Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

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Milan, Italy – 4 December 2025

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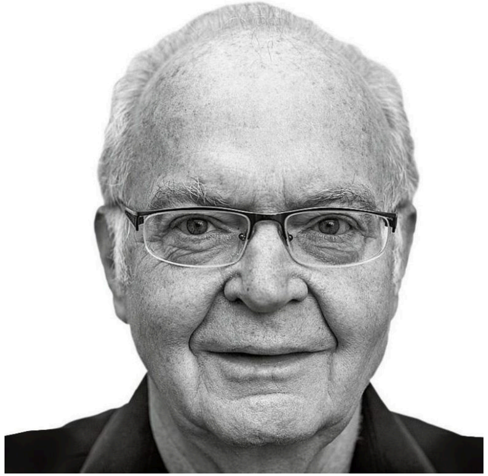
Slides: federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf

Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.



In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [2], [3].

Compiler optimizations[2] are such transformations (say *meaning-preserving mappings* [5]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [6], [7], such compilers are referred to as **optimizing compilers**.

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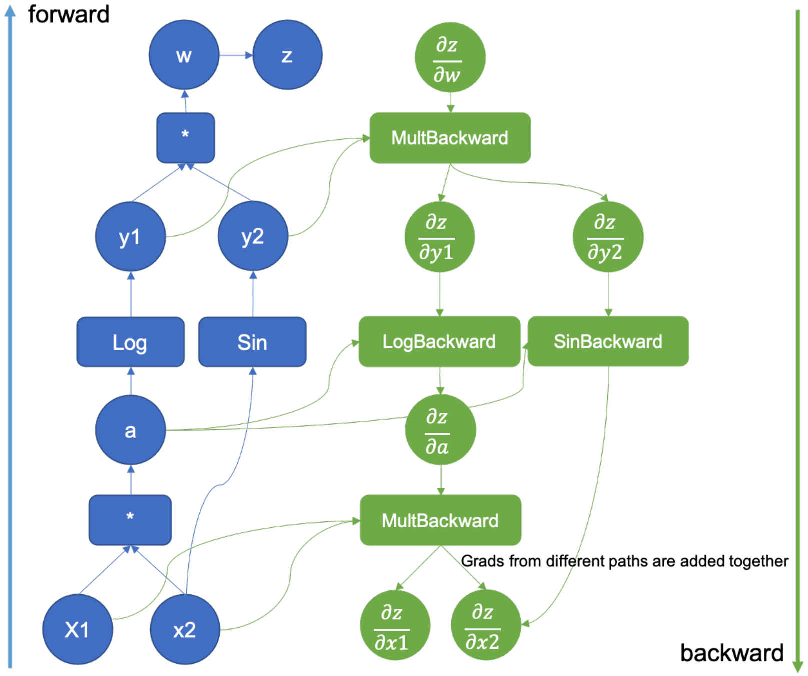
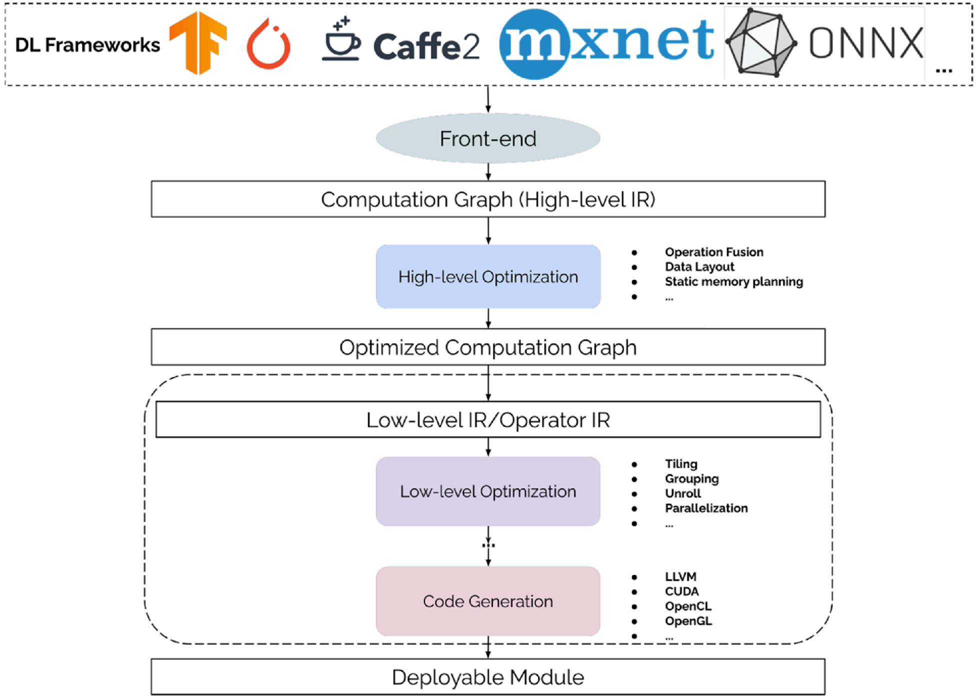
Machine Learning Framework are Just Optimizing Compilers[5]





[5] TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use **LLVM** [8]. Li *et al.*, [9] compiled a survey on ML compilers.

Peephole Optimizations in x86-64 (cf. [10], [11])

```asm
; x = x * 2, s.t. x: i32
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imul     eax, 2
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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

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; x = x << 1, s.t. x: i32
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```asm
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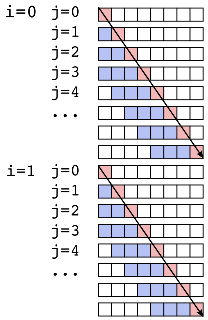
The `mov` instructions are redundant and can be **pruned** as well!

Loop Nest Optimizations — Loop Tiling (cf. [12], [13])

```cpp
for (int i=0; i<n; ++i) {
  for (int j=0; j<m; ++j) {
      c[i][j] = a[i] * b[j];
  }
}
```



The vector b **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

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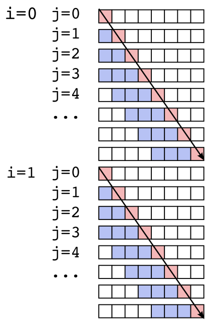

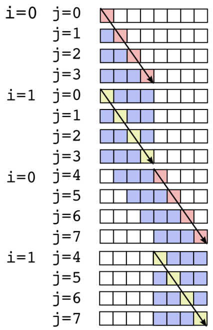

The inner loop works on a **tile** of `b` that fits into the cache.

```cpp
for (int jj = 0; jj < m; jj += TILE_SIZE)
    for (int i = 0; i < n; ++i)
        for (int j = jj; j < MIN(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
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A. Vladimirov, Session 10

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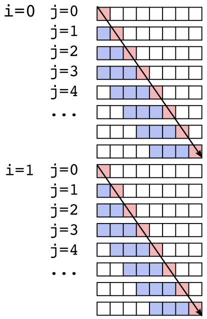

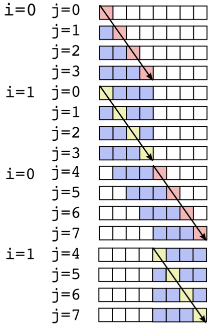

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    for (int j = jj; j < MIN(jj + TILE_SIZE, m); ++j)
      c[i][j] = a[i] * b[j];
```

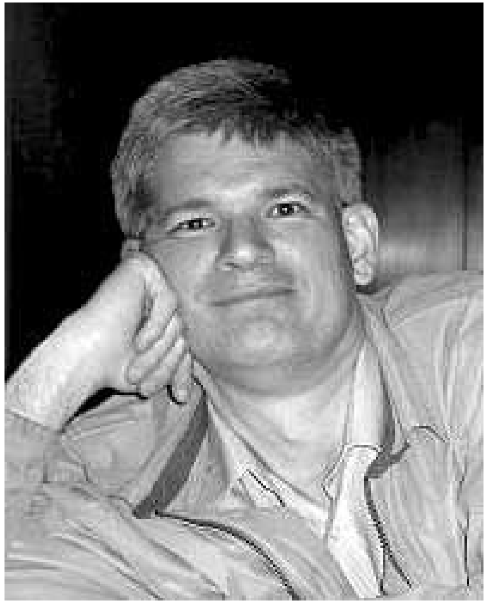
Careful readers may notice that, in this version, the values for the array `a` will be read `m / TILE_SIZE`!

```cpp
for (int ii = 0; ii < n; ii += TILE_SIZE_I)
 for (int jj = 0; jj < m; jj += TILE_SIZE_J)
  for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)
   for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)
    c[i][j] = a[i] * b[j];
```

Tail Call/Recursion Optimization (cf. [2], [14], [15])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [16]:

In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.



From Recursion to Iteration (cf. [17])

$$\boldsymbol{f}(x) = \begin{cases} b(x_0) \text{ if } x = x_0 \\ a(x, \boldsymbol{f}(d(x))) \text{ otherwise} \end{cases}$$

$s.t.$ a, b, and so on may denote any pieces of code.

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To transform recursive function $\boldsymbol{f}$ into iterative form, we need to:

1. Identifies an increment $\oplus$ to the argument of $\boldsymbol{f}$, *id est*, $x' = x \oplus y$ such that $x = prev(x')$, where $prev$ is based on the arguments of the recursive call. In this case, $prev(x) = d(x)$ and, if d^{-1} exists, $x \oplus y = d^{-1}(x)$, can be plugged in for y.
2. Derives an incremental program $\boldsymbol{f}'(x, r)$ that computes $\boldsymbol{f}(x)$ using an accumulator r of $\boldsymbol{f}(prev(x))$.
3. Forms an iterative version that initializes using the base case of $\boldsymbol{f}$ and iteratively applies $\boldsymbol{f}'$ until reaching the desired argument.

$$
\begin{aligned}
&\boldsymbol{f}(x) = \{\\
&\quad x_1 = x_0; r = b(x_0);\\
&\quad \textbf{while } (x_1 \neq x)\{\\
&\quad\quad x_1 = d^{-1}(x_1);\\
&\quad\quad r = a(x_1, r);\\
&\quad \}\\
&\quad \textbf{return } r;\\
&\}
\end{aligned}
$$

Note that, when a is in the form $a(a_1(x), y)$ and a is associative, we do not need d^{-1} and x_1.

Tail-recursive Factorial Function

```cpp
int fact(int n) {
    if (n == 0) {
        return 1;
    }
    return n * fact(n - 1);
}
```

The replacement of $n * ((n-1) * (n-2))$ by $(n * (n-1)) * (n-2)$ is valid due to the **associativity** of multiplication. In the general form:

$$
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&\qquad r = b(x_0); \\
&\qquad \textbf{while } (x \neq x_0)\{ \\
&\qquad\qquad r = a(r, a_1(x)); \\
&\qquad\qquad x = d(x); \\
&\qquad \} \\
&\qquad \textbf{return } r; \\
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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"`clang -O3 -S -emit-llvm fact.c -o -`" produces something like:

```llvm
define i32 @fact(i32 %n)  {
entry:
  %cmp3 = icmp eq i32 %n, 0
  br i1 %cmp3, label %return, label %if.else
if.else:
  %n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]
  %acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]
  %sub = add nsw i32 %n.tr5, -1
  %mul = mul nsw i32 %n.tr5, %acc.tr4
  %cmp = icmp eq i32 %sub, 0
  br i1 %cmp, label %return, label %if.else
return:
  %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]
  ret i32 %acc.tr.lcssa
}
```

The basic blocks related to loop vectorization (for performing SIMD operations) have been omitted for clarity

What About Fibonacci? To Hell With Multiple Recursions!

```cpp
int fib(int n) {
    if (n <= 1) {
        return n;
    }
    return fib(n - 1) + fib(n - 2);
}
```

The LLVM Optimized Version (but human readable)

```cpp
int fib(int n) {
    if (n < 2) {
        return n;
    }
    int acc = 0;
loop: /* while (1) { */
        int call = fib(n - 1);
        acc = call + acc;
        if (n < 4) goto ret; /* return acc + (n - 2); */
        n = n - 2;
        goto loop; /* } */
ret:
    return acc + (n - 2);
}
```

Note that, the following LLVM IR is a fixed-point representation of the `fib` function; observable by the output of "`opt -passes="default<O3>" -S fib-O3.ll -o -`"

```llvm
define i32 @fib(i32 %n) {
entry:
  %cmp6 = icmp slt i32 %n, 2
  br i1 %cmp6, label %return, label %if.end
if.end:
  %n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]
  %accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]
  %sub = add nsw i32 %n.tr8, -1
  %call = tail call i32 @fib(i32 %sub)
  %sub1 = add nsw i32 %n.tr8, -2
  %add = add nsw i32 %call, %accumulator.tr7
  %cmp = icmp samesign ult i32 %n.tr8, 4
  br i1 %cmp, label %return, label %if.end
return:
  %accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]
  %n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]
  %accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa
  ret i32 %accumulator.ret.tr
}
```

The Y. Annie Liu's Incrementalization

In 1990, Liu *et al.* have done extensive research on **Incrementalization** [17], [18], [19], [20].

Even in presence of multiple recursions, in [17, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (cf. slide "*From Recursion to Iteration*").

But the Step 2. builds upon the principles of [18] and [19] — which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

Conclusions

The papers are a little bit old-fashioned, and the key aspect of deriving the incremental program in presence of multiple recursions is **opaque**.

Last week I wrote an email to Liu asking for clarification, and two days ago I received a kind reply directing me to [21] (2024)—which confirms my understanding of the situation.

Despite her work, we are trying to understand whether it is really possible to rely exclusively on static analysis steps to automatically perform the transformation in a "general" context.

Thank You!

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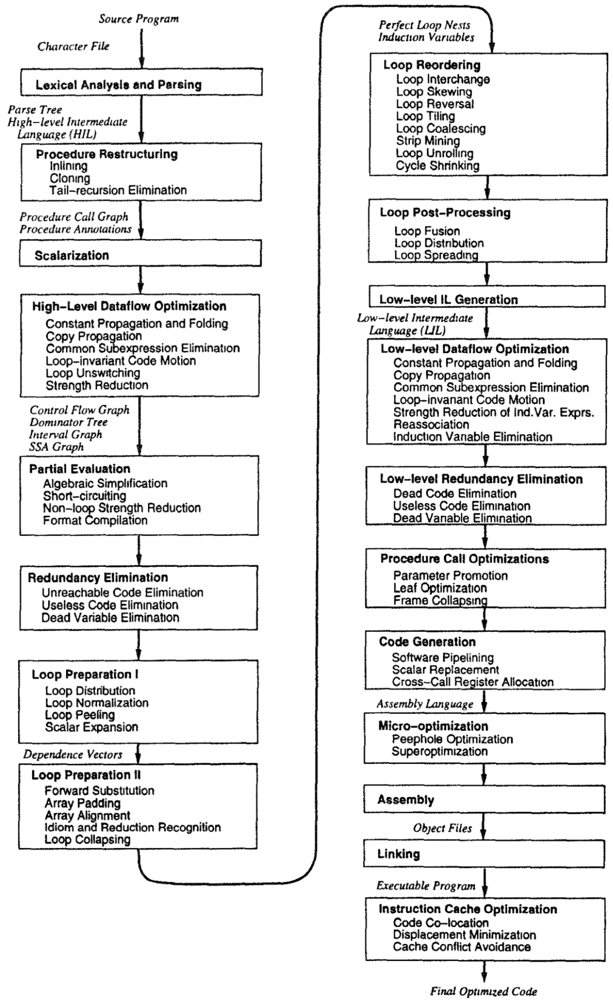
Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Bacon *et al.*, CSUR 1994 [22]