DEPARTAMENTO DE COMPUTACION
Facultad de Ciencias Exactas y Naturales - UBA

Guia 3

2do cuatrimestre 2024 — Algoritmos y Estructuras de Datos I

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1.1. Ejercicio 1

Calcular las siguientes expresiones, donde a, b son variables reales, i una variable entera y A es una secuencia de reales.

a) $\texttt{def}(a+1) \equiv True$

b) $\texttt{def}(a/b) \equiv b \neq 0$

c) $\texttt{def}(\sqrt{a/b}) \equiv b \neq 0 \wedge_L a/b \geq 0$

d) $\texttt{def}(A[i]+1) \equiv 0 \leq i < |A|$

e) $\texttt{def}(A[i+2]) \equiv 0 \leq i+2 < |A| \equiv -2 \leq i < |A|-2$

f) $\texttt{def}(0 \leq i \leq |A| \wedge_L A[i] \geq 0) \equiv i \neq |A|$

1.2. Ejercicio 2

Calcular las siguientes precondiciones más débiles, donde a, b son variables realies, i una variable entera y A es una secuencia de reales.

a)

$$\begin{aligned} wp(\mathbf{a := a{+}1;\ b := a/2}, b \geq 0) &\equiv wp(\mathbf{a := a{+}1}, wp(\mathbf{b := a/2}, b \geq 0)) \\ &\equiv wp(\mathbf{a := a{+}1}, \texttt{def}(a/2) \wedge_L a/2 \geq 0) \\ &\equiv wp(\mathbf{a := a{+}1}, a \geq 0) \\ &\equiv \texttt{def}(a+1) \wedge_L a+1 \geq 0 \\ &\equiv \boxed{a \geq -1} \end{aligned}$$

b)

$$\begin{aligned} wp(\mathbf{a := A[i]\ +\ 1;\ b := a{*}a}, b \neq 2) &\equiv wp(\mathbf{a := A[i]\ +\ 1}, wp(\mathbf{b := a{*}a}, b \neq 2)) \\ &\equiv wp(\mathbf{a := A[i]\ +\ 1}, a * a \neq 2) \\ &\equiv wp(\mathbf{a := A[i]\ +\ 1}, |a| \neq \sqrt{2}) \\ &\equiv \texttt{def}(A[i]+1) \wedge_L |A[i]+1| \neq \sqrt{2} \\ &\equiv \boxed{0 \leq i < |A| \wedge_L A[i] \neq -1 \pm \sqrt{2}} \end{aligned}$$

c)

$$\begin{aligned} wp(\mathbf{a := A[i]\ +\ 1;\ a := b{*}b}, a \geq 0) &\equiv wp(\mathbf{a := A[i]\ +\ 1}, wp(\mathbf{a := b{*}b}, a \geq 0)) \\ &\equiv wp(\mathbf{a := A[i]\ +\ 1}, b * b \geq 0) \\ &\equiv wp(\mathbf{a := A[i]\ +\ 1}, |b| \geq 0) \\ &\equiv wp(\mathbf{a := A[i]\ +\ 1}, True) \\ &\equiv \boxed{0 \leq i < |A|} \end{aligned}$$

d)

$$\begin{aligned} wp(\mathbf{a := a{-}b;\ b := a{+}b}, a \geq 0 \wedge b \geq 0) &\equiv wp(\mathbf{a := a{-}b}, wp(\mathbf{b := a{+}b}, a \geq 0 \wedge b \geq 0)) \\ &\equiv wp(\mathbf{a := a{-}b}, a \geq 0 \wedge a+b \geq 0) \\ &\equiv a-b \geq 0 \wedge (a-b)+b \geq 0 \\ &\equiv \boxed{a \geq b \wedge a \geq 0} \end{aligned}$$

1.3. Ejercicio 3

Sea $Q \equiv (\forall j : \mathbb{Z})\ (0 \leq j < |A| \rightarrow_L A[j] \geq 0)$. Calcular las siguientes precondiciones más débiles, donde i es una variable entera y A es una secuencia de enteros.

a)

$$\begin{aligned}
wp(\mathbf{A[i]\ :=\ 0}, Q) &\equiv wp(\mathbf{A\ :=\ setAt(A,\ i,\ 0)}, Q) \\
&\equiv 0 \le i < |A| \wedge_L Q^{A}_{setAt(A,i,0)} \\
&\equiv 0 \le i < |A| \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L setAt(A,i,0)[j] \ge 0) \\
&\equiv 0 \le i < |A| \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L (i = j \rightarrow 0 \ge 0) \wedge (i \ne j \rightarrow A[i] \ge 0)) \\
&\equiv \boxed{0 \le i < |A| \wedge_L (\forall j : \mathbb{Z})\ ((0 \le j < |A| \wedge i \ne j) \rightarrow_L A[j] \ge 0)}
\end{aligned}$$

b)

$$\begin{aligned}
wp(\mathbf{A[i+2]\ :=\ 0}, Q) &\equiv 0 \le i + 2 < |A| \wedge_L Q^{Q}_{setAt(A,i+2,0)} \\
&\equiv \ldots \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L (i + 2 = j \rightarrow 0 \ge 0) \wedge (i + 2 \ne j \rightarrow A[j] \ge 0)) \\
&\equiv \boxed{-2 \le i < |A| - 2 \rightarrow_L (\forall j : \mathbb{Z})\ ((0 \le j < |A| \wedge i + 2 \ne j) \rightarrow_L A[j] \ge 0)}
\end{aligned}$$

c) HACER!

d)

$$\begin{aligned}
wp(\mathbf{A[i]\ :=\ 2 * A[i]}, Q) &\equiv wp(\mathbf{A\ :=\ setAt(A,\ i,\ 2 * A[i])}, Q) \\
&\equiv 0 \le i < |A| \wedge_L Q^{A}_{setAt(A,i,2*A[i])} \\
&\equiv \ldots \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L (i = j \rightarrow 2 * A[i] \ge 0) \wedge (i \ne j \rightarrow A[j] \ge 0)) \\
&\equiv \ldots \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L (i = j \rightarrow A[j] \ge 0) \wedge (i \ne j \rightarrow A[j] \ge 0)) \\
&\equiv \boxed{0 \le i < |A| \wedge_L (\forall j : \mathbb{Z})\ (0 \le j < |A| \rightarrow_L A[j] \ge 0)}
\end{aligned}$$

e) HACER!

1.4. Ejercicio 4

Para los siguientes pares de programas S y postcondiciones Q

- Escribir la precondición más débil $P = wp(\mathbf{S}, Q)$
- Mostrar formalmente que la P elegida es correcta

a) $S \equiv$

```
if (a < 0)
    b := a
else
    b := -a
endif
```

$Q \equiv (b = -|a|)$

$$\begin{aligned}
wp(\mathbf{S}, Q) &\equiv (a < 0 \wedge wp(\mathbf{b\ :=\ a}, Q)) \vee (a \ge 0 \wedge wp(\mathbf{b\ :=\ -a}, Q)) \\
&\equiv (a < 0 \wedge a = -|a|) \vee (a \ge 0 \wedge -a = -|a|) \\
&\equiv a < 0 \vee a \ge 0 \\
&\equiv True
\end{aligned}$$

b) HACER!

c) HACER!

d) $S \equiv$

```
if (i > 1)
    s[i] := s[i-1]
else
    s[i] := 0
endif
```

$Q \equiv (\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L s[j] = s[j-1])$

$$wp(\mathbf{S}, Q) \equiv (i > 1 \land wp(\mathbf{s[i]\ := s[i-1]}, Q)) \lor (i \leq 1 \land wp(\mathbf{s[i]\ := 0}, Q))$$

$$\begin{aligned} wp(\mathbf{s[i]\ := s[i\ -\ 1]}, Q) &\equiv (0 \leq i < |s| \land 0 \leq i-1 < |s|) \land_L Q^s_{setAt(s,i,s[i-1])} \\ &\equiv 1 \leq i < |s| \land_L \\ &(\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L setAt(s,i,s[i-1])[j] = setAt(s,i,s[i-1])[j-1]) \\ \\ &\equiv 1 \leq i < |s| \land_L \\ &(\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L (i = j \rightarrow s[j-1] = s[j-1]) \land \\ &(i = j-1 \rightarrow s[j] = s[j-2]) \land ((i \neq j \land i \neq j-1) \rightarrow s[j] = s[j-1])) \\ \\ &\equiv 1 \leq i < |s| \land_L \\ &(\forall j : \mathbb{Z})\ (1 \leq j < |s| \land_L (i = j-1 \rightarrow s[j] = s[j-2]) \land \\ &((i \neq j \land i \neq j-1) \rightarrow s[j] = s[j-1])) \end{aligned}$$

$$\begin{aligned} wp(\mathbf{s[i]\ := 0}, Q) &\equiv 0 \leq i < |s| \land_L Q^s_{setAt(s,i,0)} \\ &\equiv 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L setAt(s,i,0)[j] = setAt(s,i,0)[j-1]) \\ &\equiv 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L \\ &(i = j \rightarrow 0 = s[j-1]) \land (i = j-1 \rightarrow s[j] = 0) \land ((i \neq j \land i \neq j-1) \rightarrow s[j] = s[j-1])) \end{aligned}$$

$$\begin{aligned} wp(\mathbf{S}, Q) \equiv\ &(1 < i < |s| \land_L (\forall j : \mathbb{Z})\ (1 \leq j < |s| \land_L (i = j-1 \rightarrow s[j] = s[j-2]) \land \\ &((i \neq j \land i \neq j-1) \rightarrow s[j] = s[j-1]))) \lor \\ &(i \leq 1 \land 0 \leq i < |s| \land_L (\forall j : \mathbb{Z})\ (1 \leq j < |s| \rightarrow_L (i = j \rightarrow 0 = s[j-1]) \land \\ &(i = j-1 \rightarrow s[j] = 0) \land ((i \neq j \land i \neq j-1) \rightarrow s[j] = s[j-1]))) \end{aligned}$$

e) HACER!

f) HACER!

1.5. Ejercicio 5

Para las siguientes especificaciones:

- Poner nombre al problema que resuelven
- Escribir un programa S sencillo en SmallLang, sin ciclos, que lo resuelva
- Dar la precondición más débil del programa escrito con respecto a la postcondición de su especificación

a) `proc sumaIesimoElemento` (in s: $seq\langle\mathbb{Z}\rangle$, in i: $\mathbb{Z}$, inout a: $\mathbb{Z}$)

`requiere` $\{0 \leq i < |s| \land_L a = \sum_{j=0}^{i-1} s[j]\}$

`asegura` $\{a = \sum_{j=0}^{i} s[j]\}$

```
func sumaIesimoElemento(s <int>, i int, a int) {
    a := a + s[i];
    return a
}
```

$$
\begin{aligned}
wp(\mathbf{a := a + s[i]}, a = \sum_{j=0}^{i} s[j]) &\equiv 0 \leq i < |s| \wedge_L a + s[i] = \sum_{j=0}^{i} s[j] \\
&\equiv \boxed{0 \leq i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[j]}
\end{aligned}
$$

b) HACER!

c) HACER!