University of Strathclyde, Department of Mathematics and Statistics

MM102 Applications of Calculus
Exercises for Week 3

1. Sketch the finite region bounded by following curves and the x-axis. Hence find the volume generated when this region is rotated through 360° about the **x-axis**:

   (a) $y = \sqrt{4 + 3x - x^2}$;

   (b) $y = 2x + 1, \qquad x = 0, \qquad x = 2$;

   (c) $y = \sin x, \qquad x = 0, \qquad x = \pi$.

2. Sketch the finite region bounded by the following pairs of curves and find the points of intersection. Hence find the volume generated when this region is rotated through 360° about the **x-axis**:

   (a) $y = x^2 + 1, \qquad y = 3 - x$;

   (b) $y = x^2 - 4x + 6, \qquad y = 4x - x^2$;

   (c) $y = x^2, \qquad y = \sqrt{x}$;

   (d) $y = 2x + 3, \qquad y = x^2$.

3. Sketch the finite region bounded by the following curves. Hence find the volume generated when this region is rotated through 360° about the **y-axis**:

   (a) $y = x^2 - 3x + 4, \qquad y = 0, \qquad x = 1, \qquad x = 3$;

   (b) $y = x + 1, \qquad y = 0, \qquad x = 1, \qquad x = 2$;

   (c) $y = \sin x, \qquad y = 0, \qquad x = 0, \qquad x = \pi$;

   (d) $y = \dfrac{1}{x}, \qquad y = 0, \qquad x = 1, \qquad x = 2$;

   (e) $y = x^2 + 1, \qquad y = -x^2 - 1, \qquad x = 0, \qquad x = 1$.

4. Find the arc length of the following curves:

   (a) $y = \dfrac{1}{8}x^2 - \ln x, \qquad x \in [1, 4]$;

   (Note that the expression under the square root that appears in the integral is a complete square.)

   (b) $y = e^x, \qquad x \in \left[0, \dfrac{1}{2}\ln 3\right]$;

   (Hint: for the integral use the substitution $u = \sqrt{1 + e^{2x}}$, i.e. $x = \dfrac{1}{2}\ln(u^2 - 1)$. Use the latter relation to obtain the connection between $\mathrm{d}x$ and $\mathrm{d}u$.)

(c) $y = 2x^{3/2}, \qquad x \in [0,1]$;

(d) $y = \ln(\cos x), \qquad x \in \left[0, \frac{\pi}{4}\right]$.

(Hint: for the integral, use the relation $\sec^2 x = \tan^2 x + 1$.)

5. Find the surface area when the following curve is rotated through 360° about the x-axis:

(a) $y = \sqrt{2x+1}, \qquad x \in [1,7]$;

(b) $y = \sqrt{x}, \qquad x \in [0,1]$.

(Hint for (a) and (b): write the integrand as a single square root.)