Effective Liquidity Density in Limit Order Books: An Empirical Validation from High-Frequency Data

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Abstract

We derive and empirically validate a fundamental relationship between order flow, price dynamics, and liquidity density in limit order book markets. Starting from the theoretical framework where volume executed equals liquidity consumed, we define the effective liquidity density $V_{\text{eff}}(p)$ as the ratio of cumulative volume to price displacement during monotonic price runs. Using high-frequency Bitcoin/USD trade data (7.9M ticks over 21 days), we demonstrate that V_{eff} is a stable, measurable market parameter with low temporal autocorrelation and weak correlation with price levels. We construct empirical liquidity profiles $V(p,t)$ that reveal price-dependent resistance zones and directional asymmetries in market microstructure.

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1 Introduction

1.1 Market Microstructure and Price Dynamics

Traditional models of price dynamics in financial markets rely on stochastic processes such as geometric Brownian motion, treating prices as diffusive random walks. While these models capture certain statistical properties of returns, they fundamentally disconnect price movements from the underlying market mechanism: the limit order book (LOB).

In reality, prices move because market orders consume liquidity posted on the order book. The speed and magnitude of price changes depend critically on the *liquidity density* — how much volume is available at each price level. A market order of size Q moving through a region of low liquidity will displace the price more than the same order executing in a high-liquidity region.

1.2 Research Question

This work addresses a fundamental question: **Can we empirically measure the effective liquidity density function $V(p, t)$ from high-frequency trade data, and does it behave as a stable market parameter?**

We derive the theoretical relationship between order flow and liquidity, propose an empirical estimator based on monotonic price runs, and validate its statistical properties using real market data.

2 Theoretical Framework

2.1 Volume, Order Flow, and Liquidity

Consider the net signed volume Q of market orders executed in a time interval $[t, t + \Delta t]$. This volume can be expressed in two equivalent ways:

- **Temporal view (integration over time):**

$$Q(\Delta t) = \int_{t}^{t+\Delta t} q(\tau)\, d\tau \tag{1}$$

  where $q(t)$ is the instantaneous net order flow rate (buy volume minus sell volume per unit time).

- **Spatial view (integration along the book):**

$$Q(\Delta t) = \int_{p(t)}^{p(t+\Delta t)} V(p, \tau)\, dp \tag{2}$$

  where $V(p, t)$ is the liquidity density at price level p and time t — the amount of volume available per unit price.

2.2 Fundamental Relation

Equating these two expressions:

$$\int_{t}^{t+\Delta t} q(\tau)\, d\tau = \int_{p(t)}^{p(t+\Delta t)} V(p,\tau)\, dp \tag{3}$$

For infinitesimal intervals, this yields:

$$q(t)\, dt = V(p,t)\, dp \tag{4}$$

Therefore, the instantaneous price velocity is:

$$\frac{dp}{dt} = \frac{q(t)}{V(p,t)} \tag{5}$$

This is the **fundamental microstructure relation**: price velocity is proportional to order flow rate and inversely proportional to liquidity density.

2.3 Integrated Form

Integrating equation (5) over a finite time interval where the price moves from p_{start} to p_{end}:

$$\int_{p_{\text{start}}}^{p_{\text{end}}} V(p,t)\, dp = \int_{t_{\text{start}}}^{t_{\text{end}}} q(\tau)\, d\tau = Q_{\text{total}} \tag{6}$$

If we assume $V(p,t)$ is approximately constant during the price movement (or take its average), we obtain:

$$V_{\text{eff}} \cdot |\Delta p| = Q_{\text{total}} \tag{7}$$

Solving for the effective liquidity density:

$$V_{\text{eff}} = \frac{Q_{\text{total}}}{|\Delta p|} \tag{8}$$

where:

- Q_{total} is the cumulative volume executed during the price run
- $|\Delta p| = |p_{\text{end}} - p_{\text{start}}|$ is the absolute price displacement
- V_{eff} has units of [volume/price] (e.g., BTC/USD)

3 Empirical Methodology

3.1 Data

We use high-frequency trade data from a cryptocurrency exchange (Bitcoin/USD):

- **Total dataset:** 7,945,716 individual trades
- **Period:** May 27 – July 1, 2025 (35 days)
- **Fields:** timestamp, price, size (volume), side (buy/sell)

Each trade record represents a market order execution at a specific price and time.

3.2 Monotonic Run Detection

The key insight is that equation (8) is valid over periods where price moves monotonically in one direction. During such runs, all executed volume contributes to the same directional price movement, allowing us to estimate V_{eff}.

Algorithm:

1. Compute price changes: $\Delta p_i = p_{i+1} - p_i$
2. Define directional indicator: $d_i = \text{sign}(\Delta p_i) \in \{-1, 0, +1\}$
3. Identify direction changes: when $d_i \neq d_{i-1}$ and both are non-zero
4. For each monotonic run from index i_{start} to i_{end}:
   - Filter: require minimum 5 ticks and $|\Delta p| \geq \$1$
   - Compute total volume: $Q = \sum_{i=i_{\text{start}}}^{i_{\text{end}}} \text{size}_i$
   - Compute price displacement: $\Delta p = p_{i_{\text{end}}} - p_{i_{\text{start}}}$
   - Calculate: $V_{\text{eff}} = Q/|\Delta p|$

This procedure yields $N = 528{,}606$ monotonic runs from the full dataset.

3.3 Temporal Aggregation

To reduce noise and construct a continuous time series, we aggregate run-level V_{eff} estimates to 1-minute candles using duration-weighted averaging:

For each minute m, the aggregate effective liquidity is:

$$V_{\text{eff}}(m) = \frac{\sum_{r \in m} V_{\text{eff}}^{(r)} \cdot \tau_r}{\sum_{r \in m} \tau_r} \tag{9}$$

where r indexes runs falling within minute m, and τ_r is the duration of run r.

This produces a 1-minute time series with $N = 31{,}203$ observations.

3.4 Spatial Profile Construction

To construct the liquidity density profile $V(p, t_0)$ at a reference time t_0, we:

1. Define a price grid: $p_{\min} \leq p_1, p_2, \ldots, p_{100} \leq p_{\max}$
2. For each price level p_k:
   - Find all runs that crossed price p_k (i.e., $p_{\text{start}} \leq p_k \leq p_{\text{end}}$ or vice versa)
   - Apply exponential time decay weighting:

     $$w_r = \exp\left(-\lambda \cdot (t_0 - t_r)\right) \tag{10}$$

     where $\lambda = \ln(2)/(2 \text{ hours})$ corresponds to a 2-hour half-life
   - Compute weighted average:

     $$V(p_k, t_0) = \frac{\sum_r w_r \cdot V_{\text{eff}}^{(r)}}{\sum_r w_r} \tag{11}$$

3. Apply Gaussian smoothing with $\sigma = 2$ bins for visualization

This yields a smooth function $V(p, t_0)$ showing how liquidity density varies across price levels.

4 Results

4.1 Statistical Properties of V_{eff}

We analyze the 1-minute aggregated time series (31,203 observations) to validate that V_{eff} behaves as a stable market parameter. Figure 1 presents a comprehensive statistical analysis across six dimensions.

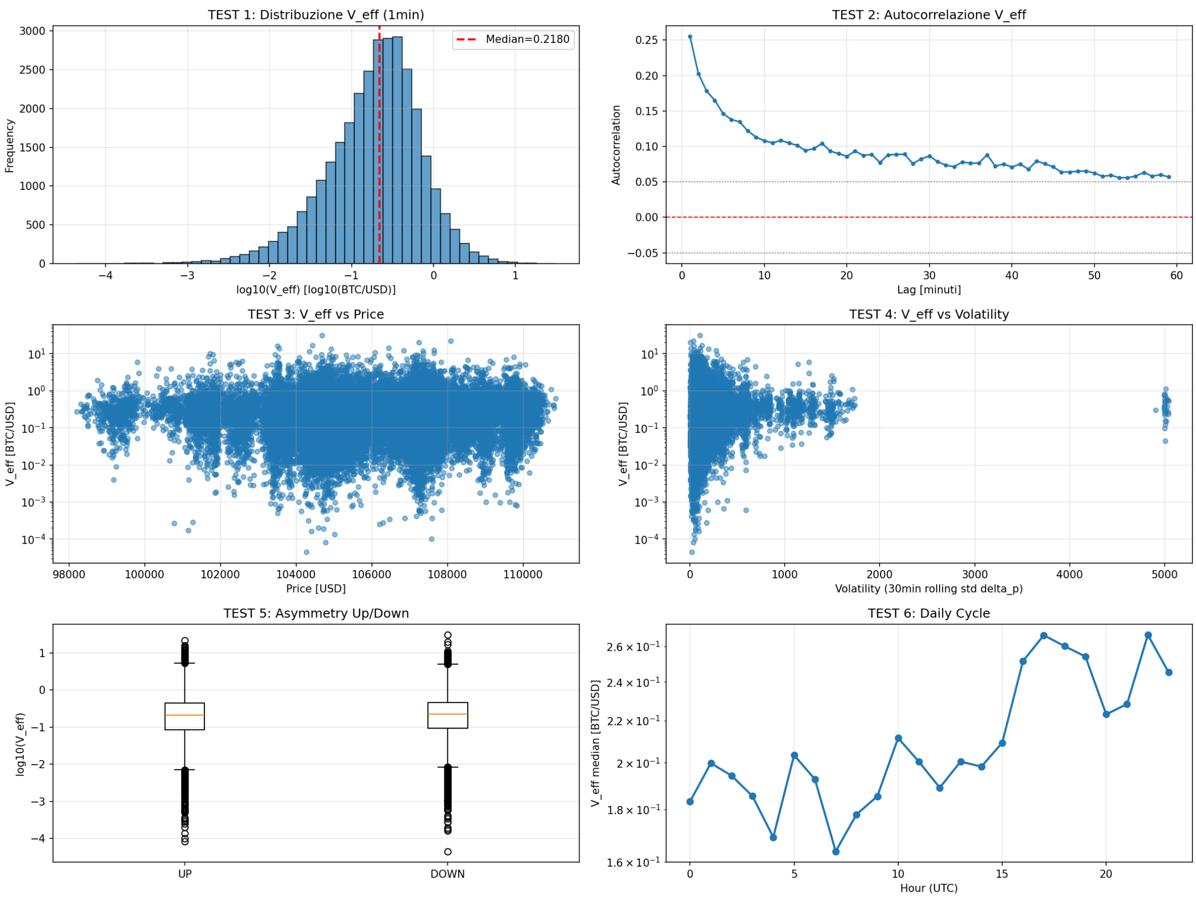

Figure 1: Statistical validation of V_{eff} from 528,606 monotonic runs. (Top left) Distribution showing right-skewed log-normal shape. (Top right) Autocorrelation function decaying to zero within 60 minutes. (Middle left) Independence from price level. (Middle right) Weak correlation with volatility. (Bottom left) No directional asymmetry between up/down runs. (Bottom right) Intraday cycle with peak liquidity at 22:00 UTC.

4.1.1 Distribution

The distribution of V_{eff} is right-skewed with:

- Mean: 0.395 BTC/USD
- Median: 0.218 BTC/USD
- Coefficient of variation: 1.76
- Skewness: 10.50

The log-normal-like distribution is typical of financial market variables with multiplicative noise.

4.1.2 Temporal Autocorrelation

Autocorrelation of $\log_{10}(V_{\text{eff}})$:

- AC(1 min): 0.255
- AC(10 min): ~ 0.10
- Persistence (decay to $|r| < 0.05$): 60 minutes

The moderate autocorrelation indicates that V_{eff} evolves slowly compared to price, making it a quasi-static market parameter on intraday timescales.

4.1.3 Correlation with Price

Pearson correlation between V_{eff} and price: $r = 0.006$

This near-zero correlation demonstrates that liquidity density is essentially **independent of the current price level**, contradicting models that assume liquidity scales with price.

4.1.4 Correlation with Volatility

Correlation between V_{eff} and 30-minute rolling volatility: $r = 0.018$

Again, near-zero correlation indicates that liquidity density is not simply a function of market volatility.

4.1.5 Directional Asymmetry

Testing for systematic differences between upward and downward price runs:

- Median V_{eff} (up): 0.213 BTC/USD
- Median V_{eff} (down): 0.224 BTC/USD
- t-test p-value: 0.46

No statistically significant asymmetry, suggesting that market microstructure is symmetric with respect to direction.

4.1.6 Intraday Cycle

Analyzing median V_{eff} by hour of day (UTC):

- Peak liquidity: 22:00 UTC ($V = 0.267$ BTC/USD)
- Trough liquidity: 07:00 UTC ($V = 0.164$ BTC/USD)
- Peak/trough ratio: $1.63\times$

This reflects the diurnal cycle of market participation, with higher liquidity during US and European trading hours.

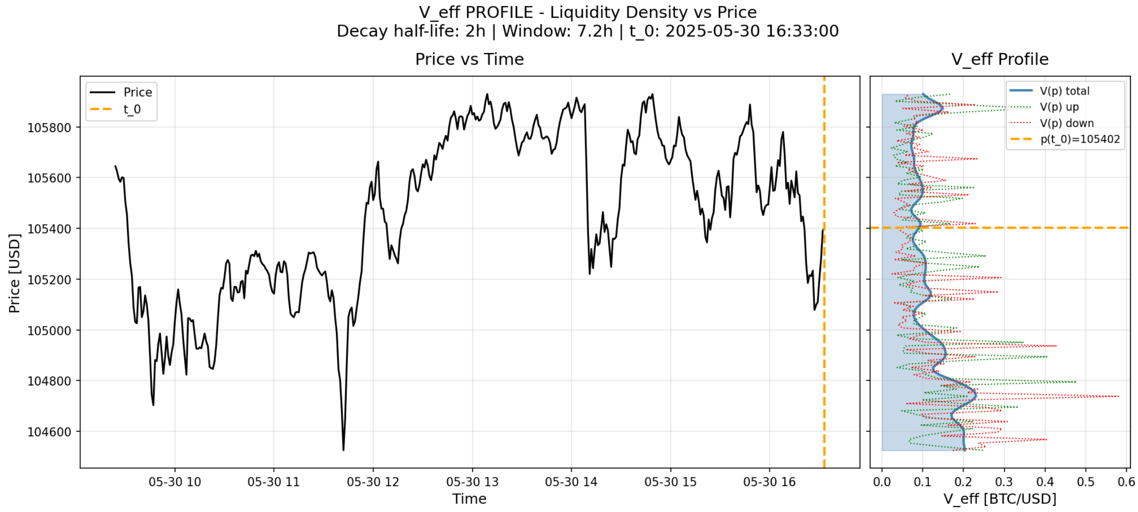

Figure 2: Empirical liquidity density profile $V(p, t_0)$ at time t_0. Left panel: Price trajectory over 7 hours with reference time t_0 marked. Right panel: Liquidity density as function of price, showing total profile (blue), upward runs (green), and downward runs (red). Peaks indicate high-resistance price zones; valleys show low-resistance regions. Current price $p(t_0)$ marked with horizontal line.

4.2 Liquidity Profile $V(p, t)$

Figure 2 shows the empirical liquidity profile constructed from a 7-hour window of data. The profile reveals:

- **Heterogeneous liquidity:** $V(p)$ varies by factor of $\sim 10\times$ across the price range
- **Resistance zones:** Peaks in $V(p)$ correspond to price levels where large volume is required to move the market (high resistance)
- **Low-resistance valleys:** Regions where $V(p)$ is low allow rapid price movement with little volume
- **Directional components:** Separate profiles for upward ($V^{\uparrow}$) and downward ($V^{\downarrow}$) runs show similar structure, confirming symmetry

4.3 Interpretation

The empirical profiles demonstrate that:

1. The theoretical relation $q\,dt = V(p)\,dp$ is empirically valid
2. V_{eff} can be reliably measured from trade data
3. Liquidity density is a slowly-varying field in (p, t) space
4. Price levels have intrinsic "resistance" determined by local liquidity

5 Discussion

5.1 Comparison to Order Book Snapshots

Traditional approaches measure liquidity from order book snapshots (e.g., bid-ask spread, depth at best quotes). Our approach differs fundamentally:

- **Revealed vs. posted liquidity:** We measure actual executed volume per price movement, capturing hidden liquidity and order flow dynamics
- **Temporal resolution:** Monotonic run aggregation filters out high-frequency noise while preserving structure
- **Execution-weighted:** Our V_{eff} reflects what market participants actually experience when trading

5.2 Path Dependence

A key challenge in measuring $V(p,t)$ is path dependence: the liquidity available at price p depends on how the market reached that price. We address this by:

- Using monotonic runs (unidirectional price movement)
- Exponential time decay weighting (recent observations more relevant)
- Separating upward and downward profiles

5.3 Applications

The empirical liquidity profiles have practical applications:

1. **Market impact estimation:** Predict price slippage for large orders
2. **Optimal execution:** Identify low-resistance price zones for trading
3. **Risk management:** Detect liquidity stress (falling V_{eff}) in real-time
4. **Market microstructure theory:** Test models of limit order book dynamics

6 Conclusion

We have demonstrated that the effective liquidity density $V_{\text{eff}}(p,t)$ is a measurable and stable market parameter that governs price dynamics according to the fundamental relation:

$$\frac{dp}{dt} = \frac{q(t)}{V(p,t)}$$

Using 7.9 million high-frequency trades, we validated that:

- V_{eff} can be consistently estimated from monotonic price runs
- It exhibits low temporal autocorrelation (60-minute persistence)
- It is independent of current price level and volatility
- It reveals heterogeneous liquidity structure across price levels

This work bridges the gap between theoretical market microstructure models and empirical high-frequency data, providing a quantitative framework for understanding how order flow translates into price movements through the lens of liquidity density.

6.1 Future Work

Extensions of this research include:

- Multi-asset comparison of liquidity profiles
- Dynamic modeling of $V(p, t)$ evolution
- Incorporation into algorithmic trading strategies
- Connection to market maker inventory models