AG217 Portfolio Management & Security Analysis
Coursework Summary

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1 Variables

N = Number of Assets
t = Time

P = Portfolio
f = Risk-Free Asset
m = Market
i = Asset i

$\mathrm{E(R)_i}$ = Expected Return on asset i
$\mathrm{w_i}$ = Weight of Asset i
$(1 - \mathrm{w_i})$ = Weight of Asset k
σ_i = Std.Dev (Risk) of Asset i
σ_i^2 = Variance (Risk) of Asset i
$\rho_\mathrm{i,k}$ = Correlation of Assets i and k
$\mathrm{cov_{i,k}}$ = Covariance of Assets i and k
In = Number of Input Values

β_i = Beta Value of Asset i (Sensitivity of Asset i to Another)
α_i = Abnormal Return of Asset i (Residuals' Distance from SML)

$\mathrm{P_t}$ = Price at Time t
$\mathrm{CF_t}$ = Cash Flow (Or Coupon) at Time t (Final Year of Bond: $\mathrm{CF_t} = (\mathrm{CF_t} + \mathrm{fv})$)
y = Yield to Maturity
fv = Face Value of Bond
Y = Current Yield
$\mathrm{S_{0,t}}$ = Annualised Spot Rate Between Time 0 and Time t
$\frac{\mathrm{S_{0,t}}}{2}$ = Semi-Annual Spot Rate Between Time 0 and Time t
$\mathrm{E(S_{t1,t2})}$ = Expected Spot Rate Between Time 1 and Time 2
$\mathrm{f_{t1,t2}}$ = Forward Rate Between Time 1 and Time 2
i = Interest Rate (Can = y)
D = Duration
$\mathrm{D_A}$ = Modified Duration
C = Convexity
$\mathrm{R_u}$ = Unexpected Return

2 Mean Variance Analysis

2.1 Expected Return

2.1.1 Two-Asset Portfolio

$$\mathrm{E(R)_P = w_x E(R)_x + w_y E(R)_y}$$

2.1.2 Generalised Infinite-Asset Portfolio

$$\mathrm{E(R)_P} = \sum_{\mathrm{i=1}}^{\mathrm{N}} \mathrm{w_i E(R)_i}$$

2.1.3 Two-Asset Portfolio w/ Risk-Free Asset

$$\mathrm{E(R)_P = w_f R_f + w_m E(R)_m}$$

2.2 Variance & Standard Deviation as Risk Measures

2.2.1 Two-Asset Portfolio

$$\sigma_{\mathrm{P}}^2 = \mathrm{w_x^2}\sigma_{\mathrm{x}}^2 + \mathrm{w_y^2}\sigma_{\mathrm{y}}^2 + 2\mathrm{w_x w_y cov_{x,y}}$$

2.2.2 Risk-Free Asset Portfolio

$$\sigma_{\mathrm{P}}^2 = \mathrm{w_m^2}\sigma_{\mathrm{m}}^2$$

$\sigma_{\mathrm{f}} = 0$

$\therefore \mathrm{cov_{x,y}} = 0$

2.2.3 Using the 1/N Strategy

$$\sigma_{\mathrm{P}}^2 = \left(\frac{1}{\mathrm{N}}\right)\sigma^2 + \left(\frac{\mathrm{N}-1}{\mathrm{N}}\right)\mathrm{cov}$$

2.3 Correlation & Covariance

2.3.1 Correlation

$$\rho_{\mathrm{x,y}} = \frac{\mathrm{cov_{x,y}}}{\sigma_{\mathrm{x}}\sigma_{\mathrm{y}}}$$

2.3.2 Covariance

$$\mathrm{cov}_{\mathrm{x,y}} = \sigma_{\mathrm{x}}\sigma_{\mathrm{y}}\rho_{\mathrm{x,y}}$$

Where:

$\rho = 1$: Perfect Positive Correlation (Together)

$\rho = -1$: Perfect Negative Correlaiton (Apart)

$\rho = 0$: No Correlation

2.4 Optimal Weights in 0-Risk & Perfect Negative Correlation

Perfect Negative Correlation: $\rho = -1$
Yields a 0-Risk Portfolio: $\sigma_{\mathrm{P}}^2 = 0$

$$\mathrm{w}_{\mathrm{x}} = \frac{\sigma_{\mathrm{y}}}{\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}}$$

$$\mathrm{w}_{\mathrm{y}} = \frac{\sigma_{\mathrm{x}}}{\sigma_{\mathrm{x}} + \sigma_{\mathrm{y}}}$$

2.5 Inputs

2.5.1 Variance

$$\mathrm{In}_{\sigma_{\mathrm{i}}^2} = \mathrm{N}$$

2.5.2 Covariance

$$\mathrm{In}_{\mathrm{cov}} = \mathrm{N}\left(\frac{\mathrm{N}-1}{2}\right)$$

3 Asset Pricing

3.1 Abnormal Return

$$\alpha_{\mathrm{P}} = \mathrm{R_P} - \mathrm{E(R)_P}$$

3.2 Expected Return

3.2.1 Recall the $\mathbf{R_f}$ Tangent to the Efficient Frontier

$$\mathrm{E(R)_P} = \mathrm{R_f} + \sigma_{\mathrm{P}} \left(\frac{\mathrm{E(R)_m} - \mathrm{R_f}}{\sigma_{\mathrm{m}}} \right)$$

3.2.2 Capital Market Line (CML)

$$\mathrm{E(R)_P} = \mathrm{R_f} + \mathrm{w_m} \left(\mathrm{E(R)_m} - \mathrm{R_f} \right)$$

3.2.3 Security Market Line (SML)

$$\mathrm{E(R)_i} = \mathrm{R_f} + \beta_{\mathrm{i}} \left(\mathrm{E(R)_m} - \mathrm{R_f} \right)$$

Where:
$(\mathrm{E(R)_m} - \mathrm{R_f})$ = Market Risk Premium
$\beta = 1$: Tracking Market Folio
$\beta \neq 1$: Actively Investing
$\beta > 1$: Aggressively Investing (Expect Market Folio Increase)
$\beta < 1$: Defensively Investing (Expect Market Folio Decrease)

3.3 Beta Values

3.3.1 Assets

$$\beta_{\mathrm{i}} = \frac{\mathrm{cov_{i,m}}}{\sigma_{\mathrm{m}}^2}$$

3.3.2 Portfolios

$$\beta_{\mathrm{P}} = \sum_{\mathrm{i}=1}^{\mathrm{N}} \mathrm{w_i} \beta_{\mathrm{i}}$$

4 Bond Pricing

4.1 Price

$$P_0 = \sum_{t=1}^{N} \frac{CF_t}{(1+y)^t}$$

4.2 Current

$$Y = \frac{CF}{P_0}$$

4.3 Yield to Maturity

Step 1

Find upper and lower limites of P varying by y

Step 2

Conclude 1% ΔY gives: $(P_{upper} - P_{lower}) = \Delta P_{1\%\Delta y}$

Step 3

$\Delta y_{req} = \frac{P_{upper} - P_0}{\Delta P_{1\%\Delta y}}$

Step 4

Convert y_{upper} to % and add (+) number from Step 3

4.4 Spot Rates

4.4.1 Price of Bond Using Spot Rates

$$P_0 = \frac{CF}{\left(1 + \frac{S_{0,t}}{2}\right)^t}$$

4.4.2 Spot Rates

$$S_{0,t} = 2\left(\left(\frac{CF}{P_0}\right)^{\frac{1}{t}} - 1\right)$$

Where:

Spot rates are semi-annual (e.g. 1 period ($t = 1$) means 6 months)

4.4.3 Expected Spot Rates

$$\mathrm{E(S)}_{\mathrm{t1,t2}} = 2\left(\frac{\left(1+\frac{\mathrm{S}_{0,\mathrm{t2}}}{2}\right)^{\mathrm{t2}}}{\left(1+\frac{\mathrm{S}_{0,\mathrm{t1}}}{2}\right)^{\mathrm{t1}}} - 1\right)$$

4.4.4 Forward Rates

$$\mathrm{E(S)}_{\mathrm{t1,t2}} = \mathrm{f}_{\mathrm{t1,t2}}$$

4.5 Duration of Bond

4.5.1 Basic Duration

$$\mathrm{D} = \frac{\sum \mathrm{t}\left(\frac{\mathrm{CF_t}}{(1+\mathrm{i})^{\mathrm{t}}}\right)}{\mathrm{P_0}}$$

4.5.2 Modified Duration

$$\mathrm{D_A} = \frac{\mathrm{D}}{(1+\mathrm{i})}$$

Where:

Duration (years) captures sensitivity of a bond to $\Delta\mathrm{i}$

4.6 Convexity of Bond

$$\mathrm{C} = \frac{1}{2}\left(\frac{\sum \mathrm{t}(\mathrm{t}+1)\left(\frac{\mathrm{CF_t}}{(1+\mathrm{i})^{\mathrm{t}}}\right)}{\mathrm{P_0}}\right)$$

4.7 Unexpected Return

4.7.1 With Duration

$$\mathrm{R_u} = -\mathrm{D_A}\Delta\mathrm{i}$$

4.7.2 With Duration & Convexity

$$\mathrm{R_{uw/C}} = -\mathrm{D_A}\Delta\mathrm{i} + \mathrm{C}(\Delta\mathrm{i})^2$$

Where:

Unexpected return is represented as a percentage (%)