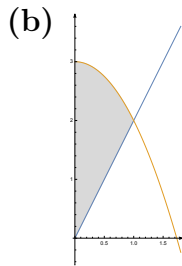


# MM102 Applications of Calculus

## Answers to the exam in May 2017

1. (a) (i)  $2 \ln(x^2 + 4) + \frac{1}{2} \arctan \frac{x}{2} - 2 \ln |x - 1| + C$

(ii)  $\frac{9\pi}{4}$



$$V = \frac{7\pi}{6}$$

2. (a)  $\frac{dy}{dx} = -\frac{2x \sin y + 5x^4 y^3}{x^2 \cos y + 3x^5 y^2}$

(b)  $\frac{dy}{dx} = \frac{1+t}{2t} e^{t-t^2}$

(c) The distance is  $d(x) = \frac{x^2}{x-f}$ .

This function has a minimum at  $x = 2f$ , where the value is  $d(2f) = 4f$ .

(d) (i)  $\text{dom}(f) = \{x \in \mathbb{R} : x \neq 1\}$

(ii) Vertical asymptote:  $x = 1$

Horizontal asymptote:  $y = 0$

(iii) Local maximum at  $x = 3$  with  $f(3) = \frac{1}{4}$

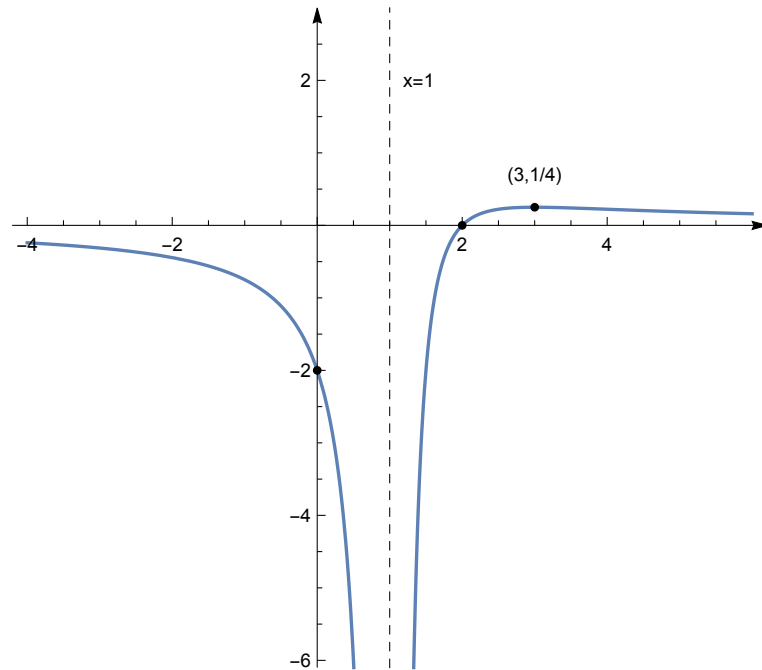
The function is increasing on the interval  $(1, 3)$ .

The function is decreasing on the intervals  $(-\infty, 1)$  and  $(3, \infty)$ .

(iv) Point of intersection with the  $y$ -axis:  $(0, -2)$

Point of intersection with the  $x$ -axis:  $(2, 0)$

(v)



3. (a)  $4 \operatorname{cis}\left(\frac{2\pi}{9}\right), 4 \operatorname{cis}\left(\frac{8\pi}{9}\right), 4 \operatorname{cis}\left(-\frac{4\pi}{9}\right)$

(b)  $a = -\frac{1}{32}, b = -\frac{1}{16}, c = \frac{1}{32}, d = \frac{1}{16}$

(c)  $P(z) = (z+1)(z-3-4i)(z-3+4i)$

(d)  $\frac{1}{2} \ln 18 + i\left(-\frac{3\pi}{4} + 2k\pi\right), \quad k \in \mathbb{Z}$

4. (a) (i) Integrating factor:  $I(x) = x^{-1/4}$

(ii)  $y = x^{1/4}(\ln x + C)$

(b)  $y = (28x^4 - x^3)^{1/3}$

(c)  $y = Ae^{3x} + Be^{-2x} + 2xe^{3x} + x + 3$