First Meeting Presentation

Project A2

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Politecnico of Milano
Bayesian Statistics course

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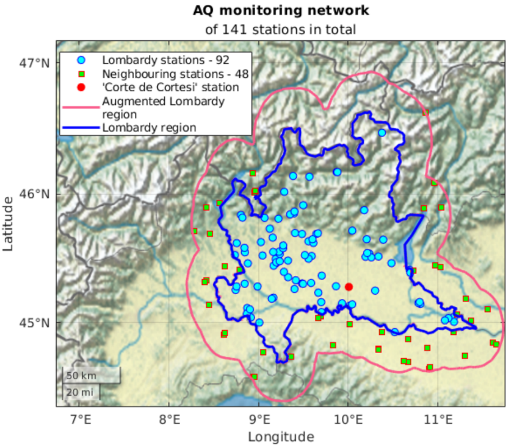
Presentation Flow

The goal of the project

Goal: Clustering weekly data of one year of PM10 (plus covariates)
Dataset: AGRIMONIA project, at
`https://zenodo.org/records/7563265`

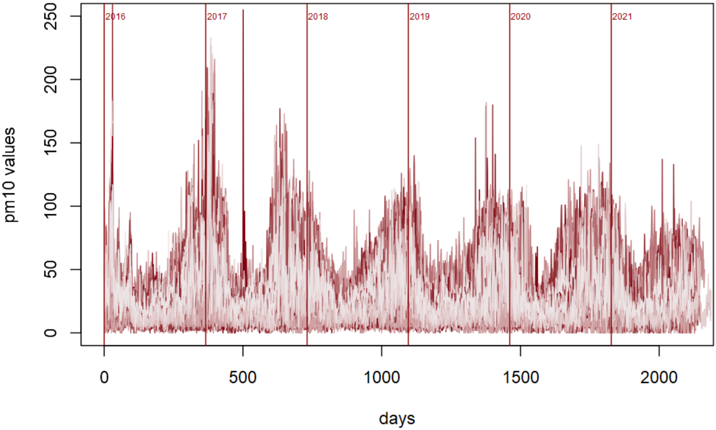
Spatial Exploration

We have 141 stations, which recorded data for 6 years (from 01/01/2016 to 31/12/2021).



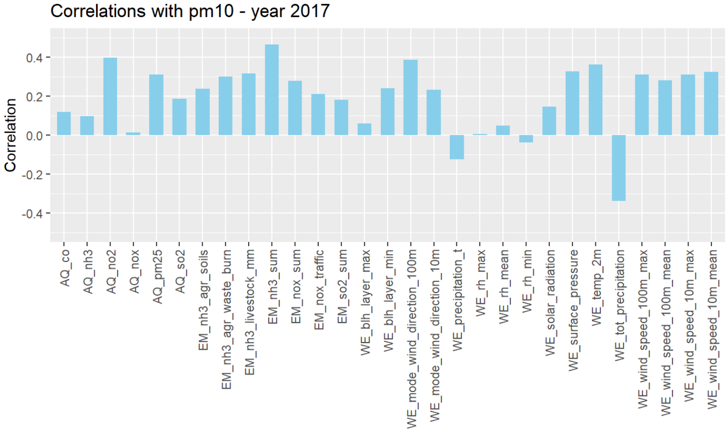
Temporal Exploration

We have 141 stations, which recorded data
for 6 years (from 01/01/2016 to 31/12/2021).



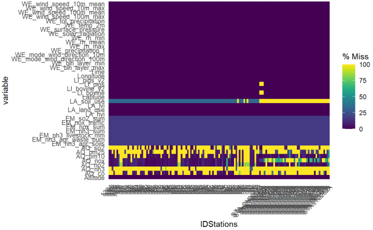
Correlation: pm10 and others

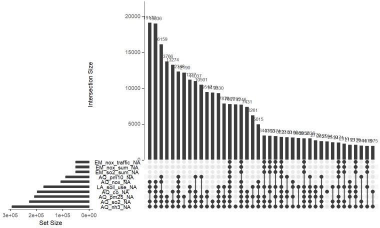
We compute the spearman's correlation index between the pm10 and the other covariates, which in the functional framework quantifies with a value in [-1,1] the tendency of 2 r.v. X_t and Y_t to be perfect monotone functions one of each other



Missing value exploration

Plots to identify a general pattern on missing values





Combinations of variables with the most missing values

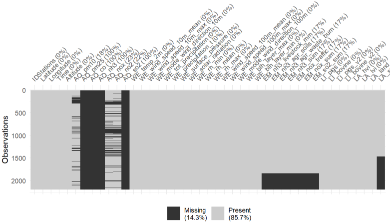
Missing value exploration

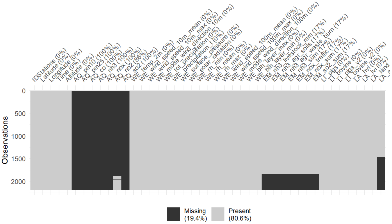
Plot of missing data divided by station to see particular patterns to help select variables

Possible to remove some stations that are not measuring PM10 values

Some columns present missing values in most columns, so we can remove corresponding covariates

Some missing observations concentrated in specific periods such as during last years





Missing value exploration

```
 [1] "1274"        "1800"       "1801"       "501"        "504"
 [6] "514"         "529"        "539"        "544"        "545"
[11] "551"         "552"        "573"        "588"        "602"
[16] "603"         "606"        "607"        "626"        "652"
[21] "656"         "657"        "665"        "672"        "682"
[26] "694"         "696"        "698"        "699"        "700"
[31] "702"         "707"        "STA.IT1518A" "STA.IT1751A" "STA.IT1924A"
[36] "STA.IT2282A"
```

First selection to remove non informative stations, then count of missing values for column, removing those above a chosen threshold

```
             IDStations                  Latitude
                      0                         0
              Longitude                      Time
                      0                         0
               Altitude                   AQ_pm10
                      0                     12719
                AQ_pm25                     AQ_co
                 136792                    136560
                 AQ_nh3                    AQ_nox
                 217540                     72988
                 AQ_no2                    AQ_so2
                  21266                    156205
             WE_temp_2m    WE_wind_speed_10m_mean
                      0                         0
  WE_wind_speed_10m_max WE_mode_wind_direction_10m
                      0                         0
  WE_tot_precipitation        WE_precipitation_t
                      0                         0
   WE_surface_pressure        WE_solar_radiation
                      0                         0
              WE_rh_min                WE_rh_mean
                      0                         0
              WE_rh_max   WE_wind_speed_100m_mean
                      0                         0
 WE_wind_speed_100m_max WE_mode_wind_direction_100m
                      0                         0
       WE_blh_layer_max          WE_blh_layer_min
                      0                         0
    EM_nh3_livestock_mm         EM_nh3_agr_soils
                  38325                     38325
  EM_nh3_agr_waste_burn               EM_nh3_sum
                  38325                     38325
         EM_nox_traffic               EM_nox_sum
                  38325                     38325
             EM_so2_sum                  LI_pigs
                  38325                      6576
              LI_bovine               LI_pigs_v2
                   6576                         0
           LI_bovine_v2                   LA_hvi
                      0                         0
                 LA_lvi              LA_land_use
                      0                         0
            LA_soil_use
                 136656
```

Models: a complex task

Considering the nature of the data, our models should account for different levels of information:

- spatial context
- temporal context
- covariates

which is a not-so-trivial task.
Now we see the general incremental idea to build such models.

Purely spatial model

- We have n distinct locations $\boldsymbol{s}_1, \ldots, \boldsymbol{s}_n$, where $\boldsymbol{s}_i = (\text{lat}, \text{long})$.
- There we record data y_i and (possibly) covariates $\boldsymbol{x}_i$, for $i = 1, \ldots, n$. The goal is to define a model for partitioning them into k groups.
- So we define $\rho = \{S_1, \ldots, S_k\}$ the cluster set variable (with $S_h \subseteq \{1, \ldots, n\}$ for $h = 1, \ldots, k$).
  An equivalent formulation is possible through some cluster indicator variables $c_1, \ldots, c_n$; where $c_i = h \iff i \in S_h$ for $i = 1, \ldots, n$.
- In general, the law for ρ follows a spatial Product Partition Model (sPPM):

$$p_\rho(\tilde{\rho}) \propto \prod_{h=1}^{k_n} C(\tilde{S}_h, \boldsymbol{s}_h^\star)$$

  where $\tilde{\rho} = \{\tilde{S}_1, \ldots, \tilde{S}_k\}$, $\boldsymbol{s}_h^\star = \{\boldsymbol{s}_i : i \in \tilde{S}_h\}$ and $C(\tilde{S}_h, \boldsymbol{s}_h^\star)$ is a cohesion function.

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Spatial and temporal model

We have n distinct locations $\boldsymbol{s}_1, \dots, \boldsymbol{s}_n$, where $\boldsymbol{s}_i = (\text{lat, long})$.
There we record data y_i and (possibly) covariates $\boldsymbol{x}_i$, for $i = 1, \dots, n$.
The goal is to define a model for partitioning them into k_t groups, with t spanning over $1, \dots, T$.
So we define $\rho_t = \{S_{1,t}, \dots, S_{k_t,t}\}$ the cluster set variable (with $S_{h,t} \subseteq \{1, \dots, n\}$ for $h = 1, \dots, k_t$).
In general, the law for ρ_t follows a spatial Product Partition Model (sPPM) updated to account for the time relation (stPPM); meaning that we need a formulation of a joint probability model for $\rho_1, \dots, \rho_T$.
This update can be explicited for example by

- supposing a Markov Chain structure, letting ρ_t depend just on ρ_{t-1};
- introducing some cluster reallocation variable $\gamma_{i,t} \in \{0, 1\}$.

Model 1

Garritt L. Page, Fernando A. Quintana, David B. Dahl (2022)
Dependent Modeling of Temporal Sequences of Random Partitions. Journal of Computational and Graphical Statistics, 31:2, 614-627.

$$Y_{it}|\boldsymbol{\mu}_t^\star, \boldsymbol{\sigma}_t^{2\star}, \boldsymbol{c}_t \overset{\text{ind}}{\sim} \mathcal{N}(\mu_{c_{it}t}^\star, \sigma_{c_{it}t}^{2\star}) \quad i = 1, \dots, n \quad \text{and} \quad t = 1, \dots, T$$

$$(\mu_{jt}, \sigma_{jt}) \,|\, \theta_t, \tau_t^2 \overset{\text{ind}}{\sim} \mathcal{N}(\theta_t, \tau_t^2) \times \mathcal{U}(0, A_\sigma) \quad j = 1, \dots, k_t$$

$$(\theta_t, \tau_t) \overset{\text{iid}}{\sim} \mathcal{N}(\phi_0, \lambda^2) \times \mathcal{U}(0, A_\tau) \quad t = 1, \dots, T$$

$$(\phi_0, \lambda) \sim \mathcal{N}(m_0, s_0^2) \times \mathcal{U}(0, A_\lambda)$$

$$\{\boldsymbol{c_t}, \dots, \boldsymbol{c_T}\} \sim \text{tRPM}(\boldsymbol{\alpha}, M) \quad \text{with } \alpha_t \overset{\text{iid}}{\sim} \text{Beta}(a_\alpha, b_\alpha)$$

Adding covariates

Given a partition, now we can easily design models which also account for covariates. For example we can update the previous model into

$$Y_{it}|\beta_t^\star, \boldsymbol{\sigma}_t^{2\star}, \boldsymbol{c}_t \overset{\text{ind}}{\sim} \mathcal{N}(\boldsymbol{x}_{it}^T \beta_{c_{it}t}^\star, \sigma_{c_{it}t}^{2\star})$$
$$(\beta_{jt}, \sigma_{jt}) \,|\, \theta_t, \tau_t^2 \overset{\text{ind}}{\sim} \mathcal{N}(\theta_t, \tau_t^2) \times \mathcal{U}(0, A_\sigma)$$
$$\vdots$$

or we can further characterize the time dependance with some AR($\cdot$) model

$$Y_{it}|Y_{it-1}, \beta_t^\star, \boldsymbol{\sigma}_t^{2\star}, \boldsymbol{c}_t \overset{\text{ind}}{\sim} \mathcal{N}(\boldsymbol{x}_{it}^T \beta_{c_{it}t}^\star + Y_{it-1}\eta_i, \sigma_{c_{it}t}^{2\star}(1-\eta_i)^2)$$
$$Y_{i1}|\beta_1^\star, \boldsymbol{\sigma}_1^{2\star}, \boldsymbol{c}_1 \sim \mathcal{N}(\boldsymbol{x}_{i1}^T \beta_{c_{i1}1}^{2\star}, \sigma_{c_{i1}1}^{2\star})$$
$$\vdots$$

Model 2

Mozdzen A., Cremaschi A., Cadonna A., Guglielmi A., Kastner G. (2022)
Bayesian modeling and clustering for spatio-temporal areal data: An application to Italian unemployment. Spatial Statistics 52, 100715.

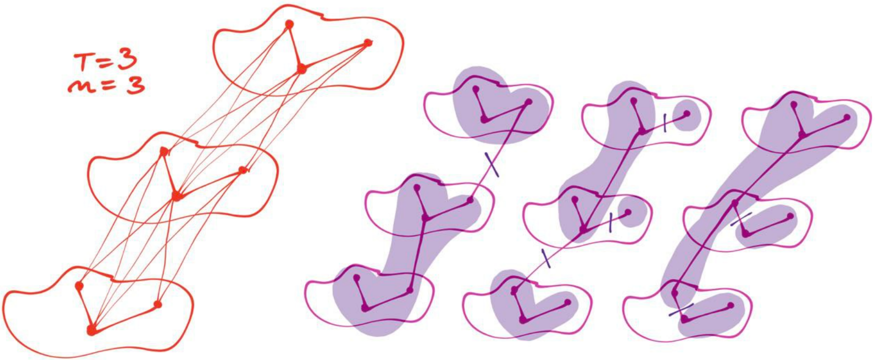
$$
\begin{aligned}
Y_{it} | \boldsymbol{x}_{it}, \beta^\star_{s_i}, w_{it}, \sigma^2, s_i &\overset{\text{ind}}{\sim} \mathcal{N}(\boldsymbol{x}_{it}^T \beta^\star_{s_i} + w_{it}, \sigma^2) \\
\boldsymbol{w}_t | \boldsymbol{w}_{t-1}, \boldsymbol{\xi}^\star_{\boldsymbol{s}}, \boldsymbol{s}, \tau^2, \rho, W &\sim \mathcal{N}_I(\text{diag}(\boldsymbol{\xi}^\star_{\boldsymbol{s}}) \boldsymbol{w}_{t-1}, \tau^2 Q(\rho, W)^{-1}) \\
\boldsymbol{w}_1 | \tau^2, \rho, W &\sim \mathcal{N}_I(\boldsymbol{0}, \tau^2 Q(\rho, W)^{-1}) \\
\sigma^2 &\sim \text{Inv-Gamma}(a_{\sigma^2}, b_{\sigma^2}) \\
\tau^2 &\sim \text{Inv-Gamma}(a_{\tau^2}, b_{\tau^2}) \\
\rho &\sim \text{Beta}(\alpha_\rho, \beta_\rho) \\
\boldsymbol{s} | \alpha &\sim \text{PólyaUrn}(\boldsymbol{s} | \alpha) \\
\alpha &\sim \text{Gamma}(a_\alpha, b_\alpha) \\
\boldsymbol{\phi}^\star_1, \dots, \boldsymbol{\phi}^\star_{K_I} | \boldsymbol{\mu}_{\boldsymbol{\beta}}, \Sigma_{\boldsymbol{\beta}}, \alpha_\xi, \beta_\xi &\overset{\text{iid}}{\sim} \mathrm{P}_0, \qquad \boldsymbol{\phi}^\star_j = (\boldsymbol{\beta}^\star_j, \boldsymbol{\xi}^\star_j) \qquad \mathrm{j} = 1, \dots, \mathrm{K}_I \\
\mathrm{P}_0(d\boldsymbol{\phi}^\star) &= \mathcal{N}_{p+1}(d\boldsymbol{\beta}^\star | \boldsymbol{\mu}_{\boldsymbol{0}}, \Sigma_0) \text{Beta}_{(-1,1)}(d\xi^\star | \alpha_\xi, \beta_\xi)
\end{aligned}
$$

Model 3

Leonardo V. Teixeira, Renato M. Assunção, Rosangela H. Loschi (2019)
Bayesian Space-Time Partitioning by Sampling and Pruning Spanning Trees.
Journal of Machine Learning Research 20, 85, 1–35.

This model works on a graph structure, which incorporates together space and time. That is, from data Y_{jt} for $j = 1, \ldots, n$ and $t = 1, \ldots, T$, we now move to Y_i for $i \in I = \{1, \ldots, nT\}$ by stacking T times the spatial map. So we have a graph $\mathcal{G} = (V, E)$ of nT nodes and edges built according to time and space connections.
The idea is to search a partition $\boldsymbol{\pi} = \{\mathcal{G}_1, \ldots, \mathcal{G}_c\}$ of I (with $\mathcal{G}_k$ subgraphs for $\mathcal{G}$), on randomly selected spanning trees $\mathcal{T}$ of $\mathcal{G}$, on which we set cluster-specific parameters $\beta_{\mathcal{G}_1}, \ldots, \beta_{\mathcal{G}_c}$.

Model 3



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Journal of Machine Learning Research 20, 85, 1–35.

$$
\begin{aligned}
Y_i | \mathcal{T}, \boldsymbol{\pi}, \beta_{\mathcal{G}_k} &\overset{\text{iid}}{\sim} f(Y_i | \beta_{\mathcal{G}_k}; \boldsymbol{x}_i) \quad i \in \mathcal{G}_k \\
\beta_{\mathcal{G}_1}, \ldots, \beta_{\mathcal{G}_c} | \mathcal{T}, \boldsymbol{\pi} &\sim \prod_{k=1}^{c} f(\beta_{\mathcal{G}_k}) \\
p(\boldsymbol{\pi} = \{\tilde{\mathcal{G}}_1, \ldots, \tilde{\mathcal{G}}_c\} | \mathcal{T}) &\sim \prod_{k=1}^{c} \kappa(\tilde{\mathcal{G}}_k) \\
\mathcal{T} &\sim \mathcal{U}(\text{St}(\mathcal{G}))
\end{aligned}
$$

Expected workflow

- PM10 covariates analysis (physical/chemical relation)
- First models implementation and comparison
- Implementation of variations of those simple models, or more complex models from literature
- Gif/Video interactive plots for displaying results

References

Garritt L. Page, Fernando A. Quintana, David B. Dahl (2022)
Dependent Modeling of Temporal Sequences of Random Partitions. Journal of Computational and Graphical Statistics, 31:2, 614-627.

Mozdzen A., Cremaschi A., Cadonna A., Guglielmi A., Kastner G. (2022)
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Bayesian Space-Time Partitioning by Sampling and Pruning Spanning Trees. Journal of Machine Learning Research 20, 85, 1–35.

Paragraphs of Text

Sed iaculis dapibus gravida. Morbi sed tortor erat, nec interdum arcu. Sed id lorem lectus. Quisque viverra augue id sem ornare non aliquam nibh tristique. Aenean in ligula nisl. Nulla sed tellus ipsum. Donec vestibulum ligula non lorem vulputate fermentum accumsan neque mollis.

> *Sed diam enim, sagittis nec condimentum sit amet, ullamcorper sit amet libero. Aliquam vel dui orci, a porta odio.*
> *— Someone, somewhere...*

Nullam id suscipit ipsum. Aenean lobortis commodo sem, ut commodo leo gravida vitae. Pellentesque vehicula ante iaculis arcu pretium rutrum eget sit amet purus. Integer ornare nulla quis neque ultrices lobortis.

Lists

Bullet Points and Numbered Lists

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus
  - Lorem ipsum dolor sit amet, consectetur adipiscing elit
  - Nam cursus est eget velit posuere pellentesque
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero

1. Nam cursus est eget velit posuere pellentesque
2. Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block Title

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue.

Example Block Title

Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan.

Alert Block Title

Pellentesque sed tellus purus. Class aptent taciti sociosqu ad litora torquent per conubia nostra, per inceptos himenaeos.

Suspendisse tincidunt sagittis gravida. Curabitur condimentum, enim sed venenatis rutrum, ipsum neque consectetur orci.

Multiple Columns

Subtitle

Heading

1. Statement
2. Explanation
3. Example

Lorem ipsum dolor sit amet, consectetur adipiscing elit. Integer lectus nisl, ultricies in feugiat rutrum, porttitor sit amet augue. Aliquam ut tortor mauris. Sed volutpat ante purus, quis accumsan dolor.

Table

Subtitle

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Table: Table caption

Figure

Figure: Space for a possible image.

Definitions & Examples

Definition

A prime number is a number that has exactly two divisors.

Example

- 2 is prime (two divisors: 1 and 2).
- 3 is prime (two divisors: 1 and 3).
- 4 is not prime (three divisors: 1, 2, and 4).

You can also use the `theorem`, `lemma`, `proof` and `corollary` environments.

Theorem, Corollary & Proof

Theorem (Mass–energy equivalence)

$E = mc^2$

Corollary

$x + y = y + x$

Proof.

$\omega + \phi = \epsilon$ $\square$

Equation

$$\cos^3\theta = \frac{1}{4}\cos\theta + \frac{3}{4}\cos 3\theta \tag{1}$$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Slide without title.

Citing References

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2022, Kennedy, 2023].

References

John Smith (2022)
Publication title
Journal Name 12(3), 45 – 678.

Annabelle Kennedy (2023)
Publication title
Journal Name 12(3), 45 – 678.

Acknowledgements

Smith Lab

- Alice Smith
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- Yuan

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- Norwegian Government

The End

Questions? Comments?