Notazione: $\,^{-}\!\!\sqrt{x} := \dfrac{1}{\sqrt{x}}$

Stati

Principio 1

$$\mathcal{S} \mapsto \mathcal{H}$$

$$\Sigma \mapsto \hat{\psi} := \{\lambda \left|\psi\right\rangle \mid \lambda \in \mathbb{C}\backslash\{0\}\}$$

Funzione d'onda e densità di probabilità

$$P(x) = \frac{|\psi(x)|^2}{||\psi(x)||^2}$$

$$P(x) \geq 0, \qquad \int \mathrm{d}x P(x) = 1$$

Trasformata di Fourier

$$\widetilde{\psi}(p) = \,^{-}\!\!\sqrt{2\pi\hbar} \int \mathrm{d}x \psi(x) e^{-\frac{ipx}{\hbar}}$$

$$P(p) = \frac{|\psi(p)|^2}{||\psi(p)||^2}$$

Basi generalizzate

$$\left|x\right\rangle = \xi_x(x) = \delta(x - x_0)$$

$$\left|p\right\rangle = v_p(x) = \,^{-}\!\!\sqrt{2\pi\hbar}\, e^{\frac{ipx}{\hbar}}$$

$$\langle x_0 | x_0' \rangle = \delta(x_0 - x_0')$$

$$\langle p_0 | p_0' \rangle = \delta(p_0 - p_0')$$

Osservabili

Posizione e impulso

$$X\psi(x) = x\psi(x)$$

$$P\psi(x) = -i\hbar \frac{\mathrm{d}\psi(x)}{\mathrm{d}x}$$

$$[X, P] = i\hbar$$

Principio 2

$$\mathcal{A} \mapsto A$$

$$\langle \mathcal{A} \rangle_\Sigma = \frac{\left\langle\psi\right| A \left|\psi\right\rangle}{\langle \psi | \psi \rangle}$$

$$\Delta A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$

Principio 3

$$A \left|a\right\rangle = a \left|a\right\rangle$$

$$\sigma(\mathcal{A}) = \sigma(A)$$

Principio 4

$$w(a_k) = \frac{\left|\langle a_k | \psi \rangle\right|^2}{||\psi||^2}$$

$$\left|\psi\right\rangle = \sum_{k=1}^{N} c_k \left|a_k\right\rangle$$

$$w(a_k) = \frac{\left|c_k\right|^2}{||\psi||^2}$$

$$w(a_k) = \sum_{i=1}^{d_k} \frac{\left|\langle a_{k,i} | \psi \rangle\right|^2}{||\psi||^2}$$

$$\left|\psi\right\rangle = \sum_{k=1}^{N} \sum_{i=1}^{d_k} c_k^i \left|a_k\right\rangle$$

$$w(a_k) = \sum_{i=1}^{d_k} \frac{\left|c_k^i\right|^2}{||\psi||^2}$$

$$\mathrm{d}w(a) = \rho(a)\mathrm{d}a = \frac{\left|\langle a | \psi \rangle\right|^2}{||\psi||^2}$$

$$\left|\psi\right\rangle = \int \mathrm{d}a\, c(a) \left|a\right\rangle$$

$$\rho(a) = \frac{\left|c(a)\right|^2}{||\psi||^2}$$

Proiettori e misure

Definizione di proiettore $(P, \mathcal{D}(P))$

P è proiettore $\Leftrightarrow P^\dagger = P \wedge P^2 = P$

$P_q = \sum_i \left|\varphi_i\right\rangle \left\langle\varphi_i\right|$, $\{\left|\varphi_i\right\rangle\}$ base ON di $\mathcal{H}_q$

Principio 6

$$\left|\psi\right\rangle \to \left|\psi'\right\rangle = \frac{P_{a_k} \left|\psi\right\rangle}{\sqrt{\left\langle\psi\right| P_{a_k} \left|\psi\right\rangle}}$$

Osservabili compatibili

$\mathcal{A}, \mathcal{B}$ compatibili $\Leftrightarrow [A, B] = 0$

$\mathcal{A}, \mathcal{B}$ compatibili $\Leftrightarrow$ hanno una base di autovettori comuni