DEPARTMENT OF MATHEMATICS & STATISTICS
MM102 APPLICATIONS OF CALCULUS
Complex Numbers: Exercise Sheet for Week 5

1. Find the solutions of the following quadratic equations (for $z \in \mathbb{C}$).

(a) $z^2 - 4z + 5 = 0$, **(b)** $z^2 + 6 = 0$,

(c) $z^2 + 24z + 26 = 0$, **(d)** $3z^2 - z + 3 = 0$.

2. If $z_1 = 2 + 3i$ and $z_2 = 3 - 4i$ express the following expressions in the form $a + ib$ where a and b are real.

(a) $z_1 - 2z_2$, **(b)** z_1^2, **(c)** $z_1 z_2$, **(d)** $\overline{z}_1 z_2$, **(e)** $\dfrac{z_1}{z_2}$.

3. Express the following in the form $a + ib$ where a and b are real.

(a) $(1 + 2i) + (-3 + 6i)$ **(b)** $(4 - 2i) - (6 - 7i)$ **(c)** $2(3 + 2i) - 3(1 - 3i)$

(d) $i(4 - i)$ **(e)** $(2 + i)(6 - 2i)$ **(f)** $(3 + i)(4 - 11i)$

(g) $(1 - i)(6 + 3i)$ **(h)** $2(1 + 3i) - (3 - 7i)(2 + 6i)$ **(i)** $(8 + 2i)(-6 - 4i)(3 - 2i)$

(j) $\dfrac{3 + 4i}{4 - 3i}$ **(k)** $\dfrac{2 + 3i}{7 - i}$ **(l)** $\dfrac{(1 + i)(2 + 3i)}{1 - i}$

(m) $\dfrac{1}{4 - 3i} + \dfrac{1}{4 + 3i}$ **(n)** $\dfrac{1 + i}{2 + i} + \dfrac{3 - i}{1 - i}$ **(o)** $\dfrac{10i}{1 + 3i}$

4. Solve the following equations for $z \in \mathbb{C}$:

(a) $z^2 + 4z + 7 = 0$, **(b)** $z^2 + 4iz + 7 = 0$.

5. By equating the real and imaginary parts, solve the following equations for the real numbers x and y.

(a) $x + iy - 4i = 3y - 2ix + 9$, **(b)** $x + iy = \dfrac{1}{x - iy} + 2$, **(c)** $\dfrac{1 - x + 2iy}{2x - iy} = 1 - 3i$.

6. Draw an Argand diagram to represent the following complex numbers.

(a) $2 + 2i$ **(b)** $4 - 4i$ **(c)** $-3i$ **(d)** $-\sqrt{3} + i$

(e) $\dfrac{1}{-\sqrt{3} + i}$ **(f)** -5 **(g)** $-2 - \sqrt{12}i$.

7. Let $z_1 = 1 + 2i$, $z_2 = 3 - i$ and $z_3 = -3i$. Draw an Argand diagram to represent the following complex numbers.

(a) $z_1 - \overline{z}_2$, **(b)** $z_1 z_3$, **(c)** $\dfrac{z_2}{z_3}$, **(d)** $\dfrac{1}{z_2}$, **(e)** $z_2 z_3 - z_1$.

8. Draw Argand diagrams to show the regions in the complex plane that satisfy the following relationships.

(a) $|z| = 1,$ **(b)** $\arg z = 1,$ **(c)** $|z| < 2,$ **(d)** $-\dfrac{\pi}{4} \leq \arg z \leq \dfrac{\pi}{3},$

(e) $|z| > 3$ and $0 \leq \arg z \leq \dfrac{\pi}{4},$ **(f)** $\mathrm{Re}\,(z) > 3,$ **(g)** $\dfrac{1}{2} < |z| < 4.$

9. Find the modulus and principal value of the argument of the following complex numbers:

(a) $1 + i,$ **(b)** $\sqrt{3} - i,$ **(c)** $-4 - 4\sqrt{3}\,i,$ **(d)** $-6,$

(e) $2i,$ **(f)** $-4 + 4i,$ **(g)** $\cos\left(\dfrac{\pi}{12}\right) + i\sin\left(\dfrac{\pi}{12}\right),$ **(h)** $\cos\left(\dfrac{\pi}{4}\right) - i\sin\left(\dfrac{\pi}{4}\right).$

10. Find the polar form of the following complex numbers using the principal value of the argument in each case.

(a) $2 - \sqrt{12}i$ **(b)** $3 + 4i$ **(c)** -12 **(d)** $-128 - 128i$

(e) $6 + 6i$ **(f)** $4i$ **(g)** $-2i$ **(h)** $-3 + \sqrt{3}i.$

11. Calculate the modulus, argument and principal value of $z = 3 + 2i$. Also, express z is polar form using the principal value of the argument.

12. Write the following numbers in the form $x + iy$ where $x,\ y \in \mathbb{R}$.

(a) $3\ \mathrm{cis}\left(\dfrac{15}{4}\pi\right),$ **(b)** $6\ \mathrm{cis}\left(\dfrac{11}{6}\pi\right),$ **(c)** $2\ \mathrm{cis}\,(7\pi).$

13. Use the polar form to find the modulus of $\dfrac{(6-i)^2(12+5i)}{-7-24i}.$