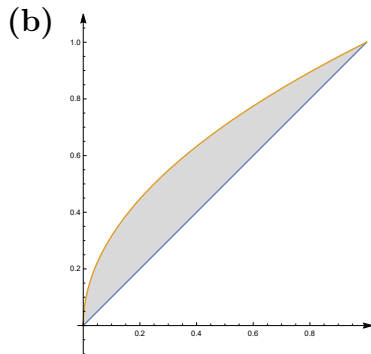


# MM102 Applications of Calculus

## Answers to the exam in May 2019

1. (a) (i)  $\frac{1}{3} \sin^3 x - \frac{3}{5} \sin^5 x + \frac{3}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$

(ii)  $-6\sqrt{3} + \frac{5\pi}{3} + 8$



$$V = \frac{\pi}{6}$$

2. (a)  $\frac{dy}{dx} = -\frac{e^y + 3x^2 \sin y}{xe^y + x^3 \cos y}$ ; tangent:  $y = -\frac{1}{2}(x-1)$

(b)  $p_{2,e}(x) = e^2 + 3e(x-e) + \frac{5}{2}(x-e)^2$ ;  $R_{2,e}(x) = \frac{1}{3\xi}(x-e)^3$  with  $\xi$  between  $x$  and  $e$ .

(c) (i)  $\text{dom}(f) = \{x \in \mathbb{R} : x \neq 1\}$

(ii) Vertical asymptote:  $x = 1$

Slant asymptote:  $y = x + 1$

(iii) Local maximum at  $x = -1$  with  $f(-1) = -2$ ;

local minimum at  $x = 3$  with  $f(3) = 6$ .

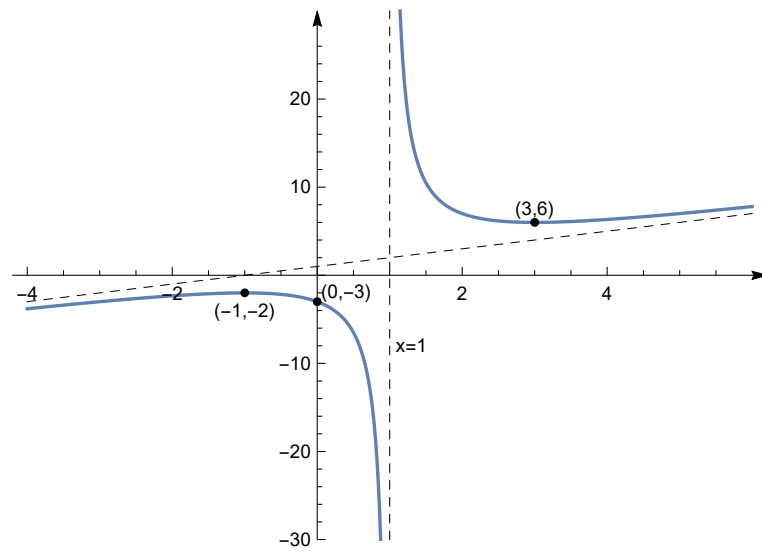
The function is increasing on the intervals  $(-\infty, -1)$ ,  $(3, \infty)$ .

The function is decreasing on the intervals  $(-1, 1)$  and  $(1, 3)$ .

(iv) Point of intersection with the  $y$ -axis:  $(0, -3)$

There is no point of intersection with the  $x$ -axis.

(v)



3. (a) (i)  $12 \operatorname{cis}\left(-\frac{5\pi}{6}\right)$

(ii)  $12^{1/3} \operatorname{cis}\left(-\frac{5\pi}{18}\right), \quad 12^{1/3} \operatorname{cis}\left(\frac{7\pi}{18}\right), \quad 12^{1/3} \operatorname{cis}\left(-\frac{17\pi}{18}\right)$

(b)  $a = \frac{1}{8}, \quad b = -\frac{1}{2}, \quad c = \frac{3}{8}$

(c)  $3 - i, \quad 3 + i, \quad -1 - 2i, \quad -1 + 2i$

4. (a) (i) Integrating factor:  $I(x) = x^2 - 1$

(ii)  $y = \frac{(1 + 2x)^{3/2}}{3(x^2 - 1)} + \frac{C}{x^2 - 1}$

(b)  $y = x - \frac{2x}{\ln x + 2}$

(c)  $y = A + Be^x - 5 \sin x + 5 \cos x - x$