

MM102 Applications of Calculus

Answers to the exam in May 2016

1. (a) (i) $\frac{3\pi - 8}{32}$

(ii) $4 - 2\sqrt{3} + \frac{5\pi}{6}$

(b) $\frac{9\pi}{2}$

2. (a) $p_{2,1}(x) = -1 - (x - 1) + \frac{\pi^2}{2}(x - 1)^2$

$R_{2,1}(x) = \frac{-3\pi^2 \cos(\pi\xi) + \pi^3 \xi \sin(\pi\xi)}{6}(x - 1)^3$ with ξ between x and 1

(b) (i) $\text{dom}(f) = \{x \in \mathbb{R} : x \neq 0 \text{ and } x \neq 2\}$

(ii) Vertical asymptotes: $x = 0, x = 2$

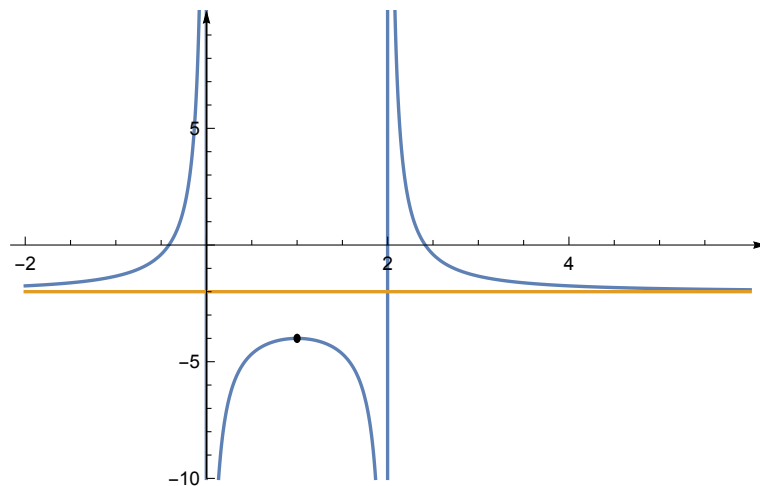
Horizontal asymptote: $y = -2$

(iv) Local maximum at $x = 1$ with $f(1) = -4$

The function is increasing on the intervals $(-\infty, 0)$ and $(0, 1)$.

The function is decreasing on the intervals $(1, 2)$ and $(2, \infty)$.

(v)



The function f has two zeros.

3. (a) $a = \frac{1}{16}, b = -\frac{3}{16}, c = \frac{1}{8}$

(b) $2 \operatorname{cis}\left(\frac{5\pi}{24}\right), 2 \operatorname{cis}\left(\frac{17\pi}{24}\right), 2 \operatorname{cis}\left(-\frac{19\pi}{24}\right), 2 \operatorname{cis}\left(-\frac{7\pi}{24}\right)$

(c) $1 \pm 3i, 3 \pm i$

(d) $\ln(\sqrt{2}) + \frac{3\pi i}{4}$

4. (a) (i) Integrating factor: $I(x) = \frac{1}{\sqrt{x}}$

(ii) $y = 2x^{7/2} + Cx^{1/2}$

(b) $y = -x \left(1 + \frac{1}{1 + \ln x} \right)$

(c) $y = A + Be^{-4x} + e^{2x} + x(2x - 1)$