Your Optimizing Compiler is not Optimizing Enough. To hell with Multiple Recursions!

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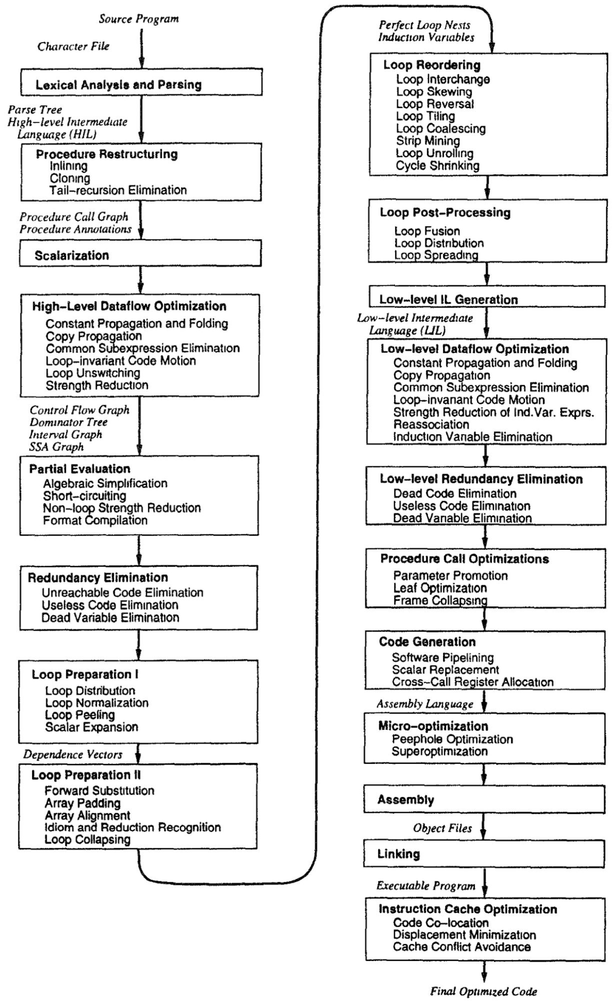
Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.

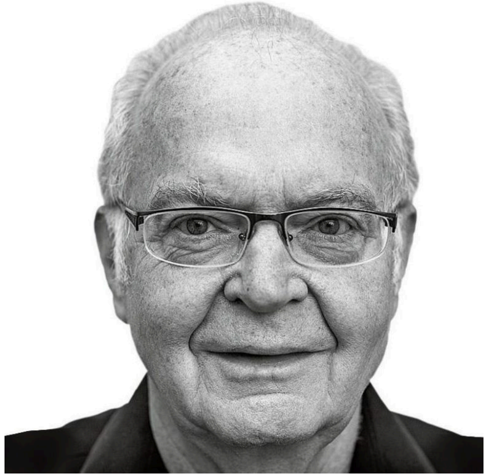


Bacon *et al.*, CSUR 1994 [1]

Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [2]:

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.



In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations are such transformations (say *meaning-preserving mappings* [5]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [6], [7], such compilers are referred to as **optimizing compilers**.

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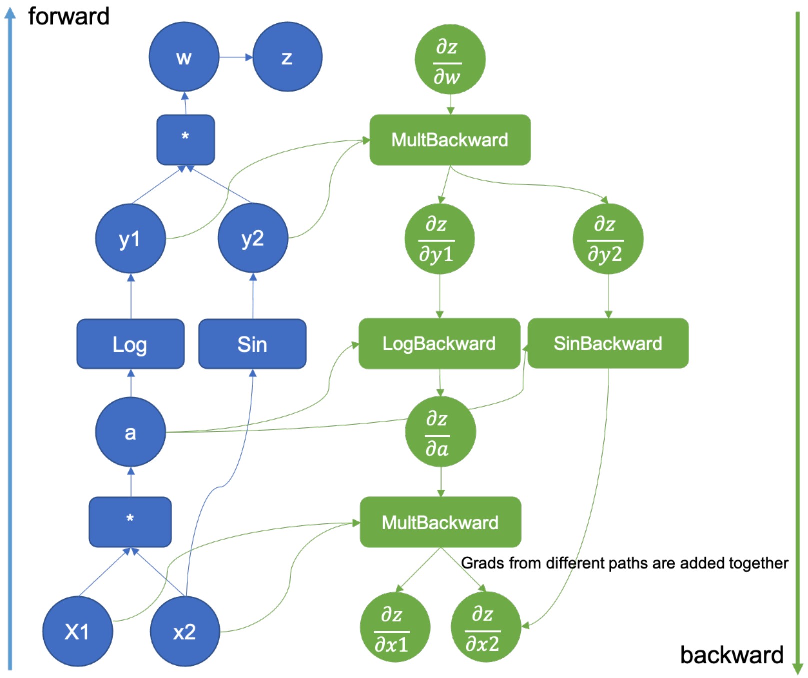
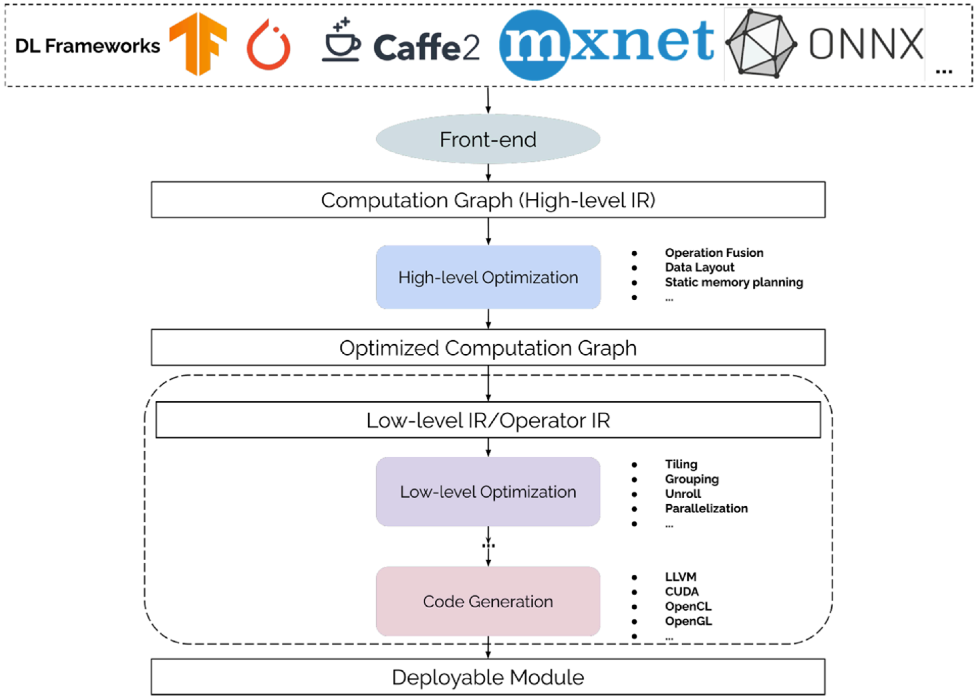
Machine Learning Framework are Just Optimizing Compilers[2]





[2] Li *et al.*, CSUR 2020, [8] compiled a comprehensive survey on deep learning compilers.

Peephole Optimizations in x86-64 (cf. [9], [10])

```asm
; x = x * 2  x: i32
mov     eax, dword ptr [rbp - 4]
imul    eax, 2
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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

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The `mov` instructions are redundant and can be **pruned** as well!

Loop Nest Optimizations – Loop Tiling (cf. [11], [12])

C++

```cpp
for (int i=0; i<n; ++i) {
  for (int j=0; j<m; ++j) {
    c[i][j] = a[i] * b[j];
  }
}
```

The vector `b` **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

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```cpp
int TS = 16; // Tile Size
for (int jj=0; jj<m; jj+=TS) {
  for (int i=0; i<n; ++i) {
      for (int j=jj; j<MIN(jj + TS, m); ++j) {
          c[i][j] = a[i] * b[j];
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}
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The inner loop works on a **tile** of b that fits into the cache.

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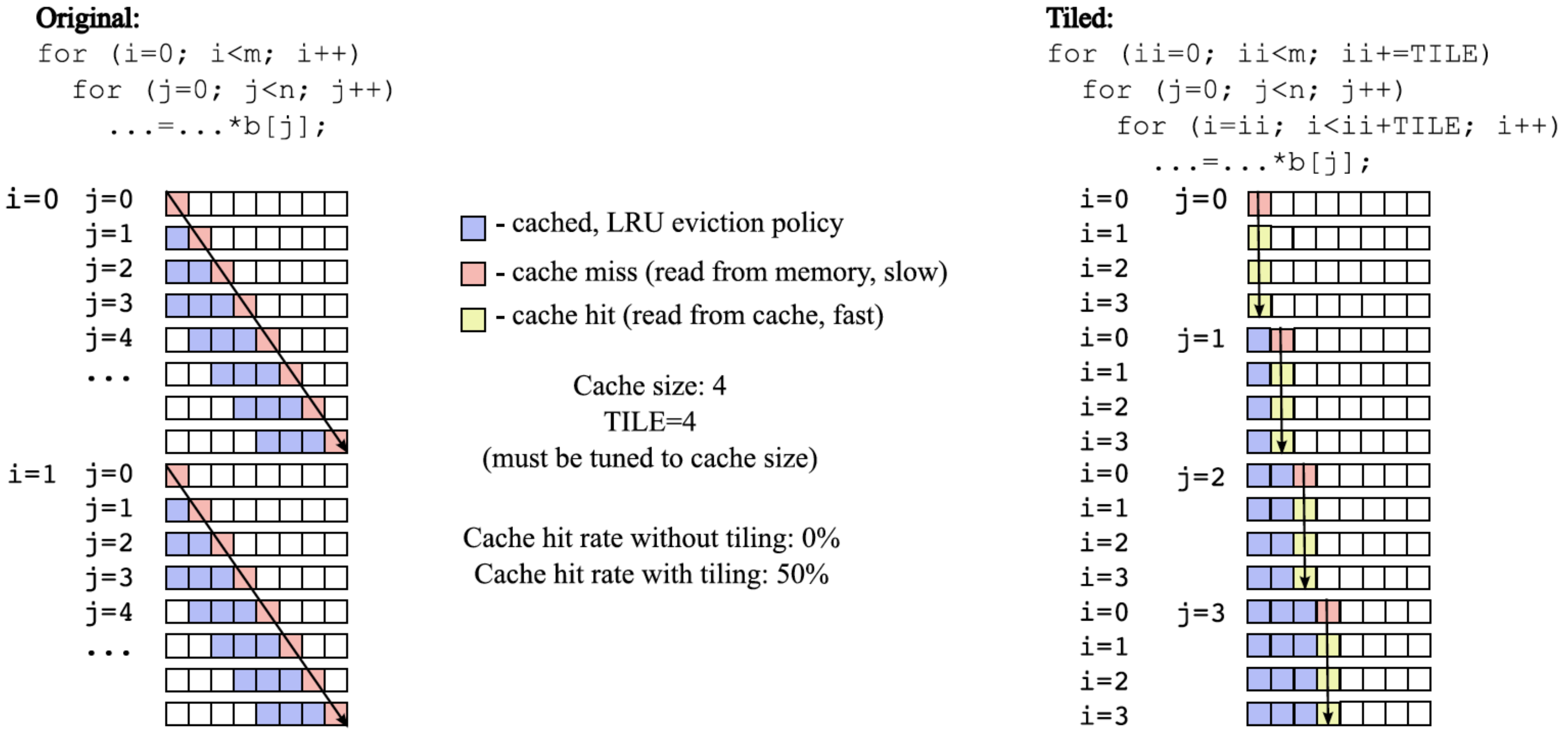
The inner loop works on a **tile** of b that fits into the cache.

But, the values for the array a will be read m / TS times!

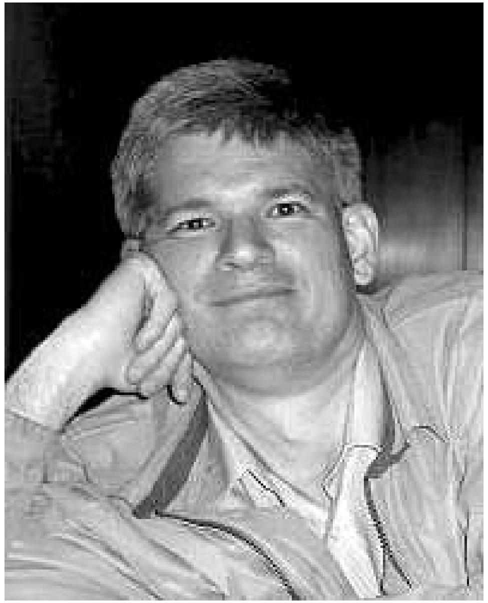
Loop Tiling Visualization



Tail Call/Recursion Optimization (cf. [3], [13], [14])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [15]:

In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.



From Recursive to Iterative Functions (cf. [16])

$$\boldsymbol{f}(x) = \begin{cases} b(x_0) \text{ if } x = x_0 \\ a(x, \boldsymbol{f}(d(x))) \text{ otherwise} \end{cases}$$

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To transform recursive function $\boldsymbol{f}$ into iterative form, we need to:

1. Identifies an increment $\oplus$ to the argument of $\boldsymbol{f}$, i.e., $x' = x \oplus y$ such that $x = \mathit{prev}(x')$, where prev is based on the arguments of the recursive call. In this case, $\mathit{prev}(x) = d(x)$ and, if d^{-1} exists, $x \oplus y = d^{-1}(x)$, can be plugged in for y.
2. Derives an incremental program $\boldsymbol{f}'(x, r)$ that computes $\boldsymbol{f}(x)$ using an accumulator r of $\boldsymbol{f}(\mathit{prev}(x))$.
3. Forms an iterative version that initializes using the base case of $\boldsymbol{f}$ and iteratively applies $\boldsymbol{f}'$ until reaching the desired argument.

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\begin{array}{l}
\boldsymbol{f}(x) = \{ \\
\quad x_1 = x_0; r = b(x_0); \\
\quad \text{while } (x_1 \neq x)\{ \\
\qquad x_1 = d^{-1}(x_1); \\
\qquad r = a(x_1, r); \\
\quad \} \\
\quad \text{return } r; \\
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\end{array}
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Note that, when a is in the form $a(a_1(x), y)$ and a is associative, we do not need d^{-1} and x_1.

Tail-recursive Factorial Function

```cpp
int fact(int n) {
    if (n == 0) {
        return 1;
    }
    return n * fact(n - 1);
}
```

The replacement of $n * ((n-1) * (n-2))$ by $(n * (n-1)) * (n-2)$ is valid due to the **associativity** of multiplication.

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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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```llvm
define i32 @fact(int)(i32 %n)  {
entry:
  %cmp3 = icmp eq i32 %n, 0
  br i1 %cmp3, label %return, label %if.else  ; label %if.else.preheader
; if.else.preheader -> vector.ph -> vector.body -> middle.block -> if.else.preheader7
if.else:
  %n.tr5 = phi i32 [ %sub, %if.else ], [ %n.tr5.ph, %if.else.preheader7 ]
  %acc.tr4 = phi i32 [ %mul, %if.else ], [ %acc.tr4.ph, %if.else.preheader7 ]
  %sub = add nsw i32 %n.tr5, -1
  %mul = mul nsw i32 %n.tr5, %acc.tr4
  %cmp = icmp eq i32 %sub, 0
  br i1 %cmp, label %return, label %if.else
return:
  %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %4, %middle.block ], [ %mul, %if.else ]
  ret i32 %acc.tr.lcssa
}
```

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