University of Strathclyde Glasgow

Department of Mathematics and Statistics

MM102 APPLICATIONS OF CALCULUS

Monday, 8 May 2017 **9:30 – 11:30 a.m.**

Duration: 2 hours

Attempt ALL questions.
Use of a calculator is NOT permitted.
Answers will receive credit only if supported by appropriate working.

1. **(a)** Evaluate the following integrals

$$\textbf{(i)} \quad \int \frac{2x^2-3x-9}{(x^2+4)(x-1)}\,\mathrm{d}x\,, \qquad \textbf{(ii)} \quad \int_2^5 \sqrt{-x^2+4x+5}\,\mathrm{d}x.$$

(8, 7 marks)

(b) Sketch the finite region in the first quadrant that is bounded by the curves

$$y = 2x, \qquad y = 3 - x^2, \qquad x = 0.$$

Hence find the volume of the solid that is obtained when this region is rotated through 360° about the $\boldsymbol{y}$**-axis**.

(5 marks)

Qu. 2 ON NEXT SHEET

2. **(a)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x and y given that

$$x^2 \sin y + x^5 y^3 = 1.$$

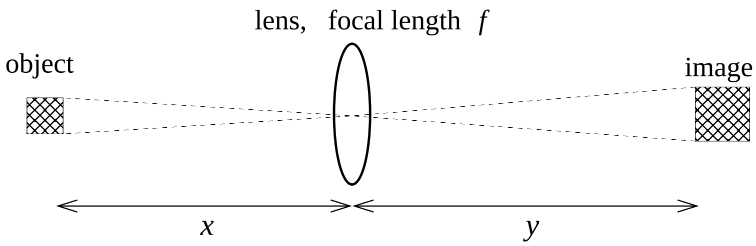
(3 marks)

(b) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of the parameter t when x and y are given by

$$x = e^{t^2}, \qquad y = te^t \qquad (t > 0).$$

(3 marks)

(c) Consider a thin lens with given focal length $f > 0$.



If a small object is placed at a distance x in front of the lens with $x > f$, then the distance y of the image behind the lens satisfies

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{f}.$$

Express the distance between the image and the object, $x + y$, in terms of x and the constant f only. Hence show that the minimum distance between the image and the object is $4f$.

(6 marks)

(d) Consider the function

$$f(x) = \frac{x-2}{(x-1)^2}.$$

(i) Determine the natural domain of f.

(ii) Find all the asymptotes of f.

(iii) Find the position and the nature of the stationary points and calculate the values of the function f at these stationary points.
Moreover, determine where the function is increasing and where it is decreasing.

(iv) Find the points of intersection of the graph with the x-axis and the y-axis.

(v) Use this information to sketch the graph of f.
Draw the asymptotes and label the stationary point(s) and points of intersection with the axes.

(1, 3, 6, 1, 2 marks)

Qu. 3 ON NEXT SHEET

3. **(a)** Find the three distinct cube roots of $32(-1+\sqrt{3}i)$.
(Give the roots in polar form using the principal value of the argument in each case.)

(4 marks)

(b) **Use de Moivre's Theorem** to find constants a, b c and d such that

$$\cos^4(\theta)\sin^2(\theta) \;=\; a\cos(6\theta)+b\cos(4\theta)+c\cos(2\theta)+d.$$

(6 marks)

(c) Express the cubic polynomial $P(z) = z^3 - 5z^2 + 19z + 25$ as the product of three linear factors.

(4 marks)

(d) Find all solutions of $\mathrm{e}^{4z} = -3 - 3i$.
(Express your answer in the form $a + ib$, where a, $b \in \mathbb{R}$.)

(2 marks)

Qu. 4 ON NEXT SHEET

4. **(a)** Consider the first order, linear differential equation

$$x\,\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{1}{4}y \;=\; 2x^{1/4},$$

where $x > 0$.

(i) What is the integrating factor for the differential equation?

(ii) Find the General Solution of the differential equation.
(Express your solution y explicitly as a function of x.)

(2, 3 marks)

(b) Find the General Solution of the differential equation equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{4y^3 + x^3}{3xy^2} \qquad \text{(where } x,\ y > 0\text{)}$$

which satisfies $y(1) = 3$.
(Express your solution for y explicitly as a function of x.)

(7 marks)

(c) Find the General Solution of the second order, linear differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 6y \;=\; 10\mathrm{e}^{3x} - 6x - 19.$$

(7 marks)

Total number of marks: 80

END OF PAPER

(ML/GMcK)