DIVISION ALGEBRAS AND SUPERSYMMETRY

AN INTRODUCTION

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In this talk we will investigate the correlation between division algebras, trialities and supersymmetry. These topics are of fundamental importance in current research in physics, where the aim is to create a mathematical coherent and univocal description for a theory of everything.

Being a theory of everything, it requires everything: a lot of knowledge, creativity and combined actions of different areas of mathematics and physics, in order to really give a contribution. In particular, higher category theory, Lie groups and Lie algebras, global analysis, C^*- algebras, as well as a pure algebraic version of K-Theory, based on the work of Atiyah, Quillen, Grothendieck and Moore, are strongly recommended.

However, the basic idea lies here in this talk.

In the first part, division algebras and their properties are analysed using the geometrical approach in Hatcher [H1], in order to prove Hurwitz theorem via Adams Theorem.

In the second part, the connection with SYMT is explored, making use of the light-cone formulation, to establish the following one-to-one correspondences:

$\exists$ SYMT d = 3, 4, 6, 10 $\longleftrightarrow$ normed trialities for n = 1, 2, 4, 8 $\longleftrightarrow$ division algebras $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, $\mathbb{O}$.

The treatment of the correlation between division algebras and SYM-theories will follow [ES] and will require the concepts of triality and alternativity.

A detailed, mostly self-contained, discussion of these topics is provided in my notes for this talk.

References

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