

From Separate Compilation to Sound Language Composition

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The development of programming languages involves complex theoretical and practical challenges, particularly when addressing modularity and reusability through language extensions. While language workbenches aim to enable modular development under the constraints of the language extension problem, one critical constraint—separate compilation—is often relaxed due to its complexity. However, this relaxation undermines artifact reusability and integration with common dependency systems. A key difficulty under separate compilation arises from managing attribute grammars, as extensions may introduce new attributes that invalidate previously generated abstract syntax tree structures. Existing approaches, such as the use of dynamic maps in the `NeverLang` workbench, favor flexibility at the cost of compile-time correctness, leading to potential runtime errors due to undefined attributes. This work addresses this issue by introducing `nlgcheck`, a theoretically sound static analysis tool based on data-flow analysis for the `NeverLang` language workbench. `nlgcheck` detects potential runtime errors—such as undefined attribute accesses—at compile time, preserving separate compilation while maintaining strong static correctness guarantees. Experimental evaluation using mutation testing on `NeverLang`-based projects demonstrates that `nlgcheck` effectively enhances robustness without sacrificing modularity or flexibility and with a level of performance that does not impede its adoption in daily development activities.

CCS Concepts: • **Theory of computation** → **Program analysis**; • **Software and its engineering** → **Compilers**.

Additional Key Words and Phrases: language composition, separate compilation, modular language design, sound compositional semantics, domain-specific languages, language frameworks, attribute grammars, static analysis

ACM Reference Format:

Federico Bruzzone, Walter Cazzola, and Luca Favalli. 2026. From Separate Compilation to Sound Language Composition. *ACM Trans. Program. Lang. Syst.* 1, 1 (January 2026), 43 pages. <https://doi.org/XXXXXXX.XXXXXXX>

1 Introduction

Development of programming languages is a complex activity that requires mastery of both theoretical and practical concepts spanning aspects such as grammars, parsers, optimization techniques, architectures, and software engineering. This is true for both general-purpose languages (GPL) and domain-specific languages (DSL) although with varying degrees of complexity and different

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ACM 1558-4593/2026/1-ART

<https://doi.org/XXXXXXX.XXXXXXX>

challenges: where GPLs are usually larger and more complex projects overall, DSLs require deep knowledge of the application domain and often involve more interaction with domain experts that may or may not coincide with the end users.

Although each programming language is unique and has been designed for a specific purpose [15], it has been observed [56] that many provide similar language constructs among them, due to recurring modeling patterns and abstractions [22]. In this context, the challenge for language designers is to take advantage of such commonalities by reusing any existing software artifacts as much as possible. In this regard, the *language extension problem* [52] is a popular variant of the even more popular *expression problem* [75], modified to suit the needs and constraints of programming languages and their extension. Among the tools for language implementation, *language workbenches* [34] strive to enable purely modular development and composition techniques by adhering to the constraints of the language extension problem and by providing the means to perform language composition at several levels of granularity [10, 30].

Despite the efforts made to achieve this result, one among the constraints of the language extension problem is often (if not always) disregarded. In the authors' own words [52]: «the constraint of *separate compilation* usually impacts other non-functional properties (...). Consequently, it can be worthwhile to relax the separate compilation constraint in order to comply with other non-functional properties». While the increase in complexity related to separate compilation is undeniable, we believe that relaxing this constraint can severely hinder the actual reusability of language artifacts: for instance, without separate compilation, language extension is only possible when source code is available and some among the most popular repositories for dependency management such as Maven and Cargo only ship binaries [15], thus limiting the possibility to reuse language libraries from the web. Among the complexities brought by separate compilation, we believe the most pressing one to be the implementation of attribute grammars [48]: in most language workbenches, the compiler generates data structures for the abstract syntax tree (AST) nodes that may appear in the language under development. Then, semantic actions [1] can be associated with each node to implement the semantics. However, to properly create the data structure for each AST node, it is necessary to foresee all the grammar attributes that may be *inherited* or *synthesized* (§2.1) by that node, as each attribute will need a corresponding field within that data structure. It is apparent that when separate compilation is permitted, a language construct may be extended with more semantic actions at a later time, possibly adding more attributes, thus rendering the AST data structures obsolete. Some language workbenches with separate compilation such as Neverlang [16, 21, 68] simply ignore this problem by using generic and dynamic maps to implement grammar attributes instead of generating specific data structures for its AST. While this solution greatly improves the composability among artifacts thanks to loose coupling, it denies any compile-time composition correctness guarantee, reducing the robustness of programming languages developed this way and increasing the likelihood of defects in the composition to go unnoticed and become evident only at runtime. Since the 2000s, as de Moor et al. [26] pointed out, “the use of an attribute that has not been defined” is a common source of runtime errors in attribute grammar-based systems—a problem that could be mitigated by an appropriate type system or static analysis (§2.2).

With this in mind, in the Neverlang world, an attempt had already been made by developing a type system and its type inference algorithm [19]. This approach suffered from two main limitations: (i) separate compilation was not supported, and (ii) it was not fully modularizable. These issues arose because, in both cases, it needed to see the entire composition of language constructs. It is therefore of particular interest to determine to which extent the runtime errors which can arise in a Neverlang-based project can also be detected statically, without losing the separate compilation constraint and without tightening the coupling among language constructs. Thus, the development

of a tool that performs such static checks and warns developers about potentially erroneous runtime instructions becomes a natural objective. With such a tool, we believe language workbenches can achieve the best of both worlds by keeping a loose level of coupling while maintaining static correctness and without relaxing the separate compilation constraint of the language extension problem.

In this work, we present a generic approach to achieve this goal based on data-flow analysis and an implementation thereof for the `Neverlang` language workbench, dubbed `nlgcheck`. We show that, for realistic scenarios, `nlgcheck` can bridge the gap with the static analysis capabilities offered by compilers without separate compilations by navigating a representation of all possible ASTs that can be generated for a given grammar. Meanwhile, `nlgcheck` can even outmatch the capabilities typically offered by compilers because it can prevent any null pointer exceptions that may occur when an attribute is generated within conditional statements and specific code branches and then retrieved elsewhere. Finally, we complement our contribution with an experiment performed on `Neverlang`-based projects by estimating the capability of `nlgcheck` to detect errors through mutation testing.

The remainder of this paper is structured as follows. Section 2 provides the necessary background, introducing the attribute grammars, the program dependence analysis, and the `Neverlang` language workbench. Section 4 presents our main contribution: a generic, data-flow-based approach for statically checking attribute usage in the context of language composition with separate compilation, detailing the theoretical underpinnings and the rationale behind our method. Section 5 describes the implementation of our approach in the context of the `Neverlang` language workbench. Section 6 evaluates the effectiveness of `nlgcheck` through experiments on real-world `Neverlang`-based projects, including a mutation testing campaign to assess its error-detection capabilities. Section 7 discusses related work, comparing our approach to existing solutions. Finally, Section 8 concludes the paper, summarizing our findings and outlining directions for future research. An appendix provides additional technical details and supporting material.

2 Background

2.1 Attribute Grammars

A formal extension of context-free grammars is the concept of *attribute grammars* (AGs), introduced by Knuth [48] as a way to integrate semantics into the syntactic structure of a language. In his seminal paper “Semantics of Context-Free Languages”, the productions of a grammar are enriched with attributes and semantic equations that evaluate these attributes based on parse tree context—enabling systematic semantic analysis and evaluation within grammar-based systems. Attributes allow the transmission of information between different parts of the abstract syntax tree (AST). Just as parsing strategies are typically divided into *bottom-up* and *top-down* approaches, attributes are categorized according to the direction in which their values propagate: *synthesized* or *inherited* [1, 49], respectively. To formally define the attributes, let $G = \langle \Sigma, N, S, \Pi \rangle$ be a grammar, where

- Σ is a set of terminal symbols,
- N is a set of non-terminal symbols,
- $S \in N$ is the distinguished start symbol.
- Π is a set of productions of the form $A \rightarrow \alpha$, where $A \in N$ and $\alpha \in (N \cup T)^*$, and

Synthesized Attributes. A *synthesized attribute* is an attribute whose value is computed from the values of the attributes of its children nodes and propagated upward in the tree. Formally, if

$A \rightarrow \alpha_1 \dots \alpha_n \in P$ is a production, then a synthesized attribute s of A can be defined as:

$$A.s = f(\alpha_{j_1}.s_1, \alpha_{j_2}.s_2, \dots, \alpha_{j_m}.s_m)$$

where f is a function that computes the value of $A.s$ based on the synthesized attributes s_1, s_2, \dots, s_m of its children nodes $\{\alpha_{j_1}, \alpha_{j_2}, \dots, \alpha_{j_m}\} \subseteq \{\alpha_1, \alpha_2, \dots, \alpha_n\}$.

Inherited Attributes. As stated by Knuth [49], P. Wegner introduced the *inherited attribute* concept. Its value is computed from the values of the attributes of its parent and/or siblings nodes and propagated downward in the tree. Formally, for a production $A \rightarrow \alpha_1 \dots \alpha_n \in P$, an inherited attribute i of α_k ($1 \leq k \leq n$) can be defined as:

$$\alpha_k.i = g(A.i_1, \dots, A.i_p, \alpha_{l_1}.s_1, \dots, \alpha_{l_q}.s_q)$$

where g is a function that computes $\alpha_k.i$ based on the inherited attributes i_1, \dots, i_p of the parent node A and the synthesized attributes s_1, \dots, s_q of some sibling nodes $\{\alpha_{l_1}, \dots, \alpha_{l_q}\} \subseteq \{\alpha_1, \dots, \alpha_n\} \setminus \{\alpha_k\}$.

Right-attributed grammars (R-AGs) are a class of AGs where all attributes are synthesized—no inherited attributes are used. They can be incorporated into both bottom-up and top-down parsing strategies. On the other hand, *left-attributed grammars* (L-AGs) are a strict superset of R-AGs that allows the evaluation of attributes in a single left-to-right traversal of the AST. Any R-AG can be transformed into an equivalent L-AG, but the reverse is not always true [76].

Over the years, several comprehensive works have shaped the study and application of AGs. Early collections such as “Attribute Grammars: Definitions, Systems and Bibliography” provided formal foundations, implementation frameworks, and an extensive bibliography [28]. A comprehensive survey by Paakki [59] classified modern extensions of attribute grammars—structured, modular, object-oriented, logical, and functional variants—and reviewed techniques for parallel and incremental evaluation. In parallel, foundational research addressed decidability and efficiency issues. Jazayeri et al. [41] established the inherent exponential complexity of circularity tests, while later work by Deransart et al. [27] developed more efficient algorithms to improve circularity checking and ensure consistent attribute evaluation.

2.1.1 Reference Attribute Grammars. A major limitation of canonical AGs is their handling of *non-local dependencies*, i.e., situations where a property of one syntax tree node depends on properties of nodes arbitrarily far away in the tree [38]. Before that Hedin [38] introduced *reference attribute grammars* (RAGs), several extensions to AGs were proposed to address this limitation [11, 13, 37, 40, 42–44, 74]. The necessity for such extensions arises from the fact that many semantic analyses, such as *name resolution* and *type checking*, inherently involve non-local dependencies. RAGs address this by allowing attributes to hold references to other nodes in the syntax tree, enabling direct access to their attributes regardless of their position in the tree. This capability is particularly useful for implementing complex language features, such as *scoping rules* and *inheritance hierarchies* in object-oriented languages.

2.2 Program Dependence Analysis

Foundational to many static program analyses is the concept of *program dependence analysis* (PDA). The major seminal work in this area are the Towle [67]’s PhD thesis, Banerjee [7], and Allen [5], and the papers by Wolfe [78] and Banerjee88 [8]. PDA identifies dependencies between different parts of a program, that is, a statement A is said to *depend* on another statement A if the execution of A requires information from A . These dependencies can be classified into two main categories: *control dependencies* (§2.2.2) and *data dependencies* (§2.2.3).

2.2.1 Dominance. F. Allen [4] introduced the concept of *control flow graph* (CFG) as a way to represent all possible paths that a program can take during its execution. Formally, a CFG is a directed graph $G = (V, E, e_y, e_t)$, where:

- V is a set of vertices (or nodes) representing basic blocks of code,
- E is a set of edges representing control flow between nodes,
- e_y is the entry node, and
- e_t is the exit node.

In a CFG, a node A is said to (*pre*)*dominate* another node B if every path from e_y to B must pass through A . Formally, $A \text{ dom } B$ iif:

$$\forall P \in \text{Paths}(e_y, B), A \in P$$

where $\text{Paths}(e_y, B)$ is the set of all paths from the e_y to B . The *strictly dominates* relation ($A \overline{\text{dom}} B$) is defined similarly, but it requires that $A \neq B$. The *immediate dominator* of a node B is the unique node A such that A strictly dominates B and does not strictly dominate any other node that also dominates B ; every node has an immediate dominator, except e_y [53]. Intuitively, A is the last required “checkpoint” before reaching B in the control flow. Formally, $A \text{ idom } B$ iif:

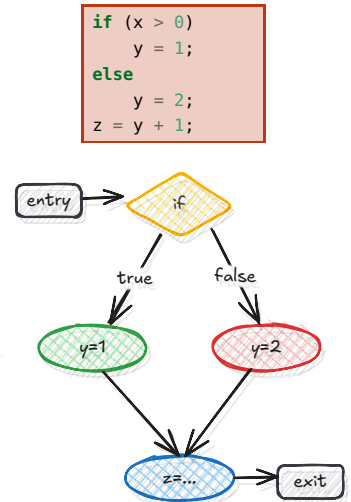
$$A \overline{\text{dom}} B \wedge \forall C \in V : (C \text{ dom } B \wedge C \neq A) \Rightarrow (A \overline{\text{dom}} C).$$

The *dominance frontier* (DF) of a node A is the set of all nodes B such that A dominates an immediate predecessor of B , but A does not strictly dominate B . Formally, the DF of A is defined as:

$$\text{DF}(A) = \{B \in V \mid \exists C \in \text{Pred}(B) : A \text{ dom } C \wedge A \overline{\text{dom}} B\}$$

where $\text{Pred}(B)$ is the set of immediate predecessors of B in the CFG. Intuitively, the DF of A is the set of nodes where the control exercised by A ends. In the Listing 1, the DF of the node $y=1$ is the set $\{z=\dots\}$ because: (1) the node $y=1$ dominates a predecessor of $z=\dots$ (itself), and (2) the node $y=1$ does not strictly dominate $z=\dots$ (since $y=2$ is another predecessor of $z=\dots$ that is not dominated by $y=1$).

Dominator trees are a tree-like structure rooted at e_y , where each node A has a child B if A is the immediate dominator of B . The *post-dominance* relation is the dual of dominance. $A \text{ dom}_{\text{post}} B$ iif $\forall P \in \text{Paths}(B, e_t), A \in P$ and so on. *Dominance* was first introduced by Prosser [61] in a 1959 study on the analysis of flow diagrams. However, Prosser did not provide an algorithm for computing dominance; a practical method for its computation was proposed ten years later by Lowry and Medlock [55]. Lengauer and Tarjan [53], by using *union-find* with path compression, developed a more efficient algorithm for computing dominator trees in *almost* linear time improving existing solutions [3, 62], then adopted by GCC. Twenty year later, Cooper et al. [23] presented a more engineered algorithm for computing them relying on the *reverse post-order* (RPO) traversal of the CFG. LLVM, on the other hand, uses the algorithm by Georgiadis et al. [36] which is more efficient in practice but has a worst-case time complexity of $O(n^2)$; it also supports incremental updates [35].



Listing 1. Dominance frontier exam-

2.2.2 Control Dependencies. *Control Dependencies* are post-dominance frontier in the CFG [24]. They refer to the dependencies that arise from the control flow of a program, such as conditional statements and loops. A statement A is said to be control dependent on another statement B if the execution of A depends on the outcome of B . Formally, $A \xrightarrow{c} B$ if the following conditions hold:

$$\exists P \in \text{Paths}(A, B) : \forall C \in P, C \neq B \Rightarrow C \rightsquigarrow A \text{ in each } P \in \text{Paths}(e_y, e_t)$$

$$\exists P \in \text{Paths}(B, e_y) : A \notin P$$

where \rightsquigarrow means that a path from C to A exists in the CFG. That is, $\forall C \in P, A \text{ dom}_{post} C$ and $B \not\text{dom}_{post} A$. Due to the impact of data dependencies on control dependencies, the two concepts are often intertwined. J. Allen et al. [6] show how control dependencies can be transformed into data dependencies using the *if-conversion* technique. This approach eliminates conditional execution by evaluating both control-flow paths unconditionally and then using a predicate to select the appropriate result, which is finally assigned to the target variable. Listing 2 illustrates a simple example of control dependency, where the statement $x = -1$ is control dependent on the if-statement.

2.2.3 Data Dependencies. Banerjee88 [8] formalized the concept of *data dependencies* within the context of program analysis. Data dependencies arise when the execution of one statement depends on the data produced by another statement. *Data hazards* are a specific type of data dependency that can occur in pipelined processors, where the execution of one instruction depends on the result of a previous instruction that has not yet completed; they can lead to *race conditions*. Three main types of data hazards are identified: *read-after-write* (RAW), *write-after-read* (WAR), and *write-after-write* (WAW). In accordance with Bernstein [9], by contrast, data dependencies can be classified into: (1) *(true)flow dependencies*, (2) *anti-dependencies*, and (3) *output dependencies*. In the following, we use the notation s_i to refer to the i -th statement in the Listing 3.

Flow Dependency. A statement s_3 is said to be *flow dependent* on another statement s_2 (denoted by $s_3 \delta^f s_2$) if s_3 uses a value produced by s_2 . RAW hazards occur when a violation of this dependency happens.

Anti-Dependency. An *anti-dependency* takes place when a statement s_4 writes to a variable that is read by another statement s_3 (denoted by $s_4 \delta^a s_3$). This can lead to WAR hazards if the order of execution is not preserved.

Output Dependency. An *output dependency* is symmetric and occurs when two statements s_4 and s_5 write to the same variable (denoted by $s_4 \delta^o s_5$). WAW hazards happens if a violation of this dependency occurs.

An additional type of data dependency is the *input dependency*, which occurs when two statements read from the same variable. Anti-dependencies and output dependencies are often grouped together as *storage-related dependencies* [1, 39]—they may be removed by renaming variables.

```

1  int f(int a) {
2      int x = 1;
3      if (a < 0) {
4          x = -1;
5      }
6      return x;
7  }
```

Listing 2. Control dependency.

```

1  int c = 0;
2  int a = 1;
3  int b = a + c;
4  c = 2;
5  c = b + 1;
```

Listing 3. Data dependency.

3 Software Frameworks

This section introduces the two main software frameworks that underpin the work described in this paper: the Neverlang language workbench and the Soot static analysis framework. Neverlang provides a modular infrastructure for language engineering, enabling the definition, composition, and extension of programming languages through reusable components. Soot, on the other hand, is a mature and widely used framework for analyzing and transforming Java programs at the bytecode and intermediate representation levels. Together, these tools form the foundation for our approach to integrating advanced program analysis capabilities into language development workflows.

3.1 Neverlang in a Nutshell

The Neverlang [16, 21, 68] language workbench¹ promotes code reusability and separation of concerns leveraging the *language-feature* approach.

The basic development unit is the **module**, as shown in line 1 of Listing 1. A module may contain a **reference syntax** and could have zero or multiple **roles**. A role, used to define the semantics, is a composition unit that defines actions that should be executed when some syntax is recognized, as defined by *syntax-directed translation* [2]. Syntax definitions are defined using *Backus-Naur form* (BNF) grammars, represented as sets of *productions* and *terminals*. Syntax definitions and semantic **roles** are tied together using **slices**. Listing 1 shows a simple example of a Neverlang module implementing a backup task of the LogLang LPL. Reference syntax is defined in lines 2–6; the *categories* (line 5) are used to generate the syntax highlighting for the IDEs. Semantic actions may be attached to a non-terminal using the production’s label as a reference, or using the position of the non-terminal in the grammar, as shown in line 8, numbering start with 0 from the top left to the bottom right. The two *String* non-terminals on the right-hand side of the *Backup* production are referenced using 1 and 2, respectively. Each **role** is a compilation phase that can be executed in a specific order, as shown in line 24. In contrast, the *BackupSlice* (lines 14–18) reveals how the syntax and semantics are tied together; choosing the **concrete syntax** from the *Backup* module (line 15), and two **roles** from two different modules (lines 16–17). Finally, the **language** can be created by composing multiple **slices** (line

```

1 module Backup {
2   reference syntax {
3     Backup: Backup ← "backup" String String;
4     Cmd: Cmd ← Backup;
5     categories : Keyword = { "backup" };
6   }
7   role(execution) {
8     0 .{
9       String src = $1.string, dest = $2.string;
10      $$FileOp.backup(src, dest);
11    }.
12  }
13 }
14 slice BackupSlice {
15   concrete syntax from Backup
16   module Backup with role execution
17   module BackupPermCheck with role permissions
18 }
19 language LogLang {
20   slices BackupSlice RemoveSlice RenameSlice
21     MergeSlice Task Main LogLangTypes
22   endemic slices FileOpEndemic PermEndemic
23   roles syntax < terminal-evaluation
24     < permissions
25     : execution
26 }

```

Fig. 1. Syntax and semantics for the backup task.

¹The term *language workbench* was coined by Martin Fowler in his 2005 article “Language Workbenches: The Killer-App for Domain Specific Languages” [34]. They are software tools that provide an integrated environment for designing, implementing, and maintaining programming languages. Language workbenches typically offer features such as syntax definition, semantic analysis, code generation, and editing support.

20). The composition in Neverlang is twofold: between modules and between slices. Thus, the grammars are merged to generate the complete language parser. On the other hand, the semantic actions are composed in a pipeline, and each **role** traverses the syntax tree in the order specified in the **roles** clause (line 24).

Since 2019 [50], Neverlang has always had an interest in providing the *editing support* for the languages developed with it. Initially, the editing support was provided through the *Eclipse IDE* plugin, which was recently replaced by the *de facto* standard LSP and DAP [14]. To provide LSP services, Neverlang leverages the TypeLang DSL—a language for type systems definition—to generate *language servers* that can be used by any editor that supports the LSP.

3.2 Soot Library

Soot [69, 70] is a long-lived Java analysis and transformation framework. Initially conceived for bytecode optimization, it has evolved into a comprehensive tool for static analysis [60, 66, 69]. A recent effort (“SootUp”) completely reimplements Soot with a more modular design, modernizing the original tool while preserving its familiar APIs [46].

Soot’s main strength lies in its intermediate representations (IRs), in particular *Jimple*, a typed three-address code designed to make control flow and data flow explicit and thus simplify static analyses. Other IRs (*Shimple* in SSA form, *Baf* for bytecode-level rewriting [64], and *Grimp* for decompilation [58]) extend its flexibility [51]. On top of these IRs, Soot offers standard analyses such as call-graph construction [46], pointer analysis (*Spark*) [54], and data-flow analysis frameworks (*Heros*) [12, 63, 65]. These enable both whole-program analyses and program transformations, with outputs ranging from optimized bytecode to instrumented programs.

Soot has been applied to a wide range of program analysis and transformation tasks. Early on it was used for Java optimizations and class file instrumentation [25], but it quickly found broad academic use. It has underpinned research in pointer analysis, concurrency analysis, symbolic execution, and the static portions of hybrid analysis techniques [51].

4 Contribution

This section presents the concept behind `nlgcheck`, which represents the bulk of our contribution.

4.1 Notation

A **grammar** is defined as a quadruple (Σ, N, S, Π) , in which Σ is a set of terminal symbols (not interesting in this context, since attributes are only attached to nonterminal symbols), N is a set of nonterminal symbols, $S \in N$ is the start nonterminal symbol (which in `Neverlang` is always named `Program` by convention), and Π is a set of production rules.

A **production** $p \in \Pi$ is written in the form $X_0 \leftarrow X_1 \cdots X_n$, with $X_i \in N$ all being nonterminal symbols, and has the meaning that during the parsing process, X_0 (which is called the **left-hand side** of the production) can be replaced by the $X_1 \cdots X_n$ sequence (called the **right-hand side** of the production). The notation $|p|$ is used to indicate the number $n + 1$ of nonterminals forming the production (one on the left-hand side and n on the right-hand side) and $p[i]$ to indicate the i -th nonterminal X_i .²

Given a production $p \in \Pi$, we will refer to the attribute named a on the i -th nonterminal of p with the notation $p[i].a$: with $p[0].a$, we indicate an attribute on the left-hand nonterminal of the

²Note that this is a simplification of the definition of production, in which the right-hand side is a sequence that can contain both terminal and nonterminal symbols. However, this does not pose a limitation in this context, since our approach focuses on the analysis of attributes attached to nonterminals.

production, and with $p[i].a$ (with $i \geq 1$), an attribute on one of the nonterminals on the right-hand side.

A **semantic action**, for the scope of this section, can be defined as a sequence of executable code attached to a production, some of which can set or access attributes associated with the production's nonterminals.³

Any attribute set or accessed by an instruction has a **type**, which is usually known during the compilation of the semantic action code. We call T the set of all available types. T is partially ordered since, given two types $t_1, t_2 \in T$, exactly one of the following is true:

- t_1 and t_2 are the same type ($t_1 = t_2$);
- t_1 is a subtype of t_2 ($t_1 <: t_2$);
- t_1 is a supertype of t_2 ($t_1 >: t_2$);
- none of the above (t_1 and t_2 are unrelated).

To refer to the type of an attribute $p[i].a$, we use the notation $\tau(p[i].a)$. More precisely, given \mathbb{N} as the set of natural numbers and I as the set of valid attribute names,⁴ we can define τ as a partial function $\tau : \Pi \times \mathbb{N} \times I \rightarrow T$ that maps the triplet (production, index of a nonterminal in the production, and attribute name) to the type of the corresponding attribute. $\tau(p[i].a) := \tau(p, i, a)$ is not defined if p has less than i nonterminals or if $p[i]$ has no attribute named a .

4.2 Verification model

After trying to solve the verification problem with a simplified model (see §A), we opted for a more complete model that closely represents the executable action. The go-to choice in this regard is the *control-flow graph* [4] (CFG).

The CFG for a semantic action we will be using in this work is an instance of the CFG presented in §2.2.1. We define a directed graph $G = (V, E)$ with $E \subseteq V \times V$ in which every vertex (or node) has a unique identifier (for the sake of simplicity, we will assume $V \subset \mathbb{N}$) and is associated with an *action*, each being a sequence of statements that forms an atomic subset of a more complex semantic action. We also define an **entry node** and an **exit node** of the CFG, representing the first and last action of the semantic action execution; the execution can follow different paths but always begins at the entry node and terminates at the exit node. The entry node and exit node are denoted as $v_{\text{entry}}(G)$ and $v_{\text{exit}}(G)$, respectively.

The semantics of each node are described through their mapping to an action by means of the $a_{\text{node}} : V \rightarrow A$ function, where A is the set of all possible instructions that can be executed in a semantic action. In this work, we consider the following instructions, grouped by category. Most instructions can be parametrized, making A an infinite set. We adopt the following notation for the parameters and their types: \mathbb{N} is the set of natural numbers, I is the set of valid attribute identifiers, and T is the set of types.⁵

Attribute actions. Attribute actions model access and manipulation of AST node attributes. We consider the following attribute actions (and their respective CFG nodes):

WriteAttr(i, n, t), with $i \in \mathbb{N}, n \in I, t \in T$

The action sets an attribute named n of type t on the i -th nonterminal of the production. If $i = 0$, then the attribute is written on the node currently being visited; otherwise, it is written to its i -th child.

³This is also a simplification, but it suffices to discuss anything relevant to the problem we are dealing with.

⁴Also called *identifiers*; for all intents and purposes, we can assume that the set of valid identifiers is a subset of all ASCII strings.

⁵For the purposes of this work, T coincides with the set of all the types in the JVM.

```

1 module componentlib.clke.expressions.TernaryOp {
2   reference syntax {
3     Op: Expression ← Expression "?" Expression ":" Expression ;
4   }
5   role (evaluation) {
6     θ .{
7       eval $Op[1];
8       if ((Boolean) $Op[1].val) {
9         eval $Op[2];
10        $Op.val = $Op[2].val;
11      } else {
12        eval $Op[3];
13        $Op.val = $Op[3].val;
14      }
15    }.
16  }
17 }

```

Listing 1. Neverlang code implementing a C-like ternary expression language feature.

ReadAttr(i, n, t), with $i \in \mathbb{N}, n \in I, t \in T$

The action accesses an attribute named n from the i -th nonterminal of the production and casts its value to type t . Please note that the previous restriction about the ability to read attributes only from child nodes and write only to the current node (see §A) is lifted, as this model does not assume postorder execution.

CopyAttr(i_d, n_d, i_s, n_s), with $i_d, i_s \in \mathbb{N}, n_d, n_s \in I$

The action accesses an attribute named n_s from the i_s -th nonterminal of the production and sets an attribute named n_d on the i_d -th nonterminal of the same production to the retrieved value.^a Please note that while the runtime behavior is identical to performing a *ReadAttr*(i_s, n_s, t) followed by a *WriteAttr*(i_d, n_d, t) with the same t and value, the difference is relevant in this context, as copying an attribute does not involve interacting with its type, and thus the type is kept even if not explicitly mentioned: using the read-write sequence of actions while lacking type information would result in using $t = \text{Object}$, thus losing relevant type information.

^aThe s and d subscripts stand for *source* and *destination*.

CheckAttrType(i, n, t), with $i \in \mathbb{N}, n \in I, t \in T$

The action accesses an attribute named n from the i -th nonterminal of the production and compares its type to t .^a This action does not modify the state nor can it cause runtime errors, but it is used to raise warnings: using **instanceof** on attribute values is considered an anti-pattern in Neverlang, as the same check could be performed more idiomatically with dedicated type guards.

^ain Java, this is done with the **instanceof** operator; similar operators are available in or equivalent in other languages.

PropagateAttrs()

Assuming that the semantic action is attached to a production with a single nonterminal on the right-hand side, thus has the form $X_0 \leftarrow X_1$, each attribute present on X_1 is copied to X_0 with the same name and type, unless already present. The code that performs such a copy is not written by the developers; instead, the copy is performed automatically by the Neverlang runtime. Thus, we attach additional nodes to the CFG representing this implicit behavior.

Evaluation actions. Evaluation actions model the control that the semantic action has over the tree-traversal order. As exemplified in Listing 1 in the context of the implementation of the ternary operator language feature, a semantic action can contain the `eval $i` construct (lines 7,

9, 12) to instruct the Neverlang runtime to suspend the execution of the semantic action, instead proceeding by evaluating the i -th child of the current AST node according to the same role. Once the eval is complete, the execution of the current semantic action is resumed (as is typically done when invoking functions in most programming languages). We consider the following attribute actions (and their respective CFG nodes):

BeginEvalMetaAction(i), with $i \in \mathbb{N}$

The action evaluates the i -th nonterminal of the production, according to the role currently being executed.

EndEvalMetaAction()

The action is resumed after visiting another node.

Concretely, these two nodes always appear jointly in the CFG: a node mapped to a *BeginEvalAction* always has exactly one outgoing edge, connected to an *EndEvalMetaAction*, which is its only incoming edge. The operation is atomic and spans over two nodes for purely practical CFG manipulation reasons (please refer to the next section for further details). Neverlang provides syntactic sugar for common behaviors, such as making a single semantic action behave as if it were used in a postorder role: this is denoted with the $\@{\dots}$ operator. Such syntactic sugar does not impact the creation of the CFG, as it is equivalent to performing eval statements for each of the right-hand side nonterminals of the production before proceeding further.

Other actions. Other actions unrelated to previous categories are the following:

NopAction()

The action does not represent any real operation; thus, the node is within the CFG for structural reasons (such as denoting branch points).

IgnoreBranchAction()

The action throws a runtime exception that would interrupt the execution of the interpreter/compiler. Since the visit cannot proceed further after throwing an exception, the branch is ignored.

Fig. 2 shows the CFG obtained from the semantic action presented in Listing 1, according to the concepts presented thus far. The label on top of Fig. 2 lists the production and the role to which the semantic action is attached. Each node is uniquely identified with a hexadecimal number. For the implementation, the CFG for a semantic action is constructed by analyzing the compiled bytecode of the Java classes generated by the Neverlang compiler using Soot, as detailed later in this section.

4.3 Modeling roles

The concept of a CFG is not limited to individual semantic actions, equivalent to *intra-procedural control-flow graphs* in traditional CFGs.

Inspecting the behavior of the whole compiler/interpreter, thus simulating the entire AST visit (role, in Neverlang), requires taking inspiration from static analysis techniques based on the *inter-procedural control-flow graph* instead. The natural choice is to combine the CFGs for the individual semantic actions—connecting their entry, exit, and eval nodes—into a unique CFG for the entire role through adequate edges.

Such information is obtained from the language grammar—e.g., the semantic action for `Stmt \leftarrow "if" Expr "then" Stmt`. To indicate that the execution flow may continue towards a semantic action whose production's head is the Expr nonterminal a *BeginEvalMetaAction*(1) node is used—i.e., a production in the `Expr \leftarrow ...` form; then, the semantic actions can be connected accordingly.

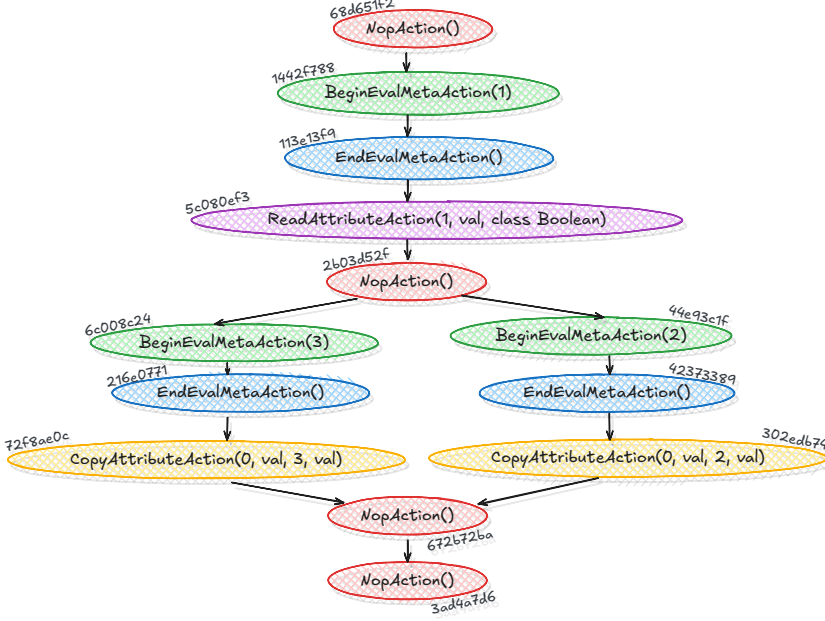


Fig. 2. Control flow graph generated by nlgcheck for the semantic action shown in Listing 1.

There are a few factors to consider when navigating the CFG built in this way.

Returning to the correct call site. In general-purpose program analysis, a given procedure may potentially be called by many others (i.e., it has many *call sites*). In our model, each semantic action equates to a procedure; thus, the call site of a semantic action G attached to a production $p : Y \leftarrow \dots$ may reside in any of the G_c semantic actions attached to productions p_c having Y on their right-hand side.

Moreover, if p_c has multiple instances of Y on the right-hand side, then the call site of G may correspond to each instance of Y in p_c . Let us consider a “caller” semantic action $G_i = (V_i, E_i)$ attached to a production p_i and a “called” semantic action $G_j = (V_j, E_j)$ attached to a production p_j . We take $v_{beginEval}, v_{endEval} \in V_i$ and $k \in \mathbb{N}$ such that

$$p_i[k] = p_j[0],$$

$$a_{node}(v_{beginEval}) = \text{BeginEvalMetaAction}(k),$$

$$a_{node}(v_{endEval}) = \text{EndEvalMetaAction}(),$$

$$(v_{beginEval}, v_{endEval}) \in E_i$$

Then, given the edge set E_r of the GCF G_r , the control flow being transferred from G_i to G_j is performed as follows:

- (1) remove $(v_{beginEval}, v_{endEval})$ from E_r (as the execution of G_j cannot be skipped);
- (2) add $(v_{beginEval}, v_{entry}(G_j))$ to E_r ;
- (3) add $(v_{exit}(G_j), v_{endEval})$ to E_r .

Since the language grammar potentially comprises many other productions $p_l \neq p_i$ such that $\exists k' : p_l[k'] = p_j[0]$, these steps are performed on each p_i, p_l, \dots , leading to several pairs of vertices in the form $v'_{beginEval}, v'_{endEval} \in V_l \subseteq V_r$ such that $(v'_{beginEval}, v_{entry}(G_j)) \in E_r$ and $(v_{exit}(G_j), v'_{endEval}) \in E_r$. Some of these vertices may never be navigated in a real interpreter; thus, we introduce *tagged*

entry and *exit edges*. We define a set L of tags, two sets $E_{entry}, E_{exit} \subset E_r$ such that $E_{entry} \cap E_{exit} = \emptyset$ and a function $l_{edge} : (E_{entry} \cup E_{exit}) \rightarrow L$. We define a *tagged entry edge* as an edge $e \in E_{entry}$ such that $l_{edge}(e) = l$ (with $l \in L$) and a *tagged exit edge* as an edge $e \in E_{exit}$ such that $l_{edge}(e) = l$. Then, we can imagine paths as well-parenthesized expressions, considering valid only those paths in which every exit edge (closing bracket) has a corresponding entry edge (opening bracket) with the same tag.

This leads to a formal definition for a well-formed CFG path:

- the empty path ϵ is a well-formed path;
- given a pair of vertices $v_a, v_b \in V_r$ connected by an untagged edge $e = (v_a, v_b)$, such that $e \in E_r$ and $e \notin (E_{entry} \cup E_{exit})$, $v_a v_b$ is a well-formed path;
- given the vertices $v_a, v_b, v_c, v_d \in V_r$ and a well-formed path $w = v_b \dots v_c$, let $e_{entry} = (v_a, v_b)$ and $e_{exit} = (v_c, v_d)$; then $v_a w v_d$ is a well-formed path if

$$e_{entry} \in E_{entry} \wedge e_{exit} \in E_{exit} \wedge l_{edge}(e_{entry}) = l_{edge}(e_{exit})$$

- given two well-formed paths w_a and w_b , their concatenation $w_a w_b$ is a well-formed path.

Finally, any prefix of a well-formed path is a *valid path*.

Two semantic actions G_i and G_j are therefore connected as follows:

- (1) remove $(v_{beginEval}, v_{endEval})$ from E_r ;
- (2) add $e_1 = (v_{beginEval}, v_{entry}(G_j))$ to E_{entry} (and thus to E_r);
- (3) add $e_2 = (v_{exit}(G_j), v_{endEval})$ to E_{exit} (and thus to E_r);
- (4) let l be a new tag ($l \notin L$) and add it to L ;
- (5) let $l_{edge}(e_1) = l_{edge}(e_2) = l$.

When the visit reaches $v_{exit}(G_j)$, only one among many outgoing edges matches the tag followed when entering the semantic action. Following such an edge returns control to the correct call site.

Context switching. Execution simulation involves keeping the state in proper data structures. The state comprises pieces of information that will be detailed later; for now, consider that the behavior of a semantic action depends on contextual information. In fact, several of the actions introduced earlier involve i parameters that refer to the AST node being visited. Thus, we extend the model with two more actions: *EnterCtx*(i) and *LeaveCtx*() (with $i \in \mathbb{N}$). The semantics are akin to procedures in low-level programming, which often involve a *prologue* and an *epilogue*: additional instructions surrounding the actual body, often used to allocate and free stack frames. Likewise, we can use a stack data structure: each item on the stack, which we call a *production context*, stores information (name and type) of the attributes of the current node's children, with the base of the stack representing the root AST node, which can also store its own attributes.

When the current semantic action sets or accesses an attribute on a right-hand-side nonterminal ($i \geq 1$) of its attached production, the operation is performed directly on the production context on top of the stack. When the current semantic action sets or accesses the left-hand-side nonterminal ($i = 0$), the operation is performed on the second-to-top production context of the stack with position i_c . This model reflects how attributes are passed at runtime between semantic actions depending on their position within the productions.

EnterCtx(i) and *LeaveCtx*() are therefore equivalent to procedure prologues and epilogues, respectively: the former pushes a new production context with $i_c = i$ to the stack, while the latter pops the production context from the top of the stack.

CFG completion. Entry and exit nodes must be defined for the entire CFG G_r . This is done through two additional nodes, both mapped to the *Nop*() action ($a_{node}(v_{entry}(G_r)) = a_{node}(v_{exit}(G_r)) = \text{Nop}()$), and then adding proper edges to connect them, respectively, with the entry and exit nodes

of each production having Program (the start symbol) as its right-hand side. Algorithm 3 in §B.1 summarizes the contents of this section by showing the complete CFG construction algorithm from a set of productions and the control-flow graphs of the respective semantic actions.

4.4 Visiting the CFG

The control-flow graph is a representation of all possible execution paths for a role: for each parse tree that can be produced according to the grammar, the CFG contains a corresponding valid path, representing the sequence of operations performed during the tree visit.

4.4.1 Execution state model. The simulated execution state is a tuple (v, w, S, C) such that

- $v \in V_r$ is the node currently being visited;
- $w \in V_r^*$ is the path followed to reach v ;
- S is the stack of edge tags;
- $C \in \mathcal{C}$ is the stack of production contexts, where \mathcal{C} is used to refer to the (infinite) set of all possible production context stacks.

Items of the edge tag stack $S = s_0 s_1 \dots s_n$ are edge tags ($s_i \in L$); moreover, if S is empty, we say that $\#pop(S)$ and $\#top(S)$. Instead, the production context stack $C = c_0 c_1 \dots c_n$ always contains at least one item c_0 (representing the AST root), and the generic c_i item represents a virtual AST node defined as $c_i = (k_i, m_i)$, a tuple where $k_i \in \mathbb{N}_{>0}$ indicates the position of the node in the child list of its parent, and $m_i : \mathbb{N}_{>0} \times I \rightarrow T$ is an attribute mapping function just like m_0 , with the only difference being that $m_i : \mathbb{N} \times I \rightarrow T$ maps a child node (represented by its index) and an attribute to the attribute type.

The traditional operations offered by the stack data structure are sufficient to express most of the actions associated with a CFG node as context-stack transformations. One notable exception is the *PropagateAttrs()* action, which needs to enumerate all the attributes within a node. We therefore extend the instruction set with a *getAll* operation defined as

$$getAll(C, i) = \begin{cases} \{(\eta, t) : ((i, \eta) \mapsto t) \in m_0\} & \text{if } C = c_0 \\ \{(\eta, t) : ((i, \eta) \mapsto t) \in m_n\} & \text{if } C = c_0 \dots c_n \text{ and } i \geq 1 \\ \{(\eta, t) : ((k_n, \eta) \mapsto t) \in m_{n-1}\} & \text{if } C = c_0 \dots c_n \text{ and } i = 0 \end{cases}$$

getAll computes a set $X \subset (I \times T)$ of name-type pairs of the attributes at the i -th position of the current node. *PropagateAttrs()* also sets the attribute name-type pairs into the current node, without replacing pre-existing mappings; this behavior is represented by the *merge* and *putAll* operations:

$$merge(m, X, i) = m \cup \{(i, \eta) \mapsto t : (\eta, t) \in (X \setminus \{(\eta', t') : ((i, \eta') \mapsto t') \in m\})\}$$

$$putAll(C, i, X) = \begin{cases} merge(m_0, X, i) & \text{if } C = c_0 \\ c_0 \dots c_{n-1}(k_n, merge(m_n, X, i)) & \text{if } C = c_0 \dots c_n \text{ and } i \geq 1 \\ c_0 \dots (k_{n-1}, merge(m_{n-1}, X, k_n))c_n & \text{if } C = c_0 \dots c_n \text{ and } i = 0 \end{cases}$$

Since the *put* only modifies at most the top of the context stack, let us introduce two more functions for brevity:

$$c_{put1}(c_{n-1}, k_n, i, \eta, t) = \begin{cases} c_{n-1} & \text{if } i \geq 1 \\ (k_{n-1}, m_{n-1} \cup \{(k_n, \eta) \mapsto t\}) & \text{if } i = 0 \end{cases}$$

$$c_{put2}(c_n, i, \eta, t) = \begin{cases} (k_n, m_n \cup \{(i, \eta) \mapsto t\}) & \text{if } i \geq 1 \\ c_n & \text{if } i = 0 \end{cases}$$

Notice that the *push* can be defined in terms of these two functions:

$$put(C, i, \eta, t) = \begin{cases} m_0 \cup \{(i, \eta) \mapsto t\} & \text{if } C = c_0 \\ c_0 \dots c_{put1}(c_{n-1}, k_n, i, \eta, t) c_{put2}(c_n, i, \eta, t) & \text{if } C = c_0 \dots c_n \end{cases}$$

4.4.2 Violations. We are now ready to define the kinds of incorrect or undesirable execution that can be detected while visiting the CFG, which we will call the set \mathcal{V} of *violations*.

MissingAttr(i, η), with $i \in \mathbb{N}$ and $\eta \in I$

This violation indicates that a semantic action accesses an attribute named η on the i -th nonterminal of its attached production that was not previously set.

BadAttrT(i, η, t_r, t_p), with $i \in \mathbb{N}$, $\eta \in I$ and $t_r, t_p \in T$

This violation indicates that a semantic action accesses an attribute named η on the i -th nonterminal of its attached production and casts it to t_r (*required* type), which was previously set to a value of type t_p (*provided* type) not assignable to t_r ($t_p \not\leq t_r$).

AttrTypeDynamicallyChecked(i, η, t), with $i \in \mathbb{N}$, $\eta \in I$ and $t \in T$

This violation indicates that a semantic action performs an instance of (or equivalent) check against t of the type of the attribute named η on the i -th nonterminal of its attached production. This violation does not cause a runtime error but is reported as a warning to suggest that the language developer use `NeverLang`'s type guards instead.

4.4.3 Mapping CFG actions to state transformations. Each of the aforementioned actions can be formally defined in terms of the transformations it performs on the state and the violations it detects: an action $a \in A$ does not manipulate or access the entire visit state, but rather just the production context stack C . We can therefore define f_{action} as a function $(C, A) \rightarrow (C, \mathcal{P}(\mathcal{V}))$ mapping a production context stack and an action to a new production context stack and a set of violations.

Here follows a list of the f_{action} with respect to the actions that can appear in the CFG:

- $f_{action}(C, WriteAttr(i, \eta, t)) = (put(C, i, \eta, t), \emptyset)$
- $f_{action}(C, ReadAttr(i, \eta, t)) = \begin{cases} (C, \{MissingAttr(i, \eta)\}) & \text{if } \nexists get(C, i, \eta) \\ (C, \{BadAttrT(i, \eta, t, get(C, i, \eta))\}) & \text{if } get(C, i, \eta) \not\leq t \\ (C, \emptyset) & \text{otherwise} \end{cases}$
- $f_{action}(C, CopyAttr(i_d, \eta_d, i_s, \eta_s)) = \begin{cases} (C, \{MissingAttr(i_s, \eta_s)\}) & \text{if } \nexists get(C, i_s, \eta_s) \\ (put(C, i_d, \eta_d, get(C, i_s, \eta_s)), \emptyset) & \text{otherwise} \end{cases}$
- $f_{action}(C, CheckAttrType(i, \eta, t)) = (C, \{AttrTypeDynamicallyChecked(i, \eta, t)\})$
- $f_{action}(C, PropagateAttrs()) = (putAll(C, 0, getAll(C, 1)), \emptyset)$
- $f_{action}(C, EnterCtx(i)) = (push(C, (i, \emptyset)), \emptyset)$
- $f_{action}(C, LeaveCtx()) = (pop(C), \emptyset)$
- $f_{action}(C, Nop()) = (C, \emptyset)$

The result of an f_{action} is made of two elements. We refer to the "next context" element of f_{action} with f_{aCtx} :

$$f_{aCtx}(C, a) = C' \iff \exists U : f_{action}(C, a) = (C', U)$$

We refer to the "violations" part of f_{action} with f_{aErr} :

$$f_{aErr}(C, a) = U \iff \exists C' : f_{action}(C, a) = (C', U)$$

Finally, we define $f_{pCtx} : (V_r^* \times C) \rightarrow C$ as the overall changes made to the production context stack during the traversal of a CFG path:

$$f_{pCtx}(w, C) = \begin{cases} C & \text{if } w = \epsilon \\ f_{pCtx}(w', f_{aCtx}(C, a_{node}(v))) & \text{if } w = vw' \end{cases}$$

LEMMA 1. f_{pCtx} can be evaluated "piecewise" on a path by composing evaluations of f_{pCtx} on parts of the path:

$$\forall w_1, w_2 \in V_r^*, C \in C, \quad f_{pCtx}(w_1 w_2, C) = f_{pCtx}(w_2, f_{pCtx}(w_1, C))$$

PROOF. This can be proven by induction on w_1 . The base case $w_1 = \epsilon$ is trivial, as by definition $f_{pCtx}(\epsilon, C) = C$. For the inductive step, given w_1 , we assume

$$\forall w_2 \in V_r^*, C' \in C, \quad f_{pCtx}(w_1 w_2, C') = f_{pCtx}(w_2, f_{pCtx}(w_1, C'))$$

and prove that

$$\forall v \in V_r, w_2 \in V_r^*, C \in C, \quad f_{pCtx}(vw_1 w_2, C) = f_{pCtx}(w_2, f_{pCtx}(vw_1, C))$$

using the definition of f_{pCtx} and the inductive hypothesis with $C' = f_{aCtx}(C, a_{node}(v))$. Please refer to §C for the complete, step-by-step proof. \square

4.4.4 Action properties. All actions, except for $EnterCtx(i)$ or $LeaveCtx()$, only depend on and affect the contents of the two topmost productions of the stack. More specifically, it can depend on the attributes present at any position in the node at the top of the stack and on those on the node at position 0 of the context right under the top of the stack. We refer to this information as the *head* of the production context stack:

$$head(C) = \begin{cases} c_0 & \text{if } C = c_0 \\ m_n \cup \{(0, \eta) \mapsto t : ((k_n, \eta) \mapsto t) \in m_{n-1}\} & \text{if } C = c_0 \dots c_n \end{cases}$$

LEMMA 2. The head of a production context stack only depends on its two topmost items:

$$\begin{aligned} \forall C_1 = \langle c_0, \dots, c_n \rangle, C_2 = \langle c'_0, \dots, c'_n \rangle \in C, \\ c_{n-1} = c'_{n-1} \wedge c_n = c'_n \implies head(C_1) = head(C_2) \end{aligned}$$

PROOF. Trivial by definition of head. \square

LEMMA 3. f_{aErr} only depends on the head of the production context stack:

$$\forall C_1, C_2 \in C, a \in A, \quad head(C_1) = head(C_2) \implies f_{aErr}(C_1, a) = f_{aErr}(C_2, a)$$

PROOF. By definition of f_{action} and a_{aErr} , if a is one among $WriteAttr$, $PropagateAttrs$, $EnterCtx$, $LeaveCtx$, or Nop , then $f_{aErr}(C, a) = \emptyset \quad \forall C$, trivially proving the property. Similarly, if $a = CheckAttrType(i, \eta, t)$, then $f_{aErr}(C, a)$ is independent of C . Finally, if $a = ReadAttr$ or $a = CopyAttr$, the contents of $f_{aErr}(C, a)$ depend on the value of a *get* operation performed on C : the value of *get* applied to a context stack $\langle c_0, \dots, c_n \rangle$ depends on either m_n or $m_{n-1}(k_n, \eta)$, both of which are equal in C_1 and C_2 under the hypothesis $head(C_1) = head(C_2)$ by definition of *head*. \square

4.4.5 Balanced action sequences. Just like tagged edges, actions performed on the context stack lend themselves to the same analogy with parenthesized expressions, although there are only two actions that are relevant in this regard: $EnterCtx(i)$ (corresponding to opening a parenthesis) and $LeaveCtx()$ (closing the parenthesis). We define the *set of balanced action sequences*, \mathcal{A}_b , with $\mathcal{A}_b \subset A^*$ as follows:

- (1) the empty sequence is balanced ($\epsilon \in \mathcal{A}_b$);
- (2) given $b \in \mathcal{A}_b$ and $a \in A \setminus (\{EnterCtx(i) : i \in \mathbb{N}\} \cup \{LeaveCtx()\})$, $ab \in \mathcal{A}_b$ and $ba \in \mathcal{A}_b$;

(3) given $a_1 = \text{EnterCtx}(i)$ for any i , $a_2 = \text{LeaveCtx}()$, and $b \in \mathcal{A}_b$, $a_1ba_2 \in \mathcal{A}_b$.

We also define the set \mathcal{A}_{pb} of *prefixes* of balanced action sequences as follows:

- (1) the empty sequence is a prefix of a balanced action sequence ($\epsilon \in \mathcal{A}_{pb}$);
- (2) given $b \in \mathcal{A}_{pb}$ and $a \in A \setminus (\{\text{LeaveCtx}()\})$, $ab \in \mathcal{A}_{pb}$ and $ba \in \mathcal{A}_{pb}$;
- (3) given $a_1 = \text{EnterCtx}(i)$ for any i , $a_2 = \text{LeaveCtx}()$, and $b \in \mathcal{A}_{pb}$, $a_1ba_2 \in \mathcal{A}_{pb}$.

LEMMA 4. $\mathcal{A}_b \subset \mathcal{A}_{pb} \subset A^*$

PROOF. Trivial by definition of \mathcal{A}_b and \mathcal{A}_{pb} . \square

4.4.6 Balanced paths. For the application of balanced action sequences in CFG paths, let us also introduce the concept of *balanced paths*—i.e., a sequence of nodes whose associated actions form a balanced action sequence. We call W_b the set of balanced paths, with $W_b \subset V_r^*$, such that:

$$W_b = \{(v_1 \dots v_n) : (a_1 \dots a_n) \in \mathcal{A}_b, \text{ with } a_i = a_{\text{node}}(v_i) \forall i\}$$

We define the set of balanced path prefixes, W_{pb} , with $W_b \subset W_{pb} \subset V_r^*$, as

$$W_{pb} = \{(v_1 \dots v_n) : (a_1 \dots a_n) \in \mathcal{A}_{pb}, \text{ with } a_i = a_{\text{node}}(v_i) \forall i\}$$

We define a generalization, top_k , of the *top* stack operation, which allows "peeking" at the k topmost items of the stack:

$$\text{top}_k(\langle c_0, \dots, c_n \rangle) = \langle c_{n-k+1}, \dots, c_n \rangle \quad \text{with } 1 \leq k \leq n+1$$

LEMMA 5. *top* and top_1 are equivalent.

PROOF. Trivial by definition of top_k . \square

LEMMA 6. *Given an integer $k > 0$ and two production context stacks sharing the same top k items, the evaluation context stacks obtained by evaluating a balanced path prefix that does not contain CopyAttr or PropagateAttrs actions share the top k items:*

$$\forall w \in W_{pb}, \forall C_1, C_2 \in \mathcal{C}, k \in \mathbb{N} \\ \text{top}_k(C_1) = \text{top}_k(C_2) \implies \text{top}_k(f_{p\text{Ctx}}(w, C_1)) = \text{top}_k(f_{p\text{Ctx}}(w, C_2))$$

PROOF. This property is proven by induction on each of the following four cases:

- (1) $w = \epsilon$;
- (2) $w = vw'$, with $w' \in W_{pb}$ and $a_{\text{node}}(v) = a$ with $a \in A \setminus \{\text{LeaveCtx}()\}$;
- (3) $w = w'v$, with $w' \in W_{pb}$ and $a_{\text{node}}(v) = a$ with $a \in A \setminus \{\text{LeaveCtx}()\}$;
- (4) $w = v_1w'v_2$ with $w' \in W_{pb}$, $a_{\text{node}}(v_1) = a_1 = \text{EnterCtx}(i)$, and $a_{\text{node}}(v_2) = a_2 = \text{LeaveCtx}()$.

Please refer to §C for the complete proof. \square

LEMMA 7. *Given two production context stacks with the same head, evaluating a balanced path prefix without CopyAttr or PropagateAttrs actions produces results with the same head.*

$$\forall w \in W_{pb}, \forall C_1, C_2 \in \mathcal{C} \\ \text{head}(C_1) = \text{head}(C_2) \implies \text{head}(f_{p\text{Ctx}}(w, C_1)) = \text{head}(f_{p\text{Ctx}}(w, C_2))$$

PROOF. This property is similar to Lemma 6 with $k = 2$. All considerations made on $\text{top}_3(C) \forall C$ also hold for the conjunction to $\text{top}_2(C)$ and $\text{head}(\text{pop}(C))$. When $\text{head}(C_1) = \text{head}(C_2)$, we have that

$$\text{head}(\text{put}(C_1, i, \eta, t)) = \text{head}(\text{put}(C_2, i, \eta, t))$$

With $C_1 = \langle c_0, \dots, c_n \rangle$ and $C_2 = \langle c'_0, \dots, c'_n \rangle$, that can be expanded as

$$\begin{aligned} \text{head}(\langle c_1, \dots, c_{\text{put1}(c_{n-1}, k_n, i, \eta, t)}, c_{\text{put2}(c_n, i, \eta, t)} \rangle) = \\ \text{head}(\langle c'_1, \dots, c'_{\text{put1}(c'_{n-1}, k'_n, i, \eta, t)}, c'_{\text{put2}(c'_n, i, \eta, t)} \rangle) \end{aligned}$$

This can be proven by definition of *head*, since $m_{put2}(c_n, i, \eta, t) = m_{put2}(c'_n, i, \eta, t)$, and by considering that the mappings for k_n contained in $m_{put1}(c_{n-1}, k_n, i, \eta, t)$ are the same as those for k'_n contained in $m_{put1}(c'_{n-1}, k'_n, i, \eta, t)$, in which m_{put1} and m_{put2} are used to denote the "m-part" of c_{put1} and c_{put2} , respectively.

Then, the proof of Lemma 6 can be adapted with $k = 2$, using *head* in place of top_2 , proving this property. \square

LEMMA 8. *Given any two production context stacks, evaluating a balanced path prefix that does not contain CopyAttr or PropagateAttrs actions, wrapped between two EnterCtx nodes, produces context stacks with the same head.*

$$\forall C_1, C_2 \in C, w \in W_{pb}, \quad v_{sep} \in V_r^* : \exists i : a_{node}(v_{sep}) = EnterCtx(i),$$

$$head(f_{pCtx}(v_{sep} w v_{sep}, C_1)) = head(f_{pCtx}(v_{sep} w v_{sep}, C_2))$$

PROOF. The proof follows the same structure as that of Lemma 6, according to the four possible definitions of a balanced path. Please refer to §C for the full proof. \square

LEMMA 9. *Given an initial production context stack $C \in C$, two balanced path prefixes $w_1, w_2 \in W_{pb}$, and a "production entry node" v_{sep} mapped to an EnterCtx action, evaluating either $v_{sep} w_1 v_{sep} w_2 v_{sep}$ or $v_{sep} w_2 v_{sep}$ on C produces resulting context stacks with the same head.*

$$\forall C \in C, w_1, w_2 \in W_{pb}, \quad v_{sep} \in V_r^* : \exists i : a_{node}(v_{sep}) = EnterCtx(i),$$

$$head(f_{pCtx}(v_{sep} w_1 v_{sep} w_2 v_{sep}, C)) = head(f_{pCtx}(v_{sep} w_2 v_{sep}, C))$$

PROOF. This property is a trivial consequence of Lemma 8. By definition of f_{pCtx} , the goal can be rewritten as

$$head(f_{pCtx}(v_{sep} w_2 v_{sep}, f_{pCtx}(v_{sep} w_1, C))) = head(f_{pCtx}(v_{sep} w_2 v_{sep}, C))$$

which is proven by applying Lemma 8 with $C_1 = f_{pCtx}(v_{sep} w_1)$, $C_2 = C$, and $w = w_2$. \square

4.4.7 Following valid paths. In §4.3, we discussed the problem of correctly modeling control-flow transfers between different semantic actions in a CFG by representing transitions that can happen at runtime, as well as the concepts of *tagged entry and exit edges* and *valid path*.

The *edge tag stack* S , previously introduced in this section as part of the state of a CFG visit, is used to keep track, at any given point of the visit, of the tagged entry edges for which a corresponding tagged exit edge has not yet been traversed.

We define the f_{edge} function as a way to drive the algorithm through valid paths according to these concepts. It determines if an edge can be followed and updates S if needed. Let L^* be the set of all possible edge tag stacks; $f_{edge} : (L^* \times E_r) \rightarrow L^*$ is defined as

$$f_{edge}(S, e) = \begin{cases} S & \text{if } e \notin (E_{entry} \cup E_{exit}) \\ push(S, l_{edge}(e)) & \text{if } e \in E_{entry} \\ pop(S) & \text{if } e \in E_{exit} \wedge l_{edge}(e) = top(S) \end{cases}$$

Therefore, f_{edge} is not defined if following the edge would lead to an invalid path. Otherwise, f_{edge} returns the updated edge tag stack after the edge is traversed.

4.4.8 Solution to the problem of infinite paths. Most practical context-free grammars are recursive. A parser built from a recursive grammar can parse an infinite set of input strings; therefore, the number of parse trees that can be generated is also infinite. Even though, according to our model, different parse trees can be mapped to the same CFG visit path, the paths themselves are potentially infinite nonetheless.⁶

Thus, it is fundamental to determine a finite subset of all possible paths to render the static analysis feasible.

Naïve solutions. By keeping track of the set of already visited nodes, one can prevent visiting the same node more than once. It can be trivially shown that this approach can easily cause inconsistency, leaving the violations presented in §4.4.2 undetected even in small CFGs: take, for instance, an if-then statement in which an attribute is set within the body when the condition is true; then, the same attribute is accessed outside. This should lead to an error, as the attribute is not set unless the condition holds. It is possible to find similar counterexamples even when keeping track of all navigated edges instead of nodes. Thus, naïve solutions do not suffice, and a more complex one must be adopted.

Complete solution. Let $W \subseteq V_r^*$ be the (potentially infinite) set of valid paths. The goal is to find a finite set $W_a \subseteq W$ such that for every node v_e , if there exist some paths from the entry node to v_e that may cause violations, then at least one such path is contained in W_a .

Let us introduce a function, f_{pErr} , that computes the set of violations detected within a path. Given a path $w \in V_r^*$ and a production context stack $C \in C$, $f_{pErr} : (V_r^* \times C) \rightarrow \mathcal{P}(\mathcal{V})$ is defined as follows:

$$f_{pErr}(w, C) = \begin{cases} \emptyset & \text{if } w = \epsilon \\ f_{aErr}(f_{pCtx}(w', C), a_{node}(v)) & \text{if } w = w'v \end{cases}$$

Notice that f_{pErr} "discards" any violations detected prior to reaching the last node; as we will show with the following properties and requirements, this does not affect the validity of the solution.

LEMMA 10. *If w is a prefix of a balanced path, then $f_{pErr}(w, C)$ only depends on the head of C .*

$$\forall w \in W_{pb}, C_1, C_2 \in C, \quad head(C_1) = head(C_2) \implies f_{pErr}(w, C_1) = f_{pErr}(w, C_2)$$

PROOF. If $w = \epsilon$, the proof is trivial, as by definition, $f_{pErr}(\epsilon, C) = \emptyset \forall C$.

If $w = w'v$, the goal is to prove that

$$f_{aErr}(f_{pCtx}(w', C_1), a_{node}(v)) = f_{aErr}(f_{pCtx}(w', C_2), a_{node}(v))$$

For the definition of a prefix of a balanced path, and since $w \in W_{pb}$, we can conclude that $w' \in W_{pb}$.

Therefore, by Lemma 7 and from the assumption $head(C_1) = head(C_2)$ follows that $head(f_{pCtx}(w', C_1)) = head(f_{pCtx}(w', C_2))$.

Applying Lemma 3 with the context stacks resulting from $f_{pCtx}(w', C_1)$ and $f_{pCtx}(w', C_2)$ as C_1 and C_2 , respectively, and $a_{node}(v)$ as a concludes the proof. \square

For conciseness, let us also define $v_0 = v_{entry}(G_r)$ and $f_e(w) = f_{pErr}(w, \langle \emptyset \rangle)$. Then:

$$\forall v_e \in V_r, \quad \exists w_e = v_0 \dots v_e : f_e(w_e) \neq \emptyset \implies \exists w'_e = v_0 \dots v_e \in W_a : f_e(w'_e) \neq \emptyset \quad (1)$$

Note that a path in W_a does not necessarily lead to a violation.⁷

⁶Two trivial examples are when a semantic action may call itself, possibly looping any number of times, or when the semantic action itself contains a loop that may eval another action any number of times.

⁷The implication is not bidirectional.

There are a few constraints that the paths must adhere to satisfy these requirements while keeping W_a finite:

- (1) A tagged entry edge is not followed if its tag is already present twice on the current edge tag stack.⁸ Formally, we define a function $f_{stack} : (V_r^* \times L^*) \rightarrow L^*$ representing how traversing a path mutates the edge tag stack.

$$f_{stack}(w, S) = \begin{cases} S & \text{if } |w| \leq 1 \\ f_{stack}(v_2 w', f_{edge}(S, (v_1, v_2))) & \text{if } w = v_1 v_2 w' \end{cases}$$

Thus, the requirement is expressed as:

$$W'_a = \{w \in V_r^* : \nexists l_1, l_2, l_3 \in f_{stack}(w, \epsilon) : l_1 = l_2 = l_3\} \quad (2)$$

Intuitively, a tag being present once on the stack tests invocation, a tag being present twice tests recursion, whereas three or more occurrences do not provide additional information.

- (2) A node is not visited again if it was previously visited with the same production context stack and edge tag stack as the current ones. Formally:

$$W_a = \{w = v_0 \dots v_n \in W'_a : \forall w' = v'_0 \dots v'_n \in W'_a \text{ such that } w' \neq w \wedge v'_n = v_n, \\ f_{pCtx}(w, \langle \emptyset \rangle) \neq f_{pCtx}(w', \langle \emptyset \rangle) \vee f_{stack}(w, \epsilon) \neq f_{stack}(w', \epsilon)\} \quad (3)$$

It can be observed that $W_a \subseteq W'_a$.

We now prove two lemmas demonstrating that under the assumption that no *CopyAttr* or *PropagateAttrs* nodes are present in the graph (which is a precondition to several properties proven thus far), visiting all the paths contained in the set W_a determined by the constraints above is sufficient to find a violation within a path, if any exists.

A limitation of this approach is that we cannot prove that the same property holds when *CopyAttr* or *PropagateAttrs* actions are used. However, we deemed this limitation acceptable for the following reasons.

- (1) While lacking a formal proof, experimentation (see §6) demonstrated that even when such actions are used, the software is able to find most violations anyway in practice.
- (2) All *PropagateAttrs* actions can always be replaced by copying relevant attributes from child nodes to parent nodes explicitly, instead of relying on this Neverlang-specific quality-of-life feature. Similarly, *CopyAttrs* actions can also be replaced by other, more explicit, actions: semantic instructions of the form $\$0.attrA = \$1.attrB$ can be replaced with $\$0.attrA = (T) \$1.attrB$, where T is the expected type of the attribute, thus modifying the CFG accordingly, substituting the single *CopyAttr*(0, attrA, 1, attrB) node with the two nodes *ReadAttr*(1, attrB, T) and *WriteAttr*(0, attrA, T) in succession. In other words, the problem is solved by rendering the type of the attribute explicit. By taking these precautions, the approach is formally sound and can always find any violations.

LEMMA 4.1. W_a as defined by (2) satisfies (1).

PROOF. Let us consider a generic node $v_e \in V_r$; we must prove that if paths from the entry node to v_e with a violation exist, then at least one of them is contained within W'_a , so that it satisfies (2). So, let's assume that

$$\exists w_e = v_0 \dots v_e : f_e(w_e) \neq \emptyset \quad (4)$$

⁸Notice that this condition is less strict than that of only following any given tagged entry edge at most twice in a path: if $e_1 \in E_{entry}$ is followed, and then in the same path $e_2 \in E_{exit}$ such that $l_{edge}(e_2) = l_{edge}(e_1)$ is also followed, e_1 can be followed again twice.

and proceed by proving that

$$\exists w'_e = v_0 \dots v_e : f_e(w'_e) \neq \emptyset \wedge w'_e \in W'_a$$

which, due to (2), can be rewritten as

$$\exists w'_e = v_0 \dots v_e : f_e(w'_e) \neq \emptyset \wedge \nexists l_1, l_2, l_3 \in f_{stack}(w'_e, \epsilon) : l_1 = l_2 = l_3$$

Proceeding by contradiction, assume the goal is false. Without loss of generality, we consider that evaluating *any* path from the entry node v_0 to v_e that leads to the detection of a violation at v_e requires pushing the same edge tag on the stack at least three times. Then:

$$\forall w'_e = v_0 \dots v_e : f_e(w'_e) = \emptyset \vee \exists l_1, l_2, l_3 \in f_{stack}(w'_e, \epsilon) : l_1 = l_2 = l_3 \quad (5)$$

With $w'_e = w_e$ and due to (4)

$$\exists l_1, l_2, l_3 \in f_{stack}(w_e, \epsilon) : l_1 = l_2 = l_3$$

or, in words, the edge tag stack necessarily contains the same tag l such that $l = l_1 = l_2 = l_3$ at least three times after traversing w_e .

We show the contradiction by proving that if l is present on the final edge tag stack exactly three or more times, then part of the path can be removed.

By construction of the CFG and by definition of f_{edge} and f_{stack} , condition (5) occurs when an entry edge $(v_1, v_2) \in E_{entry}$, such that $l_{edge}((v_1, v_2)) = l$, is traversed exactly three times in w_e without the corresponding exit edge $(v_3, v_4) \in E_{exit}$ such that $l_{edge}((v_3, v_4)) = l$ ever being traversed. Therefore, the (possibly empty) w_1, w_2, w_3, w_4 paths exist, such that

$$w_e = v_0 w_1 v_1 v_2 w_2 v_1 v_2 w_3 v_1 v_2 w_4 v_e$$

and such that none between w_2, w_3 , and w_4 contain the vertex pair $v_3 v_4$. Recall that only valid paths (as by definition provided in §4.3) are considered; thus, w_e is a valid path. It follows that w_2, w_3 , and w_4 must also be valid paths, *i.e.*, they cannot contain exit edges without a corresponding entry edge. Upon traversing each of them, the top of the edge tag stack is l ; therefore, if an exit edge had to be followed, its tag would also need to be l , but this would mean that the edge is (v_3, v_4) , which is not allowed under the current hypothesis.

It is true by CFG construction that each entry edge always starts from a node mapped to an *EnterCtx* action, and also that each exit edge always points to a node mapped to a *LeaveCtx* action. The inverse is also true, *i.e.*, all outgoing edges from an *EnterCtx* node are entry edges, and all incoming edges to a *LeaveCtx* node are exit edges.

We can therefore conclude that neither w_2, w_3 , nor w_4 can contain *LeaveCtx* nodes unless they also contain a matching *EnterCtx* node, as by definition of *balanced path prefix* given in §4.4.5; therefore, $w_2, w_3, w_4 \in W_{pb}$.

We expand $f_e(w_e)$ as follows:

$$\begin{aligned} f_e(w_e) &= && \text{(by definition of } f_e) \\ &= f_{pErr}(w_e, \langle \emptyset \rangle) = && \text{(by definition of } f_{pErr}) \\ &= f_{pErr}(v_0 w_1 v_1 v_2 w_2 v_1 v_2 w_3 v_1 v_2 w_4 v_e, \langle \emptyset \rangle) = && \text{(by Lemma 1)} \\ &= f_{pErr}(v_2 w_4 v_e, f_{pCtx}(v_0 w_1 v_1 v_2 w_2 v_1 v_2 w_3 v_1, \langle \emptyset \rangle)) = && \text{(by Lemma 1)} \\ &= f_{pErr}(v_2 w_4 v_e, f_{pCtx}(v_1 v_2 w_2 v_1 v_2 w_3 v_1, f_{pCtx}(v_0 w_1, \langle \emptyset \rangle))) \end{aligned}$$

Recall that (v_1, v_2) is an entry edge and thus:

- (1) v_1 is mapped to an *EnterCtx* action and is therefore compatible with the requirements v_{sep} in Lemma 9;

- (2) v_2 cannot be a *LeaveCtx* action—as it is the entry node of a semantic action—and therefore $v_2w_2, v_2w_3 \in W_{pb}$.

We can then apply Lemma 9 with $C = f_{pCtx}(v_0w_1, \langle \emptyset \rangle)$, $v_{sep} = v_1$, $w_1 = v_2w_2$, and $w_2 = v_2w_3$ to obtain

$$head(f_{pCtx}(v_1v_2w_2v_1v_2w_3v_1, f_{pCtx}(v_0w_1, \langle \emptyset \rangle))) = head(f_{pCtx}(v_1v_2w_3v_1, f_{pCtx}(v_0w_1, \langle \emptyset \rangle)))$$

Since v_2 is not a *LeaveCtx* action, we know also that $v_2w_4 \in W_{pb}$, whereas from the hypothesis $f_e(w_e) \neq \emptyset$, we know that $a_{node}(v_e) \neq \text{LeaveCtx}()$, as $f_{aErr}(C, \text{LeaveCtx}()) = \emptyset \forall C$; thus, $v_2w_4v_e \in W_{pb}$. We can then apply Lemma 10 to obtain

$$f_e(w_e) = f_{pErr}(v_2w_4v_e, f_{pCtx}(v_1v_2w_3v_1, f_{pCtx}(v_0w_1, \langle \emptyset \rangle)))$$

from which, by Lemma 1 and definition of f_{pErr} , follows that

$$f_e(w_e) = f_{pErr}(v_2w_4v_e, f_{pCtx}(v_0w_1v_1v_2w_3v_1, \langle \emptyset \rangle)) = f_{pErr}(v_0w_1v_1v_2w_3v_1v_2w_4v_e, \langle \emptyset \rangle)$$

$f_e(w_e) \neq \emptyset$ by hypothesis (4), from which it follows that

$$f_{pErr}(v_0w_1v_1v_2w_3v_1v_2w_4v_e, \langle \emptyset \rangle) \neq \emptyset$$

Let $w'_e := v_0w_1v_1v_2w_3v_1v_2w_4v_e$ for brevity. The previous steps proved that $f_e(w'_e) \neq \emptyset$, and w'_e only contains the edge (v_1, v_2) twice; therefore, after traversing w'_e , the tag l is only present on the edge tag stack twice. This conclusion contradicts the hypothesis of the *reductio ad absurdum* (5), proving that a generic path leading to the detection of a violation can be reduced to a path satisfying the requirement for inclusion in W'_a as specified by (2). \square

LEMMA 4.2. *With the additional restriction specified by (3), W_a still satisfies (1).*

PROOF. The goal is to prove that, given a path $w_e \in W'_a$ from the entry node v_0 to a node v_e with a detected violation ($f_e(w_e) \neq \emptyset$), then a $w'_e \in W_a$ such that $f_e(w'_e) \neq \emptyset$ also exists.

The proof is trivial when considering that the requirement for inclusion of paths in W_a (3) can be formulated as follows: given a set of paths that all share the same results of evaluating f_{pCtx} and f_{stack} , i.e., a set W_x such that

$$\exists C \in C, S \in L^*, \quad \forall w_x \in W_x \quad f_{pCtx}(w_x, \langle \emptyset \rangle) = C \wedge f_{stack}(w_x, \epsilon) = S$$

then exactly one of them is included in W_a .

In fact, a direct consequence of such a definition is that all the paths $w_x \in W_x$ also share the same value of $f_e(w_x)$, by definition of f_e . Therefore, if $\exists w_e \in W_x$ such that $f_e(w_e) \neq \emptyset$, then regardless of which of the other items is picked from W_x to fill the role of w'_e , it satisfies $f_e(w'_e) = f_e(w_x) \neq \emptyset$. \square

4.4.9 Visit algorithm. We can finally define a theoretically sound algorithm that traverses the CFG according to the properties defined thus far to detect any of the relevant violations that may occur when composing separately compiled language fragments.

Previously defined functions f_{action} and f_{edge} are used, respectively, to evolve the state and to drive the visit through valid paths only. The implementation keeps a set of states to visit as storage for paths that need to be explored further, and one of all states that have been visited to enforce path condition (3). Condition (2) is instead enforced by inspecting the edge tag stack before following an edge.

Algorithm 1 Visit of a role's CFG to find violations

```

 $\Phi := \{(v_{entry}(G_r), v_{entry}(G_r), \epsilon, \langle \emptyset \rangle)\}$ 
 $\Gamma := \emptyset$ 
 $U := \emptyset$ 
while  $\Phi \neq \emptyset$  do
  pick any  $(v, w, S, C) \in \Phi$ 
   $\Phi := \Phi \setminus \{(v, w, S, C)\}$ 
  if  $(v, C, S) \notin \Gamma$  then
     $\Gamma := \Gamma \cup (v, C, S)$ 
     $(C', U') := f_{action}(C, a_{node}(v))$ 
     $U := U \cup U'$ 
    for all  $(v, v') \in E_r$  do
       $S' := f_{edge}(S, (v, v'))$ 
      if  $\exists S' \text{ and } \nexists l_1, l_2, l_3 \in S : l_1 = l_2 = l_3$  then
         $\Phi := \Phi \cup \{(v', wv', S', C')\}$ 
      end if
    end for
  end if
end while
if  $U = \emptyset$  then
  verification succeeded
else
  violation(s) have been detected and collected in  $U$ 
end if

```

▷ Set of states to visit

▷ Set of visited states

▷ Found violations

▷ Path condition (3)

▷ Path validity and (2)

4.5 Optimization

While sound, Algorithm 1 needs performance that allows for static analysis to be performed as part of the language development process with ease. Therefore, significant effort went toward reducing the time complexity of the visit algorithm, with the goal of making the execution of `nlgcheck` adequate to be used with non-trivial languages. We will not discuss such optimizations in detail, so please refer to §6 for a performance evaluation. This section is limited to providing a brief overview of both optimizations and their rationale. Please refer to §B.2 for the optimized visit algorithm.

4.5.1 Graph deduplication. The execution time of Algorithm 1 largely depends on the size of the CFG. A reasonable optimization is therefore to reduce the size of the graph whenever possible, which in turn depends on the number of semantic actions in the language. The optimization leverages a few key empirical observations.

- It is not uncommon for a language grammar to contain productions that differ from each other only in their terminal symbols and are therefore syntactically equivalent in this analysis context. Consider, for example, pairs of productions for operators that have the same precedence, such as $\text{Expr} \leftarrow \text{Expr} + \text{Term}$ and $\text{Expr} \leftarrow \text{Expr} - \text{Term}$; the same can be said for other binary or unary operators.
- It is also not uncommon for semantic actions associated with such productions to have very similar implementations; with respect to the prior example, the implementations of addition and subtraction are extremely similar for all intents and purposes, differing only in the operator used to implement the expression. Most notably, such "similar" semantic actions

might as well be *identical* from `nlgcheck`'s standpoint because the actual operator does not affect any of the attribute types, nor is it represented in any way within the CFG.

- Modularity is one of the strongest points of language workbenches, and thus language developers are encouraged to keep semantic actions as small as possible, with the goal of improving their reusability. By adhering to this guideline, semantic actions can be as small as a single or a few lines of code, without any control flow. In this case, the CFGs simply consist of a chain of nodes, each of which has exactly one entry node and one exit node.

In summary, the actual output of converting semantic actions into CFG fragments may contain several duplicates. Therefore, the CFG building algorithm can be optimized by discarding any CFG of these duplicates. Notably, determining graph isomorphism is a well-known problem.

4.5.2 Path deduplication. Consider a generic nonterminal symbol A of the grammar. Every time the semantics of a production having A in its right-hand side—i.e., with the form $B \leftarrow \dots A \dots$ —is analyzed, the semantics of all the productions with A on the left-hand side must also be analyzed.

The language potentially contains several such productions. We empirically observed that it is frequent for the *head* of the production context stack to be the same when entering one of the semantic actions among the set of those associated with productions generating the same nonterminal. There are two possible reasons for this fact.

- (1) We found it to be more common to pass attributes from child to parent in the AST than from parent to child; therefore, when descending into the AST, when encountering *EnterCtx* nodes in a CFG path, the *head* of the context stack often contains no attributes.
- (2) In the cases in which attributes are passed from parent to child, we can expect the semantic actions associated with the child nodes to actually read these attributes; therefore, for all the paths without a violation⁹ the *head* of the context when entering a semantic action must contain all the attributes that the semantic action will read.

The idea for this optimization leverages these observations to review the structure of the CFG: for each nonterminal A , we add an "entry node" and an "exit node" that are shared among all the semantic actions attached to productions of the form $A \leftarrow \dots$. Such nodes serve as "anchor points" to determine when to access and when to update a cache storing all the production context stacks. By caching all the possible heads of the production context stack at the corresponding nonterminal exit node for each possible head of the production context stack at each nonterminal entry node, the algorithm can "jump" from the nonterminal entry node to the exit node. The goal is to avoid repeating the whole visit when a particular node is reached more than once with the same context stack.

5 Implementation

The implementation of `nlgcheck` closely matches the description provided in §4; therefore, we do not discuss it in detail. This section briefly overviews any relevant technology. Please refer to the replication package available at <https://zenodo.org/records/18352007> for the full implementation.

The `nlgcheck` tool was implemented in `Scala` to leverage the features of a functional programming language while being able to interact with the `Neverlang` API. In fact, most of the relevant information about language fragments is extracted directly from the pre-compiled class files; therefore, the choice of a JVM-based language was natural. Moreover, this made it possible to implement `nlgcheck` both as a command-line tool and as a `Gradle` plugin that can be directly included in the build file of each `Neverlang` project with minimum boilerplate. The analysis itself leveraged both the Java reflection API and `Soot` [51] to inspect the necessary class files and retrieve the following pieces of information.

⁹As is commonly done in error theory, we expect the absence of errors to be more likely than their presence.

- The list of roles, each consisting of its name and an execution mode (manual, postorder, or preorder). Each role represents a parse-tree traversal and is therefore analyzed separately by `nlgcheck`.
- The list of *modules* the language is comprised of. Each module holds information such as a set of grammar productions and any associated semantic actions. `Neverlang` slices and bundles within the language are treated similarly, by retrieving their syntax and semantics from the respective modules.
- The CFG for each semantic action. This piece of information cannot be retrieved directly from the class file; instead, we use Soot to extract a rough version of the CFG from the `apply` method of classes implementing the `SemanticAction` interface, then using a custom implementation to translate the Soot CFG to one containing nodes compliant with the actions presented in §4.

The tool also extracts other information, such as nonterminal renames, attribute mappings, and role interleaving, each representing `Neverlang`-specific syntactic sugar and quality-of-life features and therefore not directly relevant to the algorithm.

6 Evaluation

We performed a quantitative evaluation to test the applicability of `nlgcheck` beyond its theoretical soundness. In this regard, we opted for an evaluation of the results using source code mutations on pre-existing `Neverlang` languages. Traditionally, the goal of mutation testing is to assess the quality of test suites; therefore, it fits our requirement of systematically introducing structural errors within a language to verify whether `nlgcheck` is capable of detecting them. Moreover, mutation testing was part of a prior study with `Neverlang` [18] and was therefore deemed an adequate choice, especially because the `Neverlang` reflection API allows developers to easily mimic mutation operators without encompassing source code recompilation: in mutation testing, a mutation operator is a function that takes the original program and produces the mutated one. Concerning the errors that can happen when composing language fragments, a mutation operator that is extremely easy to implement and that can mimic most real-world programming errors is the random change of one or more characters within the identifier of an attribute. This is a simple yet relevant example because, due to separate compilation, `Neverlang` cannot detect such an error statically—not knowing in advance if a typo in an attribute name was indeed an error or simply an intended design choice foreseeing more language fragments that may be implemented and compiled in the future. The software and data used to perform this experiment are available at <https://zenodo.org/records/18352007>.

Experimental setup. The experimental `nlgcheck` execution flow can be summarized as follows.

- (1) Compilation of unmutated `Neverlang` sources into Java code.
- (2) Compilation of generated Java into bytecode code.
- (3) If either of the previous steps fails (*i.e.*, it produces compilation errors), the entire process is aborted.
- (4) Execution of `nlgcheck` on the compiled classes. If this fails (*i.e.*, reports one or more critical violations), execution is aborted, as `nlgcheck` was already capable of detecting a real error.
- (5) Configuration of the `Neverlang` compiler to instrument it with the mutation operator.
- (6) Perform a random mutation.
- (7) Execution of `nlgcheck` on the mutated classes.
- (8) Repetition from step 6 while keeping track of how many mutants are generated in total and how many were detected by `nlgcheck`.

Language	Generated	w/o nlgcheck		with nlgcheck		Total time
		Detected	Ratio	Detected	Ratio	
<i>expr</i>	14	0	0%	14	100%	12s
<i>numericUnitExpr</i>	77	14	18%	55	71%	1m 6s
<i>stateMachines</i>	19	0	0%	5	26%	16s
<i>logLang</i>	25	0	0%	22	88%	24s
<i>neverlangJS</i>	596	0	0%	441	74%	17m 54s
Total	731	14	2%	537	73%	19m 52s

Table 1. Results of running *nlgmutator* on various preexisting Neverlang-based projects, showing how many mutants are generated, how many are detected in the regular compilation process, and how many are detected by *nlgcheck*. Percentages are rounded to the nearest integer. The last column contains the execution time of *nlgcheck*.

Data Setup. At the time of writing, a number of Neverlang-based languages exist that can be used to evaluate *nlgcheck*. Among these, we chose a subset comprised of simple, reusable sublanguages (*expr* and *numericUnitExpr*), domain-specific languages (*stateMachines* and *logLang*), and a general-purpose programming language (*neverlangJS*).

Please consider that each of the chosen types of languages has a rationale:

- sublanguages are arguably the most fit to be analyzed with *nlgcheck*, as they are intended to be reused across several languages and therefore are pre-compiled separately from the host language;
- domain-specific languages are the main application scenario of language workbenches in general, and they can also be embedded in other languages, while also using syntactic constructs usually different from those used by general-purpose languages, leading to potential errors;
- a full-fledged general-purpose language represents a large-scale project, often developed by several developers concurrently, each possibly adopting different choices with respect to the attribute grammar; as a result, keeping track of the entire project without relaxing the separate compilation constraint may prove challenging.

Hardware setup. All tests were conducted on an Intel Core i7-4720HQ machine with a maximum clock speed of 3.6GHz and 16GB of RAM. The software consists of the GNU/Linux 6.1.12 (x86_64) operating system and OpenJDK 17.0.6. Please note that a different setup may yield different results.

6.1 Results

Table 1 summarizes the results of the experiment. For each test case, the number of generated mutants depends on the total number of attribute access expressions across all the language's semantic actions. The total number and the percentage of mutants that cause the compilation process to fail are then compared to those of mutants detected by *nlgcheck*.

Notice how, in the vast majority of cases, mutants survive the compilation process fully undetected because the Neverlang compiler does not perform any verification of attribute name matching across multiple modules. There is, however, one notable exception, *i.e.*, when an attribute appears both within the semantic action body and the type guard associated with that action. In such cases, the Neverlang compiler injects type casts within the generated Java code, and therefore, if the attribute is mutated while the guard is not, the cast is no longer injected correctly, possibly leading

to Java compilation errors.¹⁰ As reported in Table 1, such errors amount to only 18% of errors in the *numericUnitExpr* code base and only 2% overall (14 out of 731 across all experiments).

It is therefore apparent that the *Neverlang* compiler alone does not suffice to effectively capture errors in the definition of grammar attributes. In contrast, the total number of mutants detected by *nlgcheck* is 537 out of 731, thus about 73%; a significant improvement when compared to the baseline. However, one may wonder about the remaining 27% of mutants that are still undetected even when using *nlgcheck*, especially given the theoretical soundness of the approach. There are a few considerations to be made in this regard.

Read without write. First, while the mutation operator has the goal of simulating programming errors that would result in a crash at runtime, not all mutations lead to errors. Consider an attribute that is written but never read: this may indicate either a design flaw or the implementation of a sublanguage that exposes attributes to be used by modules from other sublanguages.

Unnecessary copy. Another source of undetected mutants occurs when semantic actions associated with a production of the form $A \leftarrow B$ copy an attribute from their only AST child node: as previously mentioned, attribute access to a node having exactly one child triggers a quality-of-life feature so that the AST chains are navigated from parent to child when an attribute is not found within the parent. In this particular case, copying the attribute with an incorrect name may not result in a runtime error. After performing the experiment, manual inspection of *Neverlang* code revealed such cases of unnecessary copy operations to be not uncommon, especially in the *neverlangJS* code base.

Dead code. Mutants may be undetected because the mutated code can never be reached within the CFG. We found such an example in the *numericUnitExpr* code base, which is articulated in several sublanguages, each implementing one concern of the language, such as integer arithmetic, floating-point arithmetic, unit checking, and so on. Each of these libraries also includes portions of so-called "glue code" needed to run them in stand-alone mode—mainly for testing purposes, such as a language only implementing binary operations between integer numbers. Such pieces of code consist of a definition of the Program nonterminal axiom, associated with a semantic action. Since mutation operators mutate all modules, they eventually also mutate semantic actions that are not actually used in the final language. Pieces of dead code were discovered in *neverlangJS* as well.

Non-idiomatic attribute access. Under the hood, *Neverlang* can simply be seen as a Java API for the development of programming languages. Therefore, developers can either approach *Neverlang* development in an "idiomatic" way, by writing semantic actions in *Neverlang* source files, or in a "non-idiomatic" way, by invoking Java methods and interacting with the AST through the *Neverlang* runtime API, instead of using the *Neverlang* syntax. Currently, there is no support in *nlgcheck* to translate arbitrary Java code into a CFG compatible with the requirements presented in §4; therefore, any "non-idiomatic" attribute access remains undetected even if the corresponding attribute was mutated upon it being written in a different semantic action.

Summary. We summarized the main reasons for which mutants may be undetected by *nlgcheck*. For the "read without write," "unnecessary copy," and "dead code" mutants, reporting them as errors should be considered a false positive in the analysis performed by *nlgcheck*. Neither case strictly constitutes an error, as the compiler/interpreter would not crash, but we may consider extending *nlgcheck* to output a warning in these occurrences. Instead, "non-idiomatic attribute accesses" are

¹⁰Recall that in the Java code generated by *Neverlang*, attributes are simply assumed to be Objects unless stated otherwise.

Language	Unoptimized			Opt. 1 only			Opts. 1&2
	Prods	Nodes	Time	Prods	Nodes	Time	Time
<i>expr</i>	10	61	✗	8	45	3s	3s
<i>numericUnitExpr</i>	14	159	✗	13	157	✗	7s
<i>stateMachines</i>	14	65	3s	12	59	3s	3s
<i>logLang</i>	17	112	4s	17	112	4s	4s
<i>neverlangJS</i>	230	1728	✗	197	1584	✗	20s
<i>neverlangExpressions</i>	80	3537	✗	72	3489	✗	3m 25s

Table 2. Results of running *nlgcheck* on various pre-existing Neverlang-based projects, with and without optimizations, respectively. An ✗ indicates that the process either crashed with an out-of-memory error or was manually aborted after 10 minutes. Optimization 1 (Section 4.5.1) slightly reduces the number of productions and CFG nodes; Optimization 2 (Section 4.5.2) further improves execution time without further modifications to the graph.

actual errors that are beyond the analysis capabilities of *nlgcheck*; this is the only true limitation of our approach, which should therefore be considered fully sound only when writing "idiomatic" Neverlang code, at least until the tool is extended to capture all kinds of attribute accesses.

6.2 Performance evaluation

Along with an evaluation of the ability of *nlgcheck* to detect errors, we are also interested in its performance: static program analysis is notably intensive, and if the tool does not perform fast enough, it would be infeasible to use as part of the day-to-day development process. In fact, during the development of *nlgcheck*, it quickly became apparent that the base algorithm was only usable with very small languages. This led to the optimization presented in §4.5. In particular, in this section, we use "Optimization 1" to refer to graph deduplication (Section 4.5.1) and "Optimization 2" to refer to path deduplication (Section 4.5.2).

To evaluate the final optimized implementation and the efficacy of the optimizations themselves, *nlgcheck* was tested with and without optimization on each language.

Results. Table 2 summarizes the results of the performance evaluation. For each language, the second column reports the performance with both optimizations disabled. The third column yields the results with only Optimization 1 enabled, showing how the number of productions and of CFG nodes is reduced, as well as the execution time, when applicable. Finally, the last column tool is run with both optimizations enabled; since Optimization 2 only affects the visit algorithm and not the construction of the CFG itself, its structure and size remain the same as in the previous step.

We set a time limit of 10 minutes for each task, a time frame that we deemed reasonable to perform a full analysis of the language (for instance, before a daily commit). In most instances, the unoptimized or partially optimized attempts either kept running indefinitely (thus surpassing the time limit) or failed with an out-of-memory error. Experiments that failed for either reason were marked with an ✗ symbol in Table 2.

In summary, Table 2 shows that while Optimization 1 slightly reduces the size of the CFG, in turn enabling the analysis on the previously failing *expr* case, it is not enough to make the tool suitable for use on real-world languages. The problem is eventually solved by adding Optimization 2, which makes the analysis run in a reasonable time for all the considered cases.

6.3 Threats to validity

Construct validity [77]. It is debatable whether the experiments performed within this evaluation accurately measure `nlgcheck`'s capability of detecting real errors (first experiment) and its usability (second experiment), respectively. For instance, we used only one kind of mutation operator—*i.e.*, changing an attribute name—whereas actual developer errors may vary. To counter-balance this limitation, we performed an extensive mutation testing experiment, thus mutating each attribute in the grammar at least once. Moreover, please consider that among the many errors that can occur while developing a language, `nlgcheck` is concerned with only a very limited subset. For instance, type errors are mostly irrelevant because they can be captured by the Java compiler without needing additional tooling. Similarly, errors such as forgetting an attribute or swapping two attributes can eventually be remapped to a combination of attribute renames, as with separate compilation, there is no way to determine if two similar identifiers were meant to represent two different attributes or a typing error made by the developer while writing the same attribute twice. For the second experiment, the problem of construct validity is less objective, as we arbitrarily determined a threshold of 10 minutes that some may consider either too large or too strict depending on their personal preference and development schedule. Eventually, we deemed this choice acceptable for two reasons:

- `nlgcheck` is not intended to be used on each compilation, but rather once or a few times a day, to ensure the correctness of the attribute grammar before performing a commit;
- the time limit was only relevant when the optimizations were not in place, whereas all experiments made with both optimizations were able to keep distant from the target by a large margin. Notice how this is true even for a rather complex language such as *neverlangJS*, and the only outlier is *neverlangExpressions*, which was originally developed in a deliberately complex manner—*e.g.*, by using several abstractions and attribute mappings—to stress the modularization capabilities offered by *NeverLang*. We believe that for most real development scenarios, such as domain-specific languages, the 10-minute target should never be reached, and possibly `nlgcheck` could even be included as a standard compilation phase—at least for languages in the 3 to 4-second range.

Internal validity [77]. Some of the experiments may be affected by our specific expertise with *NeverLang* and with languages developed by its means. To avoid this threat as much as possible, we included several different languages in the experimentation, making sure to include domain-specific languages, general-purpose languages, and sublanguages, each created by different developers and each preceding the existence of `nlgcheck`. This way, we should have avoided designing languages in such a way that specifically fits `nlgcheck` and its requirements. We also included results both with and without optimization to avoid any concerns about bias in the results.

External validity [77]. Even though we tried to design the static analysis approach and its proof in a general manner that should be suitable for any language workbench, it is undeniable that the idea for this work originated from a *NeverLang*-specific need, and the solution was then designed according to these needs. For instance, while we believe we adequately justified the advantages of separate compilation in §1, it must be said that separate compilation is far from common in language workbenches: *NeverLang* is, to the best of our knowledge, the only one that fully supports the compilation of each individual artifact separately. This should not be intended as a limitation of other language workbenches, but rather a design choice: most language workbenches reasonably opt for relaxing the requirement of separate compilation in favor of the guarantees provided by

joint compilation. We can only hope that this contribution encourages other language workbenches trying to support separate compilation to achieve the best of both worlds.

7 Related Work

The challenge of ensuring static correctness in AGs has been a cornerstone of language engineering since Knuth's seminal work [48]. However, as language workbenches have evolved toward modularity and separate compilation, a significant gap has emerged between structural completeness and operational safety. This section situates `nlgcheck` within the landscape of modular AG verification, advanced AG formalisms, and static analysis techniques.

Structural Verification vs. Operational Path-Sensitivity. The primary focus of traditional AG verification is Well-Definedness—ensuring that for any valid syntax tree, every attribute instance has a unique definition and no circular dependencies exist [28].

Modern systems like Silver [45, 71] and JastAdd [29] have pioneered *modular well-definedness analysis*. This allows independent modules to be verified in isolation by relying on explicit interfaces. While these tools are highly effective at ensuring structural integrity (i.e., that a definition exists in the codebase), they treat the internal logic of semantic actions as "black boxes."

In a framework like Neverlang, where semantic actions are implemented via imperative Java code and attribute access is mediated by dynamic maps, structural existence does not guarantee runtime safety. As noted in the evaluation of modular systems [45], there is a documented absence of tools that track fine-grained, path-dependent definedness. `nlgcheck` addresses this by shifting the focus from "is the attribute defined in the module?" to "is the attribute initialized before use along every possible execution path?" This path-sensitive data-flow analysis is what prevents the MissingAttr runtime error, which structural checks fundamentally overlook.

The Modularity Paradox: Flexibility vs. Static Safety. A recurring theme in the literature is the trade-off between the expressiveness of AGs and the ease of their verification.

Extensions such as reference attribute grammars [38] and higher-order attribute grammars [73] significantly increase the power of AGs by allowing the tree structure to be computed dynamically. However, systems implementing these (e.g., JastAdd) typically enforce strict static typing and require global knowledge of the tree structure to ensure safety [29].

Neverlang adopts a "feature-oriented" architecture designed for true separate compilation and binary distribution. This requires a loose coupling where attributes are not hard-coded offsets but dynamic entries in a map. While this maximizes modularity—allowing users to swap components without recompilation—it circumvents the safety nets of traditional compilers.

Mernik et al. [57] suggest that the power of domain-specific tools often necessitates a reliance on strict static discipline. `nlgcheck` serves as the essential compensatory mechanism for Neverlang's dynamic nature. By applying static analysis to the generated bytecode, it restores the safety guarantees of a strongly typed system to a highly decoupled, dynamic architecture.

Evolution of Modularity: From CAGs to Forwarding. The quest for reusability has led to several modular composition techniques:

Early work on CAGs [32] enabled the combination of smaller, independent AGs to solve complex subproblems (e.g., name analysis). Similarly, Generic Attribute Grammars [47] focused on decomposing AGs into reusable components.

This technique provides a mechanism for a node to "forward" attribute queries to a replacement subtree, aiding in modular extensibility [72].

While these approaches facilitate design-time modularity, they often struggle with deployment-time modularity (binary distribution). They typically require the entire grammar to be re-analyzed or re-compiled when modules are composed. `nlgcheck` is unique in that it validates the interaction of separately compiled binaries by analyzing the underlying data-flow of their semantic actions, ensuring that modular composition does not lead to runtime failures.

Data-Flow Analysis and Fixed-Point Iteration. The methodology of `nlgcheck` draws from classic compiler optimization and verification techniques [1].

Tools like the Extended Static Checker for Java (ESC/Java) [33] demonstrated that data-flow analysis could effectively catch null pointer exceptions and array-out-of-bounds errors in general-purpose code. `nlgcheck` applies these principles specifically to the domain of AGs.

Historically, Farrow used fixed-point evaluators to ensure value convergence in circular AGs [31]. `nlgcheck` repurposes fixed-point principles for state-space bounding. By utilizing the Soot framework [69] to analyze Java bytecode, `nlgcheck` performs an iterative analysis over the Control Flow Graph (CFG). To ensure termination in the presence of recursive semantic actions or complex loops, it implements a recursion depth bound, making the analysis of infinite execution paths computationally tractable.

8 Conclusions and Future Work

In this work, we presented a novel approach to overcome the limitations of separate compilation in language workbenches. Separate compilation offers several advantages regarding modularity and the distribution of pre-compiled artifacts through public repositories. However, this requirement is often relaxed to comply with other correctness guarantees and non-functional properties [52]. With this contribution, we aim to bridge the gap so that future language workbenches can adopt separate compilation without sacrificing safety guarantees. The static analysis approach presented in this paper is sound and effective in practice; moreover, additional optimizations make it usable on a daily basis during the development process.

Several improvements could expand this research. First, the approach should be extended beyond a single AST visit. The problem becomes significantly more complex when dealing with multiple traversals, as the state from the first traversal affects the second. Currently, the tool cannot handle this and resets the attribute set for each traversal. Other minor improvements include better error reporting, such as warnings for issues currently undetected by `nlgcheck`. In this regard, so-called “non-idiomatic” attribute accesses should either trigger a warning or, even better, be included as part of the analysis. Finally, this approach should be tested with other language workbenches to verify the degree to which the solutions proposed in this work are limited to `Neverlang` and its ecosystem.

Acknowledgments

A special thanks to Gianluca Nitti for his support in the development of `nlgcheck`.

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A Simplified verification model: postorder semantic actions

Algorithm 2 Verification of attribute accesses in postorder semantic actions execution

```

 $E := \emptyset$ 
for all  $p \in P$  do
  for  $i := 1$  to  $|p|$  do
    for all  $p' \in P$  do
      if  $p'[0] = p[i]$  then
        for all  $a \in p[i]$  do
          if  $\tau(p'[0].a)$  is not defined then
             $E := E \cup \{ \text{"} p' \text{ does not provide } a \text{ required by } p[i] \text{"} \}$ 
          else if not  $\tau(p'[0].a) \leq \tau(p[i].a)$  then
             $E := E \cup \{ \text{"type of provided } p'[0].a \text{ is incompatible with } p[i].a \text{"} \}$ 
          end if
        end for
      end if
    end for
  end for
end for
if  $E = \emptyset$  then
  verification succeeded
else
  incorrect attribute access(es) have been detected and collected in  $E$ 
end if

```

Before facing the complete verification model presented in this paper, we attempted solving the problem with a simplified model. We hereby show the limitations that ultimately lead us opting for a more complex solution. Let us consider the problem of verifying the correctness of attributes within a single semantic role in which the tree visit is performed in postorder: for each node, each of its child nodes is visited first by executing the semantic action associated with the production which generated the child node, then the same operation is repeated on the parent node itself. With this visit strategy, data can only be passed bottom-up in the tree (from child to parent node). Given this premise a semantic action attached to a production $p : X_0 \leftarrow X_1 \cdots X_n$ will read attributes from $X_1 \cdots X_n$, and then write attributes to X_0 . Although more complex behaviors are possible, we will consider semantic actions in which there is no reason to:

- read attributes on X_0 , as no semantic action which could have written them has been executed yet;
- write attributes to $X_1 \cdots X_n$ as no further semantic action will read them.

Given these restrictions, we can define that $\tau(p[0].a) = t$ if the semantic action attached to p provides an attribute a of type t , and $\tau(p[i].a) = t$ with $1 \leq i \leq |p|$ if the semantic action attached to p requires an attribute a of a type t' assignable from t ($t' \leq t$) on X_i .

Now, we can define the concept of *safe attribute access* in the scope of postorder visits. Let us consider a production $p : X_0 \leftarrow X_1 \cdots X_n$ and a semantic action attached to it which accesses the attribute $p[i].a$ (with $1 \leq i \leq n$), and with type requirement $t = \tau(p[i].a)$. We say this access is type safe iff.

$$\forall p' \in P : X_i \leftarrow Y_1 \cdots Y_n, \quad \tau(p'[0].a) \leq t$$

or, in other words, if for each production p' with X_i as its left-hand side, an attribute with the same name is provided by the semantic action attached to p' on its left-hand side nonterminal with the same type or a subtype of t .

This lead to a first, albeit limited, algorithm to verify the safety of attribute accesses, that works on semantic actions within a visit executed in postorder: the full pseudocode is shown in Algorithm 2.

A.1 Implementation aspects and limitations

An implementation of Algorithm 2 needs a way to gather information about attributes required and provided by the semantic action attached to each production for the role being analyzed. For Neverlang, there are two stages of the build process in which it is practical to access this information: during the DSL compilation, while Neverlang is performing a translation from the semantic action implementation DSL to the backend code, or after the entire compilation process by inspecting the bytecode through a bytecode analysis tool of choice. Both have been implemented in Neverlang. The former adds helper methods to the semantic action classes, returning information about required and provided attributes, in the form of index-name-type triplets; `nlgcheck` queries these methods to implement Algorithm 2.

It is important to note that this text-based approach has limitations due to it lacking any type information which is later generated by the Java compiler, and thus it requires additional syntax to properly generate the correct methods—e.g. `$0.value: Integer = ...` denotes the semantic action generating an attribute of `Integer` type. However, usage of this syntax extensions when writing semantic actions is optional, and when the developer does not provide any type hints, the attribute type is assumed to be `Object`. As we will discuss later in this section, a more “hands on” approach, inspecting the generated semantic actions bytecode directly, is preferable in general.

This approach represents a solution that was used in the early stages of this work and was successfully tested against simple languages such as the Java expression language[17], modified by adding the necessary type annotations. This allows for the language to be verified successfully, thus introducing errors in the form of incorrect attribute names and type casts or annotations.

However, such an approach has several limitations that ultimately caused us to deem it inadequate to be used in more concrete scenarios. First, the approach relies on the presence of type hints that the language developers need to manually write in the semantic actions: this requires modification and recompilation of existing modules to make them properly verifiable, and makes the code less readable due to redundant information (the Java compiler is aware of the type of each expression without needing type hints). This specific limitation does not regard the algorithm itself but rather its naive original implementation. A much more pressing matter regards the way information about required and provided attributes is extracted from semantic actions: the mere textual presence of an expression accessing an attribute is an imprecise indicator of these events actually happening at runtime. If the statement is, for example, inside a conditional block, its actual execution depends on runtime value of the condition, an aspect that cannot be captured by a simplified textual analysis. Extracting the information regarding required and provided attributes from the bytecode enables

the possibility of considering all possible execution paths. More specifically, if different paths provide the same attribute with different types, the resulting type of the attribute can be inferred as the closest common supertype. Similarly, the presence of one branch in which the attribute is not may cause indicate that the attribute is not provided in specific execution scenarios, even though the textual representation indicates otherwise. Finally, if different paths require the same attribute, but with different types, the closest common subtype must be considered. It is arguable that some of these scenarios are unlikely or may suggest improper language implementation, however please recall that this contribution aims at verifying that the composition among **precompiled** language features is feasible, even when they were not originally intended to be used jointly; therefore we believe unlikely scenarios should be taken into consideration.

Last but not least, limiting the solution to postorder visits is not a fair assumption: in any non-trivial language, semantic actions need full control with regards of how the visit must be performed. As a notable example, please consider a `while` statement in an interpreter for a GPL: a strictly postorder (or preorder) visit is not viable, because:

- the node representing the condition must be checked (and thus visited) on each iteration;
- the node representing the body may or may not be visited depending on the runtime value of the condition.

B Algorithms

This appendix contains any algorithms that were omitted from the main body of this manuscript for brevity.

B.1 Full CFG construction algorithm

Algorithm 3 shows the entire CFG construction algorithm for the complete model not limited to postorder visit.

For the sake of conciseness, this algorithm omits an implementation aspect involving semantic action guards: if multiple semantic actions with different guards are attached to the same production, their respective CFGs are merged in a single CFG representing all the possible branches which can be taken before running Algorithm 3. In case of type guards, edges connecting the entry node of the production CFGs to the entry nodes of the individual semantic actions are tagged with the required attribute types: this allows the visit algorithm to exclude paths which would never be followed at runtime because of the guards. Please refer to [20] for a full examination of guards and type guards in `NeverLang`.

B.2 Optimized CFG visit algorithm

Algorithm 1 shows the CFG visit algorithm, including the optimizations introduced in §4.5.

C Proofs

This appendix contains any proofs that were omitted from the main body of this manuscript for brevity.

LEMMA 1. *f_{pCtx} can be evaluated “piecewise” on a path by composing evaluations of f_{pCtx} on parts of the path:*

$$\forall w_1, w_2 \in V_r^*, C \in C, \quad f_{pCtx}(w_1 w_2, C) = f_{pCtx}(w_2, f_{pCtx}(w_1, C))$$

PROOF. This can be proven by induction on w_1 . The base case $w_1 = \epsilon$ is trivial, as by definition $f_{pCtx}(\epsilon, C) = C$ and the goal is reduced to an identity. For the inductive step, given w_1 , we assume

$$\forall w_2 \in V_r^*, C' \in C, \quad f_{pCtx}(w_1 w_2, C') = f_{pCtx}(w_2, f_{pCtx}(w_1, C')) \quad (6)$$

Algorithm 3 Construction of a role's CFG. $G_i = (V_i, E_i)$ denotes the CFG of the semantic action attached to the production p_i .

```

create two nodes  $v_{begin}, v_{end}$ 
 $a_{node}(v_{begin}) := a_{node}(v_{end}) := \text{Nop}()$ 
 $G_r = (V_r, E_r) := (\{v_{begin}, v_{end}\}, \emptyset)$ 
 $v_{entry}(G_r) := v_{begin}$ 
 $v_{exit}(G_r) := v_{end}$ 
for all  $p_i \in P$  do
   $V_r := V_r \cup V_i$ 
   $E_r := E_r \cup E_i$ 
  if  $p_i[0] = \text{Program}$  then
     $E_r := E_r \cup \{(v_{begin}, v_{entry}(G_i)), (v_{exit}(G_i), v_{end})\}$ 
  end if
end for
 $E_{entry} := \emptyset$ 
 $E_{exit} := \emptyset$ 
 $L := \emptyset$ 
for all  $p_i \in P$  do
  for all  $v_{beginEval} \in V_i : \exists k : a_{node}(v_{beginEval}) = \text{BeginEvalMetaAction}(k)$  do
    find  $v_{endEval} \in V_i$  such that  $(v_{beginEval}, v_{endEval}) \in E_i$ 
     $E_r := E_r \setminus \{(v_{beginEval}, v_{endEval})\}$ 
    for all  $p_j \in P : p_j[0] = p_i[k]$  do
      generate an unique tag  $l \notin L$ 
       $L := L \cup \{l\}$ 
       $e_{entry} := (v_{beginEval}, v_{entry}(G_j))$ 
       $e_{exit} := (v_{exit}(G_j), v_{endEval})$ 
       $E_{entry} := E_{entry} \cup \{e_{entry}\}$ 
       $E_{exit} := E_{exit} \cup \{e_{exit}\}$ 
       $l_{edge}(e_{entry}) := l_{edge}(e_{exit}) := l$ 
    end for
     $a_{node}(v_{beginEval}) := \text{EnterCtx}(k)$ 
     $a_{node}(v_{endEval}) := \text{LeaveCtx}()$ 
  end for
end for
 $E_r := E_r \cup E_{entry} \cup E_{exit}$ 

```

and we must prove that

$$\forall v \in V_r, w_2 \in V_r^*, C \in C, \quad f_{pCtx}(vw_1w_2, C) = f_{pCtx}(w_2, f_{pCtx}(vw_1, C))$$

By definition,

$$f_{pCtx}(vw_1w_2, C) = f_{pCtx}(w_1w_2, f_{aCtx}(C, a_{node}(v))) =$$

applying the inductive hypothesis (6) with $C' = f_{aCtx}(C, a_{node}(v))$,

$$= f_{pCtx}(w_2, f_{pCtx}(w_1, f_{aCtx}(C, a_{node}(v)))) =$$

and finally, applying once more the definition of f_{pCtx} , this time in “reverse” for the vw_1 path,

$$= f_{pCtx}(w_2, f_{pCtx}(vw_1, C))$$

Algorithm 4 Optimized visit of a role's CFG, with cache and jump logic (replaces Algorithm 1)

```

 $\Phi := \{(v_{entry}(G_r), v_{entry}(G_r), \epsilon, \langle \emptyset \rangle, \epsilon)\}$  ▷ Set of states to visit
 $\Gamma := \emptyset$  ▷ Set of visited states
 $U := \emptyset$  ▷ Found violations
 $f_c := \emptyset$  ▷ Cache for jumps
while  $\Phi \neq \emptyset$  do
  pick any  $(v, w, S, C, S_c) \in \Phi$  and remove it from  $\Phi$ 
  if  $(v, C, S) \notin \Gamma$  then ▷ Path condition (3)
     $\Gamma := \Gamma \cup (v, C, S)$ 
     $(C', U') := f_{action}(C, a_{node}(v))$ 
     $U := U \cup U'$ 
    for all  $(v, v') \in E_r$  do
       $S' := f_{edge}(S, (v, v'))$ 
      if  $\exists S'$  and  $\nexists l_1, l_2, l_3 \in S : l_1 = l_2 = l_3$  then ▷ Path validity and (2)
        if  $(v, v') \in E_{entry}$  then ▷ Then  $v'$  is a nonterminal entry
          if  $\exists f_c(v', head(C))$  then
             $(v_{out}, C_{outs}, C_{taken}) := f_c(v', head(C))$ 
            for all  $C_{out} \in C_{outs}$  do
               $\Phi := \Phi \cup \{(v_{out}, wv', S', merge_h(C, C_{out}), push(S_c, head(C)))\}$ 
            end for
             $f_c(v', head(C)) := (v_{out}, C_{outs}, C_{taken} \cup C)$ 
          else
             $\Phi := \Phi \cup \{(v', wv', S', C', push(S_c, head(C)))\}$ 
          end if
        else if  $(v, v') \in E_{exit}$  then ▷ Then  $v$  is a nonterminal exit
          find  $v_{in}$  such that  $\exists v_x : l_{edge}((v_x, v_{in})) = l_{edge}(v, v')$ 
          if  $\exists f_c(v_{in}, top(S_c))$  then
             $(v_{out}, C_{outs}, C_{taken}) := f_c(v', head(C))$ 
            if  $head(C) \notin C_{outs}$  then
              for all  $C_t \in C_{taken}$  do
                 $\Phi := \Phi \cup \{(v', wv', S', merge_h(C_t, head(C)))\}$ 
              end for
            end if
             $f_c(v', head(C)) := (v_{out}, C_{outs} \cup \{head(C)\}, C_{taken})$ 
          else
             $f_c(v_{in}, top(S_c)) := (v, \{head(C)\}, \emptyset)$ 
          end if
         $\Phi := \Phi \cup \{(v', wv', S', C', pop(S_c))\}$ 
      else
         $\Phi := \Phi \cup \{(v', wv', S', C', S_c)\}$ 
      end if
    end if
  end for
end if
end while

```

thus proving the property. \square

LEMMA 6. *Given an integer $k > 0$ and two production context stacks sharing the same top k items, the evaluation context stacks obtained by evaluating a balanced path prefix which does not contain *CopyAttr* nor *PropagateAttrs* actions share the top k items:*

$$\forall w \in W_{pb}, \forall C_1, C_2 \in \mathcal{C}, k \in \mathbb{N} \\ \text{top}_k(C_1) = \text{top}_k(C_2) \implies \text{top}_k(f_{pCtx}(w, C_1)) = \text{top}_k(f_{pCtx}(w, C_2))$$

PROOF. Due to the hypothesis $w \in W_{pb}$, the action sequence associated with w is either one of the following cases.

- If $w = \epsilon$, the implication is trivial by definition of f_{pCtx} , as $\forall C, f_{pCtx}(\epsilon, C) = C$.
- If $w = vw'$, with $w' \in W_{pb}$ and $a_{node}(v) = a$ with $a \in A \setminus \{\text{LeaveCtx}()\}$ then the implication is proved by induction, assuming

$$\forall C_1, C_2, k, \quad \text{top}_k(C_1) = \text{top}_k(C_2) \implies \text{top}_k(f_{pCtx}(w', C_1)) = \text{top}_k(f_{pCtx}(w', C_2)) \quad (7)$$

for a fixed w' and proving that

$$\forall C_1, C_2, k, \quad \text{top}_k(C_1) = \text{top}_k(C_2) \implies \text{top}_k(f_{pCtx}(vw', C_1)) = \text{top}_k(f_{pCtx}(vw', C_2))$$

Assuming $\text{top}(C_1) = \text{top}(C_2)$ and by definition of f_{pCtx} :

$$\text{top}_k(f_{pCtx}(w', f_{aCtx}(C_1, a))) = \text{top}_k(f_{pCtx}(w', f_{aCtx}(C_2, a))) \quad (8)$$

For $a \in \{\text{ReadAttr}, \text{CheckAttrType}, \text{Nop}\}$, it is true that $f_{aCtx}(C', a) = C' \forall C'$, thus by substituting $f_{aCtx}(C_1, a) = C_1$ and $f_{aCtx}(C_2, a) = C_2$ we obtain the inductive hypothesis (7). Otherwise either $a = \text{WriteAttr}(i, \eta, t)$ or $a = \text{EnterCtx}(i)$.

In the former case, applying the definition of f_{action} for *WriteAttr*, (8) becomes

$$\text{top}_k(f_{pCtx}(w', \text{put}(C_1, i, \eta, t))) = \text{top}_k(f_{pCtx}(w', \text{put}(C_2, i, \eta, t)))$$

Then, applying the definition of *put* on $C_1 = \langle c_1, \dots, c_n \rangle$ and $C_2 = \langle c'_1, \dots, c'_n \rangle$:

$$\text{top}_k(f_{pCtx}(w', \langle c_1, \dots, c_{\text{put}1}(c_{n-1}, k_n, i, \eta, t), c_{\text{put}2}(c_n, i, \eta, t) \rangle))) = \\ \text{top}_k(f_{pCtx}(w', \langle c'_1, \dots, c_{\text{put}1}(c'_{n-1}, k'_n, i, \eta, t), c_{\text{put}2}(c'_n, i, \eta, t) \rangle)))$$

Since $\text{top}_k(C_1) = \text{top}_k(C_2)$, $\forall i \in \mathbb{N} : k \leq i \leq n, \quad c_i = c'_i$, therefore, the arguments of the $f_{pCtx}(w', \dots)$ in the two members of (8) share the same top_k , proving the inductive step.

Finally, when $a = \text{EnterCtx}(i)$, (8) can be rewritten as

$$\text{top}_k(f_{pCtx}(w', \text{push}(C_1, (i, \emptyset)))) = \text{top}_k(f_{pCtx}(w', \text{push}(C_2, (i, \emptyset))))$$

By the definitions of *push* and top_k , and the hypothesis $\text{top}_k(C_1) = \text{top}_k(C_2)$, then $\text{top}_{k+1}(\text{push}(C_1, (i, \emptyset))) = \text{top}_{k+1}(\text{push}(C_2, (i, \emptyset)))$; by applying the inductive hypothesis (7) to $\text{push}(C_1, (i, \emptyset))$ and $\text{push}(C_2, (i, \emptyset))$:

$$\text{top}_{k+1}(f_{pCtx}(w', \text{push}(C_1, (i, \emptyset)))) = \text{top}_{k+1}(f_{pCtx}(w', \text{push}(C_2, (i, \emptyset))))$$

The equality of top_{k+1} trivially implies the equality of top_k , thus proving (8).

- If $w = w'v$, the proof is similar. The same inductive hypothesis holds. According to the definition of f_{pCtx} and Lemma 1 the goal $\text{top}_k(C_1) = \text{top}_k(C_2)$ can be rewritten as:

$$\text{top}_k(f_{pCtx}(w'v, C_1)) = \text{top}_k(f_{pCtx}(w'v, C_2)) \\ \text{top}_k(f_{aCtx}(f_{pCtx}(w', C_1), a)) = \text{top}_k(f_{aCtx}(f_{pCtx}(w', C_2), a))$$

Let $C_3 = f_{pCtx}(w', C_1)$ and $C_4 = f_{pCtx}(w', C_2)$; from the inductive hypothesis (7) follows that $\text{top}_k(C_3) = \text{top}_k(C_4)$. As seen in the previous case, only *WriteAttr* and *EnterCtx* can modify

the context stack and both actions preserve the equality of the top_k , thus from the definition of f_{action} follows that

$$top_k(f_{aCtx}(C_3, a)) = top_k(f_{aCtx}(C_4, a))$$

- If $w = v_1 w' v_2$ with $w' \in W_{pb}$, $a_{node}(v_1) = a_1 = EnterCtx(i)$, and $a_{node}(v_2) = a_2 = LeaveCtx()$. The inductive hypothesis (7) still holds and the goal is to prove that

$$top_k(f_{pCtx}(v_1 w' v_2, C_1)) = top_k(f_{pCtx}(v_1 w' v_2, C_2))$$

for any two $C_1 = \langle c_0, \dots, c_n \rangle$ and $C_2 = \langle c'_0, \dots, c'_n \rangle$ such that $top_k(C_1) = top_k(C_2)$. This goal can be rewritten as:

$$\begin{aligned} top_k(f_{aCtx}(f_{pCtx}(w', f_{aCtx}(C_1, a_1)), a_2)) &= top_k(f_{aCtx}(f_{pCtx}(w', f_{aCtx}(C_2, a_1)), a_2)) \\ top_k(pop(f_{pCtx}(w', push(C_1, (i, \emptyset)))))) &= top_k(pop(f_{pCtx}(w', push(C_2, (i, \emptyset)))))) \\ top_k(pop(f_{pCtx}(w', \langle c_0, \dots, c_n, (i, \emptyset) \rangle))) &= top_k(pop(f_{pCtx}(w', \langle c'_0, \dots, c'_n, (i, \emptyset) \rangle))) \end{aligned}$$

Since $top_k(C_1) = top_k(C_2)$, $c_i = c'_i \forall i \in \{k, \dots, n\}$, and thus

$$top_{k+1}(\langle c_0, \dots, c_n, (i, \emptyset) \rangle) = top_{k+1}(\langle c'_0, \dots, c'_n, (i, \emptyset) \rangle)$$

This satisfies the premise of the inductive hypothesis (7) (notice that it holds for any k , and $k + 1$ in particular), therefore

$$top_{k+1}(f_{pCtx}(\langle c_0, \dots, c_n, (i, \emptyset) \rangle)) = top_{k+1}(f_{pCtx}(\langle c'_0, \dots, c'_n, (i, \emptyset) \rangle))$$

Applying pop to each of the two stacks removes the topmost item preserving the rest; therefore even though the equality no longer holds for top_{k+1} , it still holds for top_k , thus proving the goal. \square

LEMMA 8. *Given any two production context stacks, evaluating a balanced path prefix which does not contain CopyAttr or PropagateAttrs actions, wrapped between two EnterCtx nodes produces context stacks with the same head.*

$$\begin{aligned} \forall C_1, C_2 \in C, w \in W_{pb}, v_{sep} \in V_r^* : \exists i : a_{node}(v_{sep}) = EnterCtx(i), \\ head(f_{pCtx}(v_{sep} w v_{sep}, C_1)) = head(f_{pCtx}(v_{sep} w v_{sep}, C_2)) \end{aligned} \quad (9)$$

PROOF. The lemma is proven by proving each of the following cases.

- If $w = \epsilon$, then the goal (9) becomes

$$head(f_{pCtx}(v_{sep} v_{sep}, C_1)) = head(f_{pCtx}(v_{sep} v_{sep}, C_2))$$

By definition of f_{pCtx} :

$$head(f_{aCtx}(f_{aCtx}(C_1, a_s), a_s)) = head(f_{aCtx}(f_{aCtx}(C_2, a_s), a_s)) \quad \text{with } a_s = EnterCtx(i)$$

By definition of f_{aCtx} and f_{action} for $EnterCtx$,¹¹

$$head(push(push(C_1, (i, \emptyset)), (i, \emptyset))) = head(push(push(C_2, (i, \emptyset)), (i, \emptyset)))$$

Let $C_1 = \langle c_0, \dots, c_n \rangle$ and $C_1 = \langle c'_0, \dots, c'_n \rangle$; by definition of $push$,

$$head(\langle c_0, \dots, c_n, (i, \emptyset), (i, \emptyset) \rangle) = head(\langle c'_0, \dots, c'_n, (i, \emptyset), (i, \emptyset) \rangle) =$$

which is proven by Lemma 2.

¹¹Notice that for brevity, we will keep using the equivalence $a_s = EnterCtx(i)$ throughout this proof.

- If $w = vw'$, with $w' \in W_{pb}$ and $a_{node}(v) = a$ with $a \in A \setminus \{LeaveCtx()\}$ then we must prove that, $\forall C_1, C_2 \in C$,

$$head(f_{pCtx}(v_{sep}vw'v_{sep}, C_1)) = head(f_{pCtx}(v_{sep}vw'v_{sep}, C_2)) \quad (10)$$

Let's expand both sides of the equality using a generic C instead of C_1 and C_2 by applying Lemma 1:

$$\begin{aligned} head(f_{pCtx}(v_{sep}vw'v_{sep}, C)) &= \\ head(f_{pCtx}(vw'v_{sep}, f_{pCtx}(v_{sep}, C))) &= \\ head(f_{pCtx}(v_{sep}, f_{pCtx}(vw', f_{pCtx}(v_{sep}, C)))) &= \end{aligned}$$

by definition of f_{pCtx} and then of f_{aCtx} for $EnterCtx(i)$,

$$\begin{aligned} &= head(f_{aCtx}(f_{pCtx}(vw', f_{aCtx}(C, a_s)), a_s)) = \\ &head(push(f_{pCtx}(vw', push(C, (i, \emptyset))), (i, \emptyset))) \end{aligned}$$

The goal (10) can therefore be rewritten as

$$\begin{aligned} head(push(f_{pCtx}(vw', push(C_1, (i, \emptyset))), (i, \emptyset))) &= \\ head(push(f_{pCtx}(vw', push(C_2, (i, \emptyset))), (i, \emptyset))) \end{aligned}$$

This can be proven through Lemma 2, showing that the last two items of the context stacks passed to $head$ in the two members of the equality are equal. By definition of $push$, the last item is (i, \emptyset) at both members; also by definition of $push$ we know the second-last item is the top of the argument of $push$, therefore we must prove that

$$top(f_{pCtx}(vw', push(C_1, (i, \emptyset)))) = top(f_{pCtx}(vw', push(C_2, (i, \emptyset))))$$

By definition, vw' is a balanced path prefix because it is formed by the concatenation of a balanced path prefix w' and a node v such that $a_{node}(v) \neq LeaveCtx()$. Moreover, $top(push(C_1, (i, \emptyset))) = top(push(C_2, (i, \emptyset)))$ by definition of top and $push$. Therefore, the goal is proven by applying Lemma 6 with $k = 1$, path vw' and context stacks $push(C_1, (i, \emptyset))$ and $push(C_2, (i, \emptyset))$.

- If $w = w'v$, the proof is identical to the previous case, as the considerations made on vw' can be made on $w'v$ as well.
- If $w = v_1w'v_2$ with $w' \in W_{pb}$, $a_{node}(v_1) = a_1 = EnterCtx(j)$, and $a_{node}(v_2) = a_2 = LeaveCtx()$:

$$head(f_{pCtx}(v_{sep}v_1w'v_2v_{sep}, C_1)) = head(f_{pCtx}(v_{sep}v_1w'v_2v_{sep}, C_2)) \quad (11)$$

Let us expand both members of the equality, using C in place of C_1 and C_2 respectively:

$$\begin{aligned} head(f_{pCtx}(v_{sep}v_1w'v_2v_{sep}, C)) &= \\ head(f_{aCtx}(f_{aCtx}(f_{pCtx}(w', f_{aCtx}(f_{aCtx}(C, a_s), a_1)), a_2), a_s)) &= \end{aligned}$$

recalling that $a_s = EnterCtx(i)$ and thus $f_{aCtx}(C', a_s) = push(C', (i, \emptyset))$,

$$= head(push(f_{aCtx}(f_{pCtx}(w', f_{aCtx}(push(C, (i, \emptyset))), a_1)), a_2), (i, \emptyset)) =$$

recalling $a_1 = EnterCtx(j)$ and $a_2 = LeaveCtx()$, thus $f_{aCtx}(C', a_1) = push(C', (j, \emptyset))$ and $f_{aCtx}(C', a_2) = pop(C')$,

$$= head(push(pop(f_{pCtx}(w', push(push(C, (i, \emptyset))), (j, \emptyset))), (i, \emptyset)))$$

With $C_1 = \langle c_0, \dots, c_n \rangle$ and $C_2 = \langle c'_0, \dots, c'_n \rangle$, the goal (11) can be rewritten as

$$\begin{aligned} \text{head}(\text{push}(\text{pop}(f_{pCtx}(w', \langle c_0, \dots, c_n, (i, \emptyset), (j, \emptyset) \rangle)), (i, \emptyset))) = \\ \text{head}(\text{push}(\text{pop}(f_{pCtx}(w', \langle c'_0, \dots, c'_n, (i, \emptyset), (j, \emptyset) \rangle)), (i, \emptyset))) \end{aligned}$$

Let $C_3 = \langle c_0, \dots, c_n, (i, \emptyset), (j, \emptyset) \rangle$ and $C_4 = \langle c'_0, \dots, c'_n, (i, \emptyset), (j, \emptyset) \rangle$. Then, by definition of top_k , $\text{top}_2(C_3) = \text{top}_2(C_4)$. Applying Lemma 6:

$$\text{top}_2(f_{pCtx}(w', C_3)) = \text{top}_2(f_{pCtx}(w', C_4))$$

Thus, we must prove that

$$\text{head}(\text{push}(\text{pop}(f_{pCtx}(w', C_3)), (i, \emptyset))) = \text{head}(\text{push}(\text{pop}(f_{pCtx}(w', C_4)), (i, \emptyset)))$$

Applying pop to $f_{pCtx}(w', C_3)$ and $f_{pCtx}(w', C_4)$ produces C_5 and C_6 respectively with $\text{top}(C_5) = \text{top}(C_6)$, reducing the goal to

$$\text{head}(\text{push}(C_5, (i, \emptyset))) = \text{head}(\text{push}(C_6, (i, \emptyset)))$$

which is trivially proven by definition of push and head and by the equality $\text{top}(C_5) = \text{top}(C_6)$. \square

Received 20 February 2007; revised 12 March 2009; accepted 5 June 2009