University of Strathclyde

Department of Mathematics and Statistics

MM102 APPLICATIONS OF CALCULUS

Thursday May 21st, 2015 **9:30 - 11:30a.m.**

Duration: 2 hours

Attempt ALL questions.
Use of a calculator is NOT permitted.
Answers will receive credit only if supported by appropriate working.

1. **(a)** Evaluate the following integrals

$$\textbf{(i)} \quad \int_0^{\pi/2} \cos^4 x \, \mathrm{d}x\,, \qquad\qquad \textbf{(ii)} \quad \int \frac{x}{\sqrt{8+2x-x^2}}\,\mathrm{d}x.$$

(7, 8 marks)

(b) Find the arc length of the curve

$$y = \frac{1}{3}x^{3/2}, \qquad x \in [0,4].$$

(5 marks)

Qu. 2 ON NEXT SHEET

2. **(a)** Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ as a function of x and y given that

$$x^2y^3 + x\mathrm{e}^y = \mathrm{e}.$$

(3 marks)

(b) A particle moves on the curve

$$2x^2 + 3y^3 = 11$$

where x and y are given in cm. As the particle passes the point where $x = -2\,\mathrm{cm}$ and $y = 1\,\mathrm{cm}$, the y-coordinate is increasing at $3\,\mathrm{cm/s}$. At what rate is the x-coordinate changing? Is x increasing or decreasing?

(4 marks)

(c) Consider the function

$$f(x) = \frac{2x^2 - 3x}{x - 2}.$$

(i) Determine the natural domain of f.

(ii) Find all the asymptotes of f.

(iii) Find the points of intersection with the x-axis.

(iv) Find the position and the nature of the stationary points and calculate the values of the function f at these stationary points.
Moreover, determine where the function is increasing and where it is decreasing.

(v) Use this information to sketch the graph of f. Label the stationary points and the points of intersection with the x-axis.

(1, 3, 1, 6, 2 marks)

Qu. 3 ON NEXT SHEET

3. **(a)** Express the complex number

$$\frac{\left[2\ \mathrm{cis}\left(\frac{5\pi}{6}\right)\right]^7}{\left[4\ \mathrm{cis}\left(-\frac{\pi}{2}\right)\right]^8}$$

in polar form using the **principal value** of the argument.

(3 marks)

(b) **(i)** Express $4\sqrt{2}(-1+i)$ in polar form.

(ii) Find the three distinct cube roots of $4\sqrt{2}(-1+i)$.
(Give your answers in polar form using the principal value of the argument in each case.)

(1, 4 marks)

(c) **Use de Moivre's Theorem** to find constants a, b and c such that

$$\cos(5\theta) = a\cos^5(\theta) + b\cos^3(\theta) + c\cos(\theta).$$

(6 marks)

(d) Express the cubic polynomial $P(z) = z^3 - 8z^2 + 22z - 20$ as the product of three linear factors.

(4 marks)

(e) Express $\mathrm{Log}(-\sqrt{3} - i)$ in the form $a + ib$, where a, $b \in \mathbb{R}$.

(2 marks)

Qu. 4 ON NEXT SHEET

4. **(a)** Consider the first order, linear differential equation equation

$$x\frac{\mathrm{d}y}{\mathrm{d}x} + 3\,y \;= 4x + \frac{\mathrm{e}^x}{x^2},$$

where you may assume that $x > 0$.

(i) What is the integrating factor for the differential equation?

(ii) Find the General Solution of the differential equation.

(iii) Find the Particular Solution of the differential equation that satisfies $y(1) = 1$.

(2, 3, 1 marks)

(b) Find the General Solution of the first order, homogeneous differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} \;=\; \frac{4x^2 + y^2 + xy}{x^2},$$

where you can assume that $x > 0$ and $y \neq 0$.
(Express your solution y explicitly as a function of x.)

(7 marks)

(c) Find the General Solution of the second order, linear differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} \;-\; 7\frac{\mathrm{d}y}{\mathrm{d}x} \;+\; 12y \;=\; 5\mathrm{e}^{3x}.$$

(7 marks)

END OF PAPER

(ML/GMcK)