Your Optimizing Compiler is Not Optimizing Enough. To Hell With Multiple Recursions!

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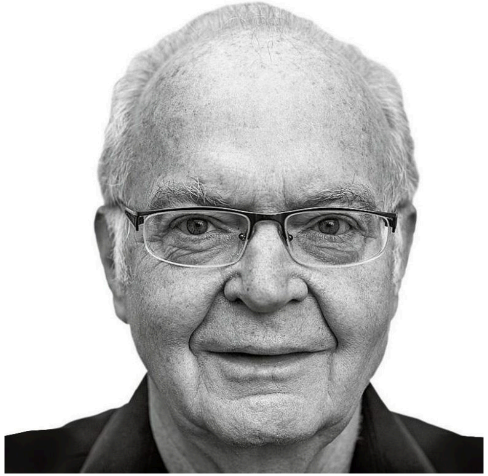
Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [1]:

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.

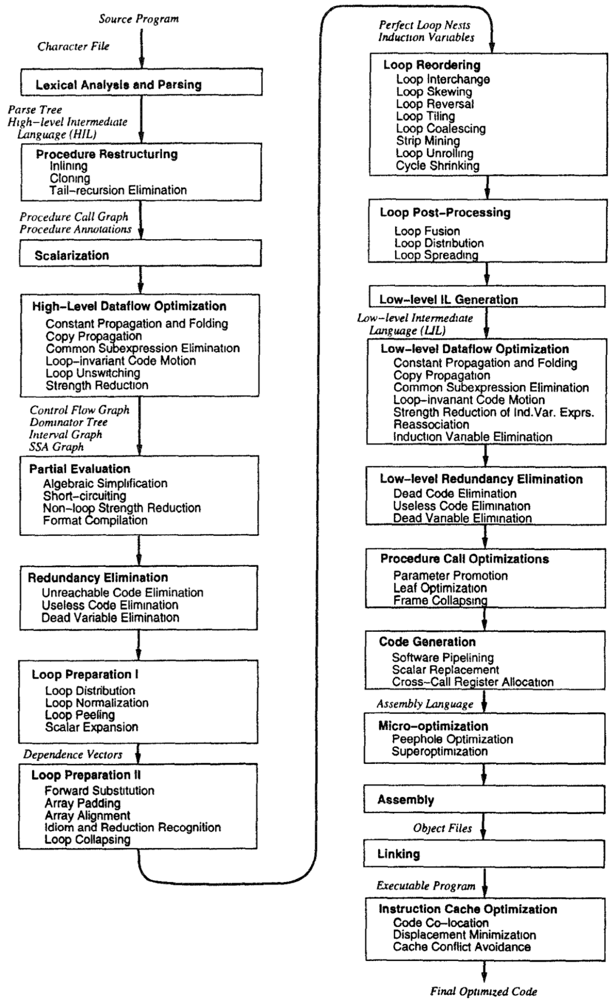


In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Bacon *et al.*, CSUR 1994 [2]

Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations[2] are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

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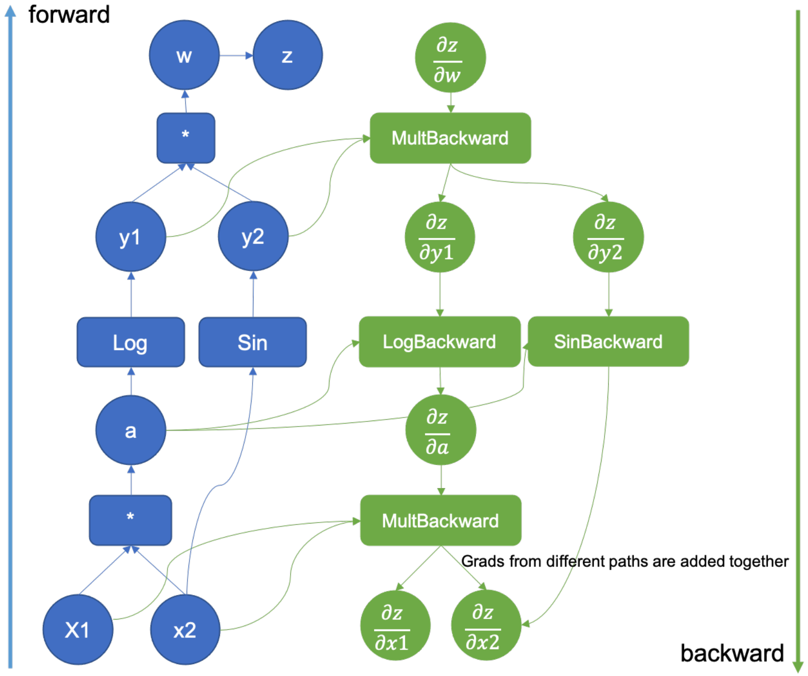
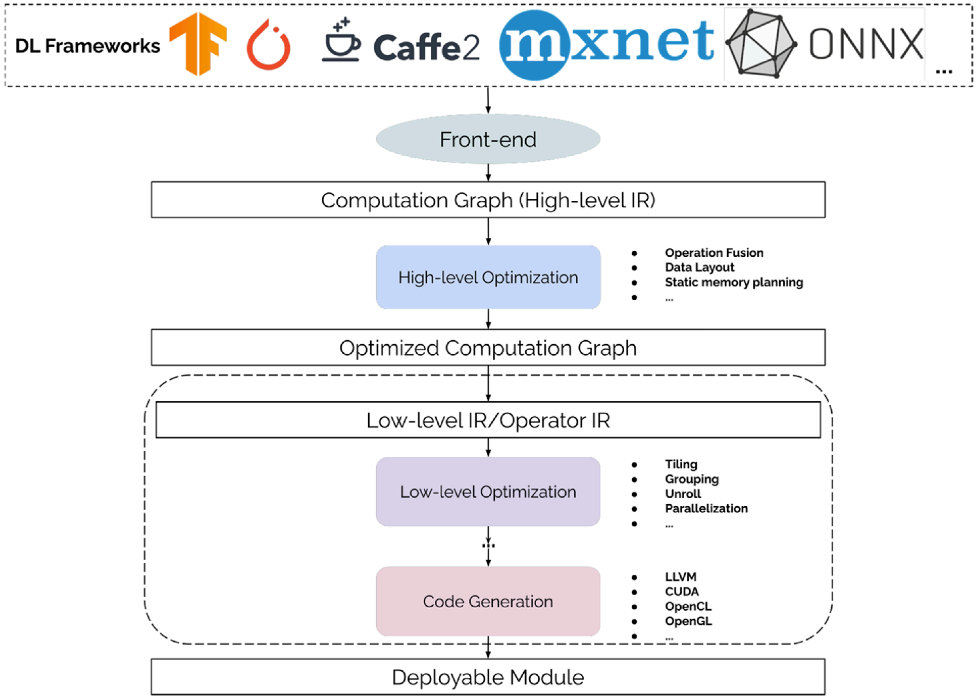
Machine Learning Framework are Just Optimizing Compilers[5]





[5] TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use **LLVM** [9]. Li *et al.*, [10] compiled a survey on ML compilers.

Peephole Optimizations in x86-64 (cf. [11], [12])

```asm
; x = x * 2, s.t. x: i32
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The optimized version replaces the multiplication by 2 with a **more efficient** binary shift operation.

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```asm
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```
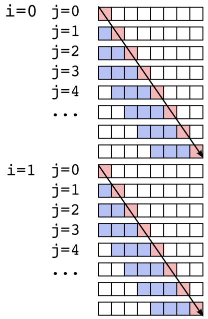

The `mov` instructions are redundant and can be **pruned** as well!

Loop Nest Optimizations — Loop Tiling (cf. [13], [14])

```cpp
for (int i=0; i<n; ++i) {
  for (int j=0; j<m; ++j) {
      c[i][j] = a[i] * b[j];
  }
}
```



The vector b **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

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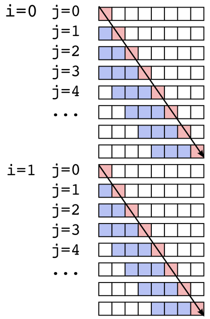

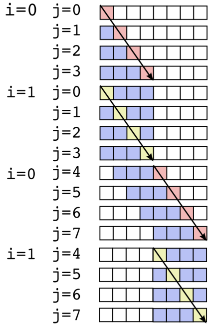

A. Vladimirov, Session 10

The inner loop works on a **tile** of b that fits into the cache.

```cpp
for (int jj = 0; jj < m; jj += TILE_SIZE)
    for (int i = 0; i < n; ++i)
        for (int j = jj; j < min(jj + TILE_SIZE, m); ++j)
            c[i][j] = a[i] * b[j];
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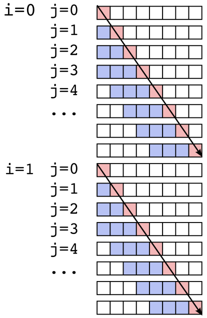

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```
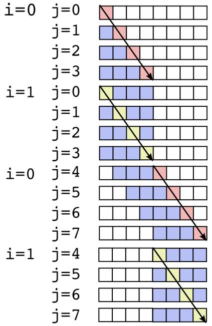

Careful readers may notice that, in this version, the values for the array a will be read m / TILE_SIZE!



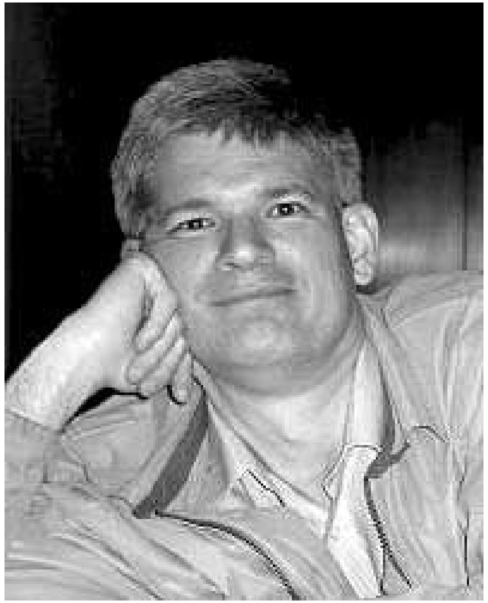

```cpp
for (int ii = 0; ii < n; ii += TILE_SIZE_I)
 for (int jj = 0; jj < m; jj += TILE_SIZE_J)
  for (int i = ii; i < MIN(n, ii + TILE_SIZE_I); i++)
   for (int j = jj; j < MIN(m, jj + TILE_SIZE_J); j++)
    c[i][j] = a[i] * b[j];
```

A. Vladimirov, Session 10

Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.



From Recursion to Iteration (cf. [18])

$$\boldsymbol{f}(x) = \begin{cases} b(x_0) \text{ if } x = x_0 \\ a(x, \boldsymbol{f}(d(x))) \text{ otherwise} \end{cases}$$

$s.t.$ a, b, and so on may denote any pieces of code.

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To transform recursive function $\boldsymbol{f}$ into iterative form, we need to:

1. Identifies an increment $\oplus$ to the argument of $\boldsymbol{f}$, *id est*, $x' = x \oplus y$ such that $x = prev(x')$, where *prev* is based on the arguments of the recursive call. In this case, $prev(x) = d(x)$ and, if d^{-1} exists, $x \oplus y = d^{-1}(x)$, can be plugged in for y.
2. Derives an incremental program $\boldsymbol{f}'(x, r)$ that computes $\boldsymbol{f}(x)$ using an accumulator r of $\boldsymbol{f}(prev(x))$.
3. Forms an iterative version that initializes using the base case of $\boldsymbol{f}$ and iteratively applies $\boldsymbol{f}'$ until reaching the desired argument.

$$
\begin{aligned}
&\boldsymbol{f}(x) = \{ \\
&\quad x_1 = x_0; r = b(x_0); \\
&\quad \textbf{while } (x_1 \neq x)\{ \\
&\qquad x_1 = d^{-1}(x_1); \\
&\qquad r = a(x_1, r); \\
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$\quad\quad x_1 = d^{-1}(x_1);$

$\quad\quad r = a(x_1, r);$

$\quad \}$

$\quad$ **return** $r;$

$\}$

Note that, when a is in the form $a(a_1(x), y)$ and a is associative, we do not need d^{-1} and x_1.

Tail-recursive Factorial Function

```
int fact(int n) {
    if (n == 0) {
        return 1;
    }
    return n * fact(n - 1);
}
```

C++

The replacement of $n * ((n-1) * (n-2))$ by $(n * (n-1)) * (n-2)$ is valid due to the **associativity** of multiplication. In the general form:

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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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"`clang -O3 -S -emit-llvm fact.c -o -`" produces something like:

The basic blocks related to loop vectorization (for performing SIMD operations) have been omitted for clarity

```llvm
define i32 @fact(i32 %n)  {
entry:
  %cmp3 = icmp eq i32 %n, 0
  br i1 %cmp3, label %return, label %if.else
if.else:
  %n.tr5 = phi i32 [ %sub, %if.else ], [ %n, %entry ]
  %acc.tr4 = phi i32 [ %mul, %if.else ], [ 1, %entry ]
  %sub = add nsw i32 %n.tr5, -1
  %mul = mul nsw i32 %n.tr5, %acc.tr4
  %cmp = icmp eq i32 %sub, 0
  br i1 %cmp, label %return, label %if.else
return:
  %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %mul, %if.else ]
  ret i32 %acc.tr.lcssa
}
```

What About Fibonacci? To Hell With Multiple Recursions!

```cpp
int fib(int n) {
    if (n <= 1) {
        return n;
    }
    return fib(n - 1) + fib(n - 2);
}
```

The LLVM Optimized Version (but human readable)

```cpp
int fib(int n) {
    if (n < 2) {
        return n;
    }
    int acc = 0;
loop: /* while (1) { */
        int call = fib(n - 1);
        acc = call + acc;
        if (n < 4) goto ret; /* return acc + (n - 2); */
        n = n - 2;
        goto loop; /* } */
ret:
    return acc + (n - 2);
}
```

Note that, the following LLVM IR is a fixed-point representation of the `fib` function; observable by the output of "`opt -passes="default<O3>" -S fib-O3.ll -o -`"

```llvm
define i32 @fib(i32 %n) {
entry:
  %cmp6 = icmp slt i32 %n, 2
  br i1 %cmp6, label %return, label %if.end
if.end:
  %n.tr8 = phi i32 [ %sub1, %if.end ], [ %n, %entry ]
  %accumulator.tr7 = phi i32 [ %add, %if.end ], [ 0, %entry ]
  %sub = add nsw i32 %n.tr8, -1
  %call = tail call i32 @fib(i32 %sub)
  %sub1 = add nsw i32 %n.tr8, -2
  %add = add nsw i32 %call, %accumulator.tr7
  %cmp = icmp samesign ult i32 %n.tr8, 4
  br i1 %cmp, label %return, label %if.end
return:
  %accumulator.tr.lcssa = phi i32 [ 0, %entry ], [ %add, %if.end ]
  %n.tr.lcssa = phi i32 [ %n, %entry ], [ %sub1, %if.end ]
  %accumulator.ret.tr = add nsw i32 %n.tr.lcssa, %accumulator.tr.lcssa
  ret i32 %accumulator.ret.tr
}
```

So your compiler is unable to *incrementalize* functions with multiple recursions?

Apparently, yes.

The Y. A. Liu's Incrementalization

In 1990, Liu *et al.* have done extensive research on **Incrementalization** [18], [19], [20], [21].

Even in presence of multiple recursions, in [18, Sect. 7], they proposed a **systematic** approach (*static analysis* and *semantic-preserving transformations*) to derive an incremental program following the three steps outlined earlier (cf. slide "*From Recursion to Iteration*").

But the Step 2. builds upon the principles of [19] and [20] — which, typically rely on user-provided knowledge or a theorem prover to derive the incremental program.

Conclusions

The papers are a little bit old-fashioned, and the key aspect of deriving the incremental program in presence of multiple recursions is **opaque**.

I wrote an email to Y.A. Liu asking for clarification last week, but I haven't received a reply yet … 🥲

Despite her work, we are trying to understand whether it is really possible to make the transformation automatically in a "general" setting.

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