Sports Eng (2010) 12:135–141
DOI 10.1007/s12283-010-0039-y

ORIGINAL ARTICLE

Parameter study using a finite element simulation of a carving Alpine ski to investigate the turn radius and its dependence on edging angle, load, and snow properties

Peter Federolf · Anton Lüthi · Markus Roos · Jürg Dual

Published online: 28 March 2010

Abstract For ski manufacturers, it is important to know how a given ski-binding system performs under different loading conditions. Important performance parameters are the ski deformation and the resulting turn radius. This study focuses on *carving turns*. The aims of this study were: (1) to investigate the dependence of the turn radius on edging angle, load on the binding, and snow hardness using a finite element (FE) simulation, and (2) to compare the results with predictions of a frequently used model introduced by Howe. The FE simulation used a quasi-static approach (similar to Howe's model), but the ski–snow interaction model incorporated the groove that forms in the snow during a carved turn. Up to edging angles of 40°, the results of the FE simulation agreed well with Howe's model. However, for large edging angles (>50°) the calculated turn radius leveled out, whereas Howe's model tends to zero. This effect was more pronounced for soft snow than for hard snow conditions. Increasing forces on the binding caused a decrease in the calculated turn radii. In summary, the FE simulation showed that particularly at large edging angles the groove in the snow needs to be considered in models of the ski–snow interaction or in computations of the turn radius.

P. Federolf (✉)
Human Performance Laboratory, University of Calgary, Calgary, Canada
e-mail: peter.federolf@kin.ucalgary.ca

P. Federolf · A. Lüthi
WSL Institute for Snow and Avalanche Research SLF, Davos, Switzerland

M. Roos
Institute of Computational Physics, Zurich University of Applied Sciences, Winterthur, Switzerland

J. Dual
Institute of Mechanical Systems, Swiss Federal Institute of Technology, Zurich, Switzerland

1 Introduction

Alpine skiing is a very popular winter sport in many mountainous regions of the world. In recent years, ski manufacturers strived to develop skis that are targeted at particular groups of users or particular applications, lady skis or deep powder skis being examples. Carving skis need a significant side-cut (SC) (Fig. 1) in order to be able to perform carved turns. For most skis, the shape of the SC is a polynomial function of the position along the ski length. However, to approximate the side-cut radius a circular shape is usually assumed. The radius of this circle is called the side-cut radius R_{SC} and may be approximated by Howe [1]:

$$R_{SC} = \frac{L^2}{8 \cdot SC}, \text{ with}$$

$$SC = \frac{S + E - 2W}{4}$$

In these equations, L denotes the effective length of the ski (i.e., the length of the ski that is in contact with the snow), S, W, and E are the ski width at the shovel, waist, and ski end, respectively. In ski racing, carving skis are developed specifically for the different disciplines. Downhill skis are long, stiff and have a minimum SC radius of 45 m, whereas slalom skis are short, less stiff and have a SC radius of about 12 m.

If the ski is set on edge and loaded as in a turn, then the ski will bend until the lower ski edge is pressed onto or into the snow surface. The ski's radius in a turn, which is

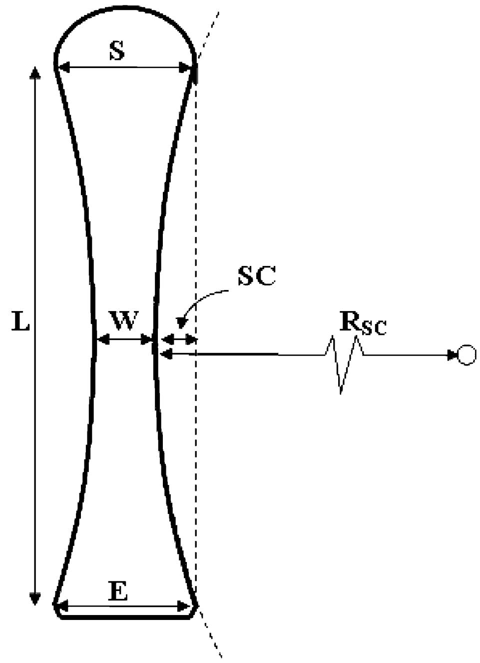

Fig. 1 Parameters characterizing the ski geometry

thought to coincide with the radius of the lower ski edge, depends, therefore, on the mechanical characteristics of the ski and the loading conditions (edging angle, load on the binding). According to Howe [1] it can be approximated if the ski's effective length L, the side-cut SC, the edging angle θ, and the penetration depth d into the snow are known:

$$R_{\mathrm{Howe}} = \frac{L^2}{8 \cdot \left({}^{\mathrm{SC}}/_{\cos\theta} + d \tan\theta \right)}$$

By equating this radius of the lower ski edge with the actual radius of the turn Howe investigated the relation of the skier's speed, his tilt angle, and other turn parameters in carved turns. For deviations from the calculated combination of carving parameters he expects either skidding or an imbalance of turning forces, which need to be compensated in order to prevent a fall.

Several other researchers explicitly refer to Howe's model [2–9] or use an analog approach in their studies [10–14]. However, this model is derived from purely static considerations and has several shortcomings: (1) In carved turns, the ski creates a groove in the snow and the rear section of the ski glides within this groove. The ski–snow interaction in Howe's model may be well suited for the situation of a skidding ski, but it does not consider the groove in the snow and is, therefore, not well suited to describe carving turns. (2) Only the edging angle and the ski's shape are taken into account. It is not possible to study the influence of the ski's stiffness or the loading conditions on the turn radius. (3) The penetration depth d of the ski into the snow has to be estimated or determined in field observations. (4) The turn radius tends to zero as the edging angle approaches 90°. In reality, turn radii of zero are impossible and it is unclear up to which edging angle the predicted turn radius is still valid.

To overcome some of these disadvantages a finite element (FE) simulation model was created specifically for a ski in the situation of a carved turn. The model and experimental validation steps are described in detail in [15, 16]. The FE model uses a similar quasi-static approach as Howe's model, but the ski–snow interaction model is adapted to the situation of a carved turn in which the rear section of the ski interacts with the groove formed by the front section of the ski.

The first aim of the current study was to investigate the dependence of the ski's turn radius in a carved turn on *edging angle*, *load on the binding*, and *snow hardness* using the FE model of a selected ski-binding system whose implementation and validation have been described by Federolf et al. [15, 16]. The second aim was to compare the results of this simulation with the predictions made by Howe's model.

2 Methods

2.1 Selected ski and binding models

The all-round carver ski *Stöckli Spirit* of the winter season 2001/2002 was selected for this parameter study. The characteristic parameters of the ski's geometry (Fig. 1) are listed in Table 1. The ski was equipped with a *Rave Powerride* binding of season 2001 manufactured by *Fritschi AG Swiss Bindings*, Reichenbach/Kandertal, Switzerland. This binding offers a high stance height of the ski boot rendering an additional damping plate unnecessary.

2.2 Summary of the FE simulation

A detailed description and discussion of the simulation model (Fig. 2) and of the validation steps conducted experimentally to confirm the simulation results were described in [15] and [16]. The following section

Table 1 Geometry of the ski used in this study

Geometry of the test ski *Stöckli Spirit*	
Effective length L	1.55 m
Width at the shovel S	117 mm
Width at the waist W	67 mm
Width at the ski end E	99.5 mm
FIS-radius R_{SC}	14.49 m

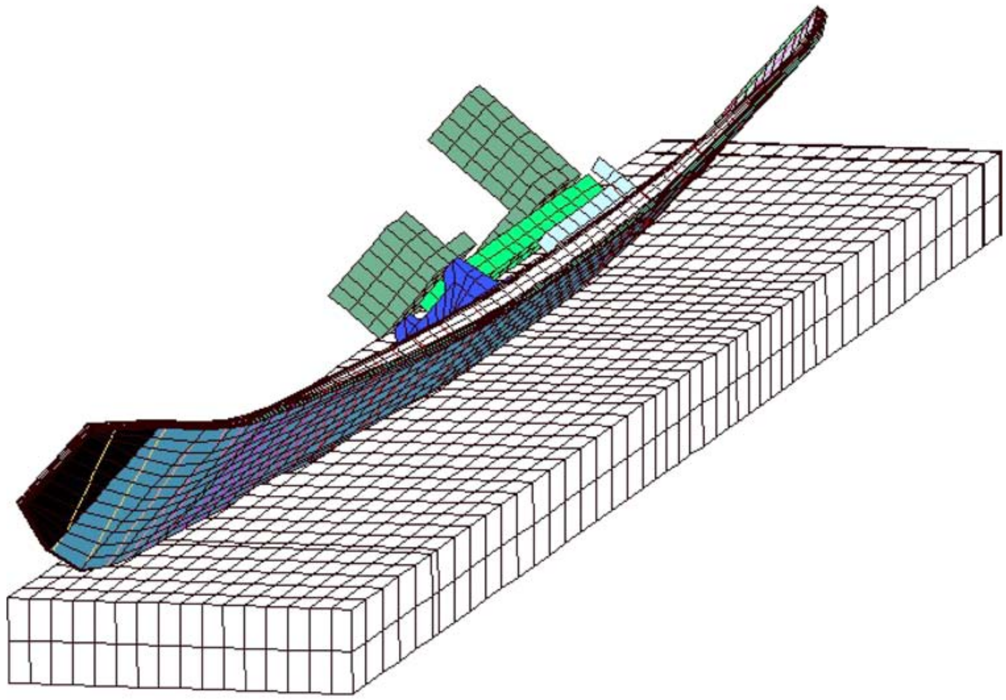

Fig. 2 FEM model of the ski in a carved turn. For reference the snow surface was indicated by adding an additional block of elements. In the actual simulation the ski–snow interaction is implemented as a boundary condition on the ski model

summarizes the most important details of this model and points out novel aspects that distinguish this simulation approach from previous models.

In the FE simulation the ski was modeled by shell elements, the binding was modeled by volume elements. The force transfer between ski and binding were modeled by internal constraints. The simulation calculated the deformation of the ski and the penetration depth into the snow for given loading conditions, which were defined using the following input parameters: force transferred from the athlete onto the binding, edging angle, snow hardness. The ski's radius in a carved turn was then derived from the calculated curvature of the lower ski edge projected onto the plane of the snow surface. The simulation assumed a steady state (no rapid change in the input variables, no system vibrations) and used a quasi-static approach in which the deformation of the ski was calculated in two nested iteration loops. In a first step, a static equilibrium was determined for a static situation in which the ski is pressed into the snow surface (analog to Howe's model). In the second step, dynamical forces were added to the simulation using D'Alemberts principle, and the boundary conditions representing the snow resistance were modified such that they incorporate the grove in the snow that forms as the ski moves forward. Two effects were taken into account when defining the ski–snow boundary conditions in the simulation of the ski–snow interaction: (1) the plasticity of the snow deformation, (2) the effect of the ski's forward movement on the formation of the groove in the snow. Hence, the ski–snow interaction model simulated the actual formation of the groove in the snow and had the rear section of the ski interact with this groove rather than with an undisturbed snow surface.

The simulation model was validated in three steps [15, 16]: (1) the implementation of the snow pressure exerted on a penetrating plate was compared with field measurements. (2) The deformation of the ski penetrating an even snow surface in a static situation was calculated and compared to the results of a static experiment. (3) The *calculated turn radii*, which were determined in the simulation for input parameters measured at specific positions during an actual carved turn, were compared to the *instantaneous radii*, which were derived from the skis' traces in the snow at the same positions at which the simulation input parameters had been determined. The validation results showed that the simulation was in general able to predict the instantaneous radius when using the input parameters determined in the experiment.

2.3 Input parameters and boundary conditions used in this study

2.3.1 Edging angle

For a ski with a given SC radius the edging angle has the highest impact on the instantaneous radius [1, 6]. In recreational skiing, edging angles of up to 60° were observed [17]. The edging angle depends on the body inclination and thus on the speed of the skier. Therefore, ski racers may even achieve edging angles of 70° and higher [16]. In this study, the turn radius was calculated for edging angles between 1° and 60°. At higher edging angles, the simulation did in most cases not converge to a solution due to limitations in the definition of the boundary conditions [15].

2.3.2 Force acting on the binding during a turn

This force depends on the skier's weight, the speed, the turn radius, and how the skier distributes his weight between left and right ski. In a carved turn, the highest forces are observed during the steering phase when the turn radius is small. For a recreational skier carving at a moderate speed the forces at the ski binding were reported to be in the range between 1,000 N and 2,000 N [18, 19].

2.3.3 Snow types

In the case of Howe's model the turn radius was calculated for two cases: (1) an icy surface with no penetration into the snow, and (2) a soft snow providing a penetration depth of 5 mm [1]. In the FEM simulation, the snow strength was implemented by defining the normal pressure acting on the ski. Field measurements on actual ski slopes [20] found that the snow's resistive pressure can be described as a stepwise linear function of the penetration depth. In the FEM simulation the snow strength was defined according to the parameters recommended for "soft", "medium", and "hard" snow types [20]. The ski's penetration depth into the snow is a result of the simulation.

3 Results

3.1 Turn radius as a function of the edging angle: comparison with Howe's model

The ski's turn radius calculated for edging angles between 1° and 60°, but otherwise constant input parameters are displayed in Fig. 3. For the same ski, binding and snow model exposed to the same loading conditions the turn radius predicted by the FE simulation fitted well to a Gaussian function of the form

$$R_{\mathrm{FEM}} = R_0 + H \cdot e^{-2\left(\frac{\theta}{w}\right)^2}$$

where R_0 is the minimum radius approached for large edging angles θ, the length $(R_0 + H)$ corresponds to the SC radius (FIS radius) of the ski and the parameter w characterizes the width of the bell-shaped Gauss curve. The parameters R_0, H, and w depend on the input conditions of the simulation (Tables 2, 3).

3.2 Influence of different load on the binding

If the force on the binding was increased from 1,000 N to 1,400 N while all other input parameters for the simulation

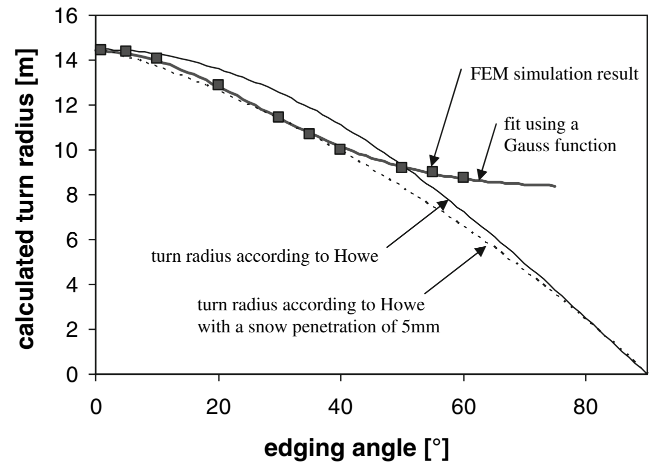

Fig. 3 Simulation results (*squares*) calculated for edging angles between 1° and 60° assuming soft snow conditions and a load of 1,000 N acting on the binding. A Gauss function was fitted to the simulation result (*thick line*). The turn radius predicted by Howe's model for no snow penetration (*thin line*) and 5 mm snow penetration (*broken thin line*) is also included

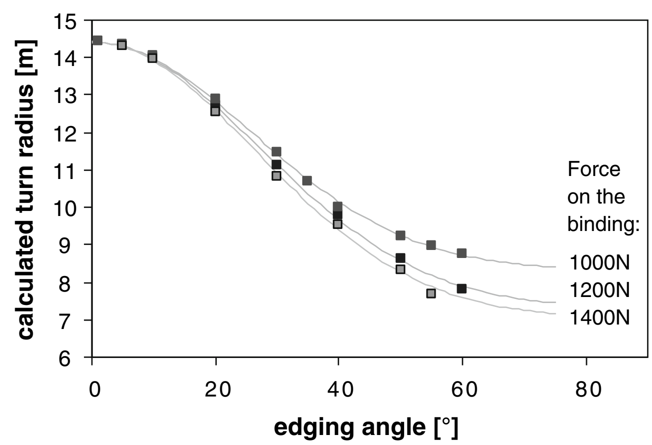

Fig. 4 Calculated turn radius as a function of the edging angle for forces on the ski binding of 1,000 N, 1,200 N and 1,400 N assuming soft snow conditions

remained the same, then the fitted Gauss curves had a similar shape (Fig. 4), only the minimum radius R_0 decreased (Table 2). Within the range of typical forces that act on a ski binding during a turn (1,000 N–2,000 N) the calculated turn radius decreased linearly with increasing force (Fig. 5).

3.3 Influence of the snow hardness on the radius

The ski radius as function of the edging angle can be fitted by a Gauss function for all three types of snow (Fig. 6). It can be noted that the characteristics of the Gauss curve for hard snow differed from those of the softer snow types. The width w of the Gauss curve for hard snow was considerably larger than that of the softer snow types, whereas the minimum radius R_0 was considerably smaller (Table 3).

Table 2 Coefficients R_0, H, and w for three different forces acting on the binding assuming soft snow conditions

Force on the binding	1,000 N	1,200 N	1,400 N
R_0	8.31	7.29	7.03
H	6.10	7.21	7.39
w	51.58	54.01	52.92

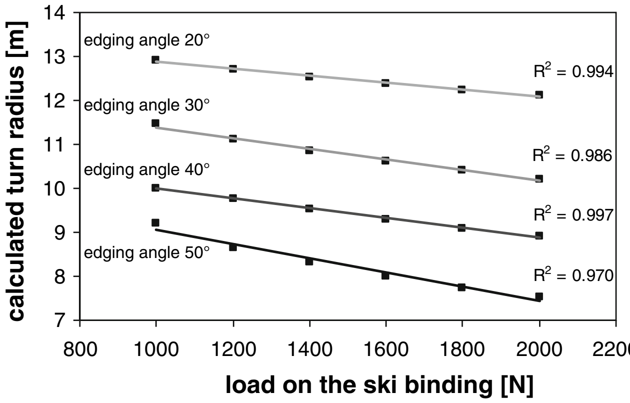

Fig. 5 Calculated turn radius as a function of the force acting on the binding for four different edging angles and soft snow conditions

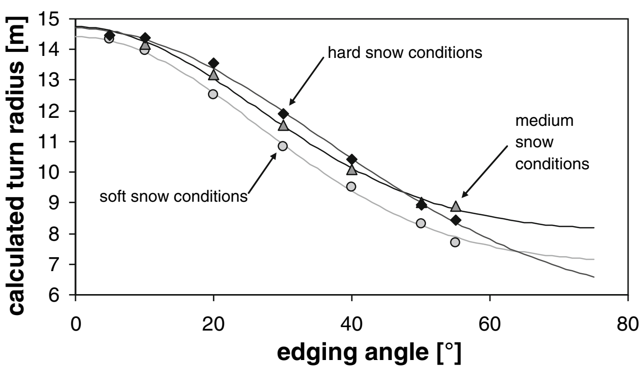

Fig. 6 Calculated turn radius as a function of the edging angle for soft, medium and hard snow conditions assuming a force of 1,400 N acting on the binding

Table 3 Coefficients R_0, H, and w for hard medium and soft snow conditions assuming a force of 1,400 N acting on the binding

Snow condition	Hard	Medium	Soft
R_0	5.58	8.06	7.03
H	9.12	6.68	7.39
w	71.45	52.34	52.92

Consequently, for edging angles between 10° and 40°, the calculated turn radius exceeded the turn radii determined for the softer snow types. For edging angles above 50°, the calculated turn radius for hard snow conditions was smaller than those calculated for medium snow. The trend for larger edging angles suggests that the smallest turn radii of a ski can be observed on hard snow.

4 Discussion

4.1 Dependence of the turn radius on edging angle, load on the binding, and snow properties

The edging angle had, as expected, the most substantial effect on the turn radius. The FE simulation used in this study was specifically developed to model carving turns and thus incorporates the formation of a groove in the snow and effects taking place due to the interaction of the ski's rear section with this groove. A consequence of this boundary condition was that the turn radius leveled out for large edging angles. The reason for this effect is the shape of the groove: the higher the edging angle, the wider is the groove that forms in the snow ($1/\cos(\theta)$ – dependence [15]). In carving turns, the rear section of the ski remains in this groove. This leads to a reduction of the bending forces acting on the rear section of the ski. Therefore, the actual turn radius of the edged ski levels out for large edging angles.

Different speed or different weight of an athlete causes an increase of the forces acting on the binding. For the ski-binding model implemented in our simulation we found a linear decrease of the turn radius with increasing force in the range between 1,000°N and 2,000°N. A heavier skier or a faster skier is thus able to carve a slightly tighter turn. However, for the ski studied here an increase in force by 100% caused a decrease in the turn radius by only between 1 and 1.5 m which corresponds to a relative change between 8 and 17%. This is important because the forces on the binding can change rapidly in actual skiing, for example, due to frequent shocks caused by unevenness of the snow surface or vibrations of the ski. The simulation shows that rapidly changing forces are not able to change the turn radius substantially and are, therefore, in most cases not able to cause mayor instabilities of the skier. The fact that the decrease of the turn radius as a function of the force fitted so well to a linear function was not expected. Outside the range investigated in this study or for other ski, binding or snow conditions linearity cannot be assumed.

Varying snow strength has a more complex impact on the turn radius. For edging angles below 40°, softer snow lets the ski penetrate deeper into the snow causing the ski to bend more and, therefore, causes a smaller turn radius. At edging angles above 40°, the groove that forms in the snow starts to substantially affect the turn radius preventing small turn radii. This effect is more pronounced for soft snow when a deep groove is created by the ski. For medium and hard snow types, when the ski's penetration depth is smaller, the impact of this effect on the turn radius is also smaller.

4.2 Comparison of the FE simulation results with Howe's model and field observations

For small edging angles ($\theta < 40°$), the result of our simulation agreed well with the result of Howe's purely static model if the same penetration depth of the ski into the snow is used in Howe's equation. However, for larger edging angles ($\theta > 50°$), the groove caused the turn radius calculated by the FE simulation to level out whereas the turn radius calculated according to Howe's model tends to zero as the edging angle approaches 90°. We suspect that Howe's model becomes increasingly unrealistic for large edging angles, particularly on soft snow types in which the ski creates a deep groove.

Although Howe's model or a similar approach is used in numerous studies, only few studies are available that tried to validated this model with field measurements. Mössner et al. [7] used a sled equipped with skis set to an edging angle of 18°. At the beginning of the experiment they found turn radii between 65 and 85% of the radius predicted by Howe. They explained this discrepancy with the snow penetration that was not included in their calculation of the turn radius. They also reported that the measured turn radius increased in the progress of the experiment. They speculated that skidding caused an increase in the turn radius, however, it seems also possible that the groove created once the ski was in motion caused an increase in turn radius.

The simulation model used in this study was validated with measurements from an actually carved turn [15]. It was found that the turn radius calculated according to Howe's model (or calculated in an early version of the simulation that did not account for the groove created in the snow) amounted to only 50% of the turn radius determined from the skis' traces. The edging angle observed in our experiments was in the range of 50°–70°. After modifying the boundary condition at the ski–snow interface such that it accounts for the formation of the groove, the turn radius calculated by the FE model and the measured radius determined from the traces agreed reasonably well. The concepts used to explain the ski–snow interaction in a carved turns have also been described by another research group who conducted highly sophisticated measurements quantifying ski deformation and ski–snow pressure distributions in actual skiing [21].

5 Conclusions and outlook

For ski manufacturers, it is important to know how a given ski-binding system performs under various conditions, e.g., at different speeds, with different athletes, or at different snow conditions. The FE simulation allows a quantitative analysis of these questions. In the current paper, the effects of edging angle, force on the binding, and snow types on the turn radius were investigated. The results of this study suggest that in carved turns the static model developed by Howe is only valid for edging angles up to about 40°. For larger edging angles, a significant groove is formed in the snow and the deformation of the ski's rear section is significantly changed due to this new boundary condition. This is particularly important because the edging angle during the steering phase in actual skiing is typically in the range of 50°–70°. In the future, it will be investigated how ski parameters, for example SC, bending stiffness, or torsional stiffness affect the turn radius.

References

1. Howe J (2001) The new skiing mechanics. McIntire Publishing, Waterford
2. Baragetti S, Lorenzi V, Riva R, Strada R, Tordini F, Zappa B (2007) Design of a fatigue ski testing bench. Int J Mater Prod Technol 30:199–215
3. Glenne B, DeRocco A, Vandergrift J (1997) The modern Alpine ski. Cold Reg Sci Technol 26:35–38
4. Kawai S, Otani H, Sakata T (2003) Coupled motion of ski and elastic foundation under ski control. JSME Int J Ser C Mech Syst Mach Elem Manufact 46:614–621
5. Lind D, Sanders SP (1996) The physics of skiing. Springer, New York
6. Müller E, Schwameder H (2003) Biomechanical aspects of new techniques in alpine skiing and ski-jumping. J Sports Sci 21:679–692
7. Mössner M, Nachbauer W, Schindelwig K (1997) Influence of the ski's sidecut on the turning radius and strain. Sportverletz Sportschaden 11:140–145
8. Müller E, Bartlett R, Raschner C, Schwameder H, Benko-Bernwick U, Lindinger S (1998) Comparisons of the ski turn techniques of experienced and intermediate skiers. J Sports Sci 16:545–559
9. Spitzenpfeil P, Mester J (1997) Carving and skiing technique: aspects of biological regulation. Sportverletz Sportschaden 11:134–136
10. Chen L, Qi Z (2009) A 2-dimensional multi rigid bodies skiing model. Multibody Syst Dyn 21:91–98
11. Hirano Y, Tada N (1996) Numerical simulation of a turning alpine ski during recreational skiing. Med Sci Sports Exerc 28:1209–1213
12. Jentschura UD, Fahrbach F (2004) Physics of skiing: the ideal-carving equation and its applications. Can J Phys 82:249–261
13. Kawai S, Yamaguchi K, Sakata T (2004) Ski control model for parallel turn using multibody system. JSME Int J Ser C 47:1095–1100
14. Sahashi T, Ichino S (2001) Carving-turn and edging angle of skis. Sports Eng 4:135–146
15. Federolf P, Roos M, Luethi A, Dual J (2010) Finite element simulation of the ski–snow interaction of an alpine ski in a carved turn. Sports Eng. doi:10.1007/s12283-010-0038-z
16. Federolf P (2005) Finite element simulation of a carving snow ski. Pro BUSINESS GmbH, Berlin

17. Yoneyama T, Scott N, Kagawa H, Osada K (2008) Ski deflection measurement during skiing and estimation of ski direction and edge angle. Sports Eng 11:3–10
18. Stricker G, Scheiber P, Lindenhofer E, Müller E (2010) Determination of forces in alpine skiing and snowboarding: validation of a mobile data acquisition system. Eur J Sport Sci 10:31–41
19. Lüthi A, Federolf P, Fauve M, Oberhofer K, Rhyner H, Ammann W, Stricker G, Schiefermüller C, Eitzlmair E, Schwameder H, Müller E (2005) Determination of forces in carving using three independent methods. In: Müller E, Bacharach D, Kilka R, Lindinger S, Schwameder H (eds), pp 11–96
20. Federolf P, JeanRichard F, Fauve M, Lüthi A, Rhyner H, Dual J (2006) Deformation of snow during a carved ski turn. Cold Reg Sci Technol 46:69–77
21. Tatsuno D, Yoneyama T, Kagawa H, Scott N, Osada K (2009) Measurement of ski deflection and ski–snow contacting pressure in an actual ski turn on the snow surface. In: Müller E, Lindinger S, Stöggl T (eds) Science and skiing iV. Meyer & Meyer Sport (UK) Ltd, Maidenhead