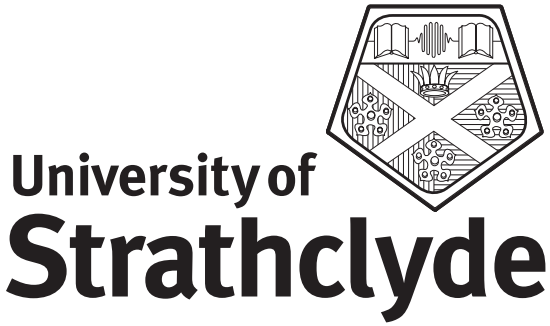

MM104/ MM106/ BM110

Topic 4: Normal Distribution

Introduction to the Inverse Normal Distribution

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Recap

In the previous set of slides we focused on finding probabilities from a normal distribution. In this lesson we are going to make use of the inverse normal distribution. This will be cases where we know the probability (area under the curve) but need to find out what value of the x axis gives that result.

Normal Distribution Calculations - Example 1

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X < x) = 0.6085$.

We need to find the value x, such that the area under the probability curve to the left of x is 0.6085. We simply use our statistical tables and select Inverse.

Statistical Tables | Probability | Inverse

Normal | t

Tail

- (●) Lower
- ( ) Upper
- ( ) Both

p

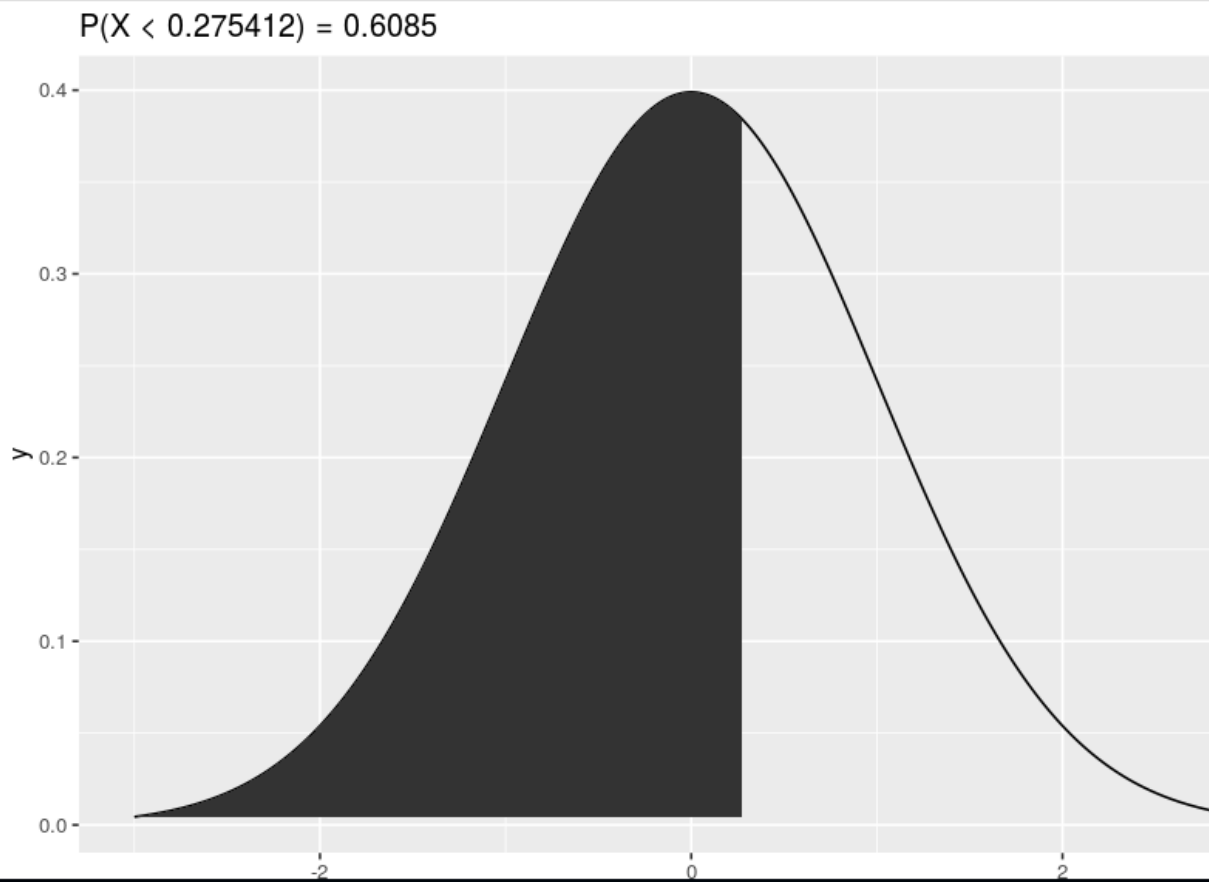
0.6085

Mean

0

sd

1



Normal Distribution Calculations - Example 1

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(X < x) = 0.6085$.

Therefore, $\mathrm{P}(X < 0.275412) = 0.6085$, so $x = \underline{0.275412}$.

Make sure you write all of the decimal places as your answer may be marked incorrectly if you are not accurate enough.

Normal Distribution Calculations - Example 2

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(-0.3 < X < x) = 0.201$.

We need to make use of the result we learned in the previous class.

$$\mathrm{P}(a < X < b) = \mathrm{P}(X < b) - \mathrm{P}(X < a)$$

Normal Distribution Calculations - Example 2

Therefore,

$$\mathrm{P}(-0.3 < X < x) = 0.201$$

$$\Rightarrow \mathrm{P}(X < x) - \mathrm{P}(X < -0.3) = 0.201$$

$$\mathrm{P}(X < -0.3) = 0.382089$$

(Use probability setting and lower tail as we have a $<$)

$$\Rightarrow \mathrm{P}(X < x) - 0.382089 = 0.201$$

$$\Rightarrow \mathrm{P}(X < x) = 0.201 + 0.382089$$

$$\Rightarrow \mathrm{P}(X < x) = 0.583089$$

(Use inverse setting and lower tail as we have a $<$)

$$\Rightarrow \mathrm{P}(X < 0.209802) = 0.583089$$

$$\underline{\underline{x = 0.209802}}$$

Normal Distribution Calculations - Example 3

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(x < X < 0.4) = 0.5696$.

Solution

$$\mathrm{P}(x < X < 0.4) = 0.5696$$
$$\Rightarrow \mathrm{P}(X < 0.4) - \mathrm{P}(X < x) = 0.5696$$
$$\mathrm{P}(X < 0.4) = 0.655422$$

(Use probability setting and lower tail as we have a $<$)

$$\Rightarrow 0.655422 - \mathrm{P}(X < x) = 0.5696$$
$$\Rightarrow 0.655422 - 0.5696 = \mathrm{P}(X < x)$$
$$\Rightarrow \mathrm{P}(X < x) = 0.655422 - 0.5696$$
$$\Rightarrow \mathrm{P}(X < x) = 0.085822$$

Normal Distribution Calculations - Example 3

Z stands for the standard Normal random variable with mean 0 and standard deviation 1. Find $\mathrm{P}(x < X < 0.4) = 0.5696$.

Solution Continued...

$$\Rightarrow \mathrm{P}(X < x) = 0.085822$$

(Use inverse setting and lower tail as we have a $<$)

$$\Rightarrow \mathrm{P}(X < -1.36694) = 0.085822$$

$$\underline{\underline{x = -1.36694}}$$

What if the mean and standard deviation change

Unfortunately as we will often be dealing with real data it is not true that every normal distribution will be a standard Normal random distribution with mean 0 and standard deviation 1.
In the questions in Topic 4.1, the mean and standard deviation are not standard, that is okay just change the numbers accordingly for the mean and standard deviation.
The next topic (4.2) will show us how to transform normal distributions to the standard normal distribution.