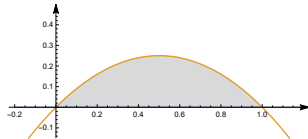# MM102 Applications of Calculus

Answers to the exam in May 2018

1. (a) (i) $-\dfrac{3}{2} - \ln 2$

   (ii) $-\dfrac{5\sqrt{3}}{2} + \dfrac{\pi}{2} + 4$

   (b)

   $V = \dfrac{\pi}{30}$

2. (a) $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{(1+t)e^t}{2t+1}, \qquad \dfrac{\mathrm{d}^2 y}{\mathrm{d}x^2} = \dfrac{(2t^2+3t)e^t}{(2t+1)^3}$

   (b) The y-coordinate is decreasing at a rate of $12\,\mathrm{cm/s}$.

   (c) (i) $\mathrm{dom}(f) = \{x \in \mathbb{R} : x \neq 1 \text{ and } x \neq -1\}$

   (ii) Vertical asymptotes: $x = 1, \quad x = -1$
   Horizontal asymptote: $y = 0$

   (iii) Local maximum at $x = \dfrac{1}{2}$ with $f\left(\dfrac{1}{2}\right) = -8$;
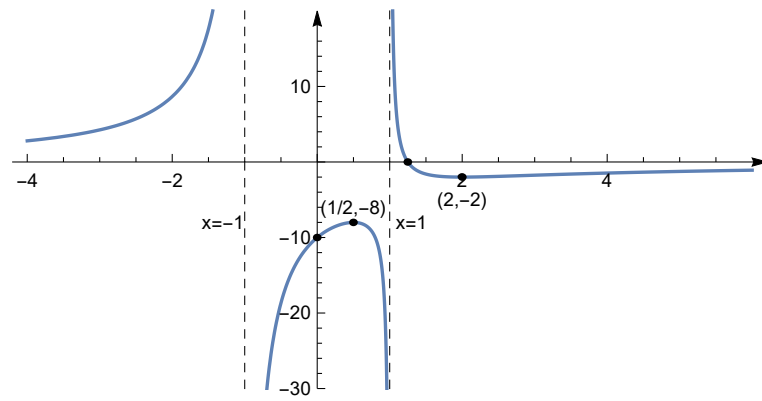   local minimum at $x = 2$ with $f(2) = -2$.
   The function is increasing on the intervals $(-\infty, -1)$, $\left(-1, \dfrac{1}{2}\right)$, and $(2, \infty)$.
   The function is decreasing on the intervals $\left(\dfrac{1}{2}, 1\right)$ and $(1, 2)$.

   (iv) Point of intersection with the y-axis: $(0, -10)$
   Point of intersection with the x-axis: $\left(\dfrac{5}{4}, 0\right)$

(v)

3. **(a)** $\dfrac{1}{\sqrt{2}}\left(-\sqrt{3}+i\right)$

(b) $a = -\dfrac{1}{16}$, $b = -\dfrac{1}{16}$, $c = \dfrac{1}{8}$

(c) $P(z) = (z+2)(z-4-i)(z-4+i)$

(d) $\dfrac{1}{2}\ln 2 + i\left(-\dfrac{\pi}{3} + k\pi\right), \quad k \in \mathbb{Z}$

4. **(a)** **(i)** Integrating factor: $I(x) = x^4$

(ii) $y = x^{-4}\left(\dfrac{2}{3}(x+1)^{3/2} + C\right)$

(b) $y = \sqrt{2}\,x(\ln x + 4)^{1/4}$

(c) $y = Ae^{3x} + Be^{-x} + 2xe^{-2x} - 2x + \dfrac{4}{3}$

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