

Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

Federico Bruzzone,¹ PhD Student

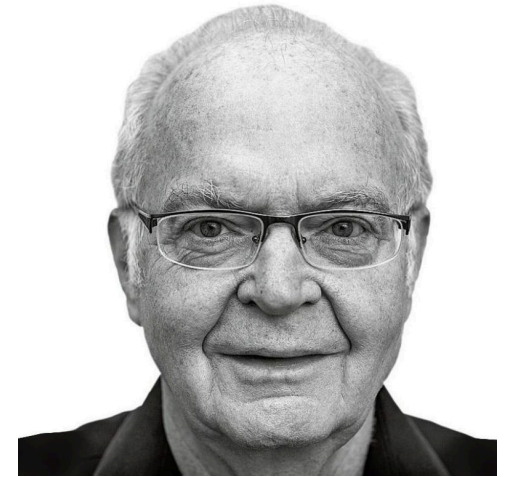
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Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.

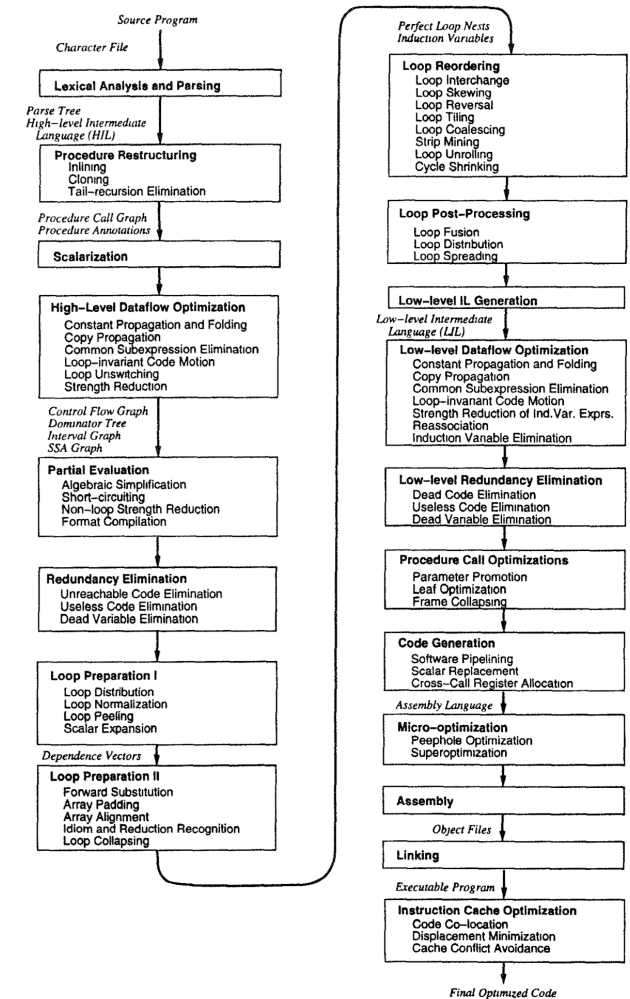


In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations² are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

²A *chronologically* sorted list of papers on compiler optimization, from the works of 1952 through the techniques of 1994, is available at [5].

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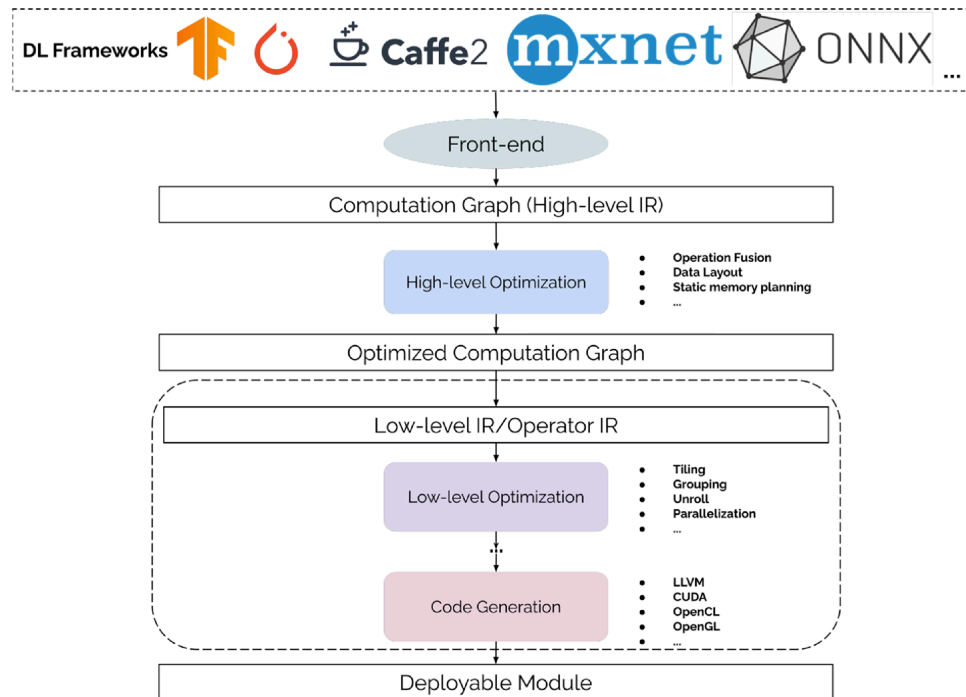
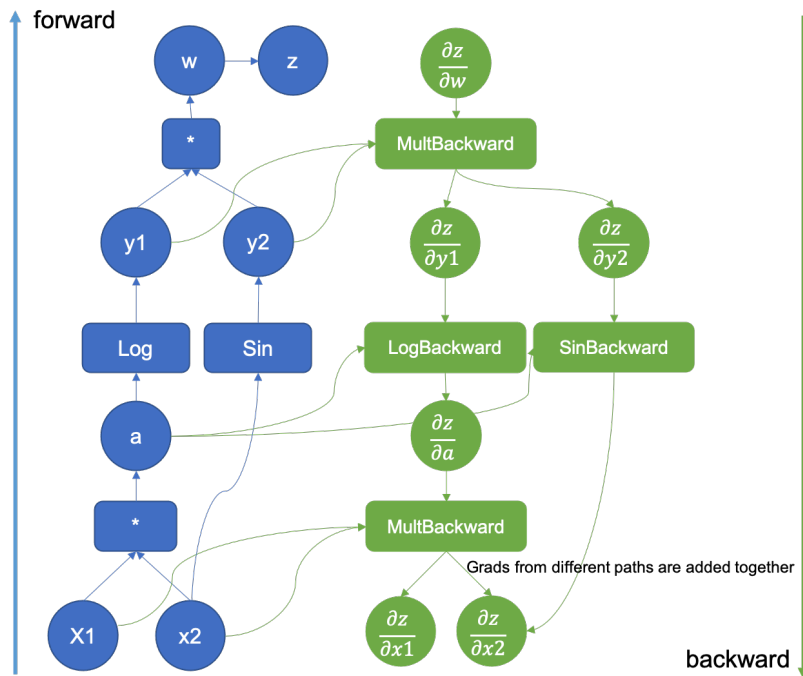
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Machine Learning Framework are Just Optimizing Compilers⁵



⁵TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use **LLVM** [9]. Li *et al.*, [10] compiled a survey on ML compilers.

Peephole Optimizations in x86-64 (cf. [11], [12])

```
1 ; x = x * 2, s.t. x: i32 asm
2 mov     eax, dword ptr [rbp - 4]
3 imul    eax, 2
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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

```
1 ; x = x << 1, s.t. x: i32    asm
2 mov     eax, dword ptr [rbp - 4]
3 shl     eax
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asm

```
1 ; x = x + 0, s.t. x: i32
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3 add     eax, 0
4 mov     dword ptr [rbp - 4], eax
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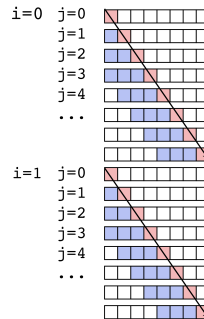
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```
1 mov     eax, dword ptr [rbp - 4]
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```

The mov instructions are redundant and can be **pruned** as well!

Loop Nest Optimizations — Loop Tiling (cf. [13], [14])

```
for (int i=0; i<n; ++i) {  
    for (int j=0; j<m; ++j) {  
        c[i][j] = a[i] * b[j];  
    }  
}
```

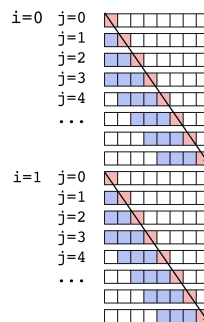


The vector **b** **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

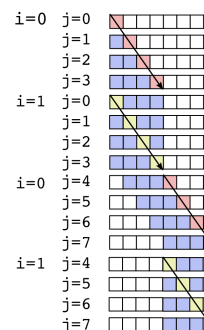
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    }  
}
```



The inner loop works on a **tile** of b that fits into the cache.

```
for (int jj = 0; jj < m; jj += TILE_SIZE) {  
    for (int i = 0; i < n; ++i)  
        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)  
            c[i][j] = a[i] * b[j];  
}
```

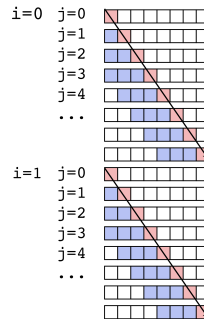


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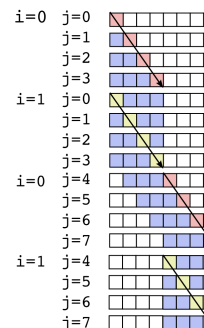
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        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)  
            c[i][j] = a[i] * b[j];  
}
```

Careful readers may notice that, in this version, the values for the array a will be read $m / \text{TILE_SIZE}$!



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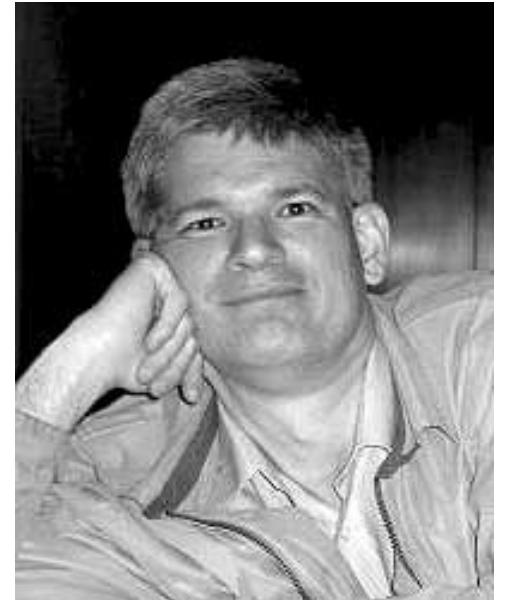
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```
for (int ii = 0; ii < n; ii += TILE_SIZE_I) {  
    for (int jj = 0; jj < m; jj += TILE_SIZE_J)  
        for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)  
            for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)  
                c[i][j] = a[i] * b[j];  
}
```


Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.



From Recursion to Iteration (cf. [18])

$$f(x) = \begin{cases} b(x_0) & \text{if } x = x_0 \\ a(x, f(d(x))) & \text{otherwise} \end{cases}$$

s.t. a, b , and so on may denote any pieces of code.

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To transform recursive function f into iterative form, we need to:

1. Identifies an increment \oplus to the argument of f , i.e., $x' = x \oplus y$ such that $x = \text{prev}(x')$, where prev is based on the arguments of the recursive call. In this case, $\text{prev}(x) = d(x)$ and, if d^{-1} exists, $x \oplus y = d^{-1}(x)$, can be plugged in for y .
2. Derives an incremental program $f'(x, r)$ that computes $f(x)$ using an accumulator r of $f(\text{prev}(x))$.
3. Forms an iterative version that initializes using the base case of f and iteratively applies f' until reaching the desired argument.

$$f(x) = \left\{ \begin{array}{l} x_1 = x_0; r = b(x_0); \\ \text{while } (x_1 \neq x) \{ \\ \quad x_1 = d^{-1}(x_1); \\ \quad r = a(x_1, r); \\ \} \\ \text{return } r; \end{array} \right\}$$

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Note that, when a is in the form $a(a_1(x), y)$ and a is associative, we do not need d^{-1} and x_1 .

Tail-recursive Factorial Function

```
int fact(int n) {  
    if (n == 0) {  
        return 1;  
    }  
    return n * fact(n - 1);  
}
```

The replacement of $n * ((n - 1) * (n - 2))$ by $(n * (n - 1)) * (n - 2)$ is valid due to the **associativity** of multiplication.

$$f(x) = \{$$
$$r = b(x_0);$$
$$\text{while } (x \neq x_0) \{$$
$$r = a(r, a_1(x));$$
$$x = d(x);$$
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$$\text{return } r;$$
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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“clang -O3 -S -emit-llvm fact.c -o -” produces the following LLVM IR:

```
define i32 @fact(i32 %n) {  
entry:  
    %cmp3 = icmp eq i32 %n, 0  
    br i1 %cmp3, label %return, label %if.else  
  
if.else:  
    %n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]  
    %acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]  
    %sub = add nsw i32 %n.tr5, -1  
    %mul = mul nsw i32 %n.tr5, %acc.tr4  
    %cmp = icmp eq i32 %sub, 0  
    br i1 %cmp, label %return, label %if.else  
  
return:  
    %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]  
    ret i32 %acc.tr.lcssa  
}
```

What About Fibonacci? To Hell With Multiple Recursions!

```
int fib(int n) {  
    if (n <= 1) {  
        return n;  
    }  
    return fib(n - 1) + fib(n - 2);  
}
```

The LLVM Optimized Version (but human readable)

```
int fib(int n) {  
    if (n < 2) {  
        return n;  
    }  
    int acc = 0;  
loop: /* while (1) { */  
    int call = fib(n - 1);  
    acc = call + acc;  
    if (n < 4) goto ret; /* return acc + (n - 2); */  
    n = n - 2;  
    goto loop; /* } */  
ret:  
    return acc + (n - 2);  
}
```

Note that, this LLVM IR is a fixed-point representation of the fib function; observable by the output of
“`opt -passes="default<03>" -S fib.ll -o -`” (it will produce the same IR as above).

```
define i32 @fib(i32 %n) {  
entry:  
    %cmp6 = icmp slt i32 %n, 2  
    br i1 %cmp6, label %return, label %if.end  
if.end:  
    %n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]  
    %accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]  
    %sub = add nsw i32 %n.tr8, -1  
    %call = tail call i32 @fib(i32 %sub)  
    %sub1 = add nsw i32 %n.tr8, -2  
    %add = add nsw i32 %call, %accumulator.tr7  
    %cmp = icmp samesign ult i32 %n.tr8, 4  
    br i1 %cmp, label %return, label %if.end  
return:  
    %accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]  
    %n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]  
    %accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa  
    ret i32 %accumulator.ret.tr  
}
```


**So, Is It Possible to
Incrementalize Functions
with Multiple Recursions?**

The Incrementalization of Y. A. Liu



In 1990, Liu *et al.* have done extensive research on **Incrementalization** [18], [19], [20], [21].

Even in presence of multiple recursions, in [18, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (slide “*From Recursion to Iteration*”).

But the Step 2. builds upon the principles of [19] and [20] — which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

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