EC315 Topics in Microeconomics with Cross-Section Econometrics Coursework Summary

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Table of Contents

1 Exam Summary

1.1 Cost-Benefit Analysis Summary

1. Purpose
2. Alternatives
3. Who
4. C/B Impacts
5. Lifetime Impacts
6. Monetize:
    - *Social Cost*: harm done to living organisms
    - *Revealed/Stated Preference*: willingness to pay or willingness to accept
        - Revealed: shown in behaviour
        - Stated: questionnaires etc.
    - *Time*:
        - Work vs leisure using wage rate
        - Travel time; how much people are willing to trade-off
    - *Lives*: life expectancy, pay, age, risks taken
    - *Natural Resources*: AONBs, surveys, investment, regulation
7. PV Discounts
    - Social discount rate
    - Intergenerational (more than 50 years)
8. NPV of Alternatives
9. Sensitivity Analysis
10. Recommend

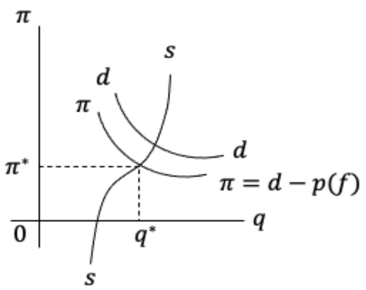
1.2 Program & Policy Evaluation Summary

$$\text{Cause} \longrightarrow \text{Intermediaries} \longrightarrow \text{Effect}$$

1. Omitted Variable Bias
    - Selection Bias: e.g. grades, income, area of ogigin
    - Selection Bias 2: e.g. effort, determination, stamina
2. Randomized Control Trial
    - Unbiased Estimator: $\bar{x} \longrightarrow \bar{\mu}$ (LLN)
    - Unbiased Estimator: randomization
    - σ^2: "how much of the result is due to chance?"
    - t-tests: causal effect; $(\bar{Y}^T - \bar{Y}^C)$
3. Regression
    - Dummy Variables: causal variable / group
    - Instrumental Variables: omitted variables (α corr. w/ ε)

1.3 Crime & Punishment Summary

1. Supply: $\pi_t = \pi_i - c_i - w_i - p_i(f_i)$
   - i = Individual
   - π_t = Net Total Payoff of Crime
   - π_i = Expected Payoff Per Offense (Minus Costs)
   - c_i = Cost Incurred if Caught
   - w_i = Wage Rate From Non-Criminal Work
   - p_i = Probability of Aprehension & Conviction
   - f_i = Punishment in Convicted
2. Normal Distribution
   - Req. $\uparrow \pi$, $\uparrow \delta$, $[\bar{x} \rightarrow$ (Right of Mean)$]$
   - Req. $\downarrow \pi$, $\downarrow \delta$, $[\leftarrow \bar{x}$(Left of Mean)$]$
   - Morals, enjoyment, risk, some demand for significantly higher payoffs etc. effect decision
3. Demand: $e_i f(v_r, v_l); q$
   - e_i = Expenditure on Protection
   - v_r = Risk of Victimization
   - v_l = Loss of Victim
   - q = Total Crime
4. Derivatives
   - $\frac{\partial e_i}{\partial v_i} > 0$: Risk $\uparrow$, Expenditure $\uparrow$
   - $\frac{\partial c_i}{\partial e_i} < 0$: Expenditure $\uparrow$, Cost $\uparrow$
   - $\frac{\partial \pi_i}{\partial c_i} < 0$: Cost $\uparrow$, Payoff $\downarrow$
5. Supply / Demand

- 

- ss = Supply of Crime
- dd = Initial Demand
- $\pi\pi$ = Demand After Government Intervention (T)
- MC of Catching Last Criminal $> MB$ [$\leftarrow \pi^*,\ q^*$]
- MC of Catching Last Criminal $< MB$ [$\pi^*,\ q^* \rightarrow$]

1.4 Exam Arithmetic Summary

1. $\pi_A = x_A p_A(x_A + x_B) - x_A$
2. $J = \pi_A + \pi_B$; $\frac{\partial J}{\partial x_A} = \frac{\partial \pi_A}{\partial x_A} + \frac{\partial \pi_B}{\partial x_B}$
3. Externalities: $\frac{\partial \pi_A}{\partial x_B}$
   - > 0: Positive: "you do $\uparrow$, my π $\uparrow$"
   - < 0: Negative: "you do $\uparrow$, my π $\downarrow$"
4. Strategic Nature: $\frac{\frac{\partial \pi_A}{\partial x_A}}{\partial x_B}$
   - > 0: Complements: "you do $\uparrow$, I do $\uparrow$"
   - < 0: Substitutes: "you do $\uparrow$, I do $\downarrow$"
5. Grim Trigger Strategy
   - 40, 50, 30
   - $\frac{40}{(1-\delta)} \geq 50 + \frac{30\delta}{(1-\delta)}$
   - $40 \geq 50 - 50\delta + 30\delta$
   - $\delta \geq \frac{1}{2}$: cooperation possible

   Tit-for-Tat Strategy
   - 40, 50, 20
   - $\frac{40}{(1-\delta)} \geq \frac{50}{(1-\delta^2)} + \frac{30\delta}{(1-\delta^2)}$
   - $40 + 40\delta \geq 50 + 20\delta$
   - $\delta \geq \frac{1}{2}$: cooperation easy

2 Game Theory

2.1 Definitions

- *Welfare Economics*: Generalising equilibriums. Competitive markets provide an incentive for firms to produce what customers want. Markets rock in fair play.
  - Theorem 1: Every competitive economy is Pareto Efficient.
  - Theorem 2: Every Pareto Efficient allocation of resources can be achieved in competitive markets (w/ appropriate redistribution between parties).
- *Pareto Efficiency*: No additional person can be made better off without making someone else worse off. There should be no government intervention. Redistribution can take place meaning there is redistribution between parties within the economy rather than externally.
- *Prisoner's Dilemma*: Pursuing your own interests leads to inefficient markets because, using the prison example, if both people choose to confess, they get full long time each. If they both lie, they get full short time. If one lies and one confesses, the one who confesses gets reduction but the liar gets full time. This is risk. Both could deny for 2 years of the other lies (gets 10 years). But then they both risk getting 8 years. If they both deny they both get the full short time (3 years). Denying is best for them both but confessing could, but only could, be best for a single one of them.
- *Rationality*: Players will choose the option with the best payoff for themselves. But back to the Joey and Phoebe, if you are choosing the best for yourself, surely the opponent must be doing the same so can you forecast? Or will they think the same and one-up you?
  - *Common Knowledge of Rationality*: Where players don't just know they possible outcomes of their decisions, they know the possible outcomes of the other's decisions. But recall the prisoner's dilemma.
- *Game Theory*: Our actions have external consequences. They effect the environment and all things around us (smoking example).
  - *Non-Cooperative Game*: In it for your own gain and only that.
  - *Information Game*: Everyone knows they are playing.
  - *Stage Game*: May be repeated (e.g. rearranging cost agreements).

- *Simultaneous Game* (Type 1): When players do not know the move of the opponent and move at the same time.
- *Sequential Game* (Type 2): When players know the move of the other player and can make their decision based on the opponent's move.

- *Imperfect/Perfect Information*: Not being able to see the others' choice. Your outcome will always depend on their choice but your decision won't. Or, you have information about their decision to look at as they have make it (historic forecasting).
- *Strategic Uncertainty* (When Simultaneous): Players must base decisions on what they think the other player will play as they do not know. But then they must consider what they think the opponent's move will be but then, the opponent will surely think they will be thinking this and so make a different move and make the same prediction about their opponent... in practice usually it comes back round to them making the first decision that you predicted they would make (Joey and Phoebe e.g.) Can lead to *Strategic Payoff* where the strategic nature of their thinking pays off and they've well forecasted the other's choice.
- *Dominant Strategy*: When there is one clear winner in the strategy you use. It takes the lead the majority or all of the time when put into the matrix. This is found through *Best Response Analysis* which is found by going through each option of B and selecting the best strategy for A to choose (repeat for all columns of B). Then repeating for B (for all rows of A). The double underlined is the dominant strategy.
- *Dominated Strategy*: When the strategy a player chooses is dominated than another strategy which would make you better-off than the one you're choosing.
- *Nash Equilibrium*: When there is a clear equilibrium between the players' *Dominant Strategies.* Means you can't *Unilaterally Deviate* to become more profitable (no incentive to deviate).
  - *Unilateral Deviation*: When there is no dominant strategy.
  - *Mixed Strategies*: Players randomise strategies on unpredictable patterns (e.g. with muscular workouts).
  - *Pure Strategies*: When the player knows for sure what option they will choose.

2.2 Simultaneous Move Game

- Sole entrant: obtains big payoff
- Multiple entrants: perhaps lacking market space
- There exists a *First Mover Advantage*

2.2.1 Pure & Mixed Strategies

- *Chicken Game*: two players heading towards each other;
  - They collide and both marginally lose out
  - One swerves and loses out bigger (chicken)
- There may be two nash equilibria
  -
		Player B	
		$E(q)$	$N(1-q)$
Player A	$E(p)$	$-50, 100$	$\underline{150}, \underline{0}$
	$E(1-p)$	$\underline{0}, \underline{100}$	$0, 0$

  - The nash equilibria are underlined. There are two
  - Pure strategies are shown through probability as seen by entering probabilities p & q above
  - For Player A (EV = Expected Value):

$$\begin{array}{l|l} EV_A^E = -50q + 150(1-q) & EV_B^E = -100p + 100(1-p) \\ EV_A^N = 0q + 0(1-q) & EV_B^N = 0p + 0(1-p) \\ -50q + 150 - 150q = 0 & -100p + 100(1-p) = 0 \\ \underline{q = \frac{3}{4}} & \underline{p = \frac{1}{2}} \end{array}$$

  - These are the probabilities of placing in the respective quartiles:

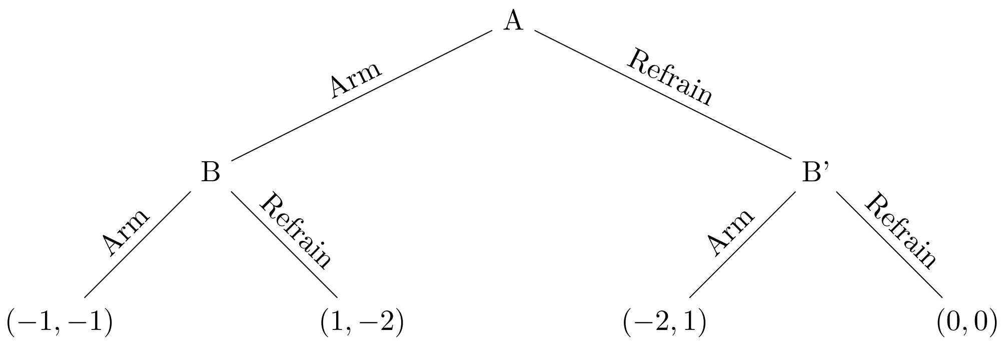
	$E\frac{3}{4}$	$N\frac{1}{4}$
$E\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$
$N\frac{1}{2}$	$\frac{3}{8}$	$\frac{1}{8}$

  - You are trying to find the option that would make you both indifferent between choosing options

2.3 Sequential Move Game

- Uses *Backward Induction*
  - Games are analysed from the end through to start
- Transforms *Normal Form* to *Extensive Form*
- Transforms *Nash Equilibrium* to *Sub-Game Nash Equilibrium*
- Not subject to *Strategic Uncertainty* (imperfect information)
  - Can observe moments
  - Hence, *Perfect Information*
  - E.g. supermarket price setting
- If the first or last mover has a *Dominant Strategy*, they'll use it

2.3.1 Game Tree



		B			
		(Arm, Arm')	(Arm, Refrain')	(Refrain, Arm')	(Refrain, Refrain')
A	Arm	$-1, -1$	$-1, -1$	$1, -2$	$1, -2$
	Refrain	$-2, 1$	$0, 0$	$-2, 1$	$0, 0$

- In simultaneous games: *Strategy* = *Action*. This isn't the case in sequential games. *Actions* are a simple move; *Strategies* are plans based on the move of the first player
- A's strategies: Arm, Refrain

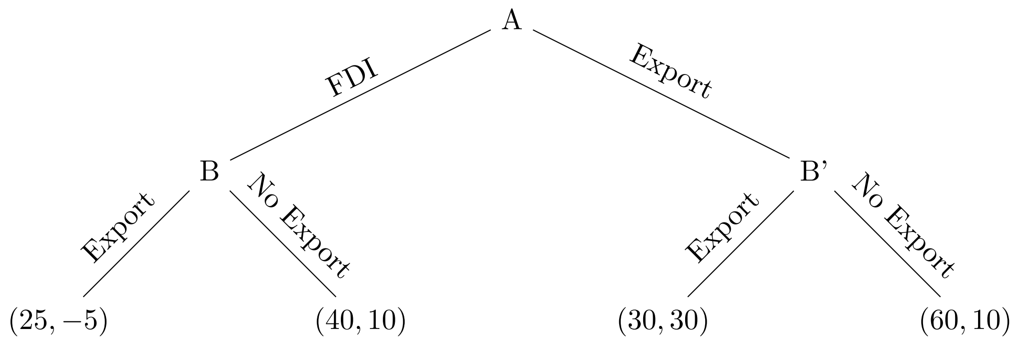
- B's strategies: (Arm, Arm'), (Arm, Refrain'). (Refrain, Arm'), (Refrain, Refrain')
- *Information Set*: don't know which two nodes you are at
- *Subgame*: the mini-looking games which B is playing under A

2.3.2 Choosing An Option

Normal:

		B	
		(Export)	(No Export)
A	FDI	$25, -5$	$40, \underline{10}$
	Export	$\underline{30}, \underline{30}$	$\underline{60}, 10$

Extensive:



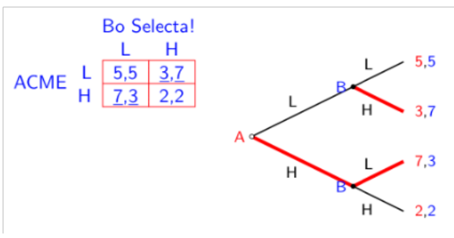
- In *Normal Form*, (Export, Export) is the *Dominant Strategy*, but there are more options:
  - (Export, Export')
  - (Export, No Export')
  - (No Export, Export')
  - (No Export, No Export')
- If A plays FDI, will B ever export?
  - (No Export, Export') allows *Incredible Threats* to be made

2.3.3 Backwards Induction

- A process used to avoid *Incredible Threats*
- If A assumes B is rational, they expect B to play $(40, 10)$ on FDI and $(30, 30)$ on Export
- *Credible Threats*:
  - Only on FDI as they could lower their payoff to punish A
  - $(25, -5)$

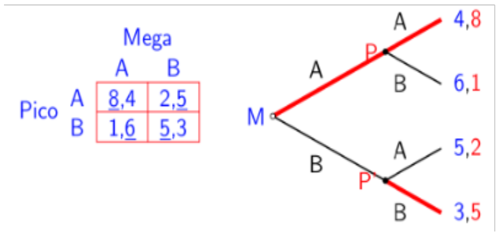
1. Start on that last stage of the game
2. Break down into two of A's options
3. Select these two *Subgame Nash Equilibria* for B on each A arm
   - (No Export, Export') are the best for B here
4. A now has a choice
   - FDI would be followed by B's No Export $(40, 10)$ $[> (25, -5)]$
   - Export would be followed by B's Export' $(30, 30)$ $[< (60, 10)]$
   - A plays FDI
5. *Nash Equilibrium* is made clear
   - {FDI,(Not Export, Export')}

2.3.4 Order Advantages

- *Commitment* in first mover vs. *Flexibility* in follower
  - Commitment has greater value in simultaneous games
  - Flexibility has greater value in sequential games
- Recall that in simultaneous games there's a first mover advantage
- *First Mover Advantage* (Simultaneous):
  - 

  - B maximises on both moves and A maximises on its one move
  - A lowers B's payoff by choosing a more profitable option for them

- *Second Mover Advantage* (Sequential):

  - 

  - M goes for overall highest payoff (6) by choosing move A
  - P has the option to choose one which greater benefits them and lowers M's expected payoff

2.3.5 Manipulating Games

- Take actions to manipulate a game? That is, guaranteeing the outcome
- *Threats & Promises*

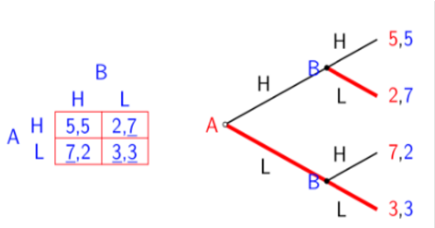
  - "If you attack, I'll fight..."
  - "If you enter, I'll enter too; making it less profitable for you..."
  - "If you work hard, I'll work harder..."
  - These lack credibility as they could be bluffs

- *Credibility*

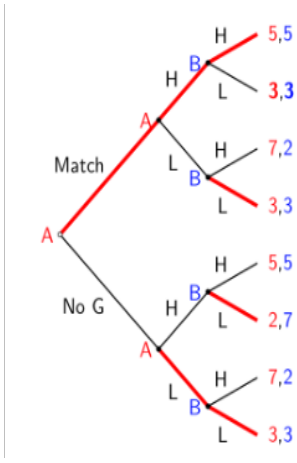
  - "If you're late, I'll set off a bomb..." (*Incredible*; bluffing / not factual?)
  - "If you're late, the timed boomb will go off..." (*Credible*; more believable / factual)
  - Changes first mover's thinking

2.3.6 Price Matching Guarantees

Standard *Subgame Perfect Nash Equilibrium*:



Offering a *Price Match Guarantee*:



- Commitment to maintain high prices
- By committing to match low prices, A changes payoffs such that it's not beneficial for B to undercut – as bigger payoff can't be seen
- Both firms end up paying high prices
- (Pricing) *Prisoner's Dilemma*

2.4 Prisoner's Dilemma

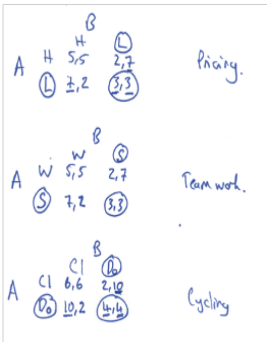
- *Cooperate* or *Defect*
- Mutual gain: *Cooperate*
- Individual incentive: *Defect*
- *Pareto Efficienct Equilibrium*
- Recall: someone has a dominant strategy where there's harm done to each other and they could be better off (Pareto Efficient). Self-interest doesn't pay off. But, games can be repeated (e.g. price re-setting), e.g. lower price than opponent now (get more custom volume), makes opponent less-off (also poor for aggregate prices), firms may form a passive collusion where they both think opponents will set low so they both set high
- Both players have dominant strategy to defect but they could have a better result when the both cooperate. Hence, when choosing best interest, harm is done to the opponent when choosing to Defect for own interest, the opponent may choose to Cooperate so have a worse outcome

		Bob	
		deny	confess
Alice	deny	3,3	10,2
	confess	2,10	8,8

Example:

1. (8,8) is Pareto Inefficient equilibrium as it is reached by both aiming for low by confessing. Could be made better off by both acting for mutual gain (3,3)
2. Pricing – non-brand loyal market, flow freely between
3. Team Work – (work vs shirk) shirk leads to more payoff as still full marks but no work done but if the other does all the work, they will get full marks but payoff will reduce due to workload
4. Clean vs Dope – (risk based) best self-interest response is to dope as highest possible payoff but the equilibrium they both do it is less than the payoff if they both don't
5. Market Share – studying marketing is a waste of time. Market is a pie, we compete over our share. Ads try to [1] inform and [2] predatory (winning market share). Start: 50/50, engage in ads to win market share. I spend money, I get

some in return but you won't gain much more market share. The opponents do this to keep up. Each keep catching back up to 50% each but both are still wasting millions on marketing. Market share isn't changing proportionately but you're still spending money



2.4.1 Externalities

- *Negative Externalities*: (own interest – doing too much) Cooperation reduces amount of work you do for the better (e.g. not over-fishing)
  - Don't see costs from defecting – too much harmful activity is done
- *Positive Externalities*: (own interest – doing too little) Cooperation says you should do more work (e.g. not doing no work in a project)
  - Don't see costs benefits form cooperating – too little of a good activity is done
- Example: Marginal Benefit vs. Marginal Cost – (1) extracting fish from the ocean makes it harder in the future (e.g. do less fishing to allow repopulation). (2) But you want more to sell now. (3) Self-interest makes it harder for others

2.4.2 Rationalise Cooperation Resolutions

- Resolutions to *Prisoner's Dilemma*
- Meet-up (verbal agreement): let's set high prices (incentive of deception however – you want him to set high prices and you want to set low)
- *Threaten*: punishment of opponent doesn't set high prices (lacking credibility as you "will do it" rather than it "will be done (automatically etc.)" – can fix credibility by using Mafia as they have more incentive to harm him)

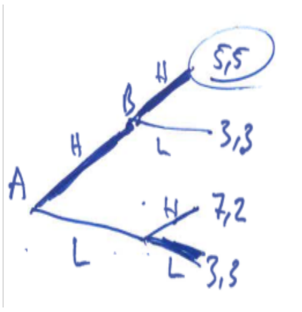
- *Reward*: offer a reward that overcomes the incentive to defect (still lacks credibility as it involves giving money – lowers your payoff. May not even believe you)

2.4.3 Behavioural Resolution

- *External Norms of Behaviour*: Think back to litter e.g.: many people won't actually litter even though it's the most beneficial for you. It conflicts with the social norm
- *Internal Norms of Behaviour*: Doing nice things for people who are nice to you (gain utility). Being bad to people but they are good to you (loss of utility). - you defect but if you care enough, you'll maybe rationalise cooperation and change mind

2.4.4 Price Matching Guarantee Resolution

- You may be undercut for opponent to gain market share from you at lower prices
- If you're offered. PMG, you will simply match prices and keep customers "he's selling at that price, can you just sell me at that too"
- Opponent now doesn't gain, just sells at lower price as no market gain
- Equilibrium of both pricing high – out of Prisoner's Dilemma



2.4.5 Dynamic Punishment Resolution

- When defecting, a player may believe they will be 'punished' in the future
- Can we achieve cooperation through fear of punishment?
  - Credible: backed with fact
  - Incredible: maybe won't happen

- *Finite Period* (T Periods): defect in last period as no more time for retaliation
  - Final period: mutual dominant strategy to defect as no future punishment
  - So: best to defect this period as well as you both will next
- *Infinite Period*: the game will continue [probability $p = 1$] so retaliation
  - Always an opportunity to punish as there's always another period
- *Impatient*: future worth less than present so defect (not caring for punishment)
- *Patient*: care more for future gain by waiting and cooperating

Discounting

- £1 from £1 today to £$1(1+r)$ tomorrow; £1 from £1 tomorrow to £$\frac{1}{(1+r)}$ today
- PV: $\frac{1}{(1+r)}, \frac{1}{(1+r)^2}, ..., \frac{1}{(1+r)^N}$; for discount rate r and discount factor $\delta = \frac{1}{(1+r)^t}$
- Hence, £X in period t is worth £$X\frac{1}{(1+r)^t}$ today
- Hence, £X in period t is worth £$X\delta^t$ today
- δ close to 1: patient; δ close to 0: impatient
- CFs: $X_0, X_1, ..., X_N$; PVs: $X_0 + \delta X_1 + \delta^2 X_2 +, ..., +\delta^N X_N$
- Infinite: $1 + \delta + \delta^2 + \delta^3 = \frac{1}{(1-\delta)}$

Example:

		B: high	B: low
A	high	600,600	170,1000
	low	1000,170	400,400

Payoff of 7 in perpetuity	Payoff of 10 today and 2 in perpetuity
$7 + 7\delta + 7\delta^2 + ... = 7\frac{1}{1-\delta}$	$10 + 2\delta + 2\delta^2 + ... = 10 + 2\delta\frac{1}{(1-\delta)}$

Trigger Strategies: Grim Trigger

- Start by cooperating
- If opponent cooperated, cooperate
- If opponent defected, defected in perpetuity

1. *Cooperate*:
   - Opponent gets 600 forever $\rightarrow 600 + 600\delta + 600\delta^2 + ... = \frac{600}{(1-\delta)}$
2. *Defect*:
   - Get 1000 now but 400 after $\rightarrow 1000 + 400\delta + 400\delta^2 + ... = 1000 + \frac{400\delta}{(1-\delta)}$
3. Answer:
   - $\frac{600}{(1-\delta)} \geq 1000 + \frac{400\delta}{(1-\delta)} \Leftrightarrow \delta \geq \frac{2}{3} (\Leftrightarrow \leq \frac{1}{2}) = 50\%$
   - $\boxed{\text{Hence, } \textit{Grim Trigger} \text{ at } \delta > \frac{2}{3} \text{ so cooperate}}$

Trigger Strategies: Tit-for-Tat

- Start by cooperating
- Play as the opponent played in the last round
- Cooperation followed by cooperation
- Defection frollowed by defection

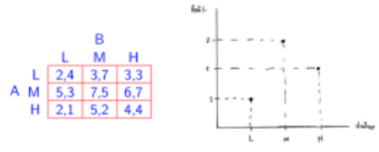
1. *Defect* in perpetuity:
   - Same as Grim Trigger: $\delta \geq \frac{2}{3}$
2. *Defect* once:
   - Get 400 now but loses 430 after $\rightarrow 400 \leq 430\delta \Rightarrow \delta \geq \frac{40}{43} (\Rightarrow \leq 0.075) = 7.5\%$
3. Answer:
   - $\boxed{\text{Hence, } \textit{TFT} \text{ at } \delta > \frac{40}{43} \text{ so cooperate}}$

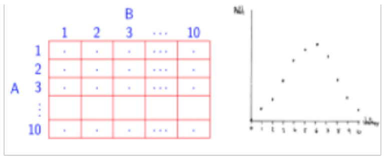
$\boxed{\text{If } \textit{Grim} \text{ works: cooperation possible; If } \textit{TFT} \text{ works: cooperation easy}}$

2.5 Games With Continuous Strategies

- This is applying maths to what we already know
- Nash equilibriums and sub-game perfect nash equilibriums remain the same
- This is applying the following more generally
- Matrix strategy: can choose any option for the expected opponent's options
  - Simultaneous: best responses and mutually consistent best responses
  - Sequential: backward induction
- Take a long time to analyse a continuous strategy using discrete sets (matrix)

2.5.1 Quantity Competition

- In a *Competitve Market*
- Firm i supplies q_i (where $\sum_{i=1}^{N} q \ \forall i\{1, ..., N\}$)
- *Inverse Demand Function*: $p(Q) = a - bQ$
- Payoff is *Profit* (π_i): $q_i p(Q) - C_i(q_i)$
- Hence: $\pi_i(q_{i=1}, q_{i=2}, ..., q_N) = q_i p(Q) - C_i(q_i)$
- There is an *Oligopoly* if several firms compete
- This method is shown in 1 to N matrices for A to respond to B:
- 

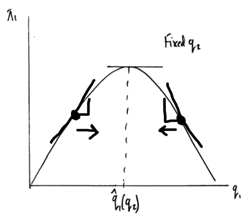
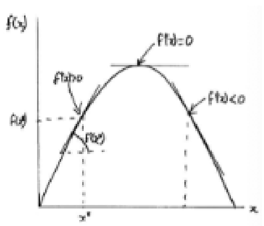
$\longrightarrow$ 

- Hence, we find a *Payoff Function* which is maximised at a point; "find the level of q_1 maximising firm A's payoff for given q_2"

2.5.2 Continuous Strategies

- Too hard to account for all the options (in this case quantities to produce)
- Recall basic math: *Function*: the level; *Derivative*: the slope of the function; *Parital Derivative*: fix a variable

Rules of Differentiation

- Woring towards *Payoff Function*
- Just like in the matrixes, fix the opponents option each time to find your best
- Hence, $y = f(x_1, x_2)$
- Fix x_2 (the adjacent firm's strategy) to observe how x_1 varies with y
- Therefore, partial derivative $\frac{\partial y}{\partial x_1}$ for fixed x_2
- Thus, best response at $\{\frac{\partial y}{\partial x_1}|_{x_2} = 0\}$ (peak of function)
- Note that, if you take the derivative on the incline of the function, you can be made better off by doing more. Take the derivative on the decline, better off by doing less
- 
- Recall the function $f(x)$:
  - $f'(x)$ positive at incline
  - $f'(x)$ negative at decline
  - $f'(x) = 0$ at stationery point (max/min)
  - 
- Recall if $f(x) = x^n$ then $f'(x) = nx^n - 1$
  - Constant: $ch(x) \longrightarrow ch'(x)$
  - Sum: $g(x) \pm h(x) \longrightarrow g'(x) \pm h'(x)$
  - Product: $g(x)h(x) \longrightarrow g(x)h'(x) + g'(x)h(x)$
  - Chain: $g(h(x)) \longrightarrow g'(h(x))h'(x)$

– Quotient: $\frac{g(x)}{h(x)} \longrightarrow \frac{g'(x)h(x) - g(x)h'(x)}{h(x)^2}$

– Log: $\ln x \longrightarrow \frac{1}{x}$

- In practice:
  - Power: $\sqrt{x} = x^{\frac{1}{2}} \longrightarrow \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$
  - Constant: $3x^2 \longrightarrow 3 \times 2x$
  - Sum: $x^2 + x^3 \longrightarrow 2x + 3x^2$
  - Product: $x^2(2x+3)^9 \longrightarrow x^2 \times 9(2x+3)^8 \times 2 + (2x+3)^9 \times 2x$
  - Chain: $x^2(2x+3)^9 \longrightarrow 9(2x+3)^8 \times 2$
  - Quotient: $\frac{x}{(1+x)} \longrightarrow \frac{(1+x)\times(1-x)\times 1}{(1+x)^2}$

2.5.3 Cournot Derivation of Payoff & Reaction (Simultaneous)

Step 1

Fix firm 2's action and find firm 1's best response to it.

- Find *Payoff Function*
- Partially derive and $= 0$ for *Best Response* with fixed q_2
- Find best q_1 for *Reaction Function*, sub for q_1^*

Step 2

Fix firm 1's action and find firm 1's best response to it.

- Find *Payoff Function*
- Partially derive and $= 0$ for *Best Response* with fixed q_1
- Find best q_2 for *Reaction Function*, sub for q_2^*

Step 3

Find meeting point of Nash Equilibrium where Reaction Functions meet; substitute to find optimal π for each firm.

Definitions

- Players: 2 firms $i = 1, 2$
- Strategies: each firm chooses quantity of q_i, for $Q = q_1 + q_2$
- Payoff: given supply choices *Marginal Cost* of c, *Price* of $P(q_1 + q_2)$, *Payoff* (profit) of $\pi_i(q_1, q_2) = q_1 p(q_1 + q_2) - cq_i$

Working Example

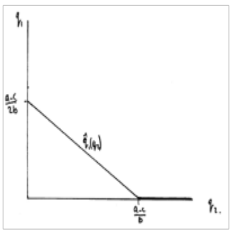
Firm 1's *Payoff Function*: $\pi_1 = q_1(a - b(q_1 + q_2)) - cq_1$
$\pi_1 = q_1(a - bq_1 - bq_2) - cq_1$
$\pi_i = aq_1 - bq_1^2 - bq_1q_2 - cq_1$

Fix q_2: $\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - c$
$[a - 2bq_1 - bq_2 - c = 0]$
$[2bq_1 = a - c - bq_2]$

Firm 1's *Reaction Function*: $[q_1 = \frac{a-c-bq_2}{2b}]...[q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2]$
Recall: q_1 is not $(-)$ as $q_2 \leq \frac{a-c}{b}$

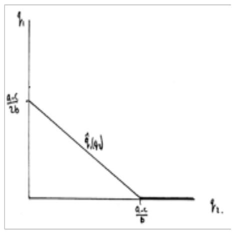
Note that *Reaction Function*: $q_1^* = \hat{q}_1(q_2) = \left\{ \begin{matrix} \frac{a-c}{2b} - \frac{1}{2}q_2 \text{if} q_2 \leq \frac{a-c}{b} \\ 0 \text{if} q_2 > \frac{a-c}{b} \end{matrix} \right\}$

Hence, *Reaction Function*: output quantity should decline as the opponent's increases; when it reaches 0, leave market; obviously no negative



Firm 2's *Payoff Function*: $\pi_2 = q_2(a - b(q_1 + q_2)) - cq_2$
Fix q_1: $\frac{\partial \pi_2}{\partial q_2} = a - 2bq_2 - bq_1 - c$
Firm 2's *Reaction Function*: $[q_2 = \frac{a-c-bq_1}{2b}]...[q_2 = \frac{a-c}{2b} - \frac{1}{2}q_1]$

Note that *Reaction Function*: $q_2^* = \hat{q}_2(q_1) = \left\{ \begin{matrix} \frac{a-c}{2b} - \frac{1}{2}q_1 \text{if} q_2 \leq \frac{a-c}{b} \\ 0 \text{if} q_1 > \frac{a-c}{b} \end{matrix} \right\}$

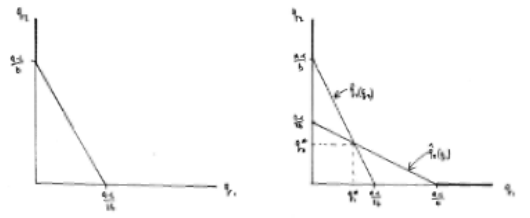


Nash Equilibrium ("The Courot Equilibrium"): flip firm 2's *Reaction Function* and overlay

Seek q_1^*, q_2^* from $q_1^* = \hat{q}_1(q_2^*)$ and $q_2^* = \hat{q}_2(q_1^*)$

"For firm 1's q which maximises its π given firm 2's q"

"For firm 2's q which maximises its π given firm 1's q"



Substitution is as follows:

From $q_1 = \frac{a-c}{2b} - \frac{1}{2}q_2$

$q_1 = \frac{a-c}{2b} - \frac{1}{2}(\frac{a-c}{2b} - \frac{1}{2}q_1)$

$q_1 = \frac{a-c}{2b} - \frac{a-c}{4b} + \frac{1}{4}q_1$

$q_1 = \frac{4b(a-c)-2b(a-c)}{8b^2} + \frac{1}{4}q_1$

$q_1 = \frac{2b(a-c)}{8b^2} + \frac{1}{4}q_1$

$q_1 = \frac{a-c}{4b} + \frac{1}{4}q_1$

$\frac{3}{4}q_1 = \frac{a-c}{4b}$

$q_1 = \frac{4}{3}\frac{a-c}{4b}$

$q_1 = \frac{4(a-c)}{12b}$

$q_1^* = \frac{a-c}{3b}$ Sub for firm 2's π

Cournot Equilibrium is as follows: Hence: $Q^* = q_1^* + q_2^*$

So: $p^* = a - bQ^*$

Thus: $\pi^* = (p^* - c)Q^*$

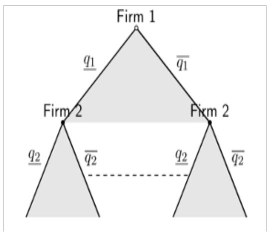
$$\boxed{q_1^* = q_2^* = \frac{a-c}{3b};\ Q^* = \frac{2(a-c)}{3b};\ p^* = \frac{a+2c}{3};\ \pi_i^* = \frac{(a-c)^2}{9b}}$$

Verify that:

Industry output between monopoly and PC: $Q^M < Q^* + Q^C$

Price between monopoly and PC: $p^M = p^* + p^C$
Industry profit between monopoly and PC: $\pi^M > {\pi_1}^* + {\pi_2}^* > 0$

2.5.4 Stakleberg Leader & Follower (Sequential)

- The leader implements the first player's reaction function intro their payoff function
- First mover advantage as leader gets higher payoff
- Recall from sequential games: backward induction
- *Linear Demand*: $P(Q) = a - bQ$
- *Constant Marginal Costs*: $C_i(q_i) = cq_i$
- *Profits*: $\pi_i(q_i, q_j) = q_i\,(a - b(q_i + q_j)) - cq_i$
- Firm 1 moves, firm 2 observes and moves
- Firm 1 is the leader, firm 2 is the follower
- Recall:
- 

Backward Induction: Step 1 (Stage 2)

- Firm 2 maximises profits given q_1
- Firm 2 uses Best Response to whatever firm 1 produces

1. Recall: $\pi_2 = q_2(a - b(q_1 + q_2)) - cq_2$ (*Payoff Function*)
2. Optimise and $= 0$: $\frac{\partial_{\pi 2}}{\partial_{q2}} = a - 2bq_2 - bq_1 - c = 0$
3. React: ${q_2}^* = \hat{q}_2\,(q_2) = \frac{a-c}{2b} - \frac{1}{2}q_1$ (*Reaction Function*)

Backward Induction: Step 2 (Stage 1)

- Firm 1 anticipates reaction of firm 2 to any decision made
- Firm 1 maximises profits given response of firm 2
- Firm 1 chooses point on firm 2's reaction function which maximises profits

1. Recall: $\pi_1(q_1, q_2) = q_1(a - b(q_1 + q_2)) - cq_1$ (*Payoff Function*)
2. It Knows: if they choose q_1 firm 2 will respond with ${q_2}^* = \hat{q}_2({q_1}^*)$
3. Firm 1 (leader): $\pi_1(q_1, \hat{q}_2(q_1)) = q_1(a - b(q_1 + \hat{q}_2(q_1))) - cq_1$
   - Therefore: $\pi_1 \left(q_1, \hat{q}_2\left(q_1\right)\right) = \frac{a-c}{2} q_1 - \frac{b}{2} q_1^2$
4. So firm 1 maximises: $\frac{\delta_{\pi 1(q1,q2)}}{\delta_{q1}} = \frac{a-c}{2} - bq_1 = 0$
   - Therefore: $q_1^L = \frac{a-c}{2b}$ (*Reaction Function*)
5. So firm 2 (follower): $q_2^F = \frac{a-c}{2b} - \frac{1}{2} q_1^L$
   - $q_2^F = \frac{a-c}{4b}$

Stakleberg Equilibrium

"Rather than an equilibrium, there is an advantage"

$$\boxed{Q^S = \frac{3(a-c)}{4b};\ \pi_1^L = \frac{(a-c)^2}{8b};\ \pi_2^F = \frac{(a-c)^2}{16b}}$$

Stakleberg vs. Cournot

Cournot: $q_1^* = q_1^* = \frac{a-c}{3b};\ Q^* = \frac{2(a-c)}{3b};\ p^* = \frac{a+2c}{3};\ \pi_1^* = \pi_2^* = \frac{(a-c)^2}{9b}$

$$\text{Stakleberg:}\quad \begin{array}{ll} q_1^L = \frac{a-c}{2b} > q_1^*; & q_2^F = \frac{a-c}{4b} < q_2^*; \\ Q^S = \frac{3(a-c)}{4b} > Q^*; & p^S = \frac{a+3c}{4} < p^*; \\ \pi_1^L = \frac{(a-c)^2}{8b} > \pi_1^*; & \pi_2^F = \frac{(a-c)^2}{16b} < \pi_2^* \end{array}$$

Hence, first mover (firm 1) advantage

2.6 Applications of Prisoner's Dilemma

- An example of a continuous game
- Rather than using *Reaction Functions*, find *Isoprofit Curves*
- This is like indifference curves for firms
- Call firms i, j; strategies x_i, x_j; payoffs $\pi(x_i, x_j)$

2.6.1 Typical Reaction Function

$$\hat{q}_i\left(q_j\right) = \frac{a-c}{2b} - \frac{1}{2}q_j$$

From *Isoprofit* contours
Equilibrium at intersection: $q_i^* = q_j^* = \frac{a-c}{3b}$

2.6.2 Maximising Joint Profit

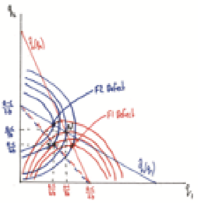
$$J\left(q_i, q_j\right) = \left(q_i + q_j\right)\left(a - b(q_i + q_j)\right) - cq_i - cq_j$$

$\frac{\partial J}{\partial q_i} = a - 2b\left(q_i + q_j\right) - c = 0 \rightarrow \widetilde{q}_i\left(q_j\right) = \frac{a-c}{2b} - q_i$
$\frac{\partial J}{\partial q_j} = a - 2b\left(q_i + q_j\right) - c = 0 \rightarrow \widetilde{q}_j\left(q_i\right) = \frac{a-c}{2b} - q_j$

Hence, assuming $q_i = q_j = \widetilde{q}$
$\widetilde{q} = \frac{a-c}{4b}$
Makes sense as: $2\left(\frac{a-c}{4b}\right) = \frac{a-c}{2b}$ (Monopoly Output)

2.6.3 Will Firms Agree?

- Will firms agree to produce at half the monopoly output?
- **No**; if firms expect you to produce more than $\widetilde{q}$;
- Best possible: $\hat{q}_i(\widetilde{q}) \rightarrow$ must expand output in excess of *Cournot Output*
- Defecting firm: *Bonanza Payoff*
- Cooperating firm: *Sucker Payoff*
- Hence, *Prisoner's Dilemma*
- $\pi^B > \frac{1}{2}\pi^M > \pi^* > \pi^S$



-

		2	
		cooperate	defect
1	cooperate	$1/2\pi^M, 1/2\pi^M$	π^S, π^B
	defect	π^B, π^S	π^*, π^*

2.6.4 Externalities

Negative

You do more, you lower my payoff (*Cournot Game*)
$\frac{\partial \pi_i}{\partial x_j} < 0$ (slope of *Payoff Function*)

Positive

You do more, you lower my payoff
$\frac{\partial \pi_i}{\partial x_j} > 0$ (slope of *Payoff Function*)

2.6.5 Strategic Nature

Strategic Substitutes

Opponent does more of their action: you optimally do less (*Reaction Function* downward)
$\left(\frac{\partial \frac{\partial \pi_i}{\partial x_i}}{\partial x_j}\right) < 0$ "with a higher x_j, the optimum is with a lower x_i"

Strategic Compliments

Opponent does more of their action: you optimally do more (*Reaction Function* upward)
$\left(\frac{\partial \frac{\partial \pi_i}{\partial x_i}}{\partial x_j}\right) > 0$ "with a higher x_j the optimum is with a higher x_i"

2.6.6 Nash Equilibrium in Games

- When non-cooperative, players optimise self-interest

- Marginal Payoff = 0: $\frac{\partial \pi_i}{\partial x_i} = 0$; $\frac{\partial \pi_j}{\partial x_j} = 0$
- Nash equilibrium at: $\hat{x}_i\left(x_j\right)$ and $\hat{x}_j(x_i)$
- Thus, nash equilibrium actions: x_i^*, x_j^*

2.6.7 Social Planner

- What happens when they 'internalise' the externality?
- *Social Planner* maximises joint payoff

$$J = \pi_i\left(x_i, x_j\right) + \pi_j\left(x_i, x_i\right)$$

Chooses x_i, x_j to maximise
$\frac{\partial J}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_j} = 0 \rightarrow \widetilde{x}_i(x_j)$
$\frac{\partial J}{\partial x_i} = \frac{\partial \pi_i}{\partial x_i} + \frac{\partial \pi_j}{\partial x_j} = 0 \rightarrow \widetilde{x}_i\left(x_j\right) \rightarrow$ these are both *Social Optimums*

2.6.8 Nash Equilibrium vs. Optimum

w/ Positive Externalities

$\frac{\partial \pi_j}{\partial x_i} > 0$
So for *Social Planner* ($\frac{\partial \pi_i}{\partial x_i}$ must be < NE)
So $\widetilde{x}_i\left(x_j\right) > \hat{x}_i\left(x_j\right)$

w/ Negative Externalities

$\frac{\partial \pi_j}{\partial x_i} < 0$
So for *Social Planner* ($\frac{\partial \pi_i}{\partial x_i}$ must be > NE)
So $\widetilde{x}_i\left(x_j\right) < \hat{x}_i\left(x_j\right)$