University of Strathclyde Glasgow

Department of Mathematics and Statistics

MM102 APPLICATIONS OF CALCULUS

Tuesday 3rd May, 2016 **9:30 - 11:30a.m.**

Duration: 2 hours

Attempt ALL questions.
Use of a calculator is NOT permitted.
Answers will receive credit only if supported by appropriate working.

1. **(a)** Evaluate the following integrals

$$\textbf{(i)} \quad \int_0^{\pi/4} \sin^4 x \, \mathrm{d}x \,, \qquad \textbf{(ii)} \quad \int_2^3 \frac{2x+1}{\sqrt{4x-x^2}} \, \mathrm{d}x.$$

(7, 8 marks)

(b) Sketch the finite region that is bounded by the curves

$$y = \sqrt{x-2}, \qquad y = 0, \qquad x = 5.$$

Hence find the volume of the solid that is obtained when this region is rotated through 360° about the ***x*-axis**.

(5 marks)

Qu. 2 ON NEXT SHEET

2. **(a)** Find the Taylor polynomial of degree 2 about $x = 1$ and the corresponding remainder term of the function

$$f(x) = x\cos(\pi x).$$

(7 marks)

(b) Consider the function

$$f(x) = \frac{-2x^2 + 4x + 2}{x^2 - 2x}.$$

(i) Determine the natural domain of f.

(ii) Find all the asymptotes of f.

(iii) Show that

$$f'(x) = -\frac{4(x-1)}{(x^2-2x)^2}.$$

(iv) Find the position and the nature of the stationary points and calculate the values of the function f at these stationary points.
Moreover, determine where the function is increasing and where it is decreasing.

(v) Use this information to sketch the graph of f.
Draw the asymptotes and label the stationary point(s).
Moreover, use the plot to determine the number of zeros of the function f.

(1, 3, 2, 4, 3 marks)

Qu. 3 ON NEXT SHEET

3. (a) Use de Moivre's Theorem to find constants a, b and c such that

$$\cos(\theta)\sin^4(\theta) \;=\; a\cos(5\theta) + b\cos(3\theta) + c\cos(\theta).$$

(6 marks)

(b) Find the four distinct fourth roots of $-8\sqrt{3} + 8i$.

(Give the roots in polar form using the **principal value** of the argument in each case.)

(5 marks)

(c) Given that $z = 1 + 3i$ is a root of the equation

$$P(z) \;=\; z^4 - 8z^3 + 32z^2 - 80z + 100 \;=\; 0,$$

find all the roots of the equation.

(6 marks)

(d) Express $\text{Log}(-1 + i)$ in the form $a + ib$, where a, $b \in \mathbb{R}$.

(2 marks)

Qu. 4 ON NEXT SHEET

4. **(a)** Consider the first order, linear differential equation

$$2x\,\frac{\mathrm{d}y}{\mathrm{d}x} - y = 12x^{7/2},$$

where $x > 0$.

(i) What is the integrating factor for the differential equation?

(ii) Find the General Solution of the differential equation.

(3,3 marks)

(b) Find the Particular Solution of the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{y^2 + 3xy + x^2}{x^2} \qquad (x > 0),$$

which satisfies $y(1) = -2$.
[Express your solution y explicitly as a function of x.]

(8 marks)

(c) Find the General Solution of the second order, linear differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + 4\frac{\mathrm{d}y}{\mathrm{d}x} = 12\mathrm{e}^{2x} + 16x.$$

(7 marks)

Total number of marks: 80

END OF PAPER

(ML/GMcK)