Assignment - Part 1

1. Use library scripts to generate p-ER random graphs and r-regular random graph. Let K denote the number of nodes.
2. Write a script to check the connectivity of a given graph.
   - algebraic method 1 (irreducibility);
   - algebraic method 2 (eigenvalue of the Laplacian matrix);
   - breadth-first search algorithm.
3. Compare the complexity as a function of K of the methods above by plotting curves of a complexity measure vs K.
4. Let $p_c(\mathcal{G})$ denote the probability that a graph $\mathcal{G}$ is connected. By running Monte Carlo simulations, estimate $p_c(\mathcal{G})$ and produce two curve plots:
   - $p_c(\mathcal{G})$ vs. p for Erdős-Rényi graphs with $K = 100$.
   - $p_c(\mathcal{G})$ vs. K, for $K \leq 100$, for r-regular random graphs with $r = 2$ and $r = 8$.

Computation job: local versus distributed run

- Let us consider a server A that has to run a computation job that takes a time $T_0 + X$, where T_0 is a fixed set-up time and X is a random variable with mean $\mathrm{E}[X]$.
- The job is applied to a file of data of size L_f.
- The output of the task is an amount of data L_o.
- If the job is split into N parallel tasks, that are run over N servers (other than A), then
  - Each task takes a time $T_0 + X_i$, where X_i is a negative exponential random variable with mean $\mathrm{E}[X_i] = \mathrm{E}[X]/N$.
  - Each server receives an amount of input data L_f/N.
  - The amount of output data produced by each server is a random variable $L_{o,i}$, uniformly distributed in $[0, 2L_o/N]$.

Communications among servers

- Data is transferred to and from server i ($i = 1, \ldots, N$) via a TCP connection between server A and server i, having average throughput given by

$$TH_i = C \frac{1/T_i}{\sum_{j=1}^{N} 1/T_j}$$

  where T_i is the RTT between the origin server A and server i and C is the capacity of each link of the DC network.
- $T_i = 2\tau h_i$, where h_i is the number of hops between server A and server i.
- The N servers selected to split the job are *the N closest servers to A in the given Data Center network topology*.
- In the following, we assume the N servers are numbered from 1 to N so that $h_1 \leq h_2 \leq \cdots \leq h_N$.

Metrics

- The **response time** R is the time elapsing since when the job is submitted to server A until the time that the output of the job is available at server A.
- In case of splitting of the job into N tasks, the output of the job is available only when *all* servers have delivered their output files to server A.

- The **Job running cost** S is defined as

$$S = \mathrm{E}[R] + \xi\, \mathrm{E}[\Theta]$$

  where $\mathrm{E}[\Theta]$ is the average server time used to run the job.
- Θ is the time that server A is used, if the job runs locally on A. Otherwise, Θ is the sum of the times that all N servers are used to run their respective tasks.

Additional details

- In building DC network topologies assume that switches with n ports are used.
  - Fat-Tree DC network topology is defined as usual.
  - Jellyfish DC network topology is build assuming that $r = n/2$, i.e., each switch devotes $n/2$ ports for connections to local servers and the other $n/2$ ports for connections to other switches.
  - The two topologies must be built so that they have the same number of servers ($n^3/4$).
- When communicating data back and forth from server A to the N servers, using TCP connections, there is some overhead involved. Specifically, the amount of bits sent through the TCP connection is given by the sum of the application job data plus an additional overhead, which is a fraction f of the original data.

Numerical values

- $C = 10$ Gbit/s.
- $\tau = 5\ \mu$s.
- $L_f = 4$ TB.
- $L_o = 4$ TB.
- $\mathrm{E}[X] = 8$ hours.
- $T_0 = 30$ sec.
- $\xi = 0.1$.
- $f = 48/1500$.
- $n = 64$.

Assignment - Part 2

1. Give a concise and accurate formal statement of the algorithm you use to evaluate the mean response time (i.e., how you conduct the statistical experiment to collect samples of R).
2. Plot the mean response time $\mathrm{E}[R]$ as a function of N for N ranging between 1 and 10000.
   - Let R_{baseline} be the response time in case only server A is used, i.e., the job is run locally on A.
   - In the plot normalize $\mathrm{E}[R]$ with respect to $\mathrm{E}[R_{\text{baseline}}]$.
3. Plot the Job running cost S as a function of N for N ranging between 1 and 10000.
   - Let S_{baseline} be the Job running cost in case only server A is used, i.e., the job is run locally on A.
   - In the plot normalize S with respect to S_{baseline}.
4. Give the numerical value of the optimal number of servers (the one that minimizes the job running cost).
5. Explain in a concise and accurate way the results you have got. Highlight the takeaways of your analysis.

Delivery of the assignment

The delivery of the assignment consists of a **max 4 pages written report** in the form of a pdf file (Font size: 12 pt).

1. **PAGE 1** – Cover Page: nickname of your group, your given and family names, and enrollment numbers.
2. **PAGE 2** – Three curve plots plus explanations: (i) Complexity versus number of nodes K, for the three connectivity checking algorithms; (ii) Probability of a connected ER random graph as a function of p for $K = 100$ nodes; (iii) Probability of a connected r-regular random graph as a function of K for $r = 2$ and 8.
3. **PAGE 3-4** – (i) Formal statement of algorithm for the evaluation of the mean response time; (ii) plot of the normalized mean response time as a function of N; (iii) plot of the normalized Job running cost as a function of N; (iv) optimal value of N that minimizes S; (v) analysis of results and takeaways. (note that this part must be done for both Fat-Tree and Jellyfish topologies)

Accuracy and conciseness of texts, as well as accuracy, readability and quality of the graphs will be fundamental in the evaluation.