

MM102 Applications of Calculus

Answers to the exam in May 2015

1. (a) (i) $\frac{3\pi}{16}$

(ii) $-3\sqrt{1 - \left(\frac{x-1}{3}\right)^2} + \arcsin \frac{x-1}{3} + C$

(b) $\frac{8}{3}(2^{3/2} - 1)$

2. (a) $\frac{dy}{dx} = -\frac{2xy^3 + e^y}{3x^2y^2 + xe^y}$

(b) The x -coordinate is increasing at a rate of $\frac{27}{8}$ cm/s.

(c) (i) $\text{dom}(f) = \{x \in \mathbb{R} : x \neq 2\}$

(ii) Vertical asymptote: $x = 2$

Slant asymptote: $y = 2x + 1$

(iii) Points of intersection with the x -axis: $(0, 0)$ and $\left(\frac{3}{2}, 0\right)$

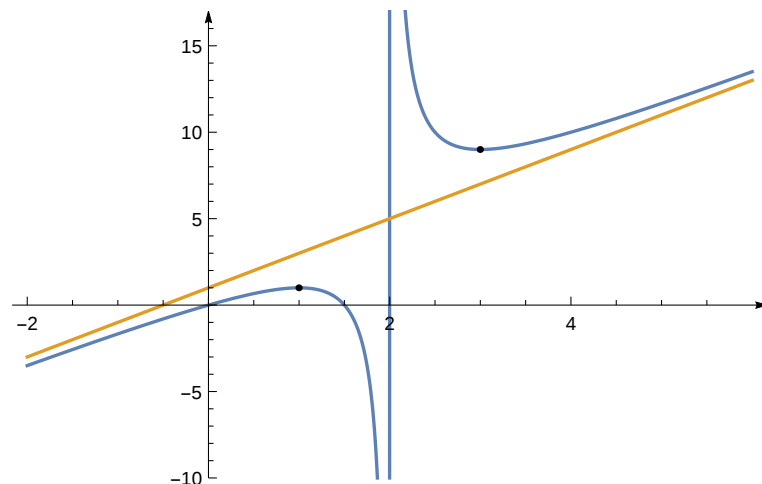
(iv) Local maximum at $x = 1$ with $f(1) = 1$

Local minimum at $x = 3$ with $f(3) = 9$

The function is increasing on the intervals $(-\infty, 1)$ and $(3, \infty)$.

The function is decreasing on the intervals $(1, 2)$ and $(2, 3)$.

(v)



3. (a) $\frac{1}{2^9} \operatorname{cis}\left(-\frac{\pi}{6}\right)$

(b) (i) $8 \operatorname{cis}\left(\frac{3\pi}{4} + 2k\pi\right), \quad k \in \mathbb{Z}$

(ii) $2 \operatorname{cis}\left(\frac{\pi}{4}\right), \quad 2 \operatorname{cis}\left(\frac{11\pi}{12}\right), \quad 2 \operatorname{cis}\left(-\frac{5\pi}{12}\right)$

(c) $a = 16, \quad b = -20, \quad c = 5$

(d) $P(z) = (z - 2)(z - 3 + i)(z - 3 - i)$

(e) $\ln 2 - \frac{5\pi i}{6}$

4. (a) (i) General solution: $y = x + \frac{e^x}{x^3} + \frac{C}{x^3}$

(ii) Particular solution: $y = x + \frac{e^x - e}{x^3}$

(b) $y = 2x \tan(2 \ln(x) + 2C)$

(c) $y = (A - 5x)e^{3x} + Be^{4x}$