

# Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

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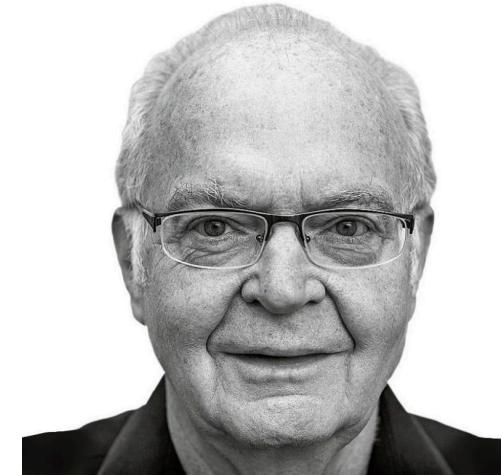
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Slides: [federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf](https://federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf)

# Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

*We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.*

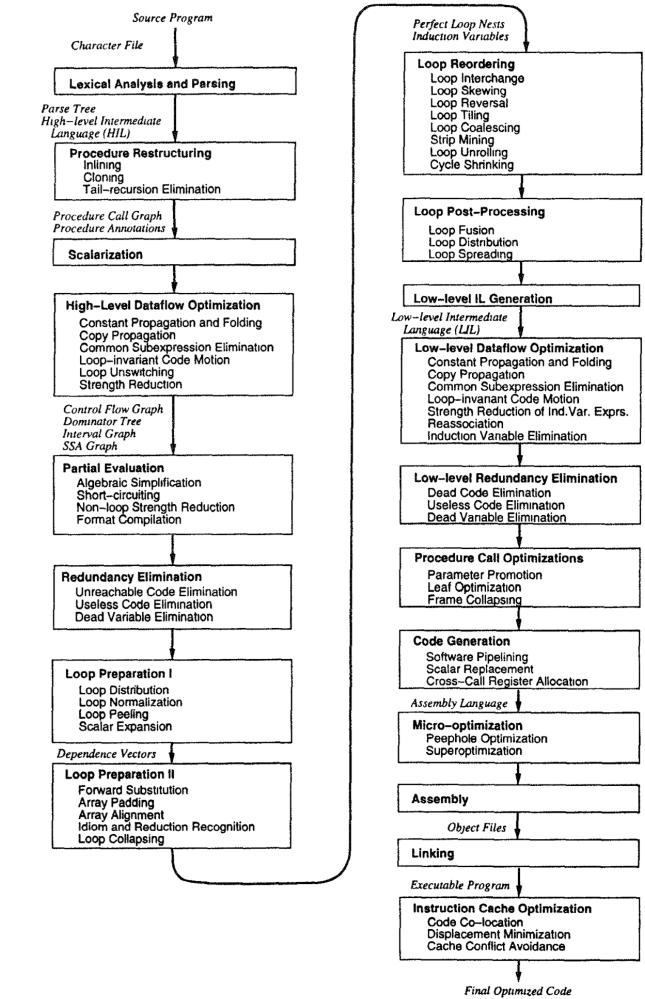


In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

# Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Bacon et al., CSUR 1994 [2]

# Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations<sup>2</sup> are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

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<sup>2</sup>A chronologically sorted list of papers on compiler optimization, from the works of 1952 through the techniques of 1994, is available at [5].

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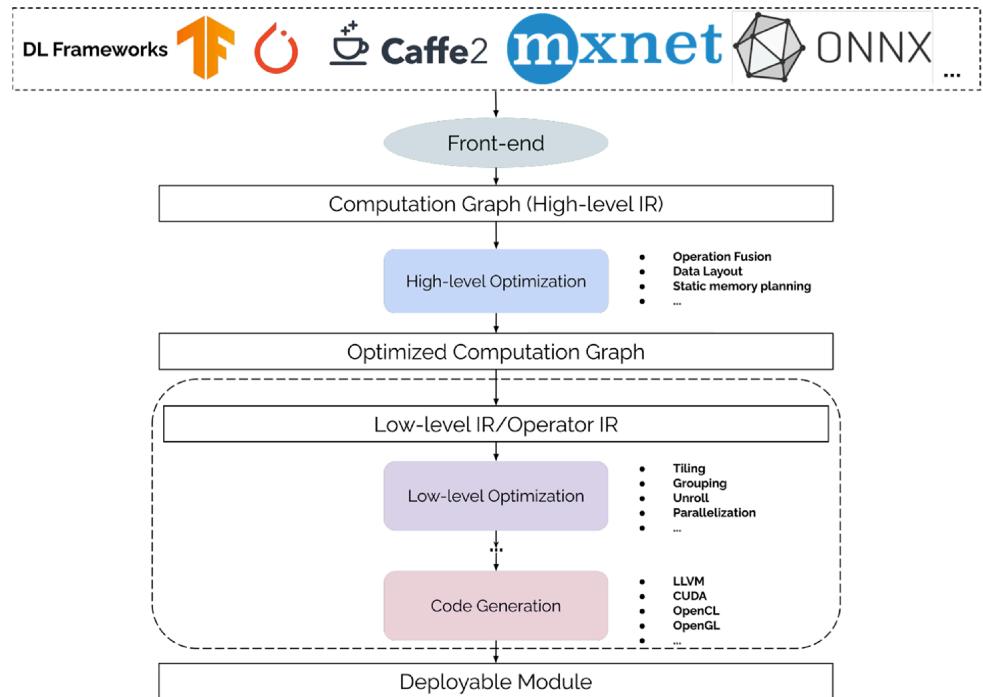
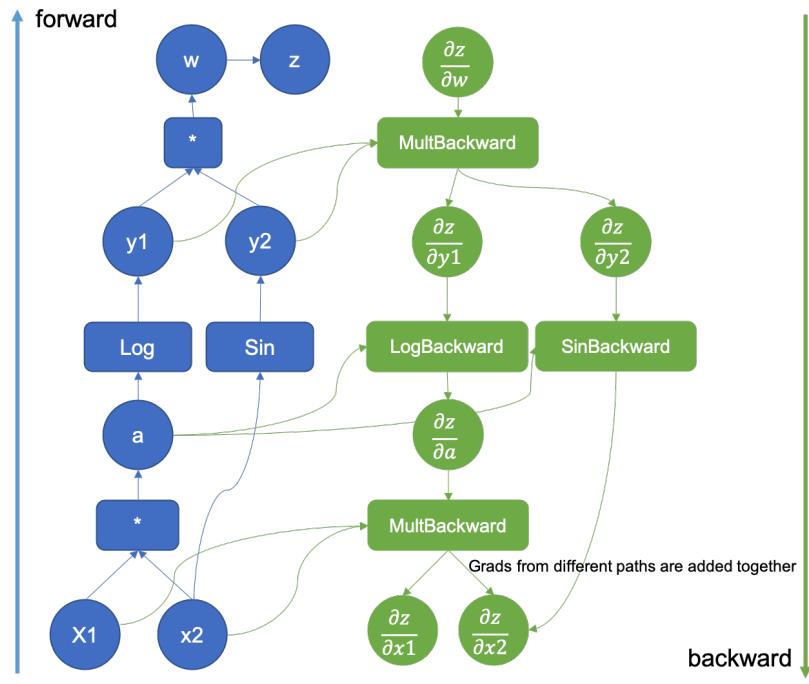
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# Machine Learning Framework are Just Optimizing Compilers<sup>5</sup>



<sup>5</sup>TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use LLVM [9]. Li *et al.*, [10] compiled a survey on ML compilers.

# Peephole Optimizations in x86-64 (cf. [11], [12])

```
1 ; x = x * 2, s.t. x: i32          asm
2 mov     eax, dword ptr [rbp - 4]
3 imul    eax, 2
4 mov     dword ptr [rbp - 4], eax
```

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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

```
1 ; x = x << 1, s.t. x: i32          asm
2 mov     eax, dword ptr [rbp - 4]
3 shl     eax
4 mov     dword ptr [rbp - 4], eax
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asm

```
1 ; x = x + 0, s.t. x: i32
```

```
2 mov     eax, dword ptr [rbp - 4]
3 add     eax, 0
4 mov     dword ptr [rbp - 4], eax
```

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```
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```

asm

The optimized version removes the **unnecessary** addition operation.

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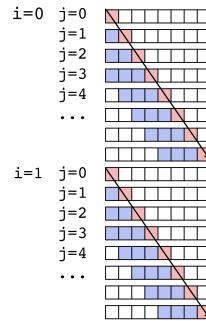
```
1 mov     eax, dword ptr [rbp - 4]
2 mov     dword ptr [rbp - 4], eax
```

asm

The `mov` instructions are redundant and can be **pruned** as well!

# Loop Nest Optimizations – Loop Tiling (cf. [13], [14])

```
for (int i=0; i<n; ++i) {    C++  
    for (int j=0; j<m; ++j) {  
        c[i][j] = a[i] * b[j];  
    }  
}
```



The vector  $b$  **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

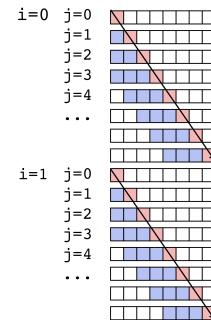
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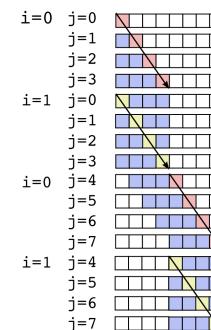
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The inner loop works on a **tile** of **b** that fits into the cache.

```
for (int jj = 0; jj < m; jj += TILE_SIZE) C++
    for (int i = 0; i < n; ++i)
        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
```



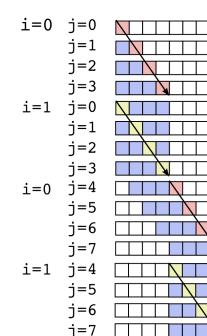
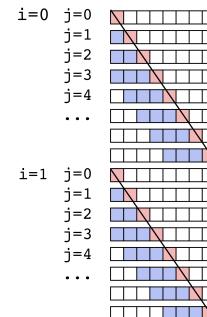
[A. Vladimirov, Session 10](#)

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        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
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```

Careful readers may notice that, in this version, the values for the array **a** will be read  $m / \text{TILE\_SIZE}$ !

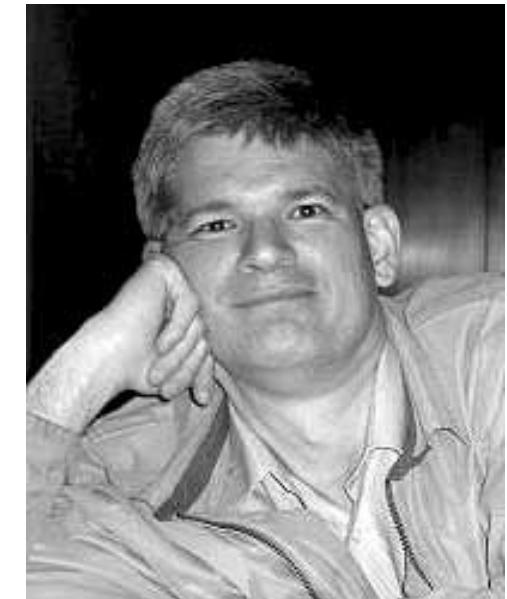
```
for (int ii = 0; ii < n; ii += TILE_SIZE_I) C++
    for (int jj = 0; jj < m; jj += TILE_SIZE_J)
        for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)
            for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)
                c[i][j] = a[i] * b[j];
```

[A. Vladimirov, Session 10](#)

# Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

*In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.*



## From Recursion to Iteration (cf. [18])

$$f(x) = \begin{cases} b(x_0) & \text{if } x = x_0 \\ a(x, f(d(x))) & \text{otherwise} \end{cases}$$

s.t.  $a$ ,  $b$ , and so on may denote any pieces of code.

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To transform recursive function  $f$  into iterative form, we need to:

1. Identifies an increment  $\oplus$  to the argument of  $f$ , *id est*,  $x' = x \oplus y$  such that  $x = \text{prev}(x')$ , where  $\text{prev}$  is based on the arguments of the recursive call. In this case,  $\text{prev}(x) = d(x)$  and, if  $d^{-1}$  exists,  $x \oplus y = d^{-1}(x)$ , can be plugged in for  $y$ .
2. Derives an incremental program  $f'(x, r)$  that computes  $f(x)$  using an accumulator  $r$  of  $f(\text{prev}(x))$ .
3. Forms an iterative version that initializes using the base case of  $f$  and iteratively applies  $f'$  until reaching the desired argument.

```

 $f(x) = \{$ 
     $x_1 = x_0; r = b(x_0);$ 
    while ( $x_1 \neq x$ ){
         $x_1 = d^{-1}(x_1);$ 
         $r = a(x_1, r);$ 
    }
    return  $r;$ 
}

```

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```
f(x) = {
    x1 = x0; r = b(x0);
    while (x1 ≠ x){
        x1 = d⁻¹(x1);
        r = a(x1, r);
    }
    return r;
}
```

Note that, when  $a$  is in the form  $a(a_1(x), y)$  and  $a$  is associative, we do not need  $d^{-1}$  and  $x_1$ .

# Tail-recursive Factorial Function

```
int fact(int n) {  
    if (n == 0) {  
        return 1;  
    }  
    return n * fact(n - 1);  
}
```

C++

The replacement of  $n * ((n - 1) * (n - 2))$  by  $(n * (n - 1)) * (n - 2)$  is valid due to the **associativity** of multiplication. In the general form:

$$f(x) = \{$$
$$r = b(x_0);$$
$$\text{while } (x \neq x_0) \{$$
$$r = a(r, a_1(x));$$
$$x = d(x);$$
$$\}$$
$$\text{return } r;$$
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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```
f(x) = {
    r = b(x₀);
    while (x ≠ x₀){
        r = a(r, a₁(x));
        x = d(x);
    }
    return r;
}
```

Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

“clang -O3 -S -emit-llvm fact.c -o -” produces something like:

The basic blocks related to loop vectorization (for performing SIMD operations) have been omitted for clarity

```
define i32 @fact(i32 %n)  {
entry:
%cmp3 = icmp eq i32 %n, 0
br i1 %cmp3, label %return, label %if.else

if.else:
%n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]
%acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]
%sub = add nsw i32 %n.tr5, -1
%mul = mul nsw i32 %n.tr5, %acc.tr4
%cmp = icmp eq i32 %sub, 0
br i1 %cmp, label %return, label %if.else

return:
%acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]
ret i32 %acc.tr.lcssa
}
```

# What About Fibonacci? To Hell With Multiple Recursions!

```
int fib(int n) {
    if (n <= 1) {
        return n;
    }
    return fib(n - 1) + fib(n - 2);
}
```

C++

## The LLVM Optimized Version (but human readable)

```
int fib(int n) {
    if (n < 2) {
        return n;
    }
    int acc = 0;
loop: /* while (1) { */
    int call = fib(n - 1);
    acc = call + acc;
    if (n < 4) goto ret; /* return acc + (n - 2); */
    n = n - 2;
    goto loop; /* } */
ret:
    return acc + (n - 2);
}
```

C++

Note that, the following LLVM IR is a fixed-point representation of the `fib` function; observable by the output of “`opt -passes="default<O3>" -S fib-03.ll -o -`”

```
; fib-03.ll
define i32 @fib(i32 %n) {
entry:
%cmp6 = icmp slt i32 %n, 2
br i1 %cmp6, label %return, label %if.end
if.end:
%n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]
%accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]
%sub = add nsw i32 %n.tr8, -1
%call = tail call i32 @fib(i32 %sub)
%sub1 = add nsw i32 %n.tr8, -2
%add = add nsw i32 %call, %accumulator.tr7
%cmp = icmp samesign ult i32 %n.tr8, 4
br i1 %cmp, label %return, label %if.end
return:
%accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]
%n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]
%accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa
ret i32 %accumulator.ret.tr
}
```

llvm

So your compiler is unable  
to *incrementalize* functions  
with multiple recursions?

Apparently, yes.

# The Y. A. Liu's Incrementalization



In 1990, Liu *et al.* have done extensive research on **Incrementalization** [18], [19], [20], [21].

Even in presence of multiple recursions, in [18, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (cf. slide “*From Recursion to Iteration*”).

But the Step 2. builds upon the principles of [19] and [20] – which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

# Conclusions

The papers are a little bit old-fashioned, and the key aspect of deriving the incremental program in presence of multiple recursions is **opaque**.

I wrote an email to Y.A. Liu asking for clarification last week, but I haven't received a reply yet ... 😢

*Despite her work, we are trying to understand whether it is really possible to make the transformation automatically in a “general” setting.*

# Bibliography

- [1] D. E. Knuth, “Structured Programming with go to Statements,” *ACM Comput. Surv.*, vol. 6, no. 4, pp. 261–301, Dec. 1974.
- [2] D. F. Bacon, S. L. Graham, and O. J. Sharp, “Compiler transformations for high-performance computing,” *ACM Computing Surveys (CSUR)*, vol. 26, no. 4, pp. 345–420, 1994.
- [3] K. D. Cooper and L. Torczon, *Engineering a Compiler*, no. . Morgan Kaufmann, 2022.
- [4] K. Kennedy and J. R. Allen, *Optimizing compilers for modern architectures: a dependence-based approach*. Morgan Kaufmann Publishers Inc., 2001.
- [5] F. Bruzzone, “Papers on Compiler Optimizations: Analysis and Transformations (1952-1994).” <https://github.com/FedericoBruzzone/papers-on-compiler-optimizations>, 2025.
- [6] R. Paige, “Future directions in program transformations,” *SIGPLAN Not.*, vol. 32, no. 1, pp. 94–98, Jan. 1997.
- [7] F. E. Allen, “Program Optimization,” *Annual Review of Automatic Programming*, vol. 5. Pergamon Press, pp. 239–307, 1966.

- [8] W. A. Wulf, “The design of an optimizing compiler,” 1973.
- [9] C. Lattner and V. Adve, “LLVM: A compilation framework for lifelong program analysis & transformation,” in *International symposium on code generation and optimization, 2004. CGO 2004.*, 2004, pp. 75–86.
- [10] M. Li *et al.*, “The deep learning compiler: A comprehensive survey,” *IEEE Transactions on Parallel and Distributed Systems*, vol. 32, no. 3, pp. 708–727, 2020.
- [11] W. M. McKeeman, “Peephole optimization,” *Commun. ACM*, vol. 8, no. 7, pp. 443–444, July 1965.
- [12] A. S. Tanenbaum, H. van Staveren, and J. W. Stevenson, “Using Peephole Optimization on Intermediate Code,” *ACM Trans. Program. Lang. Syst.*, vol. 4, no. 1, pp. 21–36, Jan. 1982.
- [13] M. Wolfe, “More iteration space tiling,” in *Proceedings of the 1989 ACM/IEEE Conference on Supercomputing*, in Supercomputing '89. Reno, Nevada, USA: Association for Computing Machinery, 1989, pp. 655–664.
- [14] M. E. Wolf and M. S. Lam, “A data locality optimizing algorithm,” *SIGPLAN Not.*, vol. 26, no. 6, pp. 30–44, May 1991.

- [15] A. V. Aho, R. Sethi, and J. D. Ullman, *Compilers: Principles, Techniques, and Tools*. Reading, Massachusetts: Addison Wesley, 1986.
- [16] S. Muchnick, *Advanced compiler design implementation*. Morgan kaufmann, 1997.
- [17] G. L. Steele, “Debunking the “expensive procedure call” myth or, procedure call implementations considered harmful or, LAMBDA: The Ultimate GOTO,” in *Proceedings of the 1977 Annual Conference*, in ACM '77. Seattle, Washington: Association for Computing Machinery, 1977, pp. 153–162.
- [18] Y. A. Liu and S. D. Stoller, “From recursion to iteration: what are the optimizations?,” in *Proceedings of the 2000 ACM SIGPLAN workshop on Partial evaluation and semantics-based program manipulation*, 1999, pp. 73–82.
- [19] Y. A. Liu, S. D. Stoller, and T. Teitelbaum, “Static caching for incremental computation,” *ACM Transactions on Programming Languages and Systems (TOPLAS)*, vol. 20, no. 3, pp. 546–585, 1998.
- [20] Y. A. Liu and T. Teitelbaum, “Systematic derivation of incremental programs,” *Science of Computer Programming*, vol. 24, no. 1, pp. 1–39, 1995.

- [21] Y. A. Liu, “CACHET: An interactive, incremental-attribution-based program transformation system for deriving incremental programs,” in *Proceedings 1995 10th Knowledge-Based Software Engineering Conference*, 1995, pp. 19–26.