UNIVERSITY OF STRATHCLYDE
DEPARTMENT OF MATHEMATICS AND STATISTICS

MM101 Introduction to Calculus

Exercises: Chapter 3

1. Write out each of the following statements using the appropriate mathematical symbols.

(i) x is 7 implies that the square of x is 49.

(ii) If y is 90 then the cosine of y° is zero.

(iii) The cube of x is 64 if and only if x is 4.

(iv) The product of two numbers a and b is zero if and only if a is zero or b is zero.

(v) x to the power 4 is 16 is equivalent to x is 2 or x is -2.

2. Insert an appropriate symbol ('=', '$\Rightarrow$' or ' $\Longleftrightarrow$ ') into the circle to produce a true statement.

(i) $\dfrac{10}{6} \quad \bigcirc \quad \dfrac{5}{3}$. (ii) $x = \dfrac{10}{6} \quad \bigcirc \quad x = \dfrac{5}{3}$.

(iii) $2x = 6 \quad \bigcirc \quad x = 3$. (iv) $x = 4 \quad \bigcirc \quad x^2 = 16$.

(v) $x^2 = 25 \quad \bigcirc \quad x = \pm 5$. (vi) $y^3 = 8 \quad \bigcirc \quad y = 2$.

(vii) $y^3 = -8 \quad \bigcirc \quad y = -2$. (viii) $y^3 = \pm 8 \quad \bigcirc \quad y = \pm 2$.

3. Indicate whether each of the following statements is true or false. Justify your answer logically.

(i) $x^2 = 4 \Leftrightarrow x = 2$ (ii) $\log_4 x = 2 \Leftarrow x = 16$

(iii) $x^2 + y \leq 5 \Rightarrow y \leq 5$ (iv) $y \geq 0$ and $x \geq 0 \Leftrightarrow x + y \geq 0$

(v) $\log_{10} x = 2 \Leftrightarrow x = 100$ (vi) $(x+y)^2 = 4 \Leftarrow x = -2 - y$

(vii) $x \leq 1 \Rightarrow x^2 \leq 1$ (viii) $2^x = 4 \Leftrightarrow x = 2$.

4. What is wrong with the following 'proof'?
Let $x = y$. Then

$$\begin{aligned} x^2 &= xy \\ \Rightarrow x^2 - y^2 &= xy - y^2 \\ \Rightarrow (x+y)(x-y) &= y(x-y) \\ \Rightarrow x + y &= y \\ \Rightarrow 2y &= y \\ \Rightarrow 2 &= 1. \end{aligned}$$

5. Show that the product of the first n even positive integers is $2^n(n!)$. [*Hint: you can do it in one line.*]

6. Using your answer to the previous question, or otherwise, show that the product of the first n odd positive integers is
$$\frac{(2n)!}{2^n(n!)}.$$

7. Use the method of proof by induction to show that $(1+x+x^2+\ldots+x^{n-1})(1-x) = 1-x^n$, for all positive integers n.

8. Prove by induction that
$$1 \times 3 + 3 \times 5 + 5 \times 7 + \ldots + (2n-1)(2n+1) = n(4n^2+6n-1)/3.$$

9. Prove by induction that
$$\frac{1}{1 \times 4} + \frac{1}{4 \times 7} + \frac{1}{7 \times 10} + \frac{1}{10 \times 13} + \ldots$$
up to n terms is $n/(3n+1)$.