MAXWELL'S EQUATIONS
VARIATIONAL FORMULATION FOR FEMS

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In this talk we will discuss Maxwell's equations, one of the fundamental sets of equations in nature, governing electrodynamics. We will go through their physical meaning and propagational properties into matter and we will investigate in great details scattering problems, that constitute an interesting and very active field of research both for experimental physics and mathematical modelling, e.g. collision of particles, optical scattering, etc..

In the second part, some basics in functional analysis are analysed in order to introduce the appropriate setting for variational formulation.

We will then define Sobolev spaces and state some theoretical results concerning their density and compactness properties. In particular, we will focus on divergence and curl based spaces of functions. Indeed, trace operators are the right tools to deal with this sets of so complex elliptic systems.

In conclusion, an application of the calculus of variations to scattering is briefly illustrated, namely the inverse scattering problem in its weak formulation, in order to motivate our discussion and display a practical implementation of the methods just introduced.

The treatment of the subject will follow the book by Monk [M]. However, for more details and technical proofs, please refer to [S] and [Z], where pdes problems, distribution and variational theories are deeply explained.
I would recommend [Sc] for the ones interested in a differential form approach.

A detailed, mostly self-contained, discussion of these topics will be provided in my notes for this talk.

References

[M] Monk, P., Finite Element Methods for Maxwell's Equations, Oxford University Press, 2003;

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[Sc] Schutz, B. F., Geometrical Methods of Mathematical Physics, Chapter 5C, Cambridge University Press, 1980;

[S] Salsa, S., Partial Differential Equations in Actions: from modelling to Theory, Springer- Verlag Italia, Milano, 2008;

[Z] Zachmanoglou, E.C., Thoe, D. W., Introduction to Partial Differential Equations with Applications, Dover Publications, Inc., New York, 1976;

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