

# Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

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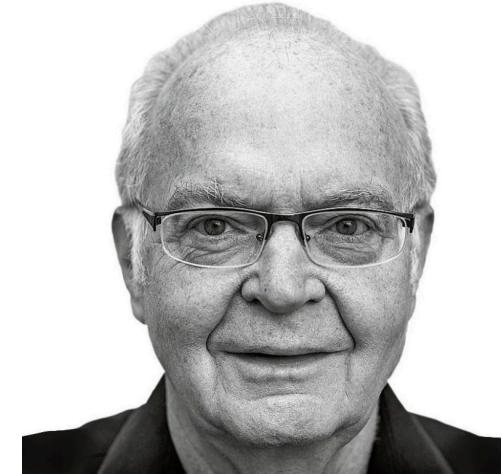
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Slides: [federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf](https://federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf)

# Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

*We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.*

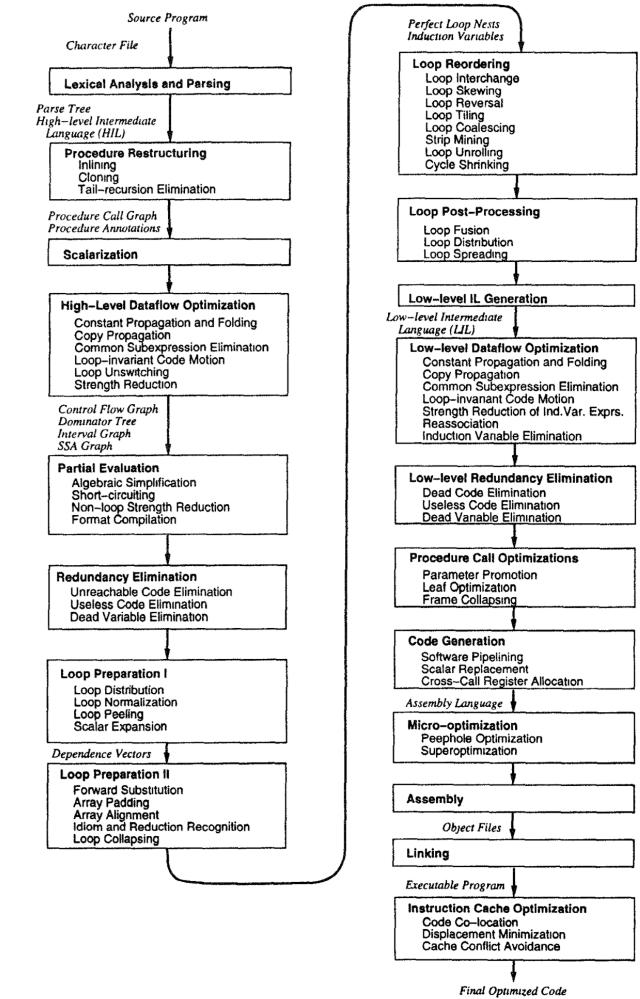


In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

# Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Bacon et al., CSUR 1994 [2]

# Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations<sup>2</sup> are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

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<sup>2</sup>A chronologically sorted list of papers on compiler optimization, from the works of 1952 through the techniques of 1994, is available at [5].

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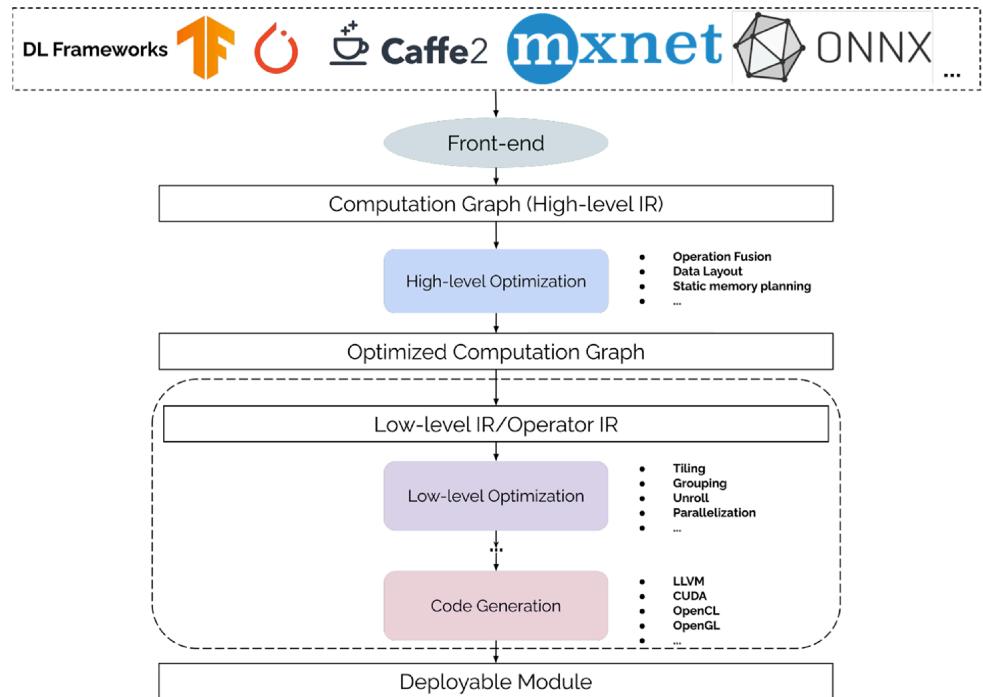
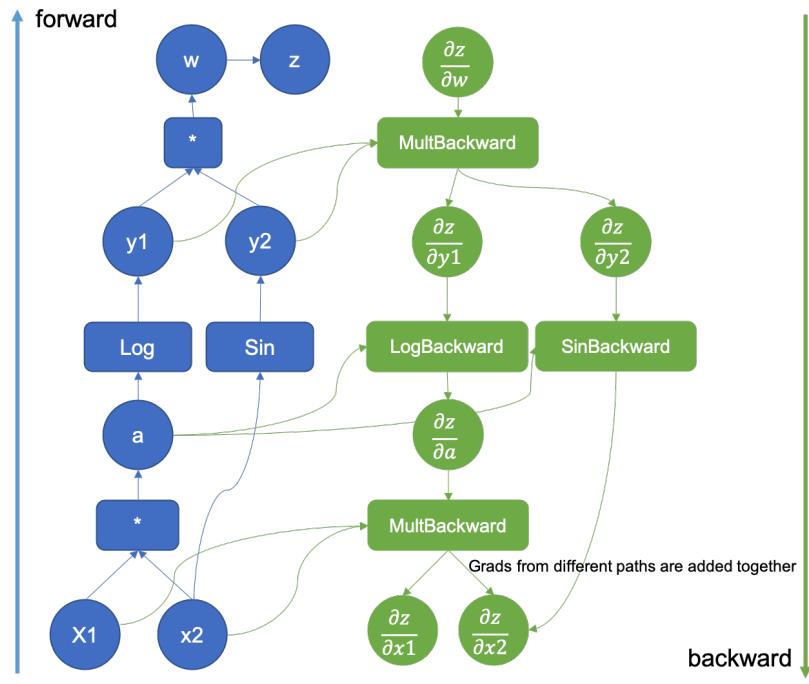
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# Machine Learning Framework are Just Optimizing Compilers<sup>5</sup>



<sup>5</sup>TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use LLVM [9]. Li *et al.*, [10] compiled a survey on ML compilers.

# Peephole Optimizations in x86-64 (cf. [11], [12])

```
1 ; x = x * 2, s.t. x: i32          asm
2 mov     eax, dword ptr [rbp - 4]
3 imul    eax, 2
4 mov     dword ptr [rbp - 4], eax
```

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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

```
1 ; x = x << 1, s.t. x: i32          asm
2 mov     eax, dword ptr [rbp - 4]
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4 mov     dword ptr [rbp - 4], eax
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asm

```
1 ; x = x + 0, s.t. x: i32
```

```
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3 add     eax, 0
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```

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asm

The optimized version removes the **unnecessary** addition operation.

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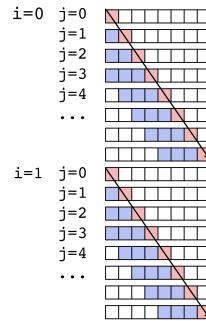
```
1 mov     eax, dword ptr [rbp - 4]
2 mov     dword ptr [rbp - 4], eax
```

asm

The `mov` instructions are redundant and can be **pruned** as well!

# Loop Nest Optimizations – Loop Tiling (cf. [13], [14])

```
for (int i=0; i<n; ++i) {    C++  
    for (int j=0; j<m; ++j) {  
        c[i][j] = a[i] * b[j];  
    }  
}
```



The vector **b** **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

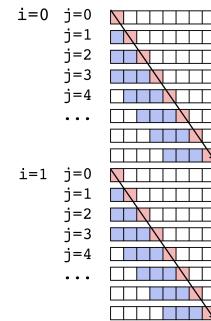
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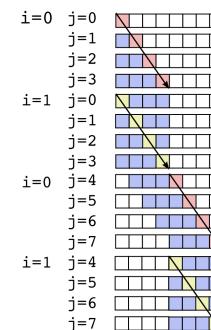
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The inner loop works on a **tile** of **b** that fits into the cache.

```
for (int jj = 0; jj < m; jj += TILE_SIZE) C++
    for (int i = 0; i < n; ++i)
        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
```



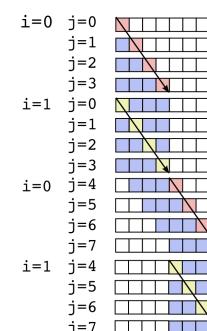
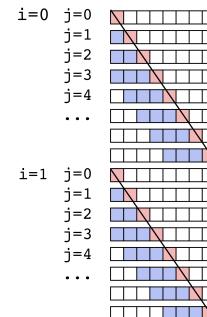
[A. Vladimirov, Session 10](#)

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        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
```

Careful readers may notice that, in this version, the values for the array **a** will be read  $m / \text{TILE\_SIZE}$ !

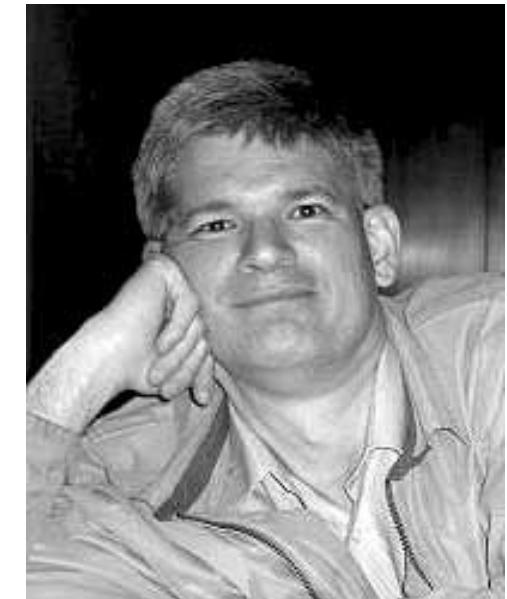
```
for (int ii = 0; ii < n; ii += TILE_SIZE_I) C++
    for (int jj = 0; jj < m; jj += TILE_SIZE_J)
        for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)
            for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)
                c[i][j] = a[i] * b[j];
```

[A. Vladimirov, Session 10](#)

# Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

*In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.*



## From Recursion to Iteration (cf. [18])

$$f(x) = \begin{cases} b(x_0) & \text{if } x = x_0 \\ a(x, f(d(x))) & \text{otherwise} \end{cases}$$

s.t.  $a$ ,  $b$ , and so on may denote any pieces of code.

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s.t.  $a, b$ , and so on may denote any pieces of code.

To transform recursive function  $f$  into iterative form, we need to:

1. Identifies an increment  $\oplus$  to the argument of  $f$ , *id est*,  $x' = x \oplus y$  such that  $x = \text{prev}(x')$ , where  $\text{prev}$  is based on the arguments of the recursive call. In this case,  $\text{prev}(x) = d(x)$  and, if  $d^{-1}$  exists,  $x \oplus y = d^{-1}(x)$ , can be plugged in for  $y$ .
2. Derives an incremental program  $f'(x, r)$  that computes  $f(x)$  using an accumulator  $r$  of  $f(\text{prev}(x))$ .
3. Forms an iterative version that initializes using the base case of  $f$  and iteratively applies  $f'$  until reaching the desired argument.

```

 $f(x) = \{$ 
     $x_1 = x_0; r = b(x_0);$ 
    while ( $x_1 \neq x$ ){
         $x_1 = d^{-1}(x_1);$ 
         $r = a(x_1, r);$ 
    }
    return  $r;$ 
}

```

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```
f(x) = {
    x1 = x0; r = b(x0);
    while (x1 ≠ x){
        x1 = d⁻¹(x1);
        r = a(x1, r);
    }
    return r;
}
```

Note that, when  $a$  is in the form  $a(a_1(x), y)$  and  $a$  is associative, we do not need  $d^{-1}$  and  $x_1$ .

# Tail-recursive Factorial Function

```
int fact(int n) {  
    if (n == 0) {  
        return 1;  
    }  
    return n * fact(n - 1);  
}
```

C++

The replacement of  $n * ((n - 1) * (n - 2))$  by  $(n * (n - 1)) * (n - 2)$  is valid due to the **associativity** of multiplication. In the general form:

$$f(x) = \{$$
$$r = b(x_0);$$
$$\text{while } (x \neq x_0) \{$$
$$r = a(r, a_1(x));$$
$$x = d(x);$$
$$\}$$
$$\text{return } r;$$
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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```
f(x) = {
    r = b(x₀);
    while (x ≠ x₀){
        r = a(r, a₁(x));
        x = d(x);
    }
    return r;
}
```

Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

C++

“clang -O3 -S -emit-llvm fact.c -o -” produces something like:

The basic blocks related to loop vectorization (for performing SIMD operations) have been omitted for clarity

```
define i32 @fact(i32 %n)  {
entry:
%cmp3 = icmp eq i32 %n, 0
br i1 %cmp3, label %return, label %if.else

if.else:
%n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]
%acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]
%sub = add nsw i32 %n.tr5, -1
%mul = mul nsw i32 %n.tr5, %acc.tr4
%cmp = icmp eq i32 %sub, 0
br i1 %cmp, label %return, label %if.else

return:
%acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]
ret i32 %acc.tr.lcssa
}
```

# What About Fibonacci? To Hell With Multiple Recursions!

```
int fib(int n) {
    if (n <= 1) {
        return n;
    }
    return fib(n - 1) + fib(n - 2);
}
```

C++

The LLVM Optimized Version (but human readable)

```
int fib(int n) {
    if (n < 2) {
        return n;
    }
    int acc = 0;
loop: /* while (1) { */
    int call = fib(n - 1);
    acc = call + acc;
    if (n < 4) goto ret; /* return acc + (n - 2); */
    n = n - 2;
    goto loop; /* } */
ret:
    return acc + (n - 2);
}
```

C++

Note that, the following LLVM IR is a fixed-point representation of the `fib` function; observable by the output of “`opt -passes="default<O3>" -S fib-03.ll -o -`”

```
define i32 @fib(i32 %n) {
entry:
    %cmp6 = icmp slt i32 %n, 2
    br i1 %cmp6, label %return, label %if.end
if.end:
    %n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]
    %accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]
    %sub = add nsw i32 %n.tr8, -1
    %call = tail call i32 @fib(i32 %sub)
    %sub1 = add nsw i32 %n.tr8, -2
    %add = add nsw i32 %call, %accumulator.tr7
    %cmp = icmp samesign ult i32 %n.tr8, 4
    br i1 %cmp, label %return, label %if.end
return:
    %accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]
    %n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]
    %accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa
    ret i32 %accumulator.ret.tr
}
```

llvm

So your compiler is unable  
to *incrementalize* functions  
with multiple recursions?

Apparently, yes.

# The Y. A. Liu's Incrementalization



In 1990, Liu *et al.* have done extensive research on **Incrementalization** [18], [19], [20], [21].

Even in presence of multiple recursions, in [18, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (cf. slide “*From Recursion to Iteration*”).

But the Step 2. builds upon the principles of [19] and [20] – which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

# Conclusions

The papers are a little bit old-fashioned, and the key aspect of deriving the incremental program in presence of multiple recursions is **opaque**.

I wrote an email to Y.A. Liu asking for clarification last week, but I haven't received a reply yet ... 😢

*Despite her work, we are trying to understand whether it is really possible to make the transformation automatically in a “general” setting.*

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