# Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

Federico Bruzzone,[1] PhD Student

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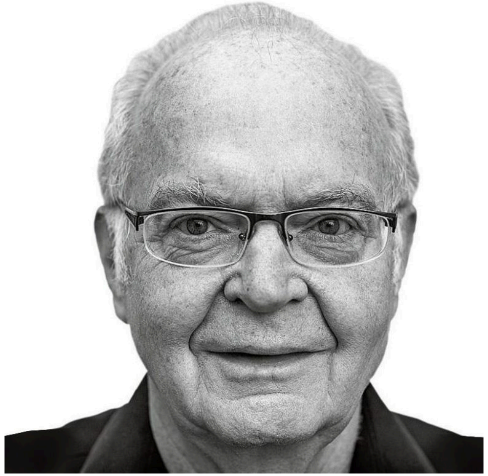
[1] ADAPT Lab – Università degli Studi di Milano,
Website: federicobruzzone.github.io,
Github: github.com/FedericoBruzzone,
Email: federico.bruzzone@unimi.it
Slides: federicobruzzone.github.io/activities/presentations/your-optimizing-compiler-is-not-optimizing-enough.pdf

# Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

*We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.*

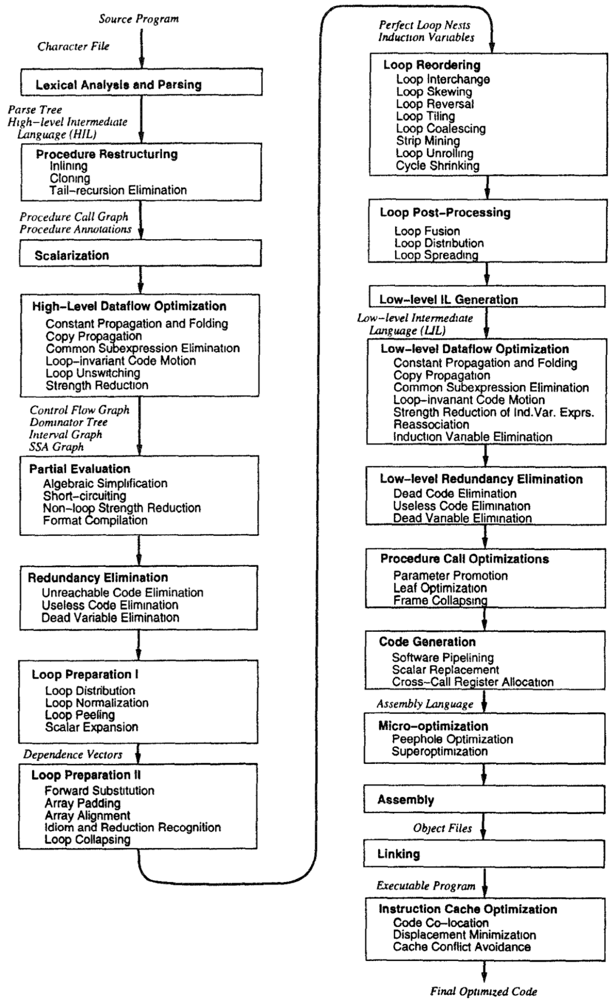


In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

# Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.




Bacon *et al.*, CSUR 1994 [2]

# Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

**Compiler optimizations**[2] are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

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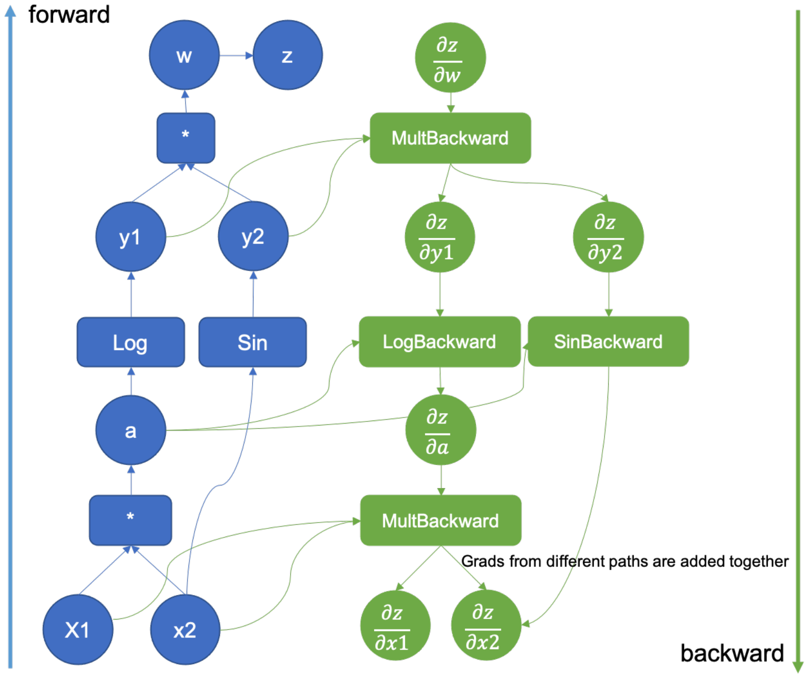
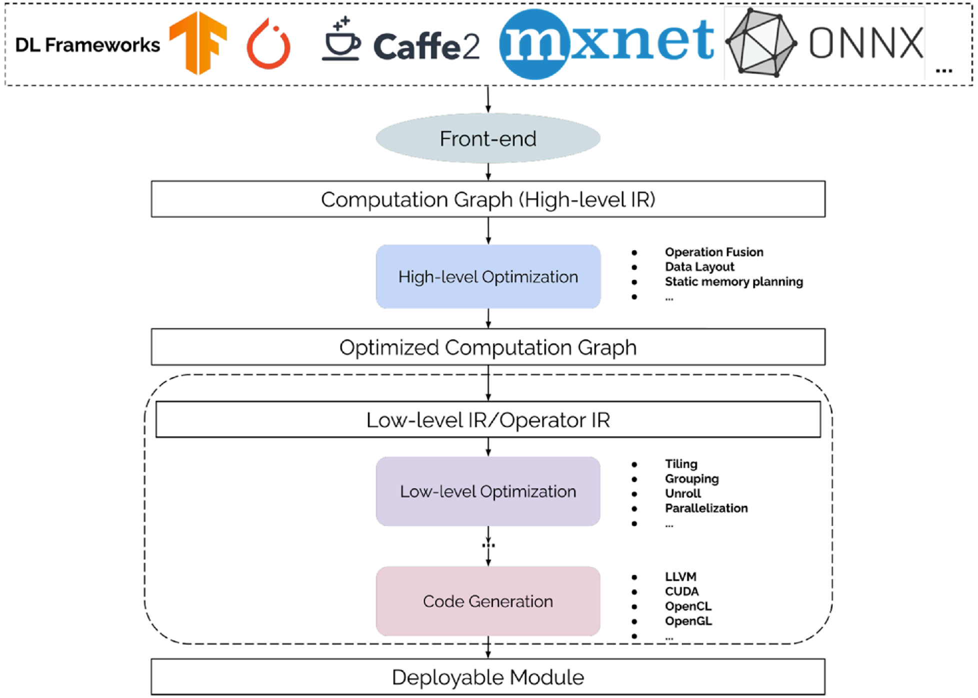
# Machine Learning Framework are Just Optimizing Compilers[5]




[5]TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use **LLVM** [9]. Li *et al.*, [10] compiled a survey on ML compilers.

# Peephole Optimizations in x86-64 (cf. [11], [12])

```asm
; x = x * 2, s.t. x: i32
mov     eax, dword ptr [rbp - 4]
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; x = x + 0, s.t. x: i32
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```
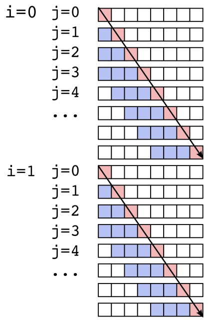

The `mov` instructions are redundant and can be **pruned** as well!

# Loop Nest Optimizations — Loop Tiling (cf. [13], [14])

```cpp
for (int i=0; i<n; ++i) {
  for (int j=0; j<m; ++j) {
      c[i][j] = a[i] * b[j];
  }
}
```




The vector `b` **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

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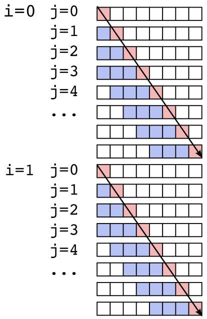


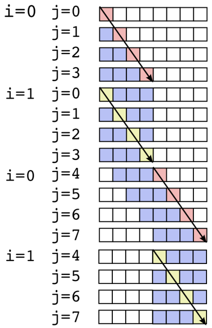


A. Vladimirov, Session 10

The inner loop works on a **tile** of b that fits into the cache.

```cpp
for (int jj = 0; jj < m; jj += TILE_SIZE)
    for (int i = 0; i < n; ++i)
        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
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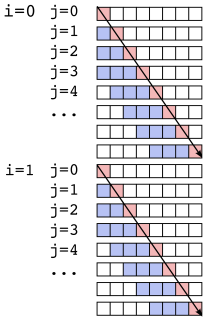


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      c[i][j] = a[i] * b[j];
```
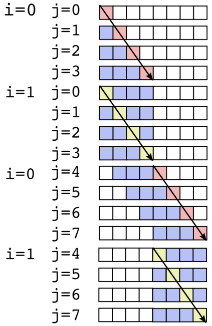

Careful readers may notice that, in this version, the values for the array a will be read m / TILE_SIZE!




```cpp
for (int ii = 0; ii < n; ii += TILE_SIZE_I)
 for (int jj = 0; jj < m; jj += TILE_SIZE_J)
  for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)
   for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)
    c[i][j] = a[i] * b[j];
```

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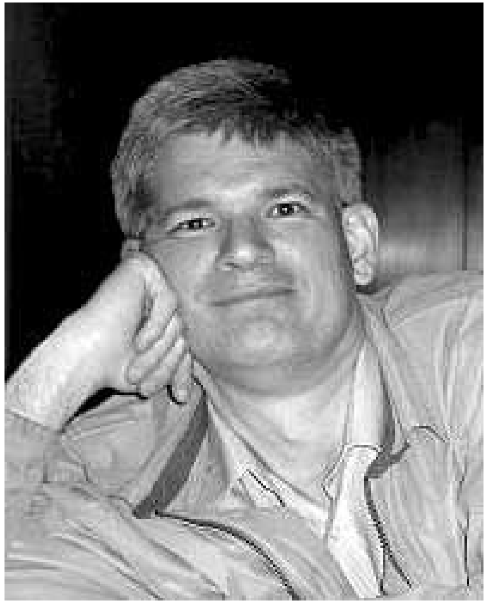
# Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

*In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.*



## From Recursion to Iteration (cf. [18])

$$\boldsymbol{f}(x) = \begin{cases} b(x_0) \text{ if } x = x_0 \\ a(x, \boldsymbol{f}(d(x))) \text{ otherwise} \end{cases}$$

$s.t.$ $a, b$, and so on may denote any pieces of code.

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To transform recursive function $\boldsymbol{f}$ into iterative form, we need to:

1. Identifies an increment $\oplus$ to the argument of $\boldsymbol{f}$, i.e., $x' = x \oplus y$ such that $x = prev(x')$, where $prev$ is based on the arguments of the recursive call. In this case, $prev(x) = d(x)$ and, if $d^{-1}$ exists, $x \oplus y = d^{-1}(x)$, can be plugged in for $y$.
2. Derives an incremental program $\boldsymbol{f}'(x, r)$ that computes $\boldsymbol{f}(x)$ using an accumulator $r$ of $\boldsymbol{f}(prev(x))$.
3. Forms an iterative version that initializes using the base case of $\boldsymbol{f}$ and iteratively applies $\boldsymbol{f}'$ until reaching the desired argument.

$\boldsymbol{f}(x) = \{$

$\quad x_1 = x_0; r = b(x_0);$

$\quad \text{while } (x_1 \neq x)\{$

$\quad\quad x_1 = d^{-1}(x_1);$

$\quad\quad r = a(x_1, r);$

$\quad \}$

$\quad \text{return } r;$

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\end{aligned}
$$

Note that, when $a$ is in the form $a(a_1(x), y)$ and $a$ is associative, we do not need $d^{-1}$ and $x_1$.

# Tail-recursive Factorial Function

```cpp
int fact(int n) {
    if (n == 0) {
        return 1;
    }
    return n * fact(n - 1);
}
```

The replacement of $n * ((n-1) * (n-2))$ by $(n * (n-1)) * (n-2)$ is valid due to the **associativity** of multiplication. In the general form:

$$
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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"`clang -O3 -S -emit-llvm fact.c -o -`" produces the following LLVM IR:

```llvm
define i32 @fact(i32 %n)  {
entry:
  %cmp3 = icmp eq i32 %n, 0
  br i1 %cmp3, label %return, label %if.else
if.else:
  %n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]
  %acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]
  %sub = add nsw i32 %n.tr5, -1
  %mul = mul nsw i32 %n.tr5, %acc.tr4
  %cmp = icmp eq i32 %sub, 0
  br i1 %cmp, label %return, label %if.else
return:
  %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]
  ret i32 %acc.tr.lcssa
}
```

# What About Fibonacci? To Hell With Multiple Recursions!

```cpp
int fib(int n) {
    if (n <= 1) {
        return n;
    }
    return fib(n - 1) + fib(n - 2);
}
```

### The LLVM Optimized Version (but human readable)

```cpp
int fib(int n) {
    if (n < 2) {
        return n;
    }
    int acc = 0;
loop: /* while (1) { */
        int call = fib(n - 1);
        acc = call + acc;
        if (n < 4) goto ret; /* return acc + (n - 2); */
        n = n - 2;
        goto loop; /* } */
ret:
    return acc + (n - 2);
}
```

Note that, this LLVM IR is a fixed-point representation of the `fib` function; observable by the output of "`opt -passes="default<O3>" -S fib.ll -o -`" (it will produce the same IR as above).

```llvm
define i32 @fib(i32 %n) {
entry:
  %cmp6 = icmp slt i32 %n, 2
  br i1 %cmp6, label %return, label %if.end
if.end:
  %n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]
  %accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]
  %sub = add nsw i32 %n.tr8, -1
  %call = tail call i32 @fib(i32 %sub)
  %sub1 = add nsw i32 %n.tr8, -2
  %add = add nsw i32 %call, %accumulator.tr7
  %cmp = icmp samesign ult i32 %n.tr8, 4
  br i1 %cmp, label %return, label %if.end
return:
  %accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]
  %n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]
  %accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa
  ret i32 %accumulator.ret.tr
}
```

# So, Is It Possible to Incrementalize Functions with Multiple Recursions?

# The Incrementalization of Y. A. Liu

In 1990, Liu *et al.* have done extensive research on **Incrementalization** [18], [19], [20], [21].

Even in presence of multiple recursions, in [18, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (slide "*From Recursion to Iteration*").

But the Step 2. builds upon the principles of [19] and [20] — which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

# Bibliography


[1] D. E. Knuth, “Structured Programming with go to Statements,” *ACM Comput. Surv.*, vol. 6, no. 4, pp. 261–301, Dec. 1974.

[2] D. F. Bacon, S. L. Graham, and O. J. Sharp, “Compiler transformations for high-performance computing,” *ACM Computing Surveys (CSUR)*, vol. 26, no. 4, pp. 345–420, 1994.

[3] K. D. Cooper and L. Torczon, *Engineering a Compiler*, no. . Morgan Kaufmann, 2022.

[4] K. Kennedy and J. R. Allen, *Optimizing compilers for modern architectures: a dependence-based approach*. Morgan Kaufmann Publishers Inc., 2001.

[5] F. Bruzzone, “Papers on Compiler Optimizations: Analysis and Transformations (1952-1994).” https://github.com/FedericoBruzzone/papers-on-compiler-optimizations, 2025.

[6] R. Paige, “Future directions in program transformations,” *SIGPLAN Not.*, vol. 32, no. 1, pp. 94–98, Jan. 1997.

[7] F. E. Allen, “Program Optimization,” *Annual Review of Automatic Programming*, vol. 5. Pergamon Press, pp. 239–307, 1966.

[8] W. A. Wulf, “The design of an optimizing compiler,” 1973.

[9] C. Lattner and V. Adve, “LLVM: A compilation framework for lifelong program analysis & transformation,” in *International symposium on code generation and optimization, 2004. CGO 2004.*, 2004, pp. 75–86.

[10] M. Li *et al.*, “The deep learning compiler: A comprehensive survey,” *IEEE Transactions on Parallel and Distributed Systems*, vol. 32, no. 3, pp. 708–727, 2020.

[11] W. M. McKeeman, “Peephole optimization,” *Commun. ACM*, vol. 8, no. 7, pp. 443–444, July 1965.

[12] A. S. Tanenbaum, H. van Staveren, and J. W. Stevenson, “Using Peephole Optimization on Intermediate Code,” *ACM Trans. Program. Lang. Syst.*, vol. 4, no. 1, pp. 21–36, Jan. 1982.

[13] M. Wolfe, “More iteration space tiling,” in *Proceedings of the 1989 ACM/IEEE Conference on Supercomputing*, in Supercomputing '89. Reno, Nevada, USA: Association for Computing Machinery, 1989, pp. 655–664.

[14] M. E. Wolf and M. S. Lam, “A data locality optimizing algorithm,” *SIGPLAN Not.*, vol. 26, no. 6, pp. 30–44, May 1991.

[15] A. V. Aho, R. Sethi, and J. D. Ullman, *Compilers: Principles, Techniques, and Tools,*. Reading, Massachusetts: Addison Wesley, 1986.

[16] S. Muchnick, *Advanced compiler design implementation*. Morgan kaufmann, 1997.

[17] G. L. Steele, “Debunking the “expensive procedure call” myth or, procedure call implementations considered harmful or, LAMBDA: The Ultimate GOTO,” in *Proceedings of the 1977 Annual Conference*, in ACM '77. Seattle, Washington: Association for Computing Machinery, 1977, pp. 153–162.

[18] Y. A. Liu and S. D. Stoller, “From recursion to iteration: what are the optimizations?,” in *Proceedings of the 2000 ACM SIGPLAN workshop on Partial evaluation and semantics-based program manipulation*, 1999, pp. 73–82.

[19] Y. A. Liu, S. D. Stoller, and T. Teitelbaum, “Static caching for incremental computation,” *ACM Transactions on Programming Languages and Systems (TOPLAS)*, vol. 20, no. 3, pp. 546–585, 1998.

[20] Y. A. Liu and T. Teitelbaum, “Systematic derivation of incremental programs,” *Science of Computer Programming*, vol. 24, no. 1, pp. 1–39, 1995.

[21] Y. A. Liu, “CACHET: An interactive, incremental-attribution-based program transformation system for deriving incremental programs,” in *Proceedings 1995 10th Knowledge-Based Software Engineering Conference*, 1995, pp. 19–26.