11 Derivatives

11.1 Use the limit definition to determine the derivatives of the following functions of x.

(a) $x^2 + 3x + 2$ (b) $\dfrac{1}{2x-1}$ (c) $\dfrac{1}{x^2+3}$ (d) $x^{1/3}$.

11.2 The symbol $\lfloor x \rfloor$ denotes the largest integer that is less or equal to x. For example, $\lfloor 1.5 \rfloor = 1$, $\lfloor \pi \rfloor = 3$, $\lfloor 42 \rfloor = 42$, and $\lfloor -3.1 \rfloor = -4$. Formally, $\lfloor x \rfloor = \max_{n \in \mathbb{Z}} \{n \leq x\}$.

Sketch the function $f(x) = \lfloor x \rfloor$ and find its derivative f'.

11.3 Prove, starting from the limit definition, that $\dfrac{\mathrm{d}x^n}{\mathrm{d}x} = n \cdot x^{n-1}$ for $n \in \mathbb{N}$.

(Hint: use the binomial theorem.)

11.4 Prove, starting from the limit definition, that for two differentiable functions f and g and a constant c:

(a) if $g(x) = f(x) + c$, then $g'(x) = f'(x)$;

(b) if $g(x) = cf(x)$, then $g'(x) = cf'(x)$;

(c) if $g(x) = f(cx)$, then $g'(x) = cf'(cx)$.

11.5 For $f(x) = x^3$ compute

(a) $f'(3)$, $f'(9)$, $f'(25)$;

(b) $f'((\sqrt{3})^2)$, $f'(3^2)$, $f'(5^2)$;

(c) $f'(a^2)$, $f'(x^2)$.

11.6 For $f(x) = x^3$ compare $f'(x^2)$ and $g'(x)$ where $g(x) = f(x^2)$.

11.7 Prove that if f is even, then $f'(x) = -f'(-x)$.

(Hint: consider $g(x) = f(-x)$ and use Exercise 11.4.)

11.8 Prove that if f is odd, then f' is even:

(a) directly, by using the limit definition;

(b) by using the results of Exercise 11.4.

11.9 Prove, using the limit definition, that if $g(x) = f(x + c)$ then $g'(x) = f'(x + c)$.

11.10 Find $f'(x)$ and $g'(x)$ for $f(x) = h(x + t)$ and $g(t) = h(x + t)$.

(Hint: use the result proved in the previous question.)

11.11 Prove that if a function f is differentiable and periodic with period p (that is, $f(x+p) = f(x)$ for all x), then f' is also periodic with period p.