are normally distributed. **Make sure you comment on the other assumptions.**

Assumption Checking - Assumption 1

The first assumption of an SPC is **the data are independent of each other.**

To check this assumption you are looking for trends in your x-bar-s chart. Generally we say that 3-4 points makes a trend.

If you find that there is a trend in your control charts it indicates that the data is not independent as the output from previous production may impact the output of subsequent production.

Assumption Checking - Assumption 2

Go back a few slides to see how to check the assumption **real valued data are approximately normally distributed and counting data may be approximated by the normal distribution.**

Assumption Checking - Assumption 3

The third assumption of an SPC is **there is no trend in the historical data used to estimate the mean and variance.**

It is unlikely that you will be able to check this assumption, as you probably will not have any data outside of your sample.

As a consequence of this, you would need to say something about this in your limitations.

Out of Control

Minitab will tell you if any of your data points are out of control from the plot below we can see that the last observation lies below the lower control limit (illustrated by the red square). You would them investigate the reasons to see if a cause can be identified for the process going out of control.