Flight Mechanics Assignment

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October 2024

1 Aerospace Task

Consider an UA with fixed wing. The competition impose to have a maximum total weight of 10 kg, where the payload weighs at least 1.25 kg. We can consider a structural mass that is approximately the 35-50 percentage of the maximum weight. The take off needs to be completed in a 10x10 m area.

- You have to choose the motors, how many and the relative batteries to allow a flight of at least 15 minutes (after landing their remaining capacity should be 20 percent), considering that the objective is to have the heaviest payload possible and the maximum speed is EAS = 50 kts.
- It's request to estimate the CG position with and without the payload, considering the stability problems and that high maneuverability is requested.
- Calculate the surface of the wing (S) and its length (b), the drone inertia tensor (I), the stability margin (sm).
- How to design flaps to facilitate takeoff within 10 meters? Write down an essential analysis focusing on the trade-off between aerodynamics and aircraft lightweight and reliability.
- Select an airfoil and discuss the reason. You can avoid this point using NACA0012, airfoil tables can be easily found on the internet

2 Development

Preliminary considerations on the shape and aerodynamic efficiency of a potential airfoil

The first step was to determine a plausible condition of aerodynamic efficiency to assess Oswald's aerodynamic efficiency factor (e) and induced drag coefficient (k). The initial assumptions were made considering the possible aerodynamic parameters for fixed-wing UAVs like wing area (S) and wingspan (b):

$$S = 0.8m^2 \tag{1}$$

$$b = (1-4)m \tag{2}$$

By changing the wingspan and thus the aspect ratio, it was easy to calculate an Oswald's factor that took into account the trade-off between wing feasibility and aerodynamic efficiency. The calculation were conducted considering the following formulas:

$$\lambda = \frac{b^2}{S} \tag{3}$$

$$e = 1.78 * (1 - 0.045 * \lambda^{0.68}) - 0.64 \tag{4}$$

$$k = \frac{1}{\pi * \lambda * e} \tag{5}$$

```
S = 0.8; % Superficie alare
b = linspace(1,4,10); % Vettore delle lunghezze alari
lambda = (b).^2./S; % Vettore dei rapporti alari
e = zeros(1,10);

% numero di Oswald
for(i=1:10)
    e(i) = (1.78.*(1 - 0.045*(lambda(i))^(0.68)) - 0.64);
end

% Coefficiente moltiplicativo della resistenza indotta
k = zeros(1,10);
for(i=1:10)
    k(i) = 1/(pi*e(i)*lambda(i));
end
```

Figure 1: MATLAB calculations

By plotting the results, it appeared that the logical choice was to assume a Oswald's coefficient of e = 0.8 and, therefore, a wingspan of b = 2,5m. The resulting induced drag coefficient was k = 0.05

Preliminary analysis based on assumptions of aerodynamic parameters

A preliminary analysis was ran on MATLAB to assess the aerodynamic condition of the UAV in order to proceed with the performance analysis. A specific range of values for the primary variable was determined to be considered in the performance analysis. In this regard, it was mandatory to decide the lift coefficient's maximum value (and the corresponding drag coefficient considering a parabolic polar curve) and the mass of the entire vehicle.

$C_{l,max}$ = 1.2 - 1.5

$C_{d,0}$ = 0.018

$mass$ = (7-10) kg

The following calculations considered the take-off equations and gave a range of values for the maximum thrust required that was taken into account in a matrix.

Corresponding drag coefficient:

$$C_d = C_{d,0} + k * {C_{l_max}}^2 \tag{6}$$

Velocity at take-off is the stall velocity and has to guarantee a lift equal to the mass of the vehicle:

$$v_{take-off} = v_{stall} = \sqrt{\frac{2 * W}{\rho * S * C_{l,max}}} \tag{7}$$

$$\rho = 1.225 \frac{kg}{m^3} \tag{8}$$

The drag at the moment of take-off:

$$D_{take-off} = \frac{1}{2} * \rho * S * {v_{stall}}^2 * C_d \tag{9}$$

The drag is not constant throughout the take-off, but it increases during the acceleration. Therefore, it is reasonable to assume a constant drag that is half of the one calculated at the moment of take-off. The take-off needs to be completed in a 10x10 m area and using a diagonal of the square we can easily calculate the distance for take-off as:

$$d_{take-off} = 10 * \sqrt{2} m \tag{10}$$

We know have the elements to calculate the acceleration of the vehicle and the thrust to achieve the stall speed in the space required.

$$a_{take-off} = \frac{{v_{stall}}^2}{2 * d_{take-off}} \tag{11}$$

$$T_{take-off} = mass * a_{take-off} - \frac{1}{2} * D_{take-off} \tag{12}$$

Considering a likely maximum value of thrust, we can dismiss the conditions that determine a maximum thrust (T = 41.75N) than exceed the one we choose. We can proceed by selecting the thrust that correspond to the maximum value of mass and the minimum value of lift coefficient from the remaining values (Figure 3).

The corresponding values of mass, lift coefficient, stall velocity, drag and acceleration are the one to consider for the following analysis. Therefore, we can now assume this values:

$$a_{take-off} = 4.244 \frac{m}{s^2} \tag{13}$$

$$T_{take-off} = 41.7633 N \tag{14}$$

$$D_{take-off} = 7.5962 N \tag{15}$$

$$v_{take-off} = 10.9562 \frac{m}{s} \tag{16}$$

$$C_{l,max} = 1.491 \tag{17}$$

$$C_{l,d} = 0.1291 \tag{18}$$

Other essential parameters for the choice of motors and batteries are the energy and power required by the maneuvers.

$$L_{take-off} = T_{take-off} * dtake_off = 590.24J \tag{19}$$

$$t_{take-off} = \sqrt{\frac{2 * d_{take-off}}{a_{take-off}}} = 2.58s \tag{20}$$

$$P_{take-off} = \frac{L_{take-off}}{t_{take_off}} = 228.78W \tag{21}$$

The remaining time of the flight is considered to be taken in a maximum efficiency condition, therefore we have to calculate the velocity which allows the required lift with the lift coefficient at maximum efficiency. The lift coefficient at maximum efficiency can be calculated as:

$$\frac{\partial E}{\partial C_l} = \frac{\partial \frac{C_l}{C_d}}{\partial C_l} = 0 \tag{22}$$

$$\frac{\partial \frac{C_l}{C_{d,0} + k * {C_l}^2}}{\partial C_l} = 0 \tag{23}$$

$$\frac{C_{d,0} + k * {C_l}^2 - 2 * k * C_l}{(C_{d,0} + k * {C_l}^2)^2} = 0 \tag{24}$$

$$C_{l,E_{max}} = \sqrt{\frac{C_{d,0}}{k}} = 0.67 \tag{25}$$

Therefore the velocity should be:

$$v_{cruise} = \sqrt{\frac{2 * W}{\rho * S * C_{l,E_{max}}}} = 16.34\frac{m}{s} \tag{26}$$

We assume to use the maximum thrust to accelerate to cruise velocity.

$$C_d = C_{d,0} + k * {C_{l,E_{max}}}^2 = 0.036 \tag{27}$$

$$D = \frac{1}{2} * \rho * S * {v_{cruise}}^2 * C_d = 4.704N \tag{28}$$

$$a = \frac{T - D}{mass} = 4.14\frac{m}{s^2} \tag{29}$$

$$\Delta t = \frac{v_{cruise} - v_{take-off}}{a} = 1.3s \tag{30}$$

The energy and power corresponding to this maneuver are:

$$\Delta s = \frac{1}{2} * a * \Delta t = 3.5m \tag{31}$$

$$L_1 = T_1 * \Delta s = 146.08J \tag{32}$$

$$P_1 = \frac{L_1}{\Delta t} = 112.4W \tag{33}$$

The batteries should allow a flight of at least 15 minutes.

$$t_{flight} = 15min = 900s \tag{34}$$

$$T_{cruise} = D_{cruise} = \frac{1}{2} * \rho * S * {v_{cruise}}^2 * C_d = 4,704N \tag{35}$$

$$P_r = T_{cruise} * v_{cruise} = 76,86W \tag{36}$$

$$L_{flight} = P_r * t_{flight} = 69177.02J \tag{37}$$

Choice of motor and battery

At this point it's possible to determine a possible motor:
AM480 3D 5-6S Freestyle Flight Plane Motor (Figure 4)

$$mass_{motor} = 0,145kg \tag{38}$$

The total energy consumed during all the stages of flight has to be 80 percent of the total capacity of the 6S batteries:

$$L_{total} = L_{take-off} + L_1 + L_{flight} = 69913.34J \tag{39}$$

$$E = L_{total} * \frac{100}{80} = 87391.68J \tag{40}$$

Remembering that E = Q*V

$$[mAh] = \frac{[J]}{3.6 * [V]} \tag{41}$$

The 6S batteries have a nominal voltage of 22.2V, therefore:

$$C = 1093mAh \tag{42}$$

Maximum Current: 55.19 A

$$C - rating = \frac{A_{max}}{C} = 50.5 \tag{43}$$

$$mass_{batteries} = 1kg \tag{44}$$

At this point we are able to assess the payload mass:

$$mass_{structural} = \frac{35}{100} * mass_{total} = 3.129kg \tag{45}$$

$$mass_{total} = mass_{structural} + mass_{motor} + mass_{batteries} + mass_{payload} = 8.9394kg \tag{46}$$

$$mass_{payload} = 4.67kg \tag{47}$$

Consideration on CG position of the UA and stability analysis

In the project stage of an aircraft, the trade-off between stability and manoeuvrability of the vehicle must be taken into account. In this regard, it is requested a high manoeuvrability and, therefore, is necessary a backward position of the center of gravity. On the other hand, it's critical the condition of stability.

It is also necessary to underline the differences in the mass distribution of the vehicle with and without the payload, but if we manage to place the center of gravity of the payload at the CG of the UA, there would be no difference in the stability analysis.

At this point, it's reasonable to choose a position for the center of gravity that allows the aircraft to have a stability margin (sm) of approximately 5 percent.

$$sm = \frac{CG - AC}{MAC} \tag{48}$$

AC being the Aerodynamic Center and MAC being the Mean Aerodynamic Chord.

We can assume a trapezoidal wing shape and, for this reason, it should be easy to geometrically calculate the Mean Aerodynamic Chord as MAC = 0.3 m. The Aerodynamic Center sits almost always at 25 percent of the Aerodynamic Chord and that means that, considering the leading edge the origin of our system, AC = 0.075 m.

$$CG = (MAC * sm) + AC = 0.09m \tag{49}$$

Brief consideration on flap design and its aerodynamic consequences

Designing flaps to facilitate take-off within a short distance requires careful balance of the aerodynamic effects of these components. Flaps increase the lift coefficient of the wing at low speed accentuating the camber of the airfoil. However they also increase drag, which needs to be minimized to allow the UAV to accelerate quickly enough. Therefore, is essential to consider a flap that increase lift without an excessive cost of more drag. Usually are recommended Fowler Flaps or Slotted Flaps for short take-offs.

Obviously, a lightweight aircraft requires less thrust to ensure the acceleration required and allows a margin on increased drag.

```
m = 200;
n = 100;
Cl_max = linspace(1.2,1.5,m);
Cd0 = 0.018;

S = 0.8;
e = 0.8;
lambda_opt = 8;
b = sqrt(lambda_opt*S);
k_opt = 0.05;

Cd = Cd0 + k_opt*Cl_max.^2;

rho = 1.225;
d = 10*sqrt(2);

mass = linspace(7,10,n);
W = 9.81.*mass;

v_stall = zeros(m,n);
D = zeros(m,n);
a = zeros(m,n);
T = zeros(m,n);

for(i=1:n)
    for(j=1:m)
        v_stall(i,j) = sqrt(2*W(i)/(rho*S*Cl_max(j)));
        D(i,j) = 1/4 * rho*(v_stall(i,j))^2*S*Cd(j);
        a(i,j) = (v_stall(i,j))^2/(2*d);

        T(i,j) = mass(i)*a(i,j) + D(i,j);
    end
end
```

Figure 2: MATLAB calculations

```
T_max = 41.75;

for(i=1:n)
    for(j=1:m)
        if(T(i,j) > T_max)
            T(i,j) = 0;
        end
    end
end

i_max = 0;
j_min = m;

for(i=1:n)
    for(j=1:m)
        if(T(i,j)~=0 && i>i_max)
            i_max = i;
            j_min = m;
            if(j_min>j)
                j_min = j;
            end
        end
    end
end
```

Figure 3: MATLAB calculations

Propeller	Throttle	Voltage (V)	Current (A)	Power (W)	RPM	Torque (N*m)	Thrust (g)	Efficiency (g/W)
T-MOTOR 13*6.5 polymer carbon	40%	21.90	5.98	130.94	5049	0.175	967	7.39
	50%	21.79	10.21	222.43	6096	0.249	1424	6.40
	55%	21.73	13.08	284.11	6629	0.305	1702	5.99
	60%	21.65	16.45	356.20	7138	0.359	1988	5.58
	65%	21.45	19.79	424.39	7569	0.405	2247	5.30
	70%	21.33	23.47	500.53	7975	0.451	2516	5.03
	75%	21.24	27.46	583.20	8363	0.499	2779	4.76
	80%	21.12	32.08	677.54	8738	0.551	3055	4.51
	90%	20.97	42.84	898.42	9513	0.663	3669	4.08
	100%	20.76	55.19	1145.52	10183	0.779	4256	3.72

Figure 4: MATLAB calculations