AG313 Summary:

Treasury Management & Derivatives

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AG313: Treasury Management & Derivatives

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AG313 Course Summary

Derivatives

1: Options

1.1: Option vs. Forward Contracts:

- Forward: Obligation to buy/sell in the future at rate
  - Future ER Safety
- Option: Right to buy/sell in future at rate
  - No Future ER Safety (may be better/worse)

1.2: Spot vs. Future/Forward Prices:

- Spot Price: immediate delivery price (S_0, S_T)
- Future/Forward Price: future delivery price (lock in price today) (F_0, F_T)
  - F_T (<) S_T: Forward is Spot grossed up at r, Spot exp. to be (>) r growth

1.3: Short vs. Long Positions:

- Short: Sells shares **now** (S_0 = Spot), buy later (S_T = Delivery)
  - Exp. fall in **future** to buy
  - Futures price up: loss
  - $\text{Profit} = (S_0 - K)$
- Long: Buy shares **now**
  - Exp. rise in **future** to sell
  - Futures price up: gain
  - $\text{Profit} = (K - S_T)$

1.4: Call vs. Put Option Types:

- **At-The-Money**: $S_T = K$
- Call: Agreement to buy at specified time and Strike Price
  - Profit **(In-The-Money)** When $S_T > K$
  - $\text{Profit} = N(S_T - K) - \text{Cost}\ \{\text{Cost} = N(C_0)\}$
- Put: Agreement to sell at specified time and Strike Price
  - Profit (**In-The-Money**) When $K > S_T$
  - $\text{Profit} = \text{Cost} - N(K - S_T)\ \{\text{Cost} = N(P_0)\}$
- European Option: exercised only on expiration
- American Option: exercised any time up-to expiration

1.5: Exchange vs. Over-the-Counter:

- Over-the-Counter: $600tn; high credit risk – high prices
  - Trades Forward Contracts
- Exchange: $60tn; more standardised and regulated
  - Trades Futures Contracts

2: Futures Market

- Regulated by 'Commodities Futures Trading Commission (CFTC)'
- **Clearing House**: always used in Futures Market to ensure payment made
- **Central Clearing Parties (CCPs)**: similar job to CH
- **Haircut**: difference between Market Value and Collateral usage of an asset
- **Bilateral Clearing**: group agree terms to trade w/ each other to minimise risk
- **Limit Order**: trader identifies worst at which trade can take place
- *Note that Futures on stock are cosh settled as opposed to w/ the underlying asset*

2.1: Forward vs. Future:
- Futures trade on Exchange Market – standardised
- Futures last shorter time than Forward Contracts
- Futures don't usually have final cash settlements
- Short: loss when futures rises – must buy at higher price to replace
- Long: gain when futures rises – made profit as share price rises

2.2: Margin 'Curtain Call' Call:
- *Broker's demand that investor adds funds to retain minimum value of fund - **daily***
- Options up to 9-months must be bought in full, post-9: margin can be taken
- The **seller** posts margin, not the buyer as they make the payment for the option
- **Margin Accounts** are adjusted for gain/loss daily
- Reduce Systematic Risk → Ensure Funds Available → Reduce Risk of Back-Out

- Margin Call when loss over: (Initial Margin (-) Maintenance Margin)
  1) If Short: ea. $1 rise in price is a $1 per unit loss – find (=) to above
  2) Add the per unit rise to the per unit price
  3) If Long: ea. $1 rise in price is a $1 per unit gain – find (=) to above
  4) Add the per unit rise to the per unit price

2.3: Corn Futures Contract:
- Initiated by: party w/ Short position – 'Notice of Intention to Deliver'
- Exchange goes through procedure of choosing party to take Long position

2.4: Hedging vs. Speculating:
- Hedging: e.g. expect volatility perhaps price rise to take Futures contract to lock in a price now
- Speculating: e.g. act upon expectation perhaps where they expect a fall in price, they'd take a Short position and buy back for profit
- *Hedgers hold Long, Speculators hold Short: ($F_T > S_T$)*

3: Forward & Futures Prices

- *Futures Price quoted as no. of US$/unit of foreign currency*
- *Lenders cannot issue instructions*
- **Investment Asset**: traded but not usually physically usable or tangible
- **Consumption Asset**: traded and usable for consumption (e.g. Copper)
  - Upper limit but no lower limit
- **Convenience Yield**: 0/(+), measures benefit of owning rather than for./fut.
  - *Having **real** value vs. **locked in F** value*
  - Investment: (=)0
  - Consumption: (+)
  - **Increase**: F as % of S decreases; more convenient to own
  - **Decrease**: F as % of S increases; more convenient to F
- **Div. Yield**: Div.'s as a % of Stock Price at t of Div. payment
- **Contango**: F_T (>) S_T abnormal

3.1: Shorting w/ Dividends:

- (1) S_0 Sell Now S_T Buy Later ($S_0 - S_T$ Gain/Share) → (2) Pays Div. ($S_0 - S_T$ – Div)

3.2: Spot to Forward Price:

$$\boxed{F_T = S_0 e^{rT}}$$

$$\boxed{F_T = (S_0 - \text{Income})e^{rT} \ \{\text{Income} = Y_t e^{-rT} + \cdots + Y_{t+n} e^{-rT}\}}$$

$$\boxed{F_T = ER_0 e^{(r_1 - r_2)T}}$$

4: Hedging Strategies w/ Futures

- *Futures delivery month should be as close as possible to purchase of asset*
- *"Tailing the Hedge"*: Corrects for daily settlement
- Hedging Futures leads to predictability

$$\boxed{\boldsymbol{Basis} = Spot_{\text{At Close}} - Futures_{\text{At Close(For Maturity)}}}$$

$$\boxed{\boldsymbol{Price\ Recieved} = Basis + \ Futures_{\text{At Purchase(For Maturity)}}}$$

$$\boxed{\boldsymbol{Optimal\ Hedge\ Ratio} = \rho_{A,B}\left(\frac{\sigma_A}{\sigma_B}\right)}$$

"Movement in S Price to Movement in F Price"

$$\boxed{\boldsymbol{Optimal\ Folios} = (\beta_{Current} - \beta_{Desired})\left(\frac{V_{Folio}}{F_0 F_N}\right)}$$

If $(+)$: Short

If $(-)$: Long

$$\boxed{\boldsymbol{P_{Total}} = w_{Hedged}P_{Hedged} + w_{NotHedged}P_{NotHedged}}$$

Where:

Given: S_0, F_0, S_T, F_T

$$P_{Hedged} = S_T - (F_T F_0)$$

$$P_{NotHedged} = S_T$$

5: Options Market

- **Recall**: *"The option, but not obligation, to buy (call) or sell (put)"*
- **Recall**:
  - Short: sell now (exp. to buy later lower)
  - Long: buy now (exp. to sell later higher)
- **Recall**: differences in *S*, *E(S)*, and *F* mean Short Call, Long Calls, Short Put, Long Put are all different
- **Option Class**: *"All Calls or Puts on a stock"*
- **Option Series**: *"All options on a certain stock* ***type****"*
- **LEAPS**: Long-Term Equity Anticipation Securities w/ long maturities

- **Stock-Split**:
  - E.g. N = 100, K = 20, 2 for 1 Split
  - Ans. N = 2(100) = 200, K = ½(20) = 10
- **Stock-Div**:
  - E.g. N = 100, K = 20, 25% Div.
  - Ans. N = 1.25(100) = 125, K = 4/5(20) = 16
- **Cash-Div**:
  - No Effect

- Option Value (=) Time Value (+) Intrinsic Value
  - At-The-Money Time Value (=) 0 so Option Value (=) Intrinsic Value
  - Call: $(S_T - K, 0)$
  - Put: $(K - S_T, 0)$

6: Option Pricing

1: Binomial Option Tree – European Put

Step 1

$$u = e^{\sigma\sqrt{\Delta t}}$$
$$d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$$
$$p = \frac{e^{r\Delta t} - d}{u - d} = \text{Risk Neutral Probability of Up Movement}$$
$$(1 - p) = \text{Risk Neutral Probability of Down Movement}$$

Step 1

$$S_{u/d} = \text{Value of Stock Upon Increase/Decrease}$$
$$S_u = Pu$$
$$S_d = Pd$$
$$S_{uu} = Pu^2$$
$$S_{ud} = Pud$$
$$S_{dd} = Pd^2$$

Step 3

$$P_{u/d} = \text{Value of Option Upon Increase/Decrease}$$
$$P_{uu} = 0$$
$$P_{ud} = K - S_{ud}$$
$$P_{dd} = K - S_{dd}$$
$$P_u = \Big((pP_{uu}) + \big((1-p)P_{ud}\big)\Big)e^{-r\Delta t}$$
$$P_d = \Big((pP_{ud}) + \big((1-p)P_{dd}\big)\Big)e^{-r\Delta t}$$
$$P_0 = \Big((pP_u) + \big((1-p)P_d\big)\Big)e^{-r\Delta t}$$

2: Converting to American Put

$$P_d = \max\{K - S_d, P_d\}\,; P_{d_A} = \text{Larger Outcome};\ P_{u_A} = \text{Remains Same}$$
$$P_{0_A} = \Big(\big(pP_{u_A}\big) + \big((1-p)P_{d_A}\big)\Big)e^{-r\Delta t}$$

7: Stock Options

- Stock Price (↑): Call (↑), Put (↓)
- Strike Price (↑): Call (↓), Put (↑)
- Volatility (↑): Call Payoff (↑), Put Payoff (↑)
- Dividends (↑): Stock Price (↓), Call (↓), Put (↑)
- Interest Rate (↑): Call (↑), Put (↓)
- Time-Maturity (↑): European Options (↑/↓)

Call Lower Bound

$$\boxed{\boldsymbol{S_0 - Ke^{-rT}}}$$

Put Lower Bound

$$\boxed{\boldsymbol{Ke^{-rT} - S_0}}$$

Put Call Parity w/o Dividend (or Interest 0)

$$\boxed{\begin{gathered}\boldsymbol{C_0 + Ke^{-rT} = P_0 + S_0} \\ \boldsymbol{C_0 + K = P_0 + S_0}\end{gathered}}$$

Put Call Parity w/ Dividend

$$\boxed{\boldsymbol{C_0 + Ke^{-rT} = P_0 + (S_0 - D)}}$$

Black & Scholes Models

$$d_1 = \frac{\ln\left(\frac{S}{K}\right) + T\left(r + \frac{\sigma^2}{2}\right)}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$\boxed{\begin{gathered}\boldsymbol{C_0 = S\big(N(d_1)\big) - Ke^{-rT}(N(d_2))} \\ \boldsymbol{C_0 = Se^{-yT}\big(N(d_1)\big) - Ke^{-rT}(N(d_2))} \\ \boldsymbol{P_0 = K\big(1 - N(d_1)\big) - Se^{-rT}\big(1 - N(d_2)\big)}\end{gathered}}$$

Treasury Management

1: Foreign Exchange Market

Domestic in Terms of Foreign; Foreign in Terms of Domestic

$$\text{Spread} = \frac{\text{Ask} - \text{Bid}}{\text{Ask}}$$

$$\text{Direct Quotation} = £/\$ = \frac{1}{\$/£}$$

$$\text{Indirect Quotation} = \$/£ = \frac{1}{£/\$}$$

$$\text{Cross Rate} = \$/£ = €/£\frac{1}{€/\$}$$

2: Interest Parity Relationships

Interest Rate Arbitrage

$$\boxed{\boldsymbol{A_n = \left(\frac{A_h}{S}\right)(1 + i_f)\big(S(1 + p)\big)}}$$

$$F = S(1 + p)$$

$$A_{h,n} = \text{Home/New Home Currency}$$
$$i_{h,f} = \text{Home/Foreign Currency}$$
$$S = \text{Spot Exchange Rate} = \text{"N of £ Per Unit of \$"}$$
$$F = \text{Forward (Locked) Exchange Rate} = \text{"N of £ Per Unit of \$"}$$
$$p = \text{Forward Premium} = \text{"Amount By Which F is ↑↓ than S"}$$

$$\text{Convert to \$: } \left(\frac{A_h}{S}\right)$$

$$\text{End of Period \$ Principal \& Interest: } \left(\frac{A_h}{S}\right)(1 + i_f)$$

$$\text{\$ Principal \& Interest Back to £: } \left(\frac{A_h}{S}\right)(1 + i_f)F$$

Interest Rate No-Arbitrage

$$\boxed{\boldsymbol{A_h(1 + i_h) = A_h(1 + i_f)(1 + p)}}$$

$$A_h(1 + i_h) = \text{Investing with Home Interest} = \text{Investing with Foreign Interest}$$

$$\therefore p = \frac{(1 + i_h)}{(1 + i_f)} - 1 \therefore p \approx i_h - i_f$$

Absolute PPP

$$\boxed{\boldsymbol{S_f^d = \frac{P_s^d}{P_s^f}}}$$

$$\text{As: } P_s^d = S_f^d * P_s^f$$

$$\text{Spot} = \text{Implied ER} = \text{Ratio of Domestic to Foreign Prices}$$
$$\text{Domestic Price (Should Be)} = \text{Foreign Price (Given } S\text{)}$$

Relative PPP (w/ Inflation)

$$\text{Adjust for Inflation:}$$
$$P_h(1+\pi_h)\ \&\ P_f(1+\pi_f)$$

$$\text{If } \pi_h > \pi_f\text{: PP is Greater When Buying Foreign Goods} \rightarrow \text{Foreign Cheaper}$$
$$\text{If } \pi_h < \pi_f\text{: PP is Greater When Buying Domestic Goods} \rightarrow \text{Domestic Cheaper}$$

$$\text{Adjust for Change in Currency:}$$
$$P_f(1+\pi_f)(1+e_f)$$

$$e_f = \%\ \text{Change Per Unit of Foreign Currency In Domestic Currency}$$

$$\text{Hence:}$$

$$\boxed{\boldsymbol{P_h(1+\pi_h) = P_f(1+\pi_f)(1+e_f)}}$$

$$\therefore e_f = \frac{P_h(1+\pi_h)}{P_f(1+\pi_f)} - 1 = \frac{(1+\pi_h)}{(1+\pi_f)} - 1$$

$$*\ \text{Given } P_h\ \&\ P_f \text{ Are Equal } *$$
$$\text{If } \pi_h > \pi_f\text{: } e_f(+)\text{: Foreign Should Appreciate} - \text{Domestic Depreciate}$$
$$\text{If } \pi_h < \pi_f\text{: } e_f(-)\text{: Foreign Should Depreciate} - \text{Domestic Appreciate}$$

$$*\ \text{For Relatively Low Inflation } *$$

$$e_f = \frac{(1+\pi_h)}{(1+\pi_f)} - 1 \approx \pi_h - \pi_f$$

3: Exchange Exposure

Variance of a Two-Asset Folio

$$\sigma_{x,y}^2 = \sigma_x^2 + \sigma_y^2 + 2(cov_{x,y})$$

Hence $\{p = \{x, y\}\}$; $\{cov_{x,y} = \rho_{x,y}\sigma_x\sigma_y\}$:

$$\boxed{\boldsymbol{\sigma_p^2 = \sigma_x^2 + \sigma_y^2 + 2\left(\rho_{x,y}\sigma_x\sigma_y\right)}}$$

Variance of a Three-Asset Folio

Where $\{p = \{x, y, z\}\}$:

$$\boxed{\boldsymbol{\sigma_p^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2 + 2\left(\rho_{x,y}\sigma_x\sigma_y\right) + 2\left(\rho_{x,z}\sigma_x\sigma_z\right) + 2\left(\rho_{y,z}\sigma_y\sigma_z\right)}}$$

Economic Exposure

$$\boxed{\boldsymbol{V_{MNC} = \sum \frac{\sum\left(E\left(CF_{j,t}\right)E\left(ER_{j,t}\right)\right)}{(1+k)^t}}}$$

Where:

$E\left(CF_{j,t}\right)$ = Expected CF in Currency j Recieved At End of Period t
$E\left(ER_{j,t}\right)$ = Expeced ER of Currency j At End of Peiod t
k = Weighted Average Cost of Capital (WACC) of MNC

4: Derivatives

Long/Sort Positions (Recall)

$$\boxed{\boldsymbol{Payoff_{Long} = (S_t - F)}}$$

$$\boxed{\boldsymbol{Payoff_{Short} = (F - S_t)}}$$

Forward Discount/Premium

$$\boxed{\boldsymbol{p = \frac{F - S}{S}}}$$

$$\boxed{\boldsymbol{F = S(1 + p)}}$$

If $\{p < 0\}$: F at Discount
If $\{p > 0\}$: F at Premium

Premium On Options

$$\boxed{\boldsymbol{C = f\left((X - S),\ T,\ \sigma^2\right)}}$$

If $(X - S)$ $(\uparrow)$; C $(\uparrow)$
If T $(\uparrow)$; C $(\uparrow)$
If $\sigma^2(\uparrow)$; C $(\uparrow)$

5: Interest Rate Risk

- $\frac{1}{100}$ of a % is 'Basis Point'
- Must Convert Period to Days

Recall

$$R = \text{Simple Interest Rate}$$

$$r = \frac{R}{m} = \text{Periodic Interest Rate}$$
$$\text{"}m\text{ periods per }n\text{"}$$

$$(1+r)^{mn} - 1 = \text{Compound Interest Rate}$$

$$EAR = (1+r)^{\frac{year}{days}} - 1$$

Duration

$$\Delta B = -DB\Delta y$$

$$B = \sum \frac{CF_t}{(1+y)^t}$$

$$\boxed{\boldsymbol{D} = \sum \boldsymbol{t}\left(\frac{\frac{\boldsymbol{CF_t}}{(\boldsymbol{1}+\boldsymbol{y})^{\boldsymbol{t}}}}{\boldsymbol{B}}\right) = \sum \boldsymbol{t}\boldsymbol{w_t}}$$

$$y = \text{Yield on Bond}$$
$$B = \text{Bond Price}$$
$$\text{"Weighted average of times of payments which is } (=) \text{ proportion of B to CF at } t\text{"}$$

$$D_{ZeroCoupon} = \text{Maturity} = T$$
$$\text{Constant Maturity: } D(\uparrow)CF(\downarrow)$$
$$\text{Constant Coupon: } D(\uparrow)T(\uparrow)$$
$$\text{Constant All Other: } D(\uparrow)y(\downarrow)$$

Forward Rate Agreements

$$\text{Payoff} = (\text{Notional Amount})(\text{LIBOR} - \text{Agreed Upon Rate})\left(\frac{\text{m}}{360}\right)$$

$$\text{Payoff} = (\text{Notional Amount})\left(((\text{LIBOR}) - \text{Agreed Upon Rate})\frac{\left(\frac{\text{m}}{360}\right)}{(1+\text{LIBOR})\left(\frac{\text{m}}{360}\right)}\right)$$

Interest Rate Option

$$\text{Payoff}_{\text{Call}} = (\text{Notional Amount})\left(Max(0, \text{LIBOR} - X)\left(\frac{m}{360}\right)\right)$$
$$\text{If LIBOR} > X\text{: Exercise}$$
$$\text{Payoff}(\uparrow)\text{LIBOR}(\uparrow)$$
$$\text{"Protection Against Rising } i\text{" (e.g. future borrowing)}$$

$$\text{Payoff}_{\text{Put}} = (\text{Notional Amount})\left(Max(0, X - \text{LIBOR})\left(\frac{m}{360}\right)\right)$$
$$\text{If LIBOR} < X\text{: Exercise}$$
$$\text{Payoff}(\uparrow)\text{LIBOR}(\downarrow)$$
$$\text{"Protection Against Falling } i\text{" (e.g. future investing)}$$

Derivatives Extensive Summary

Lecture 1: Introduction to Derivatives

- Derivative value depends on another asset
- E.g. (Derivatives): futures, forwards, swaps, options
- E.g. (Assets): equity, bonds, shares, interest rates
- Derivatives **transfer risk** in economy
- Derivative market larger than stock market (several *global-GDP)

1: How Are Derivatives Traded

- Traded on standard der. exchanges
- Over-the-counter market (OTC) – traders work for banks, fund managers – corporate treasurers contact one-another directly

1.1: Exchange-Traded Derivatives

- Standardised contracts
- Limit credit risk
- Central clearing
- Regulated exchanges
- Liquid markets

1.2: Over-the-Counter Markets

- Tailor-made contracts
- Flexibility in negotiation
- Larger than <u>exchange-traded market</u>
- Telephone and computer network of dealers
- Higher **credit-risk**

2: Forward Contracts

- **Terms**: buy/sell at future time/price
- **Market**: Over-the-Counter
- Long-term maturity available

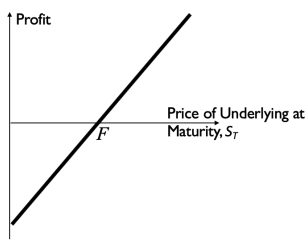
- Party agreeing to **buy** underlying asset: <u>Long Position</u>
- Party agreeing to **sell** underlying asset: <u>Short Position</u>
- *Legally binding agreement*

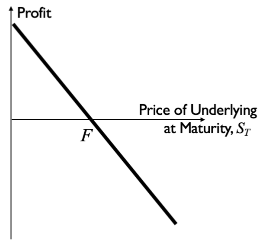
2.1: Payoffs

- F = Delivery Price of Asset @ Present
- S_t = Spot Price of Asset @ Maturity

- $(S_t - F)$ = Long Position Payoff (Buyer)
- $(F - S_t)$ = Short Position Payoff (Seller)

- From Long: From Short:





3: Futures Contracts

- **Terms**: buy/sell at future time/price
- **Market**: Exchange-traded
- Available on commodities financial assets
- Standardised contracts
- Specification:
    - What can be delivered
    - Where it can be delivered
    - When it can be delivered
- Settled daily

3: Options Contracts

- **Call Option**: right to buy underlying asset on/by date for price
- **Put Option**: right to sell underlying asset on/by date for price
- **American Option**: exercised any time up-to expiration
- **European Option**: exercised only on expiration

3.2: Definitions

- **Premium**: price options bought/sold
- **Exercise/Strike Price**: price at which right to buy/sell underlying asset of call/put is set
- **Expirations/Maturity**: date in contract on/by option must be exercised

- **Writer**: supplier/seller of option at selling price (**premium**) (Short Position)
- **Holder/Investor**: party who's acquired/bought option (Long Position)

3.3: Options vs. Futures/Forwards

- Futures/Forward is a **commitment** to buy/sell
- Options give the **right** to buy/sell
- Options traded on both **exchanges/OTC**
- Investors don't pay premium to enter futures/forwards

4: Types of Traders

- **Hedgers**: use derivatives to reduce risk they face from potential movements
- **Speculators**: use derivatives to bet on future direction of a market variable
- **Arbitrageurs**: take offsetting positions in 2+ instruments to lock profit by taking adv. of price discrepancies

4.1: Hedging

- **Long Hedge**: one future date committed to **buying** assets, fix future price by taking <u>Long Position</u> in <u>Futures</u> on asset
    - o "Hedge against possibility of price rise"
- **Short Hedge**: one future date committed to **selling** assets, fix selling price by taking <u>Short Position</u> in <u>Futures</u> on asset
    - o "Hedge against possibility of price fall"
- In both, changes in value of asset can be offset by changes in value of position in futures

4.2: Speculation

- Betting on future changes in price of an asset using derivatives
- Bank of England "broken" example video

4.3: Arbitrage

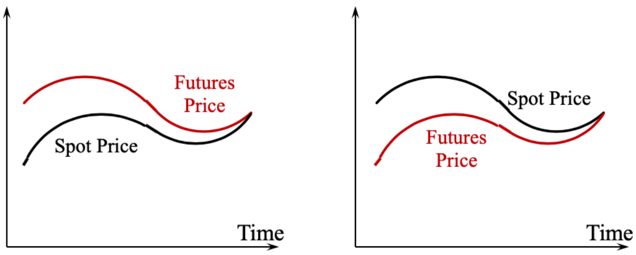
- Riskless profit: simultaneous purchase and sale of asset to profit from difference in price
- Equivalent assets trade at different prices, they buy in cheaper markets and sell in more expensive one
- Rare and don't last long
- Rational, well-formed markets: equivalent assets have same price (equilibrium: no arbitrage)

Lecture 2: Futures Markets

- Recall:
  - Long Position: Buyer
  - Short Position: Seller
  - Spot Price: Price at which can be sold immediately
  - Futures Price: Price at which can buy/sell in future
  - Basis: Difference between two
- Categories:
  - Agricultural commodities
  - Metals and minerals
  - Foreign currency
  - Financial futures

1: Terminology

1.1: Convergence



- Towards expiration: futures converges on spot
- Point of expiration: futures and spot identical

1.2: Terms

- Open Interest: total contracts outstanding
  - Equal to no. of **long** or **short** positions
- Settlement Price: price right before final bell (each day)
  - For daily settlement processes
- Volume of Trading: no. trades – contracts traded – daily

1.3: Patterns of Futures Prices

- Normal Markets: Futures prices **pos. corr.** to time to maturity
- Inverted Markets: Futures prices **neg. corr.** to time to maturity

2: Payoff from Futures (Gain/Loss)

- F_0 = Futures Price when Position Opened
- F_T = Futures Price when Position Closed
- Gain or Loss = Sell Price − Buy Price

2.1: Long Position & Short Position (Respectively)

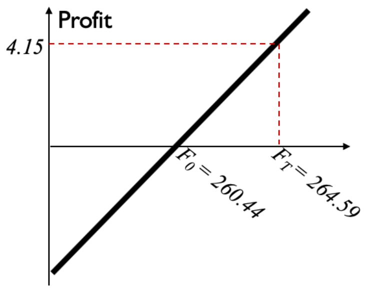
- $F_T - F_0$ = Spot @ Maturity − Original Futures
- $F_0 - F_T$ = Original Futures − Spot @ Maturity

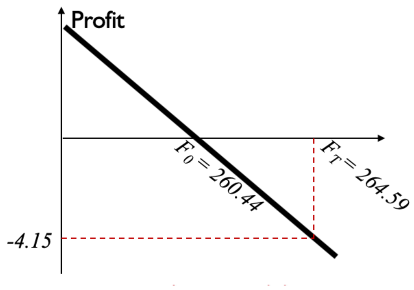
- $(F_T - F_0) * No.Contracts * Shares\ per\ Contract$
- $(F_0 - F_T) * No.Contracts * Shares\ per\ Contract$

2.2: Zero-Sum Game

- All <u>long positions</u> offset by <u>short position</u>
- Therefore aggregate profit/loss w/ futures trading over all investors = 0

- Long: Short:





3: Margin Accounts

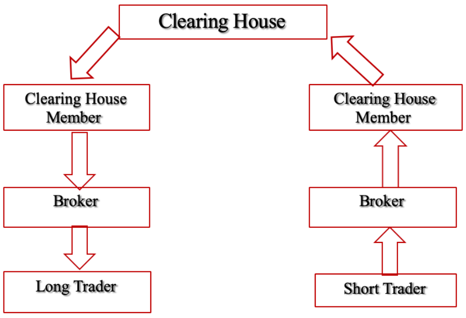
- Margin is cash/marketable securities depo'd by investors w/ their broker (interest paying account)
- Determined by var. of price of underlying asset
- Balance of <u>Margin Account</u> is reflective of <u>daily settlement</u>
- Margins minimise possibility of loss through **default** on contract

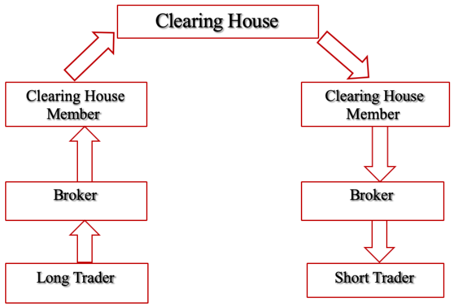
- <u>Initial Margin</u>: amount which must be deposited at entry (usually 5%-15% of total value of contract)
- <u>Marking to Market</u>: account adjusted ea. trading day for gain/loss (<u>daily settlement</u> process)
- <u>Maintenance Margin</u>: if margin falls below critical value (75% of <u>initial margin</u>), investor receives <u>margin call</u> to restore level of initial margin
- <u>Variation Margin</u>: funds deposited in <u>margin acc.</u> following a <u>margin call</u>

4: Clearing House

- The trading partner of ea. side of futures contract
- Seller of contract for long position and buyer for short position
- Obligated to deliver commodity to long pos. and pay for delivery from short
- Zero-net-position
- Improves liquidity
- Reduces uncertainty and credit risk

4.1: Margin CFs w/ Increase & Decrease in Futures Price (Respectively)





5: Types of Orders

- Market
- Limit
- Stop-Loss
- Stop-Limit
- Market-if-Touched
- Discretionary
- Time of Day
- Open
- Fill or Kill

6: Forwards vs. Futures

- Forwards
  - Private bet. Two parties
  - Non-standard contract
  - Usually one specified delivery date
  - Settled at end of contract
  - Delivery or final C settlement occurs
  - Some credit risk
- Futures
  - Exchange traded
  - Standard contract
  - Range of delivery dates
  - Settled daily
  - Contract usually closed out prior to maturity
  - Virtually no credit risk

Lecture 3: Forward & Futures Prices

1: Investment vs. Consumption Assets

- Investment Assets: held by significant numbers of people purely for investment purposes (e.g. stocks, bonds etc.)
- Consumption Assets: held primarily for consumption (e.g. oil, corn, etc.)

2: Short Selling

- Selling assets you don't own
- Broker borrows securities from another client and sells in market
- In order to close your position: buy the securities to return them
- Div.'s and other benefits must be paid to owner
- Perhaps small fee for borrowing the assets

2.1: Short Selling Example

- Short 100 shares w/ price of £100
- Close short in 3 months w/ price of £90
- £ month div. of £3 per share

- Profit = (Selling Price – Buying Price)*No. Shares – (Div.*No. Shares)
  = £700

2.2: Notation

- S_o = Spot Price Today
- F_0 = Futures or Forward Price Today
- T = Time Until Delivery
- r = fRisk − Free i for Maturity T

2.3: Assumptions

- No transaction costs
- Same tax rates for all participants
- Borrow/lend at r
- Participants take adv. of arbitrage

3: Pricing Futures: Spot-Futures Relation

- Investor requires asset at time T

1) Buy asset today at S_0 and hold until T or…
2) Enter contract today to buy asset at T at F_0

3.1: Buy Asset Today

- Cost of holding 'til *T* is **interest lost** on alternative of holding C
- **Cost 'Buy-and-Hold**' = S_0 + Interest Lost

- Cost of holding asset is reduced by any **income**, e.g. div.'s/shares
- **Cost of 'Buy-and-Hold'** = S_0 + Interest Lost – Income Received

3.2: Is There Arbitrage?

- Suppose:
  - S_0 of non-div-paying stock is £40
  - F_0 (futures price) is unknown
  - 3-month (forward price) is £43 or £39 (Arbitrage)
  - 3-month UK i is 5% p.a.

- Forward Price: $\boldsymbol{F_0 = S_0 e^{rT}}$
  - **Standard**: $F_0 = 40e^{0.05*0.25} = 40.5 \rightarrow$ £40.50
  - $F_0 > S_0$ because cost of financing the spot purchase of asset during life of project

  - **Arbitrage**: $F_0 = 40e^{0.05*0.25} = 40.5 \rightarrow$ £40.50
    - Short one share for £40
    - Invest proceeds in short sale for F_0
    - Take **long-forward** to buy share at £39 in 3 months to close short
    - Take **Riskless Profit** = £40.50 - £39 = <u>£1.50</u>

- Price Equilibrium:
  - $F_0 > S_0 e^{rT}$: arbitrageurs can buy asset and enter **short-forward** contract on asset for **Riskless Profit** = $F_0 - S_0 e^{rT}$
  - $F_0 < S_0 e^{rT}$: arbitrageurs can short sale asset and enter **long-forward** on asset for **Riskless Profit** = $S_0 e^{rT} - F_0$

4: Short-Selling Not Possible

- See Slide 13 and on

Lecture 4: Hedging Using Futures

- …

Lecture 5: Interest Rate Swaps (IRSs)

- A swap is an over-the-counter agreement
    - Two parties
    - Exchange CFs
    - Specified future times – according to rules

1: Plain Interest Swaps

- Agrees: to make fixed *i* payments on **notional principal** – given no. years
- In return: receives *i* at a floating rate on same **notional principal** – same period
    - Notional principal: used only to calculate of *i* payments – not exchanged

- **Example**:
    - *MSFT agrees to receive 6-month LIBOR and pay Intel fixed rate of 5% every 6 months for 3 years – on notional principal of £100m*
        - Microsoft: <u>Long Position</u> in floating-rate bond & <u>Short Position</u> in fixed-rate bond
        - Intel: <u>Long Position</u> in fixed-rate bond & <u>Short Position</u> in floating-rate bond

2: Uses of IRSs

- Covert Liability from:
    - Fixed-rate to floating-rate
    - Floating-rate to fixed-rate
- Convert Investment (Asset) from:
    - Fixed-rate to floating-rate
    - Floating-rate to fixed-rate

2.1: Example: Converting Liability

- MSFT borrow £100m at *i* of LIBOR + 0.1%
- Receives LIBOR on **notational principal** of £100m from Intel
- Pays 5% on **same principal**
- –
- MSFT Net Liability is **fixed-rate** *i* payment of:
    - (LIBOR + 0.1% - LIBOR + 5%) = 5.1% on £100m

3: Market Makers (Swap Dealers)

- Unlikely two companies will need to oppose positions in same swap at same time
- Large financial institutions act as <u>Market Makers</u> for swaps
- <u>Market Makers</u> prepared to enter swap w/o having offsetting/counterparty swap
- Carefully quantify and hedge risks they take

3.1: Example: Converting Liability w/ Financial Institution

- MSFT borrow £100m at *i* of LIBOR + 0.1%
- Receives LIBOR on **notational principal** of £100m from Intel
- Pays 5.015% via <u>Financial Institution</u> to Intel
- <u>Financial Institution</u> pays Intel 4.985% on £100m
- –
- MSFT Net Liability is **fixed-rate** *i* payment of:
    - (LIBOR) = 0.1% - LIBOR + 5.015% = 5.115% on £100m
- Financial Institution makes profit of:
    - 0.03% (= 5.0155 – 4.985%)

3.2: Example: Converting Assets

- MSFT bought bonds of £100m which provide 4.7% p.a. for 3 years
- Receives LIBOR on **notional principal** of £100m from Intel
- Pays Intel 5% on **same principal**
- –
- MSFT Net Interest Rate Inflow on £100m:
    - 4.7% + LIBOR – 5% = LIBOR – 0.3%

3.3: Example: Converting Assets w/ Financial Institution

- MSFT bought bonds of £100m which provide 4.7% p.a. for 3 years
- Receives LIBOR on **notional principal** of £100m from Intel
- Pays Intel 5.015% via <u>Financial Institution</u> to Intel
- <u>Financial Institution</u> pays Intel 4.985% on £100m
- –
- MSFT Net Interest Rate Inflow on £100m:
    - 4.7% + LIBOR – 5.015% = LIBOR – 0.315%
- Financial Institution makes profit of:
    - 0.03% = 5.015% – 4.985%

4: Total Gains

- Total Gain form IRS = Difference in Fixed Rates – Difference in Floating Rates
    - Slide 14 for More

5: Credit Risk

- Suppose: *i* increases right after interest rate agreement begins
- **Floating rate** payer suffers **loss** and backs out
- Loss for **fixed-rate** payer limited to difference between fixed and floating rates
- Default of **floating-rate** payer relieves **fixed-rate** payer from obligation too

6: Forward Rate Agreement (FRA) Valuation

- $L =$ Principal Underlying the Contract
- $R_{FRA} = i$ Agreed in FRA
- $R_{FL} =$ Forward LIBOR for $T_1 - T_2$
- $r =$ Risk $-$ Free i

- Value of FRA is PV of difference between *i* paid at R_{FL} and *i* which would be paid at R_{FRA}
    - $V_{FRA} = L * (R_{FL} - R_{FRA}) * (T_2 - T_i)e^{-rT_2}$

7: Interest Rate Swap Valuation

- Initially: worth close to 0
- As time goes on: equal to difference between **fixed-rate** bond and **floating-rate**

- **Floating-rate** payer: $V_{swap} = B_{fix} - B_{float}$
- **Fixed-rate** payer: $V_{swap} = B_{float} - B_{fix}$

- Alternatively: valued as portfolio of <u>Forward Rate Agreements</u>

7.1: Example: w/ Microsoft

- LIBOR rate 5%
- Risk-free rate 4%

- $V_{FRA(MSFT)} > 0$: Forward Rate $> 5.0\%$
- $V_{FRA(MSFT)} = 0$: Forward Rate $= 5.0\%$
- $V_{FRA(MSFT)} < 0$: Forward Rate $< 5.0\%$

8: Overnight Index Swaps (OIS)

- **Fixed-rate** for a period is exchanged for geometric avg. of <u>Overnight</u> rates
- Allows overnight borrowing/lending swapped at **fixed-rate**
- **Fixed-rate** in OIS referred to as "<u>Overnight Swap Rate</u>"
- Bears risk that counterparty (another bank) will default
- To compensate: LIBOR > OIS
- Example: Slide 29

9: Currency Swaps

- Exchange principal and *i* payments in one currency for principal and *i* payments in another currency
    - Convert liability in one currency to a liability in another
    - Convert investment in one currency to investment in another
    - Quick, cheap, anonymous method of restructuring balance sheet

Lecture 6: Options Markets

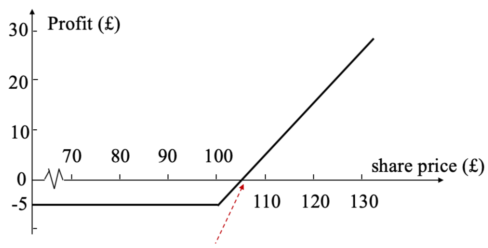
1: Options vs. Forwards/Futures

- Right to do something – does not have to exercise
- As opposed to commitment (forwards/futures)
- Forwards/futures do not require time 0 payments
- Options require time 0 payments

- Positions:
  - **Long Call**: buy right to buy *(bullish)*
  - **Long Put**: buy right to sell *(bearish)*
  - **Short Call**: sell right to buy *(bearish)*
  - **Short Put**: sell right to sell *(bullish)*

2: Long Call Option (1:4)

- C_0 = Call Option Price (e.g. £5)
- K = Strike Price (e.g. £100)
- "→" = Breakeven = C_0 + K



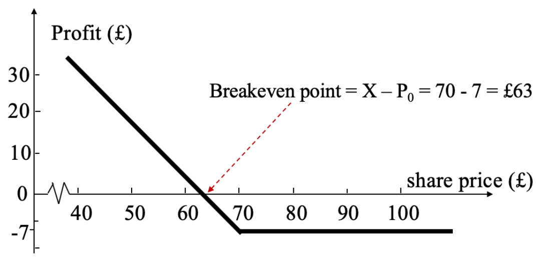
- Exercise only if share price at expiry is greater than strike price
  - If $S_T > K$: Exercise
  - If $S_T < K$: Don't Exercise

- **Example**: C = 8; K = 100
  - Profit/Loss = $\max\{S_T - K - C_0\}, - C_0$

  - Assume $S_T = 128$
    - P/L = max{128 – 100 – 8}, – 8
    - P/L = max{20, -8} = 20 so <u>Exercise</u>
  - Assume $S_T = 90$
    - P/L = max{90 – 100 – 8}, – 8
    - P/L = max{18, -8} = -8 so <u>Don't Exercise</u>
  - Assume $S_T = 105$
    - P/L = max{105 – 100 – 8}, – 8
    - P/L = max{-3, -8} = -3 so <u>Exercise to Minimise Loss</u>

3: Long Put Option (2:4)

- P_0 = Put Option Price (e.g. £7)
- K = Strike Price (e.g. £70)
- Breakeven (Shown). = K – P_0



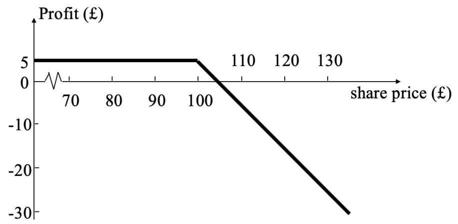
- Exercise if profit from selling asset through put is greater than market price
    - If $S_T < K$: Exercise
    - If $S_T > K$: Don't Exercise

- **Example**: P = 8; K = 100
    - Profit/Loss = $\max\{K - S_T - P_0\}, - P_0$

    - Assume $S_T = 128$
        - P/L = max{100 – 128 – 8}, – 8
        - P/L = max{-36, -8} = -8 so <u>Don't Exercise</u>
    - Assume $S_T = 90$
        - P/L = max{100 – 90 – 8}, – 8
        - P/L = max{2, -8} = 2 so <u>Exercise</u>
    - Assume $S_T = 98$
        - P/L = max{100 – 98 – 8}, – 8
        - P/L = max{-6, -8} = -6 so <u>Exercise to Minimise Loss</u>

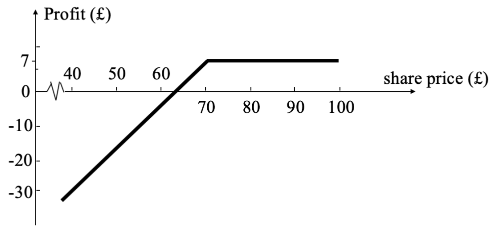
4: Short Call (3:4)

- C_0 = £5
- K = £100



5: Short Put (4:4)

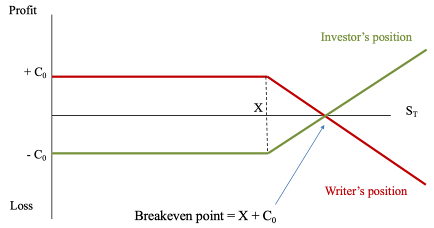
- P_0 = £7
- K = £70



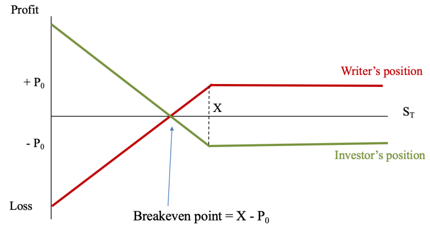
6: Call & Put Profitability

- Call Profitable: $S_T > (C_0 + K)$
- Put Profitable: $K > (P_0 + S_T)$

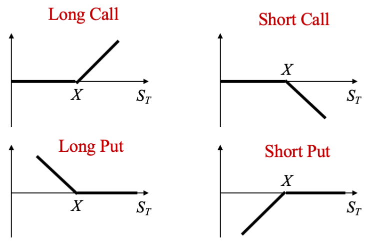
- Call Option Zero-Sum Game:



Put Options Zero-Sum Game:



- **Summary**:



7: Value At Expiration

- Call:
  - $S_T - K$ **if** $S_T > K$
  - 0 **if** $S_T \leq K$
- Put:
  - $K - S_T$ **if** $S_T < K$
  - 0 **if** $S_T \geq K$

8: Moneyness

- In-the-Money Option (profitable when):
  - Call: $S > K$
  - Put: $S < K$
- At-the-Money Option
  - $S = K$
- Out-of-the-Money Option

9: Intrinsic & Time Value

- Option Price (Premium) = Intrinsic Val. + Time Val.
- Intrinsic Value maximum of 0, option value would have it if instantly exercised
  - Call: $\max\{S - K, 0\}$
  - Put: $\max\{K - S, 0\}$
- Time Value difference between current option price and intrinsic value

10: Market Makers

- Quotes **bid** (price prepared to buy) and **ask** (price prepared to sell) prices
- **Bid-Offer Spread**: difference between **bid** and **ask**

11: Option-Like Securities

- Callable Bonds
- Convertible Bonds (and other securities)

Lecture 7: Properties of Stock Options

1: Notation & Introduction

- C_E = European Call Option Price
- P_E = European Put Option Price
- S_0 = Stock Price Today
- K = Strike (Exercise) Price
- T = Life of Option
- σ = std. dev of Stock (Volatility)
- C_A = American Call Option Price
- P_A = American Put Option Price
- S_T = Stock Price at Maturity
- D = PV of Future Divs
- r = Risk − Free Rate at T

1.1: Factors Affecting Option Prices

- Current stock price, S_0
- Strike price, K
- Time to expiration, T
- Volatility of stock price, σ
- Risk-free rate, r
- Dividends expected, D

1.2: Interaction Matrix

Variable	C_E	P_E	C_A	P_A
S_0	+	−	+	−
K	−	+	−	+
T	?	?	+	+
σ	+	+	+	+
r	+	−	+	−
D	−	+	−	+

1.3: American vs. European Options

- American worth at least as much as corresponding European
- $C_A \geq C_E$
- $P_A \geq P_E$

1.4: Upper Bounds

- **Call** can never be worth more than **underlying stock**
  - $S_0 \geq C_0$
- **Put** can never be worth more than **PV of K**
  - $PV(K) \geq P_0$

2: Call Values at T

	Investment	Value at T (In the Money) *E.g. S_T = £190 > K*	Value at T (Out of the Money) *E.g. S_T = £130 < K*
Portfolio A	Invest in S_0 = £140 **Total = S_0**	S_T = £190 **= £190**	S_T = £130 **= £130**
Portfolio B	Call (C_0 = £20) + PV of K (K = £150) **Total = C_0 + PV(K)**	*Exercise Call* $K - K + S_T$ **= £190**	*Do Not Exercise* K **= £150**

2.1: Lower Bound

- If folio B offers > folio A, worth more at t=0:
    - $C_0 + PV(K) \geq S_0$
    - $\therefore C_0 \geq S_0 - PV(K)$
    - $\therefore C_0 \geq S_0 - Ke^{-rT}$

3: Put Values at T

	Investment	Value at T (In the Money) *E.g. S_T = £130 < K*	Value at T (Out of the Money) *E.g. S_T = £190 > K*
Portfolio C	Put (P_0 = £30) Call (S_0 = £140) **Total = S_0**	*Exercise Put* K **= £150**	*Do Not Exercise* S_T **= £190**
Portfolio D	PV of K (K = £150) **Total = PV(K)**	K =150 **= £190**	K **= £150**

3.1: Lower Bound

- $P_0 + S_0 \geq PV(K)$
- $\therefore P_0 \geq PV(K) - S_0$
- $\therefore P_0 \geq Ke^{-rT} - S_0$

4: Put-Call Parity

- *"Since folio B gives equal payoffs at T as folio C, must have same payoffs at 0"*
    - $C_0 + PV(K) = P_0 + S_0$
    - Assume **continuous compounding**:
        - $\boxed{C_0 + Ke^{-rT} = P_0 + S_0}$

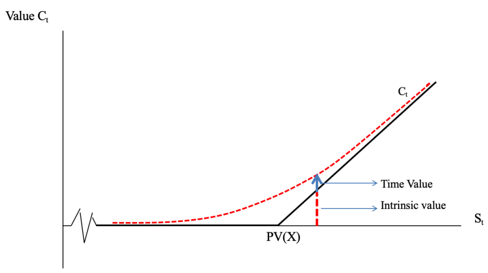
5: Early Exercise

- Chance w/ American option
- Exception where it is a non-div. paying stock (don't exercise early)
- Should **never** be exercised early if investor plans to hold stock for life of option

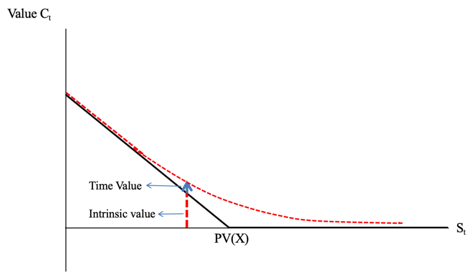
- **Do not** early exercise if:
    - No income is sacrificed
    - You delay paying the strike price
    - Holding the call provides insurance against S_0 falling below K

6: Graphical Representation

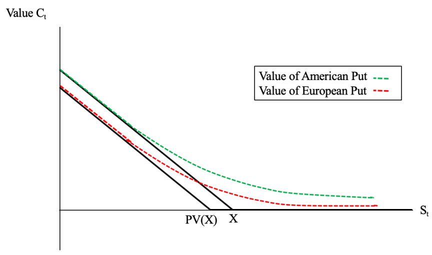
- European Call:



European Put:



- American Put:

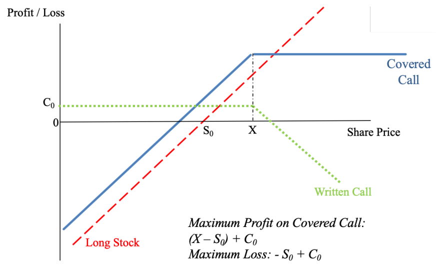


Lecture 8: Trading Strategies w/ Stock Options

- Strategies
  - Option plus underlying asset
  - Two or more options – same type (**Spread**)
  - Two or more options – different type (**Combination**)

1: Covered Call

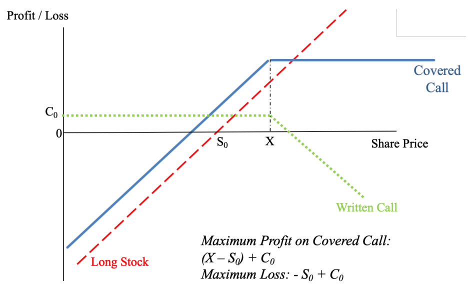
- Write call, invest in underlying asset (hold share, write call)
- Controls risk exposure – call writers
- Earn premium on holdings by giving up potential high returns – share holders



- Profit /Loss
  - Exceeds profit on simple investment for all prices up to exercise (plus call value)
  - Combination – lower profit than would be seen holding normal share for prices exceeding exercise (plus call value)
- Note: short stack and long call payoffs (Slide 6)

2: Protective Put

- Insurance against **share price fall**
  - P_0 = Put Premium
  - X – Exercise Price
  - S_0 = Share Price at 0
- If S falls below S_0, share can be sold through <u>Put Option</u> to realise ($X = S_0$)
- Investor benefits: any increase in S above S_0, guaranteed min. payoff ($S_0 – P_0$)



- Note: short stack and short put payoffs (Slide 9)

3: Spreads

- <u>Calls</u> or <u>Puts</u>
  - Bull Spreads
  - Bear Spreads
  - Box Spreads
  - Butterfly Spreads

3.1: Bull Spread

- Limits up/downside risk
- Three types:
  - Both options are **out of the money** initially
  - One is **in the money** and one **out the money** initially
  - Both options **in the money** initially
- *"Investor expects S **Increase**"*

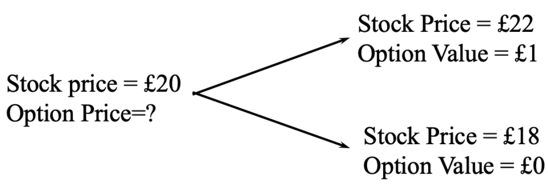
Lecture 9: Option Pricing

- **Binomial Tree**: representation of different Option paths as probabilities
  - Assumes – stock prices follow 'random walk'
  - Assumes – no arbitrage opportunity
  - As steps get smaller, model converges to Black & Scholes Merton

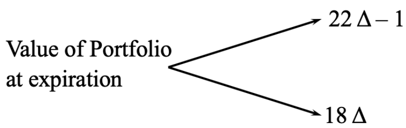
1: One-Step The Binomial Tree

1.1: Simple Tree

- <u>Call Option</u>: *"Value of European Call w/ Exercise of £21?"*



- Riskless Folio:
  - Long Position – Change in Shares
  - Short Position – 1 Call Option



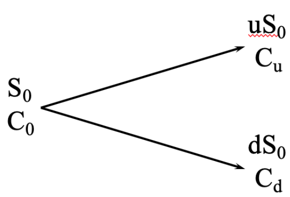
  - Riskless when: $(22\Delta - 1) = 18\Delta$
  - **Example**:
    - Stock Price = £22; $(22\Delta - 1) = 22 * 0.25 - 1 = £4.50$
    - Stock Price = £18; $18\Delta = 18 * 0.25 = £4.5$
    1) *// Riskless Folio – Earns 12% //*
    - $\boldsymbol{V_0 = 4.5e^{-0.12*0.25} = 4.367}$ $\{t = 3; T = 0.25\}$
    2) *// Value of Call Option – Stock today recall = £20 //*
    - $20\Delta - C_0 = 4.367$
    - $\boldsymbol{C_0 = 0.633}$

1.2: Notation

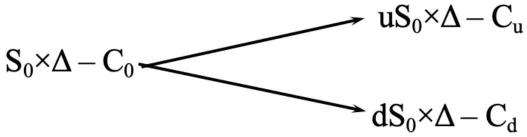
- uS_0 = Value of Stock if Increase
- dS_0 = Value of Stock if Decrease
- C_u = Value of Call Option if Increase
- C_d = Value of Call Option if Decrease

- Where: $\{u > 1; 0 < d < 1\}$

- **Generalise →**



1.3: Generalised Form

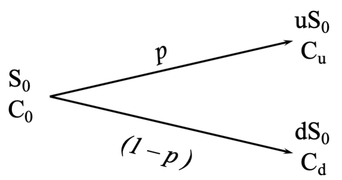


- Riskless When: $uS_0\Delta - C_u = dS_0 - C_d$

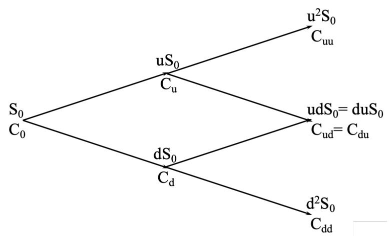
$$\boxed{\boldsymbol{\Delta = \frac{C_u - C_d}{uS_0 - dS_0}}}$$

"Ratio of Option Price change to Stock Price change"

- Value of Folio (at *T*): $uS_0\Delta - C_u$
- Value of Folio (at *T*, When Riskless): $(uS_0\Delta - C_u)e^{-rT}$
- Value of Folio (at 0): $S_0\Delta - C_0$
- **Value of Call (at 0)**: $\boldsymbol{C_0 = S_0\Delta - (uS_0\Delta - C_u)e^{-rT}}$
  - *// Substitute Δ //*
  - $C_0 = (C_u(p - C_u)(1-p))e^{-rT}$
    - $p = \frac{e^{-rT}-d}{u-d} =$ Risk Neutral Probability of Up Movement
    - $(1-p) =$ Risk Neutral Probability of Down Movement



2: Two-Step Binomial Tree



$$C_0 = ((p^2 * C_{uu}) + (2p(1-p) * C_{ud}) + (1-p)^2 * C_{dd})e^{-2r\Delta\mathrm{t}}$$
$$P_0 = ((p^2 * P_{uu}) + (2p(1-p) * P_{ud}) + (1-p)^2 * P_{dd})e^{-2r\Delta\mathrm{t}}$$

$$p = \frac{e^{r\Delta t} - d}{u - d}$$

3: Girsanov's Theorem

- σ^2 is the **same** through real-world and risk-neutral-worlds
- Can therefore measure σ^2 in real-world and use it in risk-neutral

3.1: Choosing u & d

- $u = e^{\sigma\sqrt{\Delta t}}$
- $d = e^{-\sigma\sqrt{\Delta t}} = \frac{1}{u}$

4: Probability of An Upwards Move

$$p = \frac{a - d}{u - d}$$

- $a = e^{r\Delta t}$: Non-Div. Stock
- $a = e^{(r-y)\Delta t}$: Div. Stock
- $a = e^{\left(r-r_f\right)\Delta t}$: Foreign Risk-Free Rate
- $a = 1$: Futures Contract

Lecture 10: Black & Scholes Model

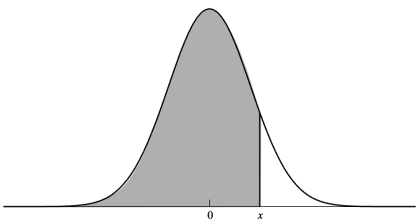
- Function of **underlying asset** and **time**
- Two parameters: **risk-free rate** and **asset volatility**
- *"The history of the variable is irrelevant"*
    - Short-selling permitted
    - No transaction costs
    - No risk arbitrage
    - R_f and volatility are constant
    - No dividends from underlying assets

1: Notation & Definitions

- C_o = Call Option Value
- P_o = Put Option Value
- S_o = Current Stock Price
- X = Exercise Price
- r = Annual Risk Free Rate
- T = Annual Time to Maturity
- σ = Annual Standard Deviation

1.1: Normal Probability

- *N(x)* is probability that a **normally distributed** variable w/ mean of 0 and std.dev of 1, is less than *x*



2: Black & Scholes: Call Option

2.1: Call Option

$$\boxed{\boldsymbol{C_0 = S_0 N(d_1) - X e^{-rT} N(d_2)}}$$

$$d_1 = \frac{\ln\left(\frac{S_0}{X}\right) + T(r + \frac{\sigma^2}{2})}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

- $N(d_1)$: risk-adjusted probability that <u>Call Option</u> will expire **in-the-money**
- $\ln\left(\frac{S}{X}\right)$: percentage amount by which <u>Call Option</u> is **in/out-of-the-money**
    - **Example**: *S* = 160, *X* = 100; Option is 60% **in-the-money** at 0.47
    - **Example**: *S* = 90; *X* = 100; Option is 10% **out-of-the-money** at -0.105

- Hence, when $N(d)$ is close to 1, there's near a 100% chance the Call will be exercised
    - If $N(d_1)$ and $N(d_2)$ are 1, Call Value will be approx. its intrinsic vlue

- When there's a yield on the underlying asset:

$$\boxed{\boldsymbol{C_0 = S_0 e^{-(r-y)T} N(d_1) - X e^{-rT} N(d_2)}}$$

2.2: Put Option

- The **Put-Call Parity**

$$\boxed{\boldsymbol{P_0 = X e^{-rT}(1 - N(d_1)) - S_0(1 - N(d_2))}}$$

2.3: Properties of Model

- As S_0 gets large: C_0 tends towards $(S_0 - Xe^{-rT})$; P_0 tends to 0
- As S_0 gets small: C_0 tends towards 0; P_0 tends to $(Xe^{-rT} - S_0)$
- As σ tends to 0: C_0 tends towards $\max\{S_0 - Xe^{-rT}, 0\}$
  C_0 tends towards $\max\{Xe^{-rT} - S_0, 0\}$

- If including **div.**, they should be the ex-div. rather than div.
    - Should be the E(reduction in S)
    - Short-Life Options: estimate div. during option life w/ decent accuracy
    - Long-Life Options: estimate div. w/ uncertainty – option pricing difficult

3: Volatility

- Estimated from historical data – suing daily R's over several months
- Volatility greater when market open
- Thus, usually measured in **trading days** rather than calendar days (252 days/yr.)
    - **Example**: April 1st → April 30th: 22 t-days: T = (22/252 = 0.0873yr.)

3.1: Implied Volatility

- σ for which Black-Scholes price = Market Price
- *Forward-looking (where normal volatility is backward)*
    - *High corr. w/ financial crisis, known as "investor fear gauge"*
- One-to-one corr. between prices and **implied volatility**
- If actual $\sigma > \sigma_{Implied}$: Option good to buy
- If actual $\sigma > \sigma_{Implied}$: Option price lower than observed

3.2: Estimated Volatility Incorrect?

- Higher σ: increases option prices; Lower σ: decreases option prices
- May be B-S biases
- Mkt. prices may be incorrect – e.g. market inefficiency
    - If market **out** of equilibrium **arbitrage** exists

4: Option Payoff at Expiration

- $X > S_T$:
  - Put exercised to receive $(\boldsymbol{X - S_T})$
  - Call not exercised
- $X < S_T$:
  - Put not exercised
  - Call exercised to receive $-(S_T - X) = (\boldsymbol{X - S_T})$

4.1: Overall Position at Expiration

- Share Value: (+) S_T
- FV of Div.: (+) $FV(D)$
- Option Payoff: (+) $(X - S_T)$
- Repay Loans: (–) $(FV(D) - X)$
- **Net:** **0**

5: Arbitrage & Biases in the Model

- Mispricing should be temporary – market should return to equilibrium
- Arbitrageurs don't need to hold until maturity
  - Buy puts at low then when equilibrium is reached, sell at high

- Based on the European option – doesn't account for early exercise however, American Calls on non-div. never early-exercise so basically same as European
- Early Put more common – div. or not. If sufficiently **in-the-money**, w/ long $(T\text{-}t)$, may be beneficial to early-exercise