UNIVERSITY OF STRATHCLYDE
DEPARTMENT OF MATHEMATICS AND STATISTICS

MM101 Introduction to Calculus

Examples 5A

1 If $P(x) = x^3 - 2x^2 - x + 1$, evaluate

$$\text{(i)} \quad P(2), \quad \text{(ii)} \quad P(-x), \quad \text{(iii)} \quad P\left(\frac{1}{x}\right), \quad \text{(iv)} \quad P(x+1).$$

$$\begin{aligned}
P(2) &= 2^3 - 2.2^2 - 2 + 1 = -1 \\
P(-x) &= (-x)^3 - 2(-x)^2 - (-x) + 1 \\
&= -x^3 - 2x^2 + x + 1 \\
P\left(\tfrac{1}{x}\right) &= \frac{1}{x^3} - \tfrac{2}{x^2} - \tfrac{1}{x} + 1 \\
P(x+1) &= (x+1)^3 - 2(x+1)^2 - (x+1) + 1 \\
&= x^3 + 3x^2 + 3x + 1 - (2x^2 + 4x + 2) - (x+1) + 1 \\
&= x^3 + x^2 - 2x - 1
\end{aligned}$$

Examples 5B

1 Let $p(x) = 3x^3 - x + 1$ and $q(x) = 2x^2 + x - 1$. Evaluate their sum and product.

$$p(x) + q(x) = (3x^3 - x + 1) + (2x^2 + x - 1) = 3x^3 + 2x^2.$$

To evaluate $p(x)q(x)$:

$$\begin{array}{lrrrrrr}
p(x) & & & 3x^3 & +0x^2 & -x & +1 \\
q(x) & & & & 2x^2 & +x & -1 \\
\hline
 & & & -3x^3 & +0x^2 & +x & -1 \\
 & & 3x^4 & +0x^3 & -1x^2 & +x & +0 \\
 & 6x^5 & +0x^4 & -2x^3 & +2x^2 & +0x & +0 \\
\hline
 & 6x^5 & +3x^4 & -5x^3 & +x^2 & +2x & -1
\end{array}$$

Hence $p(x)q(x) = 6x^5 + 3x^4 - 5x^3 + x^2 + 2x - 1$.

2 Divide $3x^4 + 4x^3 - 5x^2 + 2x + 1$ by $x^2 - 2x - 3$, identifying the quotient and the remainder in your answer.

$$\begin{array}{rrr|rrrrr}
 & & & & & 3x^2 & +10x & +24 \\
\hline
x^2 & -2x & -3 & 3x^4 & +4x^3 & -5x^2 & +2x & +1 \\
 & & & 3x^4 & -6x^3 & -9x^2 & & \\
 & & & & 10x^3 & +4x^2 & +2x & \\
 & & & & 10x^3 & -20x^2 & -30x & \\
 & & & & & 24x^2 & +32x & +1 \\
 & & & & & 24x^2 & -48x & -72 \\
\hline
 & & & & & & 80x & +73
\end{array}$$

Hence, we have

$$\frac{3x^4 + 4x^3 - 5x^2 + 2x + 1}{x^2 - 2x - 3} = 3x^2 + 10x + 24 + \frac{80x + 73}{x^2 - 2x - 3}.$$

The quotient is therefore $3x^2 + 10x + 24$ and the remainder is $80x + 73$.

[Note that we have essentially worked out that

$$3x^4 + 4x^3 - 5x^2 + 2x + 1 - (x^2 - 2x - 3)(3x^2 + 10x + 24) = 80x + 73,$$

which rearranges to the equation above.]

Examples 5C

1 Sketch the graph of $f(x) = 4x + 8$ and solve $f(x) = 0$.

Graph is straight line with slope 4 through $(0, 8)$. It cuts the x-axis at $x = -2$.

$$f(x) = 0 \Leftrightarrow 4x + 8 = 0 \Leftrightarrow 4x = -8 \Leftrightarrow x = -2.$$

So the solution set is $\{-2\}$.

2 Solve the equation $5x + 17 = 4x - 3$.

$$5x + 17 = 4x - 3 \Leftrightarrow x = -3 - 17 = -20$$

so the solution set is $\{-20\}$.

Examples 5D

1 Solve the following quadratic equations.

$$\text{(i)} \quad x^2 + 5x - 14 = 0, \qquad \text{(ii)} \quad 12x^2 - 5x = 2.$$

(i) 14 has factors 2 and 7 so

$$x^2 + 5x - 14 = 0 \Leftrightarrow (x-2)(x+7) = 0 \Leftrightarrow x = 2 \text{ or } x = -7.$$

So the solution set is $\{-7, 2\}$.

(ii) Rearrange to get $12x^2 - 5x - 2 = 0$. Then 12 has factors 2 and 6 or 3 and 4, and 2 has factors 2 and 1, so trial and error gives

$$12x^2 - 5x - 2 = 0 \Leftrightarrow (3x-2)(4x+1) = 0 \Leftrightarrow x = \frac{2}{3} \text{ or } x = -\frac{1}{4}.$$

So the solution set is $\left\{-\frac{1}{4}, \frac{2}{3}\right\}$.

Examples 5E

1 Solve
$$x^2 + 2x - 2 = 0.$$

This quadratic does not factorise. Complete the square:
$$x^2 + 2x - 2 = [(x+1)^2 - 1] - 2 = (x+1)^2 - 3$$
so
$$x^2 + 2x - 2 = 0 \Leftrightarrow (x+1)^2 - 3 = 0 \Leftrightarrow (x+1)^2 = 3.$$
Taking the square root of both sides gives
$$x + 1 = \pm\sqrt{3} \Rightarrow x = -1 \pm \sqrt{3}.$$
So the solution set is $\left\{-1 - \sqrt{3}, -1 + \sqrt{3}\right\}$.

[Note that we could have used the formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to get
$$\frac{-2 \pm \sqrt{4+8}}{2} = \frac{-2 \pm \sqrt{12}}{2} = -1 \pm \sqrt{3}.]$$

2 Solve
$$x^2 + 2x + 2 = 0.$$

This quadratic does not factorise. Complete the square:
$$x^2 + 2x + 2 = [(x+1)^2 - 1] + 2 = (x+1)^2 + 1$$
so
$$x^2 + 2x + 2 = 0 \Leftrightarrow (x+1)^2 + 1 = 0 \Leftrightarrow (x+1)^2 = -1.$$
This quadratic equation therefore has no real roots.

(Note that using the quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to get
$$\frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2}$$
leads to the same conclusion.)

Examples 5F

1 What is the remainder when $p(x) = x^3 - 4x^2 + 5x - 3$ is divided by $x - 2$?

By the Remainder Theorem, when $p(x)$ is divided by the linear factor $x-2$, the remainder is

$$p(2) = 2^3 - 4 \times 2^2 + 5 \times 2 - 3 = 8 - 16 + 10 - 3 = -1.$$

[Checking this using long division shows that

$$\frac{x^3 - 4x^2 + 5x - 3}{x - 2} = x^2 - 2x + 1 - \frac{1}{x-2}.]$$

2 If $P_n(x) = x^3 - 2x^2 - x + 2$, verify that $P_n(-1) = P_n(1) = P_n(2) = 0$. Hence factorise $P_n(x)$.

Substituting $x = -1$, $x = 1$ and $x = 2$ into $P_n(x)$ in turn gives zero in each case. So by the Factor Theorem we can write

$$P_n(x) = (x+1)(x-1)(x-2).$$

Examples 5G

1 Factorise $P_4(x) = x^4 - 4x^3 - 4x^2 - 4x - 5$.

We first 'guess' a root by trying divisors of the constant term (up to $\pm$).
Try $x = 1 : P_4(1) = 1 - 4 - 4 - 4 - 5 \neq 0$ so 1 is not a zero of P_4 and $x - 1$ is not a factor.

Try $x = -1 : P_4(-1) = 1 + 4 - 4 + 4 - 5 = 0$ so -1 is a zero of P_4 and $x + 1$ is a factor.

Divide out the factor:

$$
\begin{array}{cc|ccccc}
 & & & x^3 & -5x^2 & +x & -5 \\
\hline
x & +1 & x^4 & -4x^3 & -4x^2 & -4x & -5 \\
 & & x^4 & +x^3 & & & \\
 & & & -5x^3 & -4x^2 & -4x & -5 \\
 & & & -5x^3 & -5x^2 & & \\
 & & & & x^2 & -4x & -5 \\
 & & & & x^2 & +x & \\
 & & & & & -5x & -5 \\
 & & & & & -5x & -5 \\
\hline
 & & & & & & 0
\end{array}
$$

We see that

$$P_4(x) = (x+1)(x^3 - 5x^2 + x - 5).$$

We now repeat the process, looking for factors of

$$P_3(x) = x^3 - 5x^2 + x - 5.$$

(Note that there is no point in trying $x = 1$, as we've already checked that!)

Try $x = -1$: $P_3(x) = -1 - 5 - 1 - 5 \neq 0$ so -1 is not a zero of P_3 (i.e. it is not a repeated zero of P_4).

Try $x = 5$, gives $P_3(5) = 5^3 - 5^3 + 5 - 5 = 0$ so 5 is a zero of P_3 and $x - 5$ is a factor.

Divide out the factor:

$$\begin{array}{cc|ccccc}
 & & & x^2 & & +1 & \\
\hline
x & -5 & x^3 & -5x^2 & +x & -5 & \\
 & & x^3 & -5x^2 & & & \\
 & & & & x & -5 & \\
 & & & & x & -5 & \\
\hline
 & & & & & & 0
\end{array}$$

That is,

$$P_3(x) = (x-5)(x^2+1)..$$

As $x^2 + 1$ is irreducible, we cannot factorise any further. We therefore have

$$P_4(x) = (x+1)(x-5)(x^2+1).$$

Examples 5H

1 Find the values of a, b, c, so that

$$x^2 - 27x + 18 = a(x-1)(x-2) + b(x-2)(x-3) + cx(x+2)$$

for all values of x.

We have

$$x^2 - 27x + 18 = a(x^2 - 3x + 2) + b(x^2 - 5x + 6) + c(x^2 + 2x). \qquad (*)$$

Equate coefficients:

$$\begin{aligned}
\text{constant term:} \quad & 18 = 2a + 6b + 0 \\
x \text{ term:} \quad & -27 = -3a - 5b + 2c \\
x^2 \text{ term:} \quad & 1 = a + b + c
\end{aligned}$$

These three equations can be solved to give $a = 3, b = 2, c = -4$ (check this). However, we can avoid solving the equations by observing that $(*)$ holds for all values of x, so we can simply substitute some particular values of x instead. For example, here we have

$$\begin{aligned}
x = 2: \quad & 2^2 - 27 \times 2 + 18 = c \times 2 \times 4 \Rightarrow -32 = 8c \Rightarrow c = -4 \\
x = 1: \quad & 1 - 27 + 18 = b(-1)(-2) + c(1)(3) \Rightarrow -8 = 2b + 3c = 2b - 12 \Rightarrow b = 2 \\
x = 0: \quad & 18 = a(-1)(-2) + b(-2)(-3) \Rightarrow 18 = 2a + 6b = 2a + 12 \Rightarrow a = 3
\end{aligned}$$

2 Find the quadratic polynomial that takes the values -5 when $x = 1$, -1 when $x = -1$ and 2 when $x = 2$.

Since there are three points, the quadratic will be uniquely defined, so let

$$P_2(x) = ax^2 + bx + c.$$

Substituting in the given values gives

$$\begin{aligned}
x = 1: \quad & -5 = a + b + c \\
x = -1: \quad & -1 = a - b + c \\
x = 2: \quad & 2 = 4a + 2b + c.
\end{aligned}$$

These equations can be solved to give $a = 3, b = -2, c = -6$ (check this!). Hence the required quadratic is

$$P_2(x) = 3x^2 - 2x - 6.$$

Examples 5I

1 Solve $7x - 3 \geq 4$ for x.

$$\begin{aligned}
& 7x - 3 && \geq 4 \\
\text{add 3 to both sides} \quad & \Rightarrow 7x && \geq 7 \\
\text{multiply both sides by } \tfrac{1}{7} \quad & \Rightarrow x && \geq 1.
\end{aligned}$$

So the solution set is $\{x \in \mathbb{R} | x \geq 1\}$ or, in interval notation, $[1, \infty)$.

2 Solve $2x - 3 < 5x + 2$.

$$\begin{aligned}
& 2x - 3 && < 5x + 2 \\
\text{add } -5x + 3 \text{ to both sides} \quad & \Rightarrow -3x && < 5 \\
\text{multiply both sides by } -\tfrac{1}{3} \quad & \Rightarrow x && > -\tfrac{5}{3}.
\end{aligned}$$

So the solution set is $\left\{x \in \mathbb{R} | x > -\frac{5}{3}\right\}$ or, in interval notation, $\left(-\frac{5}{3}, \infty\right)$.

Examples 5J

1 Solve $x^2 \leq 1$.

Step 1: Rearrange to get zero on RHS:

$$x^2 \leq 1 \iff x^2 - 1 \leq 0.$$

Step 2: Factorise LHS:

$$x^2 - 1 \leq 0 \iff (x-1)(x+1) \leq 0.$$

Step 3: Analyse signs. Construct a table:
Note that the factors change sign at $x = -1$ and $x = 1$.

x values	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
$x - 1$	-	-	-	0	+
$x + 1$	-	0	+	+	+
$(x-1)(x+1)$	+	0	-	0	+

Now identify the regions where the expression is less than or equal to zero, that is, $\{x \in \mathbb{R} | -1 \leq x \leq 1\}$ (or $[-1, 1]$).

2 Solve $x^2 - 5x > -6$.

Step 1: Rearrange to get zero on RHS:

$$x^2 - 5x > -6 \iff x^2 - 5x + 6 > 0.$$

Step 2: Factorise LHS:

$$x^2 - 5x + 6 > 0 \iff (x-3)(x-2) > 0.$$

Step 3: Analyse signs. Construct a table:
Note that the factors change sign at $x = 2$ and $x = 3$.

x values	$x < 2$	$x = 2$	$2 < x < 3$	$x = 3$	$x > 3$
$x - 3$	-	-	-	0	+
$x - 2$	-	0	+	+	+
$(x-3)(x-2)$	+	0	-	0	+

Now identify the regions where the expression is greater than 0, that is, $\{x \in \mathbb{R} | x < 2 \text{ or } x > 3\}$ (or $(-\infty, 2) \cup (3, \infty)$).

3 Solve
$$\frac{1}{x-1} < \frac{1}{x+1}.$$

Noitce that if we simply multiply out the denominators we get $x+1 < x-1$ which is equivalent to $1 < -1$ suggesting the inequality can never be satisfied. But letting $x = 0$ gives $-1 < 1$, which is clearly true. The problem comes from the fact that when we multiply through we do not know whether quantities are negative or positive. We must follow the procedure outlined in the notes.

Step 1: Rearrange to get zero on RHS:
$$\frac{1}{x-1} - \frac{1}{x+1} < 0.$$

Step 2: Simplify LHS:
$$\frac{1}{x-1} - \frac{1}{x+2} = \frac{x+1-(x-1)}{(x-1)(x+1)} = \frac{2}{(x-1)(x+1)} < 0.$$

Step 3: Analyse signs.
Construct a table of signs, noting that the factors change sign at $x = -1$, and $x = 1$.

x values	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
$x+1$	-	0	+	+	+
$x-1$	-	-	-	0	+
$\frac{2}{(x-1)(x+1)}$	+	nd	-	nd	+

(here 'nd' means 'not defined'). So the solution set is $\{x \in \mathbb{R} | -1 < x < 1\}$ (or $(-1, 1)$).

4 Solve
$$\frac{1}{2x-1} > \frac{1}{x-2}.$$

Step 1: Rearrange to get zero on RHS:
$$\frac{1}{2x-1} - \frac{1}{x-2} > 0.$$

Step 2: Simplify LHS:
$$\frac{1}{2x-1} - \frac{1}{x-2} = \frac{x-2-2x+1}{(2x-1)(x-2)} = -\frac{(x+1)}{(2x-1)(x-2)} > 0$$

or, equivalently,
$$\frac{(x+1)}{(2x-1)(x-2)} < 0.$$

Step 3: Analyse signs.
Construct a table of signs, noting that the factors change sign at $x = -1$, $x = \frac{1}{2}$ and $x = 2$.

x values	$x<-1$	$x=-1$	$-1<x<\frac{1}{2}$	$x=\frac{1}{2}$	$\frac{1}{2}<x<2$	$x=2$	$x>2$
$x+1$	-	0	+	+	+	+	+
$2x-1$	-	-	-	0	+	+	+
$x-2$	-	-	-	-	-	0	+
$\dfrac{(x+1)}{(2x-1)(x-2)}$	-	0	+	nd	-	nd	+

So the solution set is $\{x \in \mathbb{R} | x < -1 \text{ or } 1/2 < x < 2\}$ (or $(-\infty, -1) \cup (\frac{1}{2}, 2)$).

Examples 5K

1 Find x if $|2-x| = 4$.

Solutions are $2-x=4$ and $-(2-x)=4 \iff 2-x=-4$ so $x=-2$ or $x=6$.

2 Find all the values of $x \in \mathbb{R}$ that satisfy the inequality $|x-1| < 2$.

This inequality says that

$$x-1 > -2 \qquad \text{and} \qquad x-1 < 2$$

so

$$x > -1 \qquad \text{and} \qquad x < 3,$$

that is, the solution interval is $(-1, 3)$. [**Note:** This is also the solution interval of $|1-x| < 2$.]

3 Solve the inequality $|2x+3| \geq 4$.

This inequality says that

$$2x+3 \leq -4 \qquad \text{or} \qquad 2x+3 \geq 4$$

so

$$2x \leq -7 \qquad \text{or} \qquad 2x \geq 1$$

and

$$x \leq -\frac{7}{2} \qquad \text{or} \qquad x \geq \frac{1}{2}.$$

Solution interval is therefore

$$\left(-\infty, -\frac{7}{2}\right] \cup \left[\frac{1}{2}, \infty\right).$$

Examples 5L

1 Solve the linear equations

$$\begin{aligned} 3x + 4y &= 5 \\ 5x - 3y &= 18 \end{aligned} .$$

Label the equations (1) and (2). Then we have

$$(1) \times 3: \quad 9x + 12y = 15, \qquad (2) \times 4: \quad 20x - 12y = 72$$

so adding these gives

$$29x = 87 \quad \Longleftrightarrow \quad x = 3.$$

Substituting this into (1) gives $y = -1$. So the solution set is $\{(3, -1)\}$.

2 Solve the nonlinear equations

$$\begin{aligned} x^2 + y^2 &= 2 \\ x^2 - y &= 0 \end{aligned} .$$

The second equation gives $y = x^2$. Substituting this into the first gives

$$x^2 + x^4 = 2 \iff x^4 + x^2 - 2 = 0 \iff (x^2 - 1)(x^2 + 2) = 0 \iff x = \pm 1.$$

So the solution set is $\{(1, 1), (-1, 1)\}$.

3 Solve the nonlinear equations

$$\begin{aligned} x^2 - 4y^2 &= 0 \\ (x - 1)(y + 2) &= 0. \end{aligned}$$

The second equation gives $x = 1$ or $y = -2$.
For $x = 1$, the first equation gives $4y^2 = 1 \Leftrightarrow y = \pm\dfrac{1}{2}$.
For $y = -2$, the first equation gives $x^2 = 16 \Leftrightarrow x = \pm 4$.

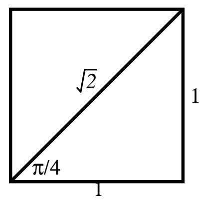
Hence there are four solutions: the solution set is
$\{(1, 1/2), (1, -1/2), (4, -2), (4, 2)\}$.

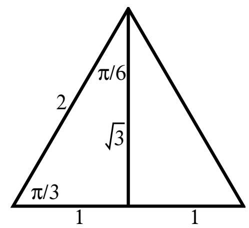
Examples 6A

1 Express the values of sine and cosine of these commonly occuring angles exactly:

$$\pi, \qquad \frac{\pi}{2}, \qquad \frac{\pi}{4}, \qquad \frac{\pi}{3}, \qquad \frac{\pi}{6}.$$

We have $\sin(\pi) = 0$, $\cos(\pi) = -1$ and $\sin\left(\frac{\pi}{2}\right) = 1$, $\cos\left(\frac{\pi}{2}\right) = 0$.





From the diagrams,

$$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \quad \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}, \qquad \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2},$$
$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}, \qquad \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}, \quad \cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}.$$

2 Find all the values of θ for which $2\sin^2(\theta) + \sin(\theta) = 1$.

The equation involves only powers of $\sin(\theta)$ so we factorise, i.e. solve the quadratic equation in $\sin(\theta)$.

$$\begin{aligned} & 2\sin^2(\theta) + \sin(\theta) - 1 \;=\; (2\sin(\theta) - 1)(\sin(\theta) + 1) \;=\; 0, \quad \text{(factorising the quadratic)} \\ \iff\; & \sin(\theta) = \frac{1}{2} \quad \text{or} \quad \sin(\theta) = -1 \\ \iff\; & \theta = n\pi + (-1)^n \frac{\pi}{6} \quad \text{or} \quad \theta = 2n\pi - \frac{\pi}{2}, \qquad n \in \mathbb{Z}. \end{aligned}$$

If solutions are required only in a particular interval, say $0 \le \theta \le 2\pi$, then we choose the appropriate values of n to get such solutions: here we get the solution set

$$\left\{ \frac{\pi}{6}, \frac{5\pi}{6}, \frac{3\pi}{2} \right\}.$$

Examples 6B

1 Without using a calculator find the *exact* value of $\sin\left(\frac{\pi}{12}\right)$.

$$\begin{aligned} \sin\left(\frac{\pi}{12}\right) &= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right) = \sin\left(\frac{\pi}{4}\right)\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{6}\right) \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2} = \frac{\sqrt{3} - 1}{2\sqrt{2}}. \end{aligned}$$

2 Show that $\cos(A+B)\cos(A-B) = \cos^2(A) - \sin^2(B)$.

$$\begin{aligned}
\text{LHS} &= (\cos(A)\cos(B) - \sin(A)\sin(B))(\cos(A)\cos(B) + \sin(A)\sin(B)) \\
&= \cos^2(A)\cos^2(B) - \sin^2(A)\sin^2(B) \\
&= \cos^2(A)(1-\sin^2(B)) - (1-\cos^2(A))\sin^2(B) \\
&= \cos^2(A) - \sin^2(B) \;=\; \text{RHS}.
\end{aligned}$$

3 Find the value of $\cos(2A)$ (i) when $\cos(A) = 0.3$; and (ii) when $\sin(A) = 0.8$.

$$\begin{aligned}
\text{(i)}\ \cos(2A) &= 2\cos^2(A) - 1 = 2\times(0.3)^2 - 1 = -0.82, \\
\text{(ii)}\ \cos(2A) &= 1 - 2\sin^2(A) = 1 - 2\times(0.8)^2 = -0.28.
\end{aligned}$$

Examples 6C

1 Show that $\dfrac{\sin(2A)}{1-\cos(2A)} = \cot(A)$.

$$\frac{\sin(2A)}{1-\cos(2A)} = \frac{2\sin(A)\cos(A)}{2\sin^2(A)} = \frac{\cos(A)}{\sin(A)} = \cot(A).$$

2 Prove that $\sin^2(\theta) + \sin^2(\theta)\tan^2(\theta) = \tan^2(\theta)$.

$$\begin{aligned}
\sin^2(\theta) + \sin^2(\theta)\tan^2(\theta) &= \sin^2(\theta)(1+\tan^2(\theta)) \\
&= \sin^2(\theta)\sec^2(\theta) \\
&= \frac{\sin^2(\theta)}{\cos^2(\theta)} \;=\; \tan^2(\theta).
\end{aligned}$$

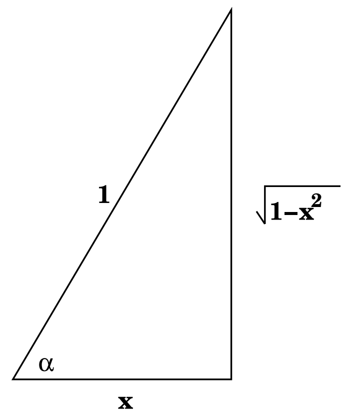
Examples 6D

1 Evaluate (i) $\arcsin\dfrac{\sqrt{3}}{2}$, (ii) $\arctan(-1)$, (iii) $\arcsin(\sin(0.75))$.

(i) $\arcsin\dfrac{\sqrt{3}}{2} = \dfrac{\pi}{3}$, (ii) $\arctan(-1) = -\dfrac{\pi}{4}$, (iii) $\arcsin(\sin(0.75)) = 0.75$.

2 If $0 < x < 1$, evaluate (i) $\cos(\arccos(x))$, (ii) $\sin(\arccos(x))$, (iii) $\tan(\arccos(x))$.

Suppose $\arccos(x) = \alpha$ so $\cos(\alpha) = x$. Clearly, $\cos(\arccos(x)) = x$.



From the diagram,

$$\sin(\alpha) = \sin(\arccos(x)) = \sqrt{1-x^2}, \qquad \tan(\alpha) = \tan(\arccos(x)) = \frac{\sqrt{1-x^2}}{x}.$$