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Applied Finite Differences

On the application of finite differences to a wide range of physical systems ruled by PDE's

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Abstract

In the beautiful and complicated world of physics it is common to come across systems whose evolution in time is utterly complicated and usually it is, as a matter of fact, an evolution ruled by partial differential equations.

It is widely accepted that those equations are considerably hard to solve, however not always is it needed to compute the exact function that solves the problem, but instead find its evolution in fixed window of time with as little error as possible.

The approach is as follows, by applying a semi-discretization to the partial differential equation (PDE) that rules the problem, a system of ordinary differential equations (ODE's) is obtained, which are significantly easier to solve for.

At this point a wide range of paths open ahead. There are several well known methods that solve this problems efficiently such as Euler, Trapeze and the θ-method, all of which will be carefully described in following chapters.

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