4 Exercise Solutions: Chapter 4

1. Refer to solutions to Ex. 1.8 for row-reduced matrices. We have

$$\begin{aligned} n(A) &= \text{dim. of soln. space of } A\boldsymbol{x} = \mathbf{0} \\ &= \text{no. of ``free variables''}. \end{aligned}$$

   (a) $A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & \frac{1}{2} \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow r(A) = 2,\ n(A) = 1.$

   (b) $A \sim \begin{bmatrix} 1 & 3 & 4 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & \frac{1}{2} \end{bmatrix} \Rightarrow r(A) = 3,\ n(A) = 1.$

   (c) $A \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow r(A) = 3,\ n(A) = 0.$

   (d) $A \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow r(A) = 3,\ n(A) = 1.$

   (e) $A \sim \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & \frac{3}{4} \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow r(A) = 3,\ n(A) = 0.$

2. (a) A is 4×4: $r(A) \leq 4,\ n(A) \geq 0$ since $r(A) + n(A) = 4.$

   (b) A is 3×5: $r(A) \leq 3,\ n(A) \geq 2$ since $r(A) + n(A) = 5.$

   (c) A is 5×3: $r(A) \leq 3,\ n(A) \geq 0$ since $r(A) + n(A) = 3.$

3. *See solns to 1.10*

   (a) $r(A) = 2$ and number of parameters $= 3 - 2 = 1$

   (b) $r(A) = 2,\ r(A|\boldsymbol{b}) = 3 \Rightarrow$ No solution.

   (c) $r(A) = r(A|\boldsymbol{b}) = 4 \Rightarrow$ unique soln. exists (No parameters).

   (d) $r(A) = r(A|\boldsymbol{b}) = 2 \Rightarrow$ Soln. exists with $4 - 2 = 2$ parameters.

   (e) $r(A) = r(A|\boldsymbol{b}) = 3 \Rightarrow$ Soln. exists with $4 - 3 = 1$ parameter.

4. (a) $r(A) = r(A|\boldsymbol{b}) \Rightarrow$ Consistent. No. params $= 3 - r(A) = 3 - 3 = 0$.
   (b) $r(A) \neq r(A|\boldsymbol{b}) \Rightarrow$ Inconsistent.
   (c) $r(A) = r(A|\boldsymbol{b}) \Rightarrow$ Consistent. No. params $= 3 - r(A) = 3 - 1 = 2$.
   (d) $r(A) = r(A|\boldsymbol{b}) \Rightarrow$ Consistent. No. params $= 9 - r(A) = 9 - 2 = 7$.
   (e) $r(A) = r(A|\boldsymbol{b}) \Rightarrow$ Consistent. No. params $= 2 - r(A) = 2 - 2 = 0$.

5. Use $r(A) + n(A) =$ number of columns in A. Find values of $n(A)$ to be
   (a) 0, (b) 1, (c) 2, (d) 7, (e) 0