EC315 Topics in Microeconomics with Cross-Section Econometrics Coursework Summary

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1 Exam Summary

1.1 Cost-Benefit Analysis Summary

1. Purpose
2. Alternatives
3. Who
4. C/B Impacts
5. Lifetime Impacts
6. Monetize:
    - *Social Cost*: harm done to living organisms
    - *Revealed/Stated Preference*: willingness to pay or willingness to accept
        - Revealed: shown in behaviour
        - Stated: questionnaires etc.
    - *Time*:
        - Work vs leisure using wage rate
        - Travel time; how much people are willing to trade-off
    - *Lives*: life expectancy, pay, age, risks taken
    - *Natural Resources*: AONBs, surveys, investment, regulation
7. PV Discounts
    - Social discount rate
    - Intergenerational (more than 50 years)
8. NPV of Alternatives
9. Sensitivity Analysis
10. Recommend

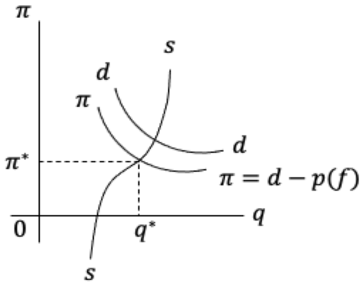
1.2 Program & Policy Evaluation Summary

$$\text{Cause} \longrightarrow \text{Intermediaries} \longrightarrow \text{Effect}$$

1. Omitted Variable Bias
   - Selection Bias: e.g. grades, income, area of ogigin
   - Selection Bias 2: e.g. effort, determination, stamina
2. Randomized Control Trial
   - Unbiased Estimator: $\bar{x} \longrightarrow \bar{\mu}$ (LLN)
   - Unbiased Estimator: randomization
   - σ^2: "how much of the result is due to chance?"
   - t-tests: causal effect; $(\bar{Y}^T - \bar{Y}^C)$
3. Regression
   - Dummy Variables: causal variable / group
   - Instrumental Variables: omitted variables (α corr. w/ ε)

1.3 Crime & Punishment Summary

1. Supply: $\pi_t = \pi_i - c_i - w_i - p_i(f_i)$
   - i = Individual
   - π_t = Net Total Payoff of Crime
   - π_i = Expected Payoff Per Offense (Minus Costs)
   - c_i = Cost Incurred if Caught
   - w_i = Wage Rate From Non-Criminal Work
   - p_i = Probability of Aprehension & Conviction
   - f_i = Punishment in Convicted
2. Normal Distribution
   - Req. $\uparrow \pi$, $\uparrow \delta$, $[\bar{x} \rightarrow$ (Right of Mean)$]$
   - Req. $\downarrow \pi$, $\downarrow \delta$, $[\leftarrow \bar{x}$(Left of Mean)$]$
   - Morals, enjoyment, risk, some demand for significantly higher payoffs etc. effect decision
3. Demand: $e_i f(v_r, v_l); q$
   - e_i = Expenditure on Protection
   - v_r = Risk of Victimization
   - v_l = Loss of Victim
   - q = Total Crime
4. Derivatives
   - $\frac{\partial e_i}{\partial v_i} > 0$: Risk $\uparrow$, Expenditure $\uparrow$
   - $\frac{\partial c_i}{\partial e_i} < 0$: Expenditure $\uparrow$, Cost $\uparrow$
   - $\frac{\partial \pi_i}{\partial c_i} < 0$: Cost $\uparrow$, Payoff $\downarrow$
5. Supply / Demand

- 

- ss = Supply of Crime
- dd = Initial Demand
- $\pi\pi$ = Demand After Government Intervention (T)
- MC of Catching Last Criminal $> MB$ $[\leftarrow \pi^*,\ q^*]$
- MC of Catching Last Criminal $< MB$ $[\pi^*,\ q^* \rightarrow]$

1.4 Exam Arithmetic Summary

1. $\pi_A = x_A p_A(x_A + x_B) - x_A$
2. $J = \pi_A + \pi_B$; $\frac{\partial J}{\partial x_A} = \frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_B}{\partial x_B}$
3. Externalities: $\frac{\partial \pi_A}{\partial x_B}$
    - > 0: Positive: "you do $\uparrow$, my π $\uparrow$"
    - < 0: Negative: "you do $\uparrow$, my π $\downarrow$"
4. Strategic Nature: $\frac{\frac{\partial \pi_A}{\partial x_A}}{\partial x_B}$
    - > 0: Complements: "you do $\uparrow$, I do $\uparrow$"
    - < 0: Substitutes: "you do $\uparrow$, I do $\downarrow$"
5. Grim Trigger Strategy
    - 40, 50, 30
    - $\frac{40}{(1-\delta)} \geq 50 + \frac{30\delta}{(1-\delta)}$
    - $40 \geq 50 - 50\delta + 30\delta$
    - $\delta \geq \frac{1}{2}$: cooperation possible

    Tit-for-Tat Strategy

    - 40, 50, 20
    - $\frac{40}{(1-\delta)} \geq \frac{50}{(1-\delta^2)} + \frac{30\delta}{(1-\delta^2)}$
    - $40 + 40\delta \geq 50 + 20\delta$
    - $\delta \geq \frac{1}{2}$: cooperation easy

2 Game Theory

2.1 Definitions

- *Welfare Economics*: Generalising equilibriums. Competitive markets provide an incentive for firms to produce what customers want. Markets rock in fair play.
  - Theorem 1: Every competitive economy is Pareto Efficient.
  - Theorem 2: Every Pareto Efficient allocation of resources can be achieved in competitive markets (w/ appropriate redistribution between parties).
- *Pareto Efficiency*: No additional person can be made better off without making someone else worse off. There should be no government intervention. Redistribution can take place meaning there is redistribution between parties within the economy rather than externally.
- *Prisoner's Dilemma*: Pursuing your own interests leads to inefficient markets because, using the prison example, if both people choose to confess, they get full long time each. If they both lie, they get full short time. If one lies and one confesses, the one who confesses gets reduction but the liar gets full time. This is risk. Both could deny for 2 years of the other lies (gets 10 years). But then they both risk getting 8 years. If they both deny they both get the full short time (3 years). Denying is best for them both but confessing could, but only could, be best for a single one of them.
- *Rationality*: Players will choose the option with the best payoff for themselves. But back to the Joey and Phoebe, if you are choosing the best for yourself, surely the opponent must be doing the same so can you forecast? Or will they think the same and one-up you?
  - *Common Knowledge of Rationality*: Where players don't just know they possible outcomes of their decisions, they know the possible outcomes of the other's decisions. But recall the prisoner's dilemma.
- *Game Theory*: Our actions have external consequences. They effect the environment and all things around us (smoking example).
  - *Non-Cooperative Game*: In it for your own gain and only that.
  - *Information Game*: Everyone knows they are playing.
  - *Stage Game*: May be repeated (e.g. rearranging cost agreements).

- *Simultaneous Game* (Type 1): When players do not know the move of the opponent and move at the same time.
- *Sequential Game* (Type 2): When players know the move of the other player and can make their decision based on the opponent's move.

- *Imperfect/Perfect Information*: Not being able to see the others' choice. Your outcome will always depend on their choice but your decision won't. Or, you have information about their decision to look at as they have make it (historic forecasting).
- *Strategic Uncertainty* (When Simultaneous): Players must base decisions on what they think the other player will play as they do not know. But then they must consider what they think the opponent's move will be but then, the opponent will surely think they will be thinking this and so make a different move and make the same prediction about their opponent... in practice usually it comes back round to them making the first decision that you predicted they would make (Joey and Phoebe e.g.) Can lead to *Strategic Payoff* where the strategic nature of their thinking pays off and they've well forecasted the other's choice.
- *Dominant Strategy*: When there is one clear winner in the strategy you use. It takes the lead the majority or all of the time when put into the matrix. This is found through *Best Response Analysis* which is found by going through each option of B and selecting the best strategy for A to choose (repeat for all columns of B). Then repeating for B (for all rows of A). The double underlined is the dominant strategy.
- *Dominated Strategy*: When the strategy a player chooses is dominated than another strategy which would make you better-off than the one you're choosing.
- *Nash Equilibrium*: When there is a clear equilibrium between the players' *Dominant Strategies.* Means you can't *Unilaterally Deviate* to become more profitable (no incentive to deviate).

  - *Unilateral Deviation*: When there is no dominant strategy.
  - *Mixed Strategies*: Players randomise strategies on unpredictable patterns (e.g. with muscular workouts).
  - *Pure Strategies*: When the player knows for sure what option they will choose.

2.2 Simultaneous Move Game

- Sole entrant: obtains big payoff
- Multiple entrants: perhaps lacking market space
- There exists a *First Mover Advantage*

2.2.1 Pure & Mixed Strategies

- *Chicken Game*: two players heading towards each other;
  - They collide and both marginally lose out
  - One swerves and loses out bigger (chicken)
- There may be two nash equilibria
  -

	Player B		
Player A		$E(q)$	$N(1-q)$
	$E(p)$	$-50, 100$	$\underline{150}, \underline{0}$
	$E(1-p)$	$\underline{0}, \underline{100}$	$0, 0$

  - The nash equilibria are underlined. There are two
  - Pure strategies are shown through probability as seen by entering probabilities p & q above
  - For Player A (EV = Expected Value):

$$
\begin{array}{l|l}
EV_A^E = -50q + 150(1-q) & EV_B^E = -100p + 100(1-p) \\
EV_A^N = 0q + 0(1-q) & EV_B^N = 0p + 0(1-p) \\
-50q + 150 - 150q = 0 & -100p + 100(1-p) = 0 \\
\underline{q = \frac{3}{4}} & \underline{p = \frac{1}{2}}
\end{array}
$$

  - These are the probabilities of placing in the respective quartiles:

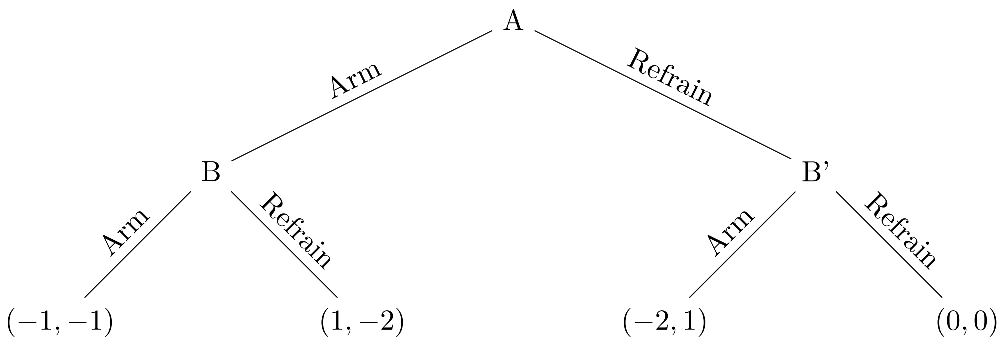
	$E\frac{3}{4}$	$N\frac{1}{4}$
$E\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$
$N\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$

  - You are trying to find the option that would make you both indifferent between choosing options

2.3 Sequential Move Game

- Uses *Backward Induction*
  - Games are analysed from the end through to start
- Transforms *Normal Form* to *Extensive Form*
- Transforms *Nash Equilibrium* to *Sub-Game Nash Equilibrium*
- Not subject to *Strategic Uncertainty* (imperfect information)
  - Can observe moments
  - Hence, *Perfect Information*
  - E.g. supermarket price setting
- If the first or last mover has a *Dominant Strategy*, they'll use it

2.3.1 Game Tree



		B			
		(Arm, Arm')	(Arm, Refrain')	(Refrain, Arm')	(Refrain, Refrain')
A	Arm	$-1,-1$	$-1,-1$	$1,-2$	$1,-2$
	Refrain	$-2,1$	$0,0$	$-2,1$	$0,0$

- In simultaneous games: *Strategy* = *Action*. This isn't the case in sequential games. *Actions* are a simple move; *Strategies* are plans based on the move of the first player
- A's strategies: Arm, Refrain

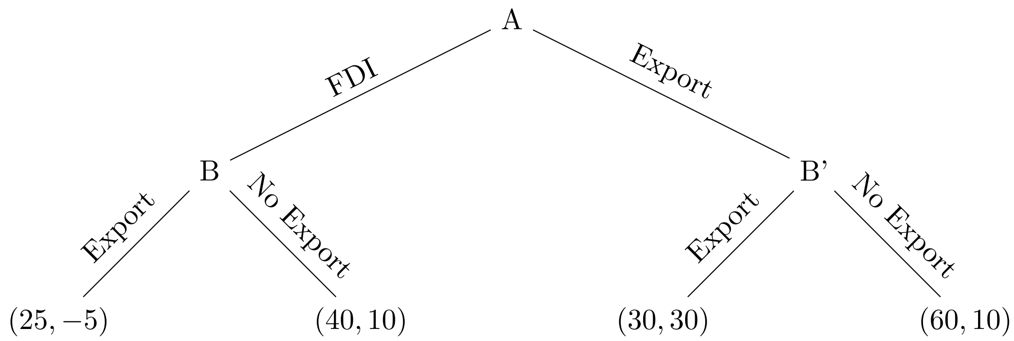
- B's strategies: (Arm, Arm'), (Arm, Refrain'). (Refrain, Arm'), (Refrain, Refrain')
- *Information Set*: don't know which two nodes you are at
- *Subgame*: the mini-looking games which B is playing under A

2.3.2 Choosing An Option

Normal:

		B	
		(Export)	(No Export)
A	FDI	$25, -5$	$40, \underline{10}$
	Export	$\underline{30}, \underline{30}$	$\underline{60}, 10$

Extensive:



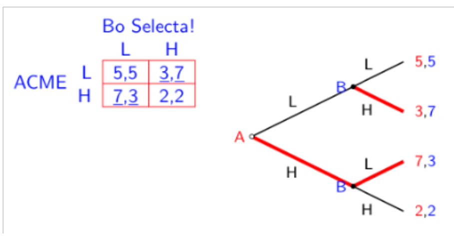
- In *Normal Form*, (Export, Export) is the *Dominant Strategy*, but there are more options:
  - (Export, Export')
  - (Export, No Export')
  - (No Export, Export')
  - (No Export, No Export')
- If A plays FDI, will B ever export?
  - (No Export, Export') allows *Incredible Threats* to be made

2.3.3 Backwards Induction

- A process used to avoid *Incredible Threats*
- If A assumes B is rational, they expect B to play $(40, 10)$ on FDI and $(30, 30)$ on Export
- *Credible Threats*:
  - Only on FDI as they could lower their payoff to punish A
  - $(25, -5)$

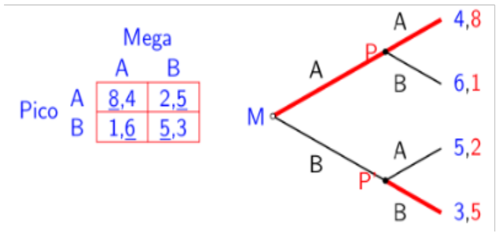
1. Start on that last stage of the game
2. Break down into two of A's options
3. Select these two *Subgame Nash Equilibria* for B on each A arm
   - (No Export, Export') are the best for B here
4. A now has a choice
   - FDI would be followed by B's No Export $(40, 10)$ $[> (25, -5)]$
   - Export would be followed by B's Export' $(30, 30)$ $[< (60, 10)]$
   - A plays FDI
5. *Nash Equilibrium* is made clear
   - {FDI,(Not Export, Export')}

2.3.4 Order Advantages

- *Commitment* in first mover vs. *Flexibility* in follower
  - Commitment has greater value in simultaneous games
  - Flexibility has greater value in sequential games
- Recall that in simultaneous games there's a first mover advantage
- *First Mover Advantage* (Simultaneous):
  - 

- B maximises on both moves and A maximises on its one move
- A lowers B's payoff by choosing a more profitable option for them

- *Second Mover Advantage* (Sequential):



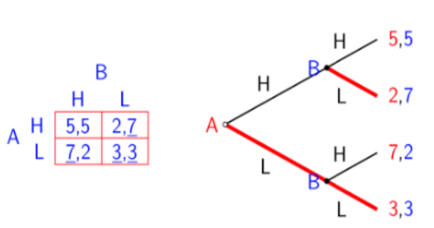
  - M goes for overall highest payoff (6) by choosing move A
  - P has the option to choose one which greater benefits them and lowers M's expected payoff

2.3.5 Manipulating Games

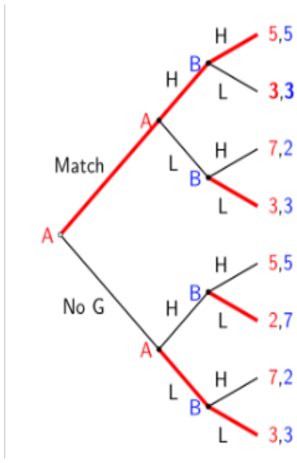
- Take actions to manipulate a game? That is, guaranteeing the outcome
- *Threats & Promises*
  - "If you attack, I'll fight..."
  - "If you enter, I'll enter too; making it less profitable for you..."
  - "If you work hard, I'll work harder..."
  - These lack credibility as they could be bluffs
- *Credibility*
  - "If you're late, I'll set off a bomb..." (*Incredible*; bluffing / not factual?)
  - "If you're late, the timed boomb will go off..." (*Credible*; more believable / factual)
  - Changes first mover's thinking

2.3.6 Price Matching Guarantees

Standard *Subgame Perfect Nash Equilibrium*:



Offering a *Price Match Guarantee*:



- Commitment to maintain high prices
- By committing to match low prices, A changes payoffs such that it's not beneficial for B to undercut – as bigger payoff can't be seen
- Both firms end up paying high prices
- (Pricing) *Prisoner's Dilemma*

2.4 Prisoner's Dilemma

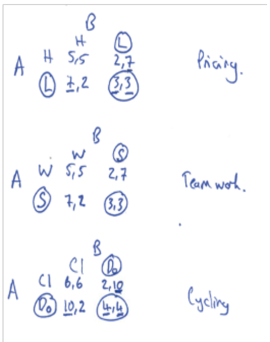
- *Cooperate* or *Defect*
- Mutual gain: *Cooperate*
- Individual incentive: *Defect*
- *Pareto Efficienct Equilibrium*
- Recall: someone has a dominant strategy where there's harm done to each other and they could be better off (Pareto Efficient). Self-interest doesn't pay off. But, games can be repeated (e.g. price re-setting), e.g. lower price than opponent now (get more custom volume), makes opponent less-off (also poor for aggregate prices), firms may form a passive collusion where they both think opponents will set low so they both set high
- Both players have dominant strategy to defect but they could have a better result when the both cooperate. Hence, when choosing best interest, harm is done to the opponent when choosing to Defect for own interest, the opponent may choose to Cooperate so have a worse outcome

		Bob	
		deny	confess
Alice	deny	3,3	10,2
	confess	2,10	8,8

Example:

1. (8,8) is Pareto Inefficient equilibrium as it is reached by both aiming for low by confessing. Could be made better off by both acting for mutual gain (3,3)
2. Pricing – non-brand loyal market, flow freely between
3. Team Work – (work vs shirk) shirk leads to more payoff as still full marks but no work done but if the other does all the work, they will get full marks but payoff will reduce due to workload
4. Clean vs Dope – (risk based) best self-interest response is to dope as highest possible payoff but the equilibrium they both do it is less than the payoff if they both don't
5. Market Share – studying marketing is a waste of time. Market is a pie, we compete over our share. Ads try to [1] inform and [2] predatory (winning market share). Start: 50/50, engage in ads to win market share. I spend money, I get

some in return but you won't gain much more market share. The opponents do this to keep up. Each keep catching back up to 50% each but both are still wasting millions on marketing. Market share isn't changing proportionately but you're still spending money



2.4.1 Externalities

- *Negative Externalities*: (own interest – doing too much) Cooperation reduces amount of work you do for the better (e.g. not over-fishing)
  - Don't see costs from defecting – too much harmful activity is done
- *Positive Externalities*: (own interest – doing too little) Cooperation says you should do more work (e.g. not doing no work in a project)
  - Don't see costs benefits form cooperating – too little of a good activity is done
- Example: Marginal Benefit vs. Marginal Cost – (1) extracting fish from the ocean makes it harder in the future (e.g. do less fishing to allow repopulation). (2) But you want more to sell now. (3) Self-interest makes it harder for others

2.4.2 Rationalise Cooperation Resolutions

- Resolutions to *Prisoner's Dilemma*
- Meet-up (verbal agreement): let's set high prices (incentive of deception however – you want him to set high prices and you want to set low)
- *Threaten*: punishment of opponent doesn't set high prices (lacking credibility as you "will do it" rather than it "will be done (automatically etc.)" – can fix credibility by using Mafia as they have more incentive to harm him)

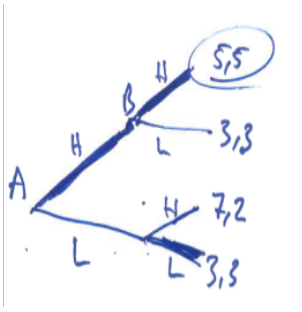
- *Reward*: offer a reward that overcomes the incentive to defect (still lacks credibility as it involves giving money – lowers your payoff. May not even believe you)

2.4.3 Behavioural Resolution

- *External Norms of Behaviour*: Think back to litter e.g.: many people won't actually litter even though it's the most beneficial for you. It conflicts with the social norm
- *Internal Norms of Behaviour*: Doing nice things for people who are nice to you (gain utility). Being bad to people but they are good to you (loss of utility). - you defect but if you care enough, you'll maybe rationalise cooperation and change mind

2.4.4 Price Matching Guarantee Resolution

- You may be undercut for opponent to gain market share from you at lower prices
- If you're offered. PMG, you will simply match prices and keep customers "he's selling at that price, can you just sell me at that too"
- Opponent now doesn't gain, just sells at lower price as no market gain
- Equilibrium of both pricing high – out of Prisoner's Dilemma



2.4.5 Dynamic Punishment Resolution

- When defecting, a player may believe they will be 'punished' in the future
- Can we achieve cooperation through fear of punishment?
  - Credible: backed with fact
  - Incredible: maybe won't happen

- *Finite Period* (T Periods): defect in last period as no more time for retaliation
  - Final period: mutual dominant strategy to defect as no future punishment
  - So: best to defect this period as well as you both will next
- *Infinite Period*: the game will continue [probability $p = 1$] so retaliation
  - Always an opportunity to punish as there's always another period
- *Impatient*: future worth less than present so defect (not caring for punishment)
- *Patient*: care more for future gain by waiting and cooperating

Discounting

- £1 from £1 today to £1$(1+r)$ tomorrow; £1 from £1 tomorrow to £$\frac{1}{(1+r)}$ today
- PV: $\frac{1}{(1+r)}, \frac{1}{(1+r)^2}, ..., \frac{1}{(1+r)^N}$; for discount rate r and discount factor $\delta = \frac{1}{(1+r)^t}$
- Hence, £X in period t is worth £$X\frac{1}{(1+r)^t}$ today
- Hence, £X in period t is worth £$X\delta^t$ today
- δ close to 1: patient; δ close to 0: impatient
- CFs: $X_0, X_1, ..., X_N$; PVs: $X_0 + \delta X_1 + \delta^2 X_2 +, ..., +\delta^N X_N$
- Infinite: $1 + \delta + \delta^2 + \delta^3 = \frac{1}{(1-\delta)}$

Example:

		B	
		high	low
A	high	600,600	170,1000
	low	1000,170	400,400

Payoff of 7 in perpetuity	Payoff of 10 today and 2 in perpetuity
$7 + 7\delta + 7\delta^2 + ... = 7\frac{1}{1-\delta}$	$10 + 2\delta + 2\delta^2 + ... = 10 + 2\delta\frac{1}{(1-\delta)}$

Trigger Strategies: Grim Trigger

- Start by cooperating
- If opponent cooperated, cooperate
- If opponent defected, defected in perpetuity

1. *Cooperate*:
   - Opponent gets 600 forever $\rightarrow 600 + 600\delta + 600\delta^2 + ... = \frac{600}{(1-\delta)}$
2. *Defect*:
   - Get 1000 now but 400 after $\rightarrow 1000 + 400\delta + 400\delta^2 + ... = 1000 + \frac{400\delta}{(1-\delta)}$
3. Answer:
   - $\frac{600}{(1-\delta)} \geq 1000 + \frac{400\delta}{(1-\delta)} \Leftrightarrow \delta \geq \frac{2}{3} (\Leftrightarrow \leq \frac{1}{2}) = 50\%$
   - $\boxed{\text{Hence, } \textit{Grim Trigger} \text{ at } \delta > \frac{2}{3} \text{ so cooperate}}$

Trigger Strategies: Tit-for-Tat

- Start by cooperating
- Play as the opponent played in the last round
- Cooperation followed by cooperation
- Defection frollowed by defection

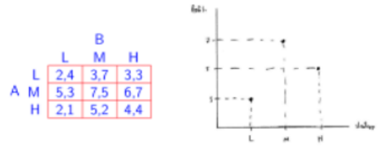
1. *Defect* in perpetuity:
   - Same as Grim Trigger: $\delta \geq \frac{2}{3}$
2. *Defect* once:
   - Get 400 now but loses 430 after $\rightarrow 400 \leq 430\delta \Rightarrow \delta \geq \frac{40}{43} (\Rightarrow \leq 0.075) = 7.5\%$
3. Answer:
   - $\boxed{\text{Hence, } \textit{TFT} \text{ at } \delta > \frac{40}{43} \text{ so cooperate}}$

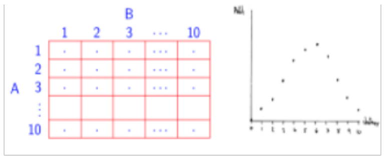
$\boxed{\text{If } \textit{Grim} \text{ works: cooperation possible; If } \textit{TFT} \text{ works: cooperation easy}}$

2.5 Games With Continuous Strategies

- This is applying maths to what we already know
- Nash equilibriums and sub-game perfect nash equilibriums remain the same
- This is applying the following more generally
- Matrix strategy: can choose any option for the expected opponent's options
  - Simultaneous: best responses and mutually consistent best responses
  - Sequential: backward induction
- Take a long time to analyse a continuous strategy using discrete sets (matrix)

2.5.1 Quantity Competition

- In a *Competitve Market*
- Firm i supplies q_i (where $\sum_{i=1}^{N} q \; \forall i\{1, ..., N\}$)
- *Inverse Demand Function*: $p(Q) = a - bQ$
- Payoff is *Profit* (π_i): $q_i p(Q) - C_i(q_i)$
- Hence: $\pi_i(q_{i=1}, q_{i=2}, ..., q_N) = q_i p(Q) - C_i(q_i)$
- There is an *Oligopoly* if several firms compete
- This method is shown in 1 to N matrices for A to respond to B:
  - 

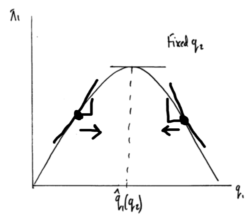
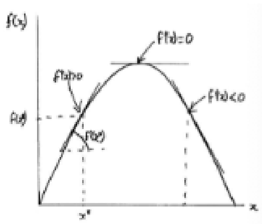
$\longrightarrow$


- Hence, we find a *Payoff Function* which is maximised at a point; "find the level of q_1 maximising firm A's payoff for given q_2"

2.5.2 Continuous Strategies

- Too hard to account for all the options (in this case quantities to produce)
- Recall basic math: *Function*: the level; *Derivative*: the slope of the function; *Parital Derivative*: fix a variable

Rules of Differentiation

- Woring towards *Payoff Function*
- Just like in the matrixes, fix the opponents option each time to find your best
- Hence, $y = f(x_1, x_2)$
- Fix x_2 (the adjacent firm's strategy) to observe how x_1 varies with y
- Therefore, partial derivative $\frac{\partial y}{\partial x_1}$ for fixed x_2
- Thus, best response at $\{\frac{\partial y}{\partial x_1}|_{x_2} = 0\}$ (peak of function)
- Note that, if you take the derivative on the incline of the function, you can be made better off by doing more. Take the derivative on the decline, better off by doing less
- 
- Recall the function $f(x)$:
  - $f'(x)$ positive at incline
  - $f'(x)$ negative at decline
  - $f'(x) = 0$ at stationery point (max/min)
  - 
- Recall if $f(x) = x^n$ then $f'(x) = nx^n - 1$
  - Constant: $ch(x) \longrightarrow ch'(x)$
  - Sum: $g(x) \pm h(x) \longrightarrow g'(x) \pm h'(x)$
  - Product: $g(x)h(x) \longrightarrow g(x)h'(x) + g'(x)h(x)$
  - Chain: $g(h(x)) \longrightarrow g'(h(x))h'(x)$

- Quotient: $\frac{g(x)}{h(x)} \longrightarrow \frac{g'(x)h(x)-g(x)h'(x)}{h(x)^2}$
- Log: $\ln x \longrightarrow \frac{1}{x}$

- In practice:
  - Power: $\sqrt{x} = x^{\frac{1}{2}} \longrightarrow \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
  - Constant: $3x^2 \longrightarrow 3 \times 2x$
  - Sum: $x^2 + x^3 \longrightarrow 2x + 3x^2$
  - Product: $x^2(2x+3)^9 \longrightarrow x^2 \times 9(2x+3)^8 \times 2 + (2x+3)^9 \times 2x$
  - Chain: $x^2(2x+3)^9 \longrightarrow 9(2x+3)^8 \times 2$
  - Quotient: $\frac{x}{(1+x)} \longrightarrow \frac{(1+x)\times(1-x)\times 1}{(1+x)^2}$

2.5.3 Cournot Derivation of Payoff & Reaction (Simultaneous)

Step 1

Fix firm 2's action and find firm 1's best response to it.

- Find *Payoff Function*
- Partially derive and $= 0$ for *Best Response* with fixed q_2
- Find best q_1 for *Reaction Function*, sub for q_1^*

Step 2

Fix firm 1's action and find firm 1's best response to it.

- Find *Payoff Function*
- Partially derive and $= 0$ for *Best Response* with fixed q_1
- Find best q_2 for *Reaction Function*, sub for q_2^*

Step 3

Find meeting point of Nash Equilibrium where Reaction Functions meet; substitute to find optimal π for each firm.

Definitions

- Players: 2 firms $i = 1, 2$
- Strategies: each firm chooses quantity of q_i, for $Q = q_1 + q_2$
- Payoff: given supply choices *Marginal Cost* of c, *Price* of $P(q_1 + q_2)$, *Payoff* (profit) of $\pi_i(q_1, q_2) = q_1 p(q_1 + q_2) - cq_i$

Working Example

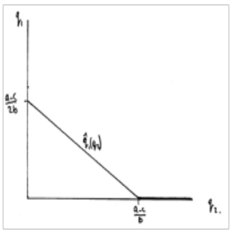
Firm 1's *Payoff Function*: $\pi_1 = q_1(a - b(q_1 + q_2)) - cq_1$
$\pi_1 = q_1(a - bq_1 - bq_2) - cq_1$
$\pi_i = aq_1 - bq_1^2 - bq_1 q_2 - cq_1$

Fix q_2: $\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c$
$[a - 2bq_1 - bq_2 - c = 0]$
$[2bq_1 = a - c - bq_2]$

Firm 1's *Reaction Function*: $[q_1 = \frac{a-c-bq_2}{2b}]$...$[q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2]$
Recall: q_1 is not $(-)$ as $q_2 \leq \frac{a-c}{b}$

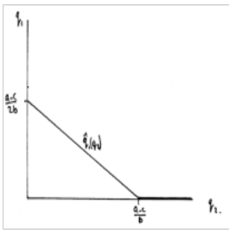
Note that *Reaction Function*: $q_1^* = \hat{q}_1(q_2) = \left\{ \begin{matrix} \frac{a-c}{2b} - \frac{1}{2}q_2 \text{if} q_2 \leq \frac{a-c}{b} \\ 0 \text{if} q_2 > \frac{a-c}{b} \end{matrix} \right\}$

Hence, *Reaction Function*: output quantity should decline as the opponent's increases; when it reaches 0, leave market; obviously no negative



Firm 2's *Payoff Function*: $\pi_2 = q_2(a - b(q_1 + q_2)) - cq_2$
Fix q_1: $\frac{\partial \pi_2}{\partial q_2} = a - 2bq_2 - bq_1 - c$
Firm 2's *Reaction Function*: $[q_2 = \frac{a-c-bq_1}{2b}]$...$[q_2 = \frac{a-c}{2b} - \frac{1}{2}q_1]$

Note that *Reaction Function*: $q_2^* = \hat{q}_2(q_1) = \left\{ \begin{matrix} \frac{a-c}{2b} - \frac{1}{2}q_1 \text{if} q_2 \leq \frac{a-c}{b} \\ 0 \text{if} q_1 > \frac{a-c}{b} \end{matrix} \right\}$

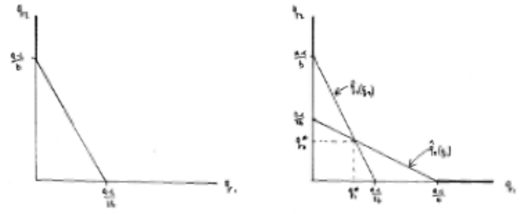


Nash Equilibrium ("The Courot Equilibrium"): flip firm 2's *Reaction Function* and overlay

Seek q_1^*, q_2^* from $q_1^* = \hat{q}_1(q_2^*)$ and $q_2^* = \hat{q}_2(q_1^*)$

"For firm 1's q which maximises its π given firm 2's q"

"For firm 2's q which maximises its π given firm 1's q"



Substitution is as follows:

From $q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2$

$q_1 = \frac{a-c}{2b} - \frac{1}{2}(\frac{a-c}{2b} - \frac{1}{2}q_1)$

$q_1 = \frac{a-c}{2b} - \frac{a-c}{4b} + \frac{1}{4}q_1$

$q_1 = \frac{4b(a-c)-2b(a-c)}{8b^2} + \frac{1}{4}q_1$

$q_1 = \frac{2b(a-c)}{8b^2} + \frac{1}{4}q_1$

$q_1 = \frac{a-c}{4b} + \frac{1}{4}q_1$

$\frac{3}{4}q_1 = \frac{a-c}{4b}$

$q_1 = \frac{4}{3}\frac{a-c}{4b}$

$q_1 = \frac{4(a-c)}{12b}$

$q_1^* = \frac{a-c}{3b}$ Sub for firm 2's π

Cournot Equilibrium is as follows: Hence: $Q^* = q_1^* + q_2^*$

So: $p^* = a - bQ^*$

Thus: $\pi^* = (p^* - c)Q^*$

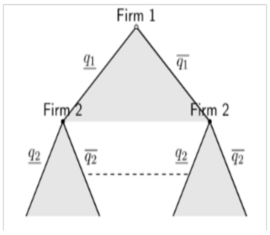
$$\boxed{q_1^* = q_2^* = \frac{a-c}{3b};\ Q^* = \frac{2(a-c)}{3b};\ p^* = \frac{a+2c}{3};\ \pi_i^* = \frac{(a-c)^2}{9b}}$$

Verify that:

Industry output between monopoly and PC: $Q^M < Q^* + Q^C$

Price between monopoly and PC: $p^M = p^* + p^C$
Industry profit between monopoly and PC: $\pi^M > \pi_1{}^* + \pi_2{}^* > 0$

2.5.4 Stakleberg Leader & Follower (Sequential)

- The leader implements the first player's reaction function intro their payoff function
- First mover advantage as leader gets higher payoff
- Recall from sequential games: backward induction
- *Linear Demand*: $P(Q) = a - bQ$
- *Constant Marginal Costs*: $C_i(q_i) = cq_i$
- *Profits*: $\pi_i(q_i, q_j) = q_i\,(a - b(q_i + q_j)) - cq_i$
- Firm 1 moves, firm 2 observes and moves
- Firm 1 is the leader, firm 2 is the follower
- Recall: 

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Backward Induction: Step 1 (Stage 2)

- Firm 2 maximises profits given q_1
- Firm 2 uses Best Response to whatever firm 1 produces

1. Recall: $\pi_2 = q_2(a - b(q_1 + q_2)) - cq_2$ (*Payoff Function*)
2. Optimise and $= 0$: $\frac{\partial_{\pi 2}}{\partial_{q2}} = a - 2bq_2 - bq_1 - c = 0$
3. React: $q_2{}^* = \hat{q}_2\,(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_1$ (*Reaction Function*)

Backward Induction: Step 2 (Stage 1)

- Firm 1 anticipates reaction of firm 2 to any decision made
- Firm 1 maximises profits given response of firm 2
- Firm 1 chooses point on firm 2's reaction function which maximises profits

1. Recall: $\pi_1(q_1, q_2) = q_1(a - b(q_1 + q_2)) - cq_1$ (*Payoff Function*)
2. It Knows: if they choose q_1 firm 2 will respond with $q_2{}^* = \hat{q}_2(q_1{}^*)$
3. Firm 1 (leader): $\pi_1(q_1, \hat{q}_2(q_1)) = q_1(a - b(q_1 + \hat{q}_2(q_1))) - cq_1$
   - Therefore: $\pi_1\left(q_1, \hat{q}_2\left(q_1\right)\right) = \frac{a-c}{2}q_1 - \frac{b}{2}q_1^2$
4. So firm 1 maximises: $\frac{\delta_{\pi 1(q1,q2)}}{\delta_{q1}} = \frac{a-c}{2} - bq_1 = 0$
   - Therefore: $q_1^L = \frac{a-c}{2b}$ (*Reaction Function*)
5. So firm 2 (follower): $q_2^F = \frac{a-c}{2b} - \frac{1}{2}q_1^L$
   - $q_2^F = \frac{a-c}{4b}$

Stakleberg Equilibrium

"Rather than an equilibrium, there is an advantage"

$$\boxed{Q^S = \frac{3(a-c)}{4b};\ \pi_1^L = \frac{(a-c)^2}{8b};\ \pi_2^F = \frac{(a-c)^2}{16b}}$$

Stakleberg vs. Cournot

Cournot: $q_1^* = q_1^* = \frac{a-c}{3b}$; $Q^* = \frac{2(a-c)}{3b}$; $p^* = \frac{a+2c}{3}$; $\pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b}$

$$\text{Stakleberg:}\quad \begin{array}{ll} q_1^L = \frac{a-c}{2b} > q_1^*; & q_2^F = \frac{a-c}{4b} < q_2^*; \\ Q^S = \frac{3(a-c)}{4b} > Q^*; & p^S = \frac{a+3c}{4} < p^*; \\ \pi_1^L = \frac{(a-c)^2}{8b} > \pi_1^*; & \pi_2^F = \frac{(a-c)^2}{16b} < \pi_2^* \end{array}$$

Hence, first mover (firm 1) advantage

2.6 Applications of Prisoner's Dilemma

- An example of a continuous game
- Rather than using *Reaction Functions*, find *Isoprofit Curves*
- This is like indifference curves for firms
- Call firms i, j; strategies x_i, x_j; payoffs $\pi(x_i, x_j)$

2.6.1 Typical Reaction Function

$$\hat{q}_i(q_j) = \frac{a-c}{2b} - \frac{1}{2}q_j$$

From *Isoprofit* contours
Equilibrium at intersection: $q_i^* = q_j^* = \frac{a-c}{3b}$

2.6.2 Maximising Joint Profit

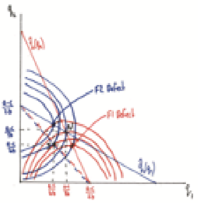
$$J(q_i, q_j) = (q_i + q_j)(a - b(q_i + q_j)) - cq_i - cq_j$$

$\frac{\partial J}{\partial q_i} = a - 2b(q_i + q_j) - c = 0 \rightarrow \widetilde{q}_i(q_j) = \frac{a-c}{2b} - q_i$
$\frac{\partial J}{\partial q_j} = a - 2b(q_i + q_j) - c = 0 \rightarrow \widetilde{q}_j(q_i) = \frac{a-c}{2b} - q_j$

Hence, assuming $q_i = q_j = \widetilde{q}$
$\widetilde{q} = \frac{a-c}{4b}$
Makes sense as: $2\left(\frac{a-c}{4b}\right) = \frac{a-c}{2b}$ (Monopoly Output)

2.6.3 Will Firms Agree?

- Will firms agree to produce at half the monopoly output?
- **No**; if firms expect you to produce more than $\widetilde{q}$;
- Best possible: $\hat{q}_i(\widetilde{q}) \rightarrow$ must expand output in excess of *Cournot Output*
- Defecting firm: *Bonanza Payoff*
- Cooperating firm: *Sucker Payoff*
- Hence, *Prisoner's Dilemma*
- $\pi^B > \frac{1}{2}\pi^M > \pi^* > \pi^S$



		2	
		cooperate	defect
1	cooperate	$1/2\pi^M, 1/2\pi^M$	π^S, π^B
	defect	π^B, π^S	π^*, π^*

2.6.4 Externalities

Negative

You do more, you lower my payoff (*Cournot Game*)
$\frac{\partial \pi_i}{\partial x_j} < 0$ (slope of *Payoff Function*)

Positive

You do more, you lower my payoff
$\frac{\partial \pi_i}{\partial x_j} > 0$ (slope of *Payoff Function*)

2.6.5 Strategic Nature

Strategic Substitutes

Opponent does more of their action: you optimally do less (*Reaction Function* downward)
$\left(\frac{\partial \frac{\partial \pi_i}{\partial x_i}}{\partial x_j}\right) < 0$ "with a higher x_j, the optimum is with a lower x_i"

Strategic Compliments

Opponent does more of their action: you optimally do more (*Reaction Function* upward)
$\left(\frac{\partial \frac{\partial \pi_i}{\partial x_i}}{\partial x_j}\right) > 0$ "with a higher x_j the optimum is with a higher x_i"

2.6.6 Nash Equilibrium in Games

- When non-cooperative, players optimise self-interest

- Marginal Payoff = 0: $\frac{\partial \pi_i}{\partial x_i} = 0$; $\frac{\partial \pi_j}{\partial x_j} = 0$
- Nash equilibrium at: $\hat{x}_i\left(x_j\right)$ and $\hat{x}_j(x_i)$
- Thus, nash equilibrium actions: x_i^*, x_j^*

2.6.7 Social Planner

- What happens when they 'internalise' the externality?
- *Social Planner* maximises joint payoff

$$J = \pi_i\left(x_i, x_j\right) + \pi_j\left(x_i, x_i\right)$$

Chooses x_i, x_j to maximise
$\frac{\partial J}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_j} = 0 \rightarrow \widetilde{x}_i(x_j)$
$\frac{\partial J}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_j} = 0 \rightarrow \widetilde{x}_i\left(x_j\right) \rightarrow$ these are both *Social Optimums*

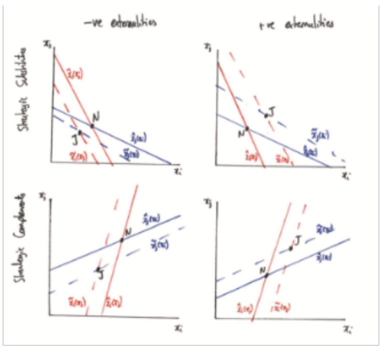
2.6.8 Nash Equilibrium vs. Optimum

w/ Positive Externalities

$\frac{\partial \pi_j}{\partial x_i} > 0$
So for *Social Planner* ($\frac{\partial \pi_i}{\partial x_i}$ must be < NE)
So $\widetilde{x}_i\left(x_j\right) > \hat{x}_i\left(x_j\right)$

w/ Negative Externalities

$\frac{\partial \pi_j}{\partial x_i} < 0$
So for *Social Planner* ($\frac{\partial \pi_i}{\partial x_i}$ must be > NE)
So $\widetilde{x}_i\left(x_j\right) < \hat{x}_i\left(x_j\right)$



3 Public Economics

3.1 Equity & Distribution

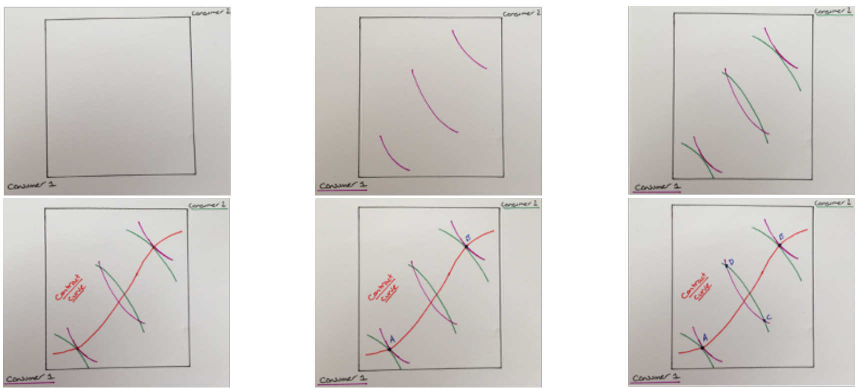
- Social Welfare Theorem
- Can the *Social Planner* decide between allocations of utility? (Efficient redistributions)
- Can lump sum taxes achieve desired outcome?
- Is *Pareto Efficiency* the desired outcome?
- Measure income and income equality

3.1.1 First & Second Welfare Theorems

1. Every Walrasian Equilibrium is pareto efficient
    - Competitive market conditions: market equilibrium is efficient
2. Every Pareto Efficient Allocation Supported by Walrasian Equilibrium
    - Competitive market conditions: efficient allocation is supported by market equilibrium

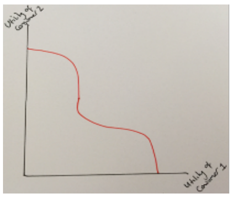
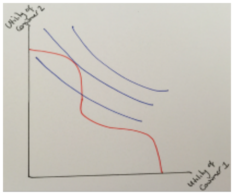
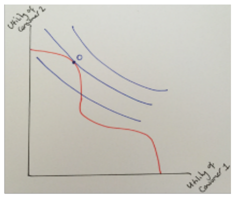
3.1.2 Edgeworth Box

1. Getting the Pareto Efficient outcome
    - "For a pareto efficient outcome, both sides can't be made better-off"
2. *Social Planner*: redistribute for efficiency



3.1.3 Social Planner

"How much of X's utility can/'should' be traded-off for more efficiency?"

3.1.4 Individual to Aggregate Utility

- *Aggregate Utility*: given individual happiness (Utility), what's the social happiness (Welfare)
- Hence, W denotes Welfare
- *Utilitarian*: $W = U_2 + U_1 + ... + U_N$
- *Rawlsian*: $W = \min\{U_2 + U_1 + ... + U_N\}$
- "If the lowest level of utility is still quite high, we're doing well"

3.1.5 How do We Achieve the Optimal Allocation?

- There are two types of people (differing in ability)
- The government (social planner) observes these abilities 'perfectly'
- s_l = Low Ability; s_h = High Ability; x = Consumption; l = Labour
- Utility Function: $U(x, l) = u(x) - v(l)$ (same for both abilities)
- $\frac{dU}{dx} > 0$ then $\frac{d^2U}{dx^2} < 0$;
- $\frac{dU}{dl} < 0$ then $\frac{d^2U}{dl^2} > 0$
- Lump Sum Taxes: individual pays tax T_i is $T_i > 0$
- At what rate do you turn additional labour into additional consumption?
- Higher ability people are more efficient at this
- Budget Constraint: $x_i = s_i l_i - T_i$
- Individual i trades-off MU of consumption with Mdis-U of labour:
- s_i: $\frac{du}{dx_i} - \frac{dv}{dl_i} = 0$
- Utilitarian to maximise total U:

- $\max\{\sum_{l,h} u(s_i l_i\,(T_i) - T_i) - v(l_i(T_i))\}$
- Government must run balanced budget:
- $T_h + T_l = 0$ where $\frac{du}{dx_h} = \frac{du}{dx_l}$

3.1.6 Measuring Inequality

1. Range of M
2. Relative Mean Deviation of M
3. Coefficient of Variation
4. Gini
5. Lorenz Curve

Range of M

$$R = \frac{M^H - M^1}{H_\mu}$$

Relative Mean Deviation of M

$$D = \frac{\sum |\mu - M^h|}{2(H-1)\mu}$$

Coefficient of Variation

$$\sigma = \sqrt{\frac{\sum (M^h - \mu)^2}{H}}$$

$$C = \frac{\sigma}{\mu(H-1)^{\frac{1}{2}}}$$

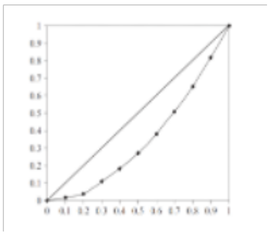
Gini

$$G = 1 - \frac{1}{H^2\mu} \begin{cases} \sum_i = 1^H \min\{M^i\} \\ \sum_j = 1^H \min\{M^j\} \end{cases}$$

$$G = 1 - \frac{1}{H^2\mu}((2H-1)M^1 + (2H-3)M^2 + (2H-5)M^3 + ... + M^H)$$

Lorez Curve

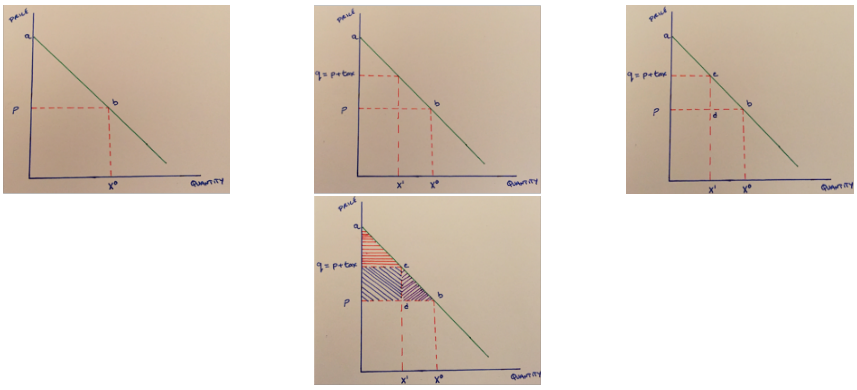
- y-axis: share of income
- x-axis: share of population

- Rank income distribution based on degree of equality
- Area below *Line of Equality* always $= \frac{1}{2}$
- $G =$ Line of Equality $-$ Lorenz Curve
- 

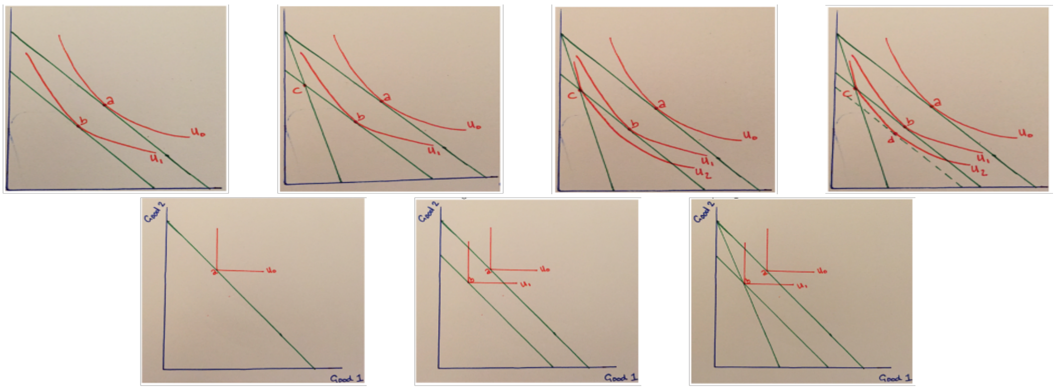
3.2 Commodity & Income Tax

3.2.1 Commodity Tax

- *Deadweight Loss* of commodity tax
- Calculating *Deadweight Tax*

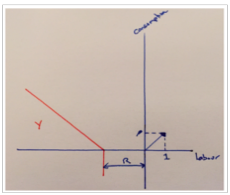


Deadweight Loss is the triangle 'edb', where:
$DWL = \frac{1}{2}((X^0 - X^1)t)$
$DWL = \frac{1}{2}\left(|\varepsilon^d|\frac{X^0}{p}t^2\right)$
Recall: $\varepsilon^d = \frac{p}{X}\frac{dX}{dp}$

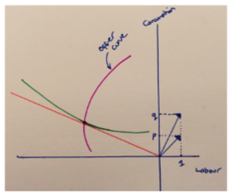


3.2.2 Optimal Taxation

- Firm
- Consumer
- Two Goods (labour l and consumption x)

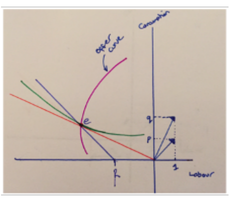


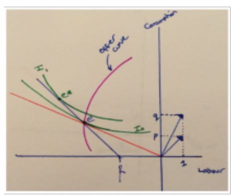
(a) Production in Robinson Crusoe World p = price; $\pi = 0$; R = Tax Revenue



(b) Production in Robinson Crusoe World $i = qx$ = Consumer Budget Constraint; q = Price Consumer Pays; p = Price Firm Recieves

Combine producer and consumer for optimal commodity taxation:



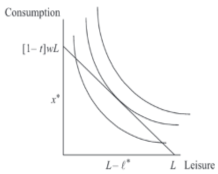


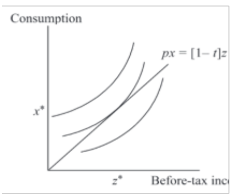
3.2.3 Modelling Consumer Choice

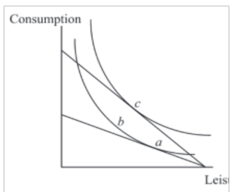
$$U = U(x, (L - \ell)) = U(x, \ell)$$

$L =$ Time
$\ell =$ Labour
$(L - \ell) =$ Leisure
$w =$ Wage
$t =$ Tax Rate
$z = w\ell =$ Pre-Tax Income
$\left(L - \frac{z^*}{w}\right) =$ Hours of Leisure
$px = w\ell(1 - t) =$ Budget Constraint
$\therefore U = U(x, \frac{z}{w})$
$\therefore px = z(1 - t)$

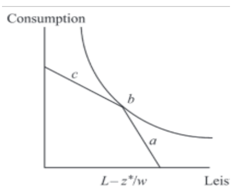
3.2.4 Graphing Income Tax

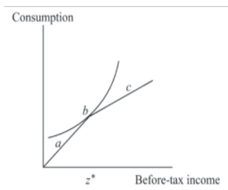






3.2.5 Graphing Income Tax w/ Thresholds





4 Cross-Section Econometrics

4.1 Descriptive Statistics

Variable Types	
Numerical	
Continuous	Infinite possible values.
	Real line or interval.
	E.g. page number on your Beamer slide show or; hh:MM:SS:ss – dd/mm.yyyy.
Discrete	Restricted possible values.
	Bound to lower and upper limit.
	E.g. number of lexer errors in TEX log when you try to use HTML shortcuts as opposed to basic Unicode shortcuts.
Categorical	
Ordinal	Ordered/sequenced and has meaning.
	E.g. gender, race, eye colour etc.; any type of group residence.
Regular	Ordered/sequenced and has no meaning.
	E.g. surname, forename.
Relationships	
Correlation	How variables move together (direction).
	Correlation $\neq$ Causation
Association	Intuitively connected but, perhaps, not correlated.
Dependence	Does correlatively rely on the movement of another variable.
Independence	Does not correlatively rely on the movement of another variable.
Data Collection	
Sample	Portion of population selected for analysis.
	Used to make assumptions and inference upon entire populations.
Population	Entirety of the group from which a sample is extracted.
Sampling Bias	
Non-Responsive	Only a small portion of a the proposed sample responded.
Voluntary Response	Irrational bias in opinions of people within sample.
Convenience	More accessibility leads to easier response. May lead to skewed results.
	E.g. a farming survey's results in Central Asia may be skewed towards commercial farming because of India (easier to survey), although subsistence farming is significantly more present in remote (harder to survey) areas such as Nepal; of which there is a larger quantity.
Observations	Conductor observes information and translates it to data (observations) from existing scenario.
Experiment	Conductor observes data from results of self-manufactured scenario.
Examining Data	
Scatter Plot	Plots scattered across a 2D x-axis/y-axis
	Allows for identification of relationship between x (explanatory axis) and y (response axis).
	E.g. linear (positive/negative), polynomial (varying by degree {quadratic, cubic, quartic, quintic, ...}) etc.
Bar Graph	Basic comparison of values (y-axis) of categories (x-axis).

Population Pyramids	Comparison of intervals (y-axis) across two categorical groups (x-axis 1, x-axis 2). Often used in human population.
Box Plot	Highlights minimum, first quartile, median, third quartile and maximum.
Distribution Moments	
Mean	$\mu = \frac{\Sigma x}{N}$ = Population Mean; $\bar{x} = \frac{\Sigma x}{n}$ = Sample Mean Common 'average' value. Supplies line of best fit in linear equations. Influenced by outliers so can be skewed.
Median	Central value in dataset. Neglects outliers, to a degree (hence, used for salary etc.) Finds quartile ranges: 0 - Q1 - Q2 - Q3 - N (IQR)
Standard Deviation	σ = Standard Deviation How much data deviates from the mean. Same units as the data.
Variance	σ^2 = Variance Fairly weighted measure of variation from the mean. Discards negatives, weights higher deviations more.
Covariance	$\text{cov}_{x,y} = \sigma_x \sigma_y \rho_{x,y}$ = Covariance Variation of members of the dataset, relative to others.
Correlation	$\rho_{x,y} = \frac{\text{cov}_{x,y}}{\sigma_x \sigma_y}$ = Correlation If ρ = 1: Perfect Positive Correlation (Together) If ρ = -1: Perfect Negative Correlation (Apart) If ρ = 0: No Correlation Correlation Matrices map all possible movements together.
Skewness	Degree of asymmetry around the mean. **Symmetric:** assume mean in centred. {mean $\approx$ median}; {skewness $\approx$ 0} **Left Skewness:** tail to left {<u>Negative Skewness</u> < 0} {mean < median}; Positive Distribution **Right Skewness:** tail to right {<u>Positive Skewness</u> > 0} {mean > median}; Negative Distribution
Kurtosis	Measure of the peak of the data; likelihood of extremes. 'Excess Kurtosis': how peaked the data is relative to the normal distribution. Excess Kurtosis = $k - 3$ (Generally, $\{k - 3 = 1\}$ is significant) **Leptokurtic:** above Normal Distribution - skinny/high tails. {Excess Kurtosis > 0; $k < 0$}; Positive Excess Kurtosis **Platykurtic:** below Normal Distribution - fat/low tails. {Excess Kurtosis < 0; $k > 0$}; Negative Excess Kurtosis **Mesokurtic:** Normal Distribution {Excess Kurtosis = 0; $k = 0$}; Normal Distribution
Modality	**Unimodal:** 1 peak **Bimodal:** 2 peaks **Multimodal:** more than 2 peaks; n peaks **Uniform:** No peaks (outcomes have equal probabilities)
Types of Economic Data	
Cross-Section	Observations of different units, over the same time period. $\mathrm{R_i}$ for $\mathrm{i} \in \{1, 2, ..., \mathrm{N}\}$; $\forall$ N unit observations. E.g. returns of 226 companies over 69/62/26.

Time-Series	Observations of the same unit, over different times periods.
	$\mathrm{R_t}$ for $\mathrm{t} \in \{1, 2, ..., \mathrm{T}\}$; $\forall$ T time periods.
	E.g. returns of 1 company over 69/62/26 - 22/66/96.
Panels	Observations of different units, over different time periods.
	$\mathrm{R_{it}}$ for $\begin{Bmatrix} i \in \{1, 2, ..., N\} \\ t \in \{1, 2, ..., T\} \end{Bmatrix}$
	E.g. returns of 226 companies over 69/62/26 - 22/66/96.

4.2 Regression Modelling Basics

Model Types
Standard Ordinary Least Squares (OLS) Multiple Regression

$$\mathrm{Y_t} = \alpha + \sum_{\mathrm{k}=1}^{\mathrm{K}} \beta_{\mathrm{k}} \mathrm{X_k} + \varepsilon_{\mathrm{t}}$$

$$\vee\ \mathrm{Y_t} = \alpha + \beta_1 \mathrm{X_1} + \beta_2 \mathrm{X_2} + ... + \beta_{\mathrm{K}} \mathrm{X_K} + \varepsilon_{\mathrm{t}}$$

For N observations; dependent (response) variable Y; k independent (explanatory) (X) variables; random error ε_{t} ($\varepsilon_i = (Y_i - \hat{Y}_i)$, each observation has a predicted $\hat{Y}_i$ on the line of best fit, directly above or below its real Y_i, distance between each of these value is the *error*) and; time t (Wooldridge, 2012). For $\mathrm{k} \in \{1, ..., \mathrm{K}\}$ and $\mathrm{t} \in \{1, ..., \mathrm{N}\}$.

β_1 is the marginal effect of $\mathrm{X_1}$ on Y, ..., β_{K} is the marginal effect of $\mathrm{X_K}$ on Y. Hence, $\beta = \frac{\Delta Y}{\Delta X} = \frac{\partial Y}{\partial X}$

This is a *Linear Regression* which uses a straight 'line of best fit' (hyperplane). For non-linear data, a *Polynomial Model* can be used to account for concave/convex data. Hence: $Y_i = \alpha + \beta_1 X_1 + \beta_2 X_2^2 + ... + \beta_K X_K^K + \varepsilon_i$

Assumptions of a standard OLS Regression:

[1] $\Sigma\varepsilon_{\mathrm{i}} \cong 0$

The sum of all residuals should approximately zero, meaning the degree to which observations outlie the line of best fit should be consistent. Errors should be minimised to satisfy min $\left\{\Sigma_{\mathrm{i}=1}^{\mathrm{N}} \varepsilon_{\mathrm{i}}^2 = \Sigma_{\mathrm{i}=1}^{\mathrm{N}} \left(\mathrm{Y_i} - \hat{\mathrm{Y}}_{\mathrm{i}}\right)^2\right\}$ for $\hat{\alpha} = \hat{\beta}_0$; $\hat{\beta} = \hat{\beta}_1$

[2] $\mathrm{var}(\varepsilon_{\mathrm{i}}) = \mathrm{E}(\varepsilon_{\mathrm{i}}^2) = \sigma^2$

All observations have constant errors, meaning that the variance of the residuals is constant. Therefore, they are homoskedastic; no need for a heteroskedasticity test (White, 1980) or the adjustment to a robust model.

[3] $\mathrm{cov}(\varepsilon_{\mathrm{i}}, \varepsilon_{\mathrm{j}}) = 0$ (for $\mathrm{i} \neq \mathrm{j}$)

Error terms uncorrelated with one another; observations are exogenous ($\mathrm{X_1}$ does not cause changes in $\mathrm{X_2}$), not endogenous. There is no need for Instrumental Variable approach. No endogeneity problem.

ARCH Stochastic Time-Series Model

$$\mathrm{Y_t} = \alpha + \mathrm{Y_{t-1}} + \varepsilon_{\mathrm{t}} \rightarrow \Delta\mathrm{Y_t} = \alpha + \varepsilon_{\mathrm{t}}$$

Referring to $\mathrm{Y_t}$, stock prices increase by an average of α each period but are otherwise unpredictable due to variation ε_{t} (Engle, 1982). Meaning that stock returns (referring to $\Delta\mathrm{Y_t}$; "a stock return is the change in stock price") are on average α but are unpredictable due to error ε_{t}.

We observe lagged variables ($\mathrm{Y_{t-1}}$) and therefore must ensure no autocorrelation between them and the dependent variable as such: $\rho = \mathrm{corr}\left(\mathrm{Y_t}, \mathrm{Y_{t-1}}\right)$. This takes form of the Autoregressive(1) (AR(1)) which allows unit-root tests which aim to identify stationarity, ensuring zero-mean, constant variance, no seasonality. Hence, testing for ($|\rho| < 1$). They're frequently referred to as tau-tests, also ensuring coefficients have a t-distribution and produce accurate t-stats and p-values. This follows:

$$\mathrm{Y_t} = \alpha + \rho \mathrm{Y_{t-1}} + \varepsilon_\mathrm{t}$$

Extended to the Autoregressive(p) (AR(p)) model for p time stamps in period T:

$$\mathrm{Y_t} = \alpha + \rho_1 \mathrm{Y_{t-1}} + ... + \rho_\mathrm{p} \mathrm{Y_{t-p}} + \varepsilon_\mathrm{t}$$

Furthermore, for modelling financial time-series, the above is transformed into the Autoregressive Conditional Heteroskedasticity (ARCH) model (Bollerslev, 1986). This accounts for return volatility and an error function as follows:

$$\mathrm{Y_t} = \alpha + \beta_1 \mathrm{X_{1t}} + ... + \beta_\mathrm{k} \mathrm{X_{k1}} + \varepsilon_\mathrm{t}$$

$$\sigma_\mathrm{t}^2 = \mathrm{var}(\varepsilon_\mathrm{t}) = \gamma_0 + \gamma_1 \varepsilon_\mathrm{t-1}^2 + ... + \gamma_\mathrm{p} \varepsilon_\mathrm{t-p}^2$$

$$\varepsilon_\mathrm{t} = f(\varepsilon_\mathrm{t-1}, \varepsilon_\mathrm{t-1}, ..., \varepsilon_\mathrm{t-p})$$

Dummy Variables

Dummy variables are used to alter a sample according to a binary criteria (Wooldridge, 2003). A dummy (D) is placed in the model with a value of 1 or 0. For example, when isolating males and females. This alters the *y-intercept* according to each in question thus, changing the premium. For example:

$$\mathrm{Y_t} = \alpha + \beta_1 \mathrm{X_1} + \tau \mathrm{D_t} + \varepsilon_\mathrm{t}$$

$\mathrm{D_t} = 0$: $\mathrm{Y_t} = \alpha + \beta_1 \mathrm{X_1} + \tau \mathrm{D_t} + \varepsilon_\mathrm{t}$;
$\mathrm{D_t} = 1$: $\mathrm{Y_t} = \alpha + \beta_1 \mathrm{X_1} + \tau + \varepsilon_\mathrm{t} = (\alpha + \tau) + \beta_1 \mathrm{X_1} + \varepsilon_\mathrm{t}$

Measures of Model Integrity

[1] $\mathbf{R^2}$ Value: Basic model integrity. The degree of explanation a selection of variables has regarding the chosen dependent variable.

[1.1] Standard R^2: Used in the case of single regression.

The R^2 value of 0.2672 suggests the explanatory variable is responsible for and can statistically explain 26.72% of the variation in Y or; variation in Y is 26.72% attributable to the explanatory variable.

[1.2] Adjusted R^2 Value: Same as the former. Used in the case of multiple regression.

The R^2 value of 0.5672 suggests the collection of explanatory variables are responsible for and can statistically explain 56.72% of the variation in Y or; variation in Y is 56.72% attributable to the collection of explanatory variables.

[2] Omitted Variable Bias: When one or more variables which could have an effect on Y are omitted from the model. This may happen when omitting variables in search of better p-values. Some explanation may be omitted.

[3] Information Criteria: One way of selecting an optimal model is by finding the one with the lowest possible Schwartz Information Criterion, Akaike Information Criteria and/or Hannan-Quinn Information Criteria.

[4] f-test: Validity of the set of variables in explaining the dependent variable. Validity of the set of instruments also, in the case of an Instrumental Variable approach.

H_0: $R^2 = 0$; the model is not statistically significant

p-value > 0.05: fail-to-reject; the model is not statistically significant
p-value < 0.05: reject; the model is statistically significant at <*relevant*> level

[5] RESET Test: Tests whether non-linear explanatories, such as polynomials and logarithms, assist in explaining the dependent variable. I.e. is your model well- specified? Hypothetically adds gamma coefficients to hypothetical polynomials and logs in the model. Try different selections of polys and logs and compare RESET results. Seek combination with highest possible p-value.

Aim to fail-to-reject (if not in desire of polys and logs):
H_0: Polynomials and logarithms do not aid explanation of Y

p-value desire flips with regards to hypothesis symmetry:
p-value > 0.05: fail-to-reject; polynomials and logarithms do aid explanation of Y
p-value < 0.05: reject; polynomials and logarithms do not aid explanation of Y

[6] Endogeneity Test: The 'Hausman Test' is often used. Random unaccounted explanatories may be correlated with ε so, the Instrumental Variable approach accounts for unknown coefficients where its variable is correlated with ε.

Endogeneity: factors in model cause ΔX, ΔX associated w/ $\Delta\varepsilon$
Exogeneity: factors in model don't cause ΔX, ΔX not associated w/ $\Delta\varepsilon$

Aim to fail-to-reject:
H_0: Explanatory variables uncorrelated with ε; no endogeneity problem

p-value desire flips with regards to hypothesis symmetry:
p-value > 0.05: fail-to-reject; no endogeneity problem present
p-value < 0.05: reject; endogeneity problem present; need IV approach

[7] Heteroskedasticity Test: The 'White Test' is often used. Often referred to as 'robustness test' finding need for robust standard errors. When there're non-constant error terms (heteroskedasticity), OLS poorly fits a suitable line through the data.

Homoskedastic Errors: constant error terms/error variance
Heteroskedastic Errors: non-constant error terms/error variance

Aim to fail-to-reject:
H_0: There is homoscedasticity present; there are constant error terms

p-value desire flips with regards to hypothesis symmetry:
p-value > 0.05: fail-to-reject; there is no heteroskedasticity present; errors are constant
p-value < 0.05: reject; there is heteroskedasticity present; errors are non-constant

[8] Unit-Root Test: This may be found under various names including 'tau-test', 'unit-root test', 'Augmented Dickie-Fuller Test' etc. However, people frequently make the same mistake with these terms as they do when referring to 'terminal', 'terminal emulator' 'command line', 'shell' and 'Bash/zsh', for example.

A 'unit-root' is a possible characteristic of stochastic trend for example, when there's a random walk. A 'unit-root' test aims to identify whether there is stationarity in a stochastic model. Stationarity requires: [1] constant mean, [2] constant variance, [3] no seasonality. The 'Dickie-Fuller Test' is the creator-based name given to a test for a unit-root, using a tau test-statistic (τ-statistic) hence, 'tau-test'. If Y is non-stationary, ϕ lacks a t-distribution so p-values/t-stats of t-tests are inaccurate. Syntax follows: 'Y is stationary/non-stationary'.

[8.1] The 'Dickie-Fuller Test' is used for single-explanatory models, commonly AR(1):
Recall: $\mathrm{Y_t} = \alpha + \rho \mathrm{Y_{t-1}} + \varepsilon_\mathrm{t}$
Testing Regression: $\Delta \mathrm{Y_t} = \alpha + \phi \mathrm{Y_{t-1}} + \varepsilon_\mathrm{t}$

$\mathrm{H_0}$: $\phi = 0 \Rightarrow \{\rho = 1 \Rightarrow \phi = (\rho - 1)\}$; there is a uni-root present

p-value > 0.05: fail-to-reject; $\{\phi = 0 \Rightarrow \rho = 1\}$; Y non-stationary; unit-root present
p-value < 0.05: reject; $\{\{-2 < \phi < 0\} \Rightarrow \{-1 < \rho < 1\}\}$; Y stationary; no unit-root

[8.2] The 'Augmented Dickie-Fuller Test' follows the same principals and is used for multiple-variable models. Variables which present stationarity can be used. Syntax amendment follows: 'Y is stationary/non-stationary about the trend of X'. The 'Augmented Dickie-Fuller Test' extends the testing methods to AR(p):

Recall: $\mathrm{Y_t} = \alpha + \rho_1 \mathrm{Y_{t-1}} + ... + \rho_\mathrm{p} \mathrm{Y_{t-p}} + \varepsilon_\mathrm{t}$
Testing Regression: $\Delta \mathrm{Y_t} = \alpha + \phi \mathrm{Y_{t-1}} + \gamma_1 \Delta \mathrm{Y_{t-1}} + ... + \gamma_\mathrm{p} \Delta \mathrm{Y_{t-p}} + \delta \mathrm{t} + \varepsilon_\mathrm{t}$

$\mathrm{H_0}$: $\phi = 0 \wedge \delta = 0$; there is a uni-root present

p-value > 0.05: fail-to-reject; Y non-stationary around trend; unit-root present
p-value < 0.05: reject; Y stationary around trend; no unit-root

4.3 Regression Results Interpretation

Coefficients		
(+)	X and Y positively related and move in the same direction.	
(−)	X and Y negatively related and move in opposing directions.	
Numerical Continuous Dependent Variable		
Continuous Explanatory	For a 1 unit change in X, there is a <*coefficient*> change in Y.	*"For a 1 unit increase in management fee, there is a* <*coefficient*> *change in hedge fund value."*
Binary Explanatory	X being Binary 1 results in a <*coefficient*> difference in Y over X being Binary 0.	*"A fund with a high-water mark has a* <*coefficient*> *different value from a fund without one."*
Binary Dummy Dependent Variable		
Continuous Explanatory	For a 1 unit change in X, the odds of Y being Binary 1 changes by <*coefficient*> units.	*"For a 1 unit increase in management fee, the odds of a hedge fund being defunct changes by* <*coefficient*> *units."*
Binary Explanatory	X being Binary 1 results in the odds of Y being Binary 1 by <*coefficient*> different units over X being Binary 0.	*"A fund with a high-water mark is* <*coefficient*> *differently probable of being defunct than a fund without one."*
Respective Numerical Examples (Coefficient = 0.226)		
"For a 1 unit change in management fee, there is a 0.226 unit increase in hedge fund value."		
"A hedge fund with a high-water mark has a 0.226 unit greater value than one without one."		
"For a 1 unit increase in management fee, the odds of a hedge fund being defunct increases by 0.226 units."		
"A hedge fund with a high-water mark is 0.226 units more probable of being defunct than one without one."		

4.4 Hypothesis Testing Basics

Hypotheses		
H_0: Null Hypothesis *Aim to Reject; may Fail-to-Reject; rarely Accept* **H_A:** Alternative Hypothesis *Favour in the case of Rejection of the Null*		
Type I Error: Rejection of null hypothesis when it is true **Type II Error:** Failure-to-reject null hypothesis when it is false		
Reduce Type I Error Risk: Reduce significance level; harder to reject null **Reduce Type II Error Risk:** Use large sample; ensuring significant spread		
Probability α: Probability of making Type I Error **Probability** β: Probability of making Type II Error		
Error Examples		
Decision	H_0 is True (Accused is Innocent)	H_0 is False (Accused is Guilty)
Reject H_0 (Accused Convicted)	**WRONG** Decision (Type I Error) Probability α	**CORRECT** Decision
Fail-To-Reject H_0 (Accused Goes Free)	**CORRECT** Decision	**WRONG** Decision (Type II Error) Probability β

4.5 t-stat & p-value Interpretation

t-stat
$$\text{t-stat} = \frac{\bar{x}-\bar{\mu}}{\left(\frac{\sigma}{\sqrt{N}}\right)}$$
$\lvert$**t-stat**$\rvert > 1.96$: reject the null hypothesis at the 5% significance level; $\lvert$**t-stat**$\rvert > 2.58$: reject the null hypothesis at the 1% significance level *Given 1000 degrees of freedom*
p-value
p-value < 0.05: reject the null hypothesis at the 5% significance level; **p-value** < 0.01: reject the null hypothesis at the 1% significance level *Given 1000 degrees of freedom*
Confidence Relevance
5% and 1% significance levels are also referred to as the 95% and 99% confidence [in rejecting the null] intervals. Correct syntax: refer to 5% and 1% significance levels when referring to p-values; 95% and 99% confidence levels when referring to hypotheses.
Error Relevance
5% and 1% levels are used to ensure accuracy and reduced probability of making a Type I error. That is, “accepting a max 5%/1% chance that you are wrong when rejecting the null”; “you are min 95%/99% confident you are right when rejecting”.