

Your Optimizing Compiler is not Optimizing Enough. To hell with Multiple Recursions!

Federico Bruzzone,¹ PhD Student

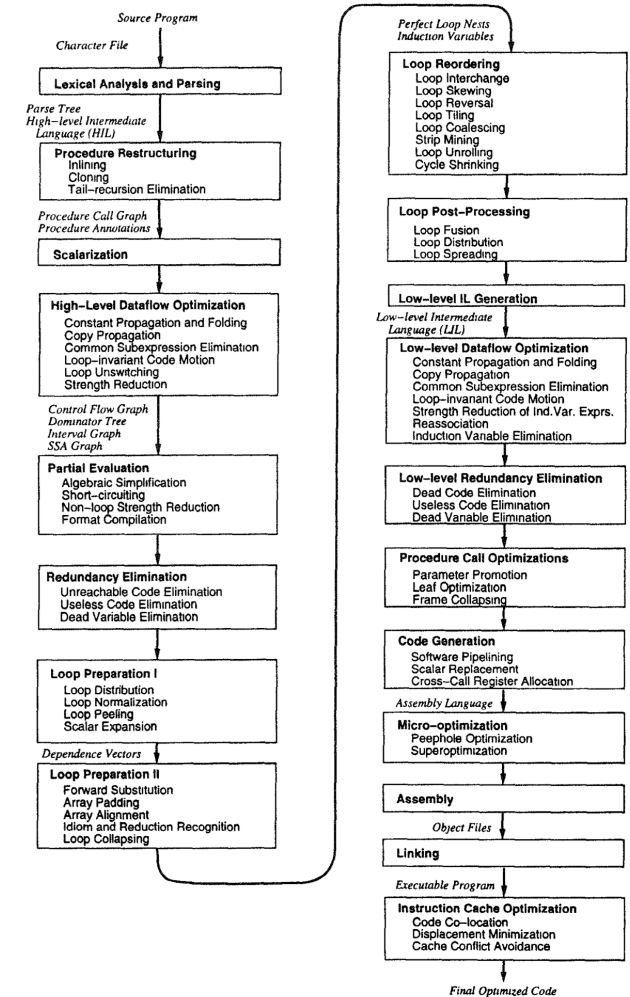
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Compilers as Musical Compositions

Compilers are frequently perceived as intricate musical compositions—like the unfinished *J. S. Bach's Art of Fugue*—where mathematical precision and logical interplay guide each part.

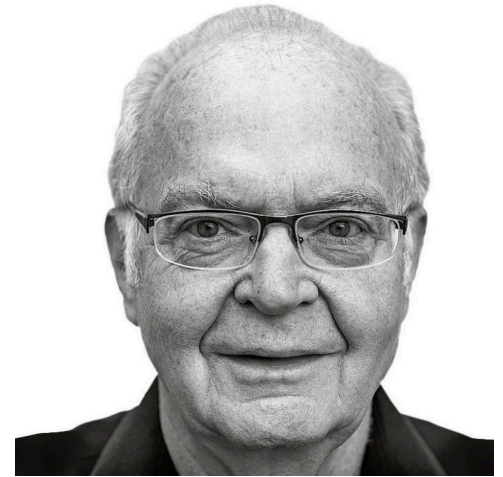
Every module enters in perfect timing, weaving together a structure that only the keenest ears can fully grasp.



Premature Optimizations

Donald E. Knuth warned in 1974 about the dangers of **premature optimization** in programming [2]:

We should forget about small efficiencies, say about 97% of the time: premature optimization is the root of all evil. Yet we should not pass up our opportunities in that critical 3%.



In the absence of either empirically measured or theoretically justified performance issues, programmers should **avoid** making optimizations based **solely** on assumptions about potential performance gains.

Optimizing Compilers

Compilers use information collected during analysis passes to guide transformations [3], [4].

Compiler optimizations² are such transformations (say *meaning-preserving mappings* [6]) applied to the input code to improve certain aspects—such as performance, resource utilization, and power consumption—without altering its observable behavior.

In accordance with the literature [7], [8], such compilers are referred to as **optimizing compilers**.

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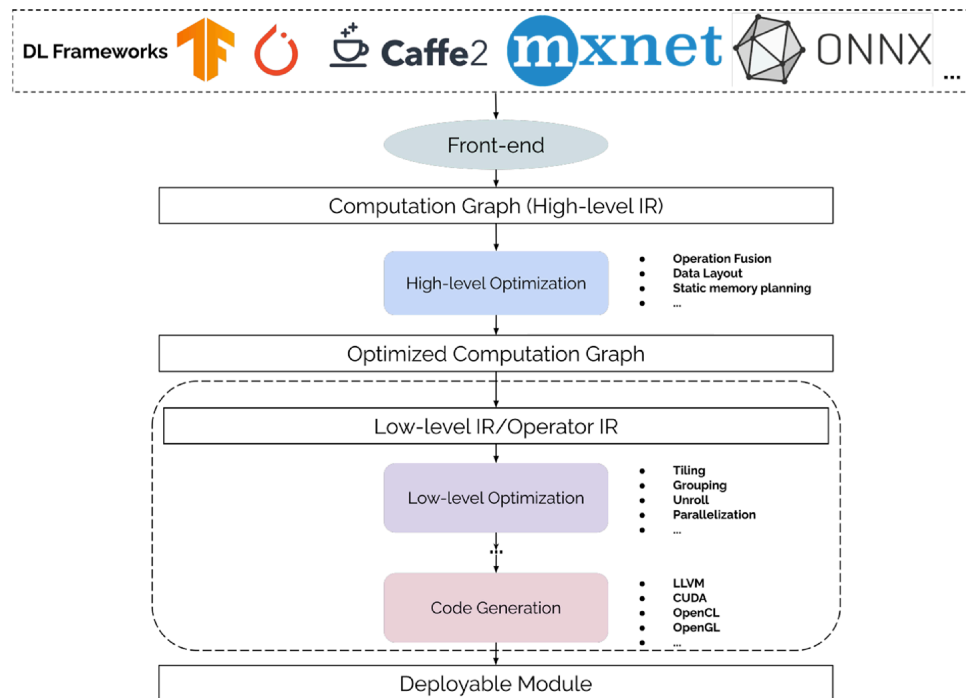
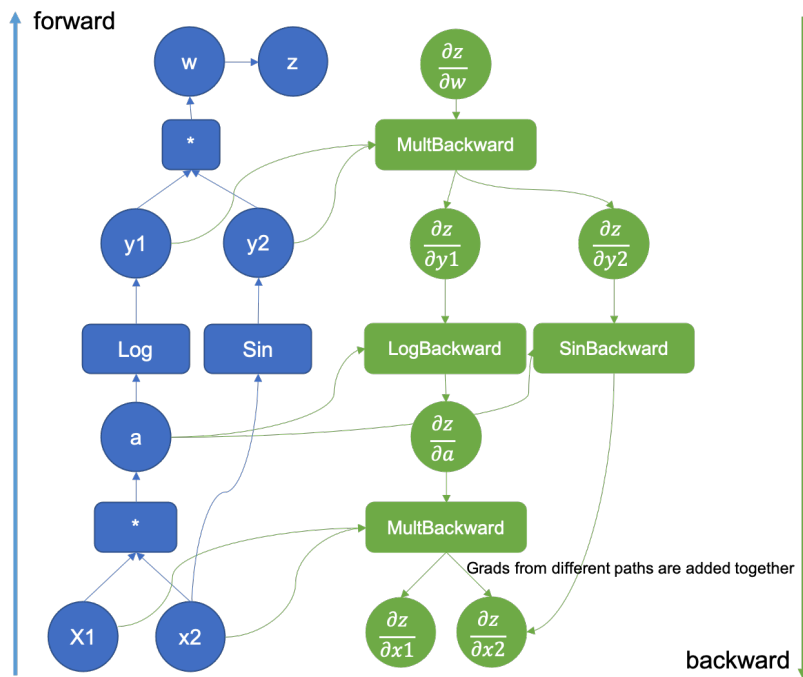
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Machine Learning Framework are Just Optimizing Compilers⁵



⁵TensorFlow XLA, NVIDIA CUDA Compiler (NVCC), MLIR, and TVM all use **LLVM** [9]. Li *et al.*, [10] compiled a survey on ML compilers.

Peephole Optimizations in x86-64 (cf. [11], [12])

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1 ; x = x * 2  x: i32  asm
2 mov     eax, dword ptr [rbp - 4]
3 imul    eax, 2
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3 add     eax, 0
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```

The mov instructions are redundant and can be **pruned** as well!

Loop Nest Optimizations — Loop Tiling (cf. [13], [14])

```
for (int i=0; i<n; ++i) {  
    for (int j=0; j<m; ++j) {  
        c[i][j] = a[i] * b[j];  
    }  
}
```

The vector **b** **may not** fit into a line of CPU cache, causing multiple cache misses during the inner loop.

It implies multiple **fetches** from the main memory, which is **slow**.

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```
int TS = 16; // Tile Size  
for (int jj=0; jj<m; jj+=TS) {  
    for (int i=0; i<n; ++i) {  
        for (int j=jj; j<MIN(jj + TS, m); ++j) {  
            c[i][j] = a[i] * b[j];  
        }  
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The inner loop works on a **tile** of **b** that fits into the cache.

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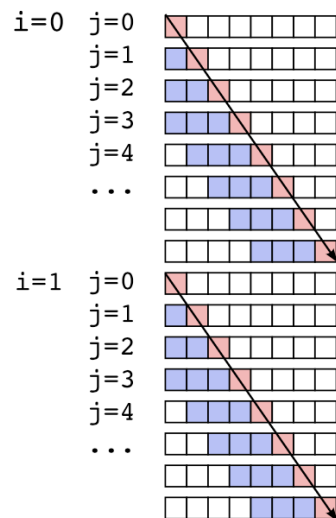
The inner loop works on a **tile** of **b** that fits into the cache.

But, the values for the array **a** will be read m / TS times!

Loop Tiling Visualization⁶

Original:

```
for (i=0; i<m; i++)  
  for (j=0; j<n; j++)  
    ...=...*b[j];
```



- - cached, LRU eviction policy
- - cache miss (read from memory, slow)
- - cache hit (read from cache, fast)

Cache size: 4

TILE=4

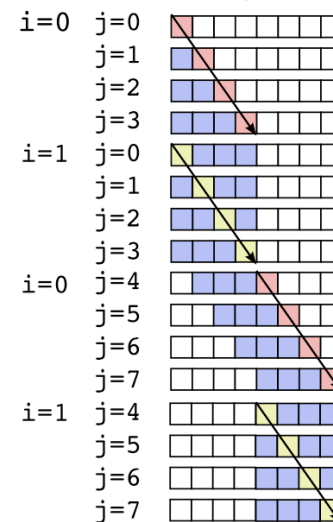
(must be tuned to cache size)

Cache hit rate without tiling: 0%

Cache hit rate with tiling: 50%

Tiled:

```
for (jj=0; jj<n; jj+=TILE)  
  for (i=0; i<m; i++)  
    for (j=jj; j<jj+TILE; j++)  
      ...=...*b[j];
```

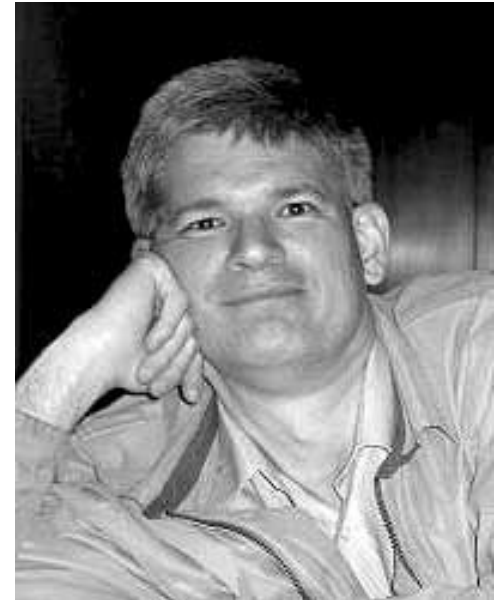


⁶[A. Vladimirov, Session 10 — Access to Caches and Memory](#)

Tail Call/Recursion Optimization (cf. [3], [15], [16])

Guy L. Steele, Jr. in 1977 observed that **tail-recursive procedure calls** can be optimized to avoid growing the call stack [17]:

In general, procedure calls may be usefully thought of as GOTO statements which also pass parameters, and can be uniformly coded as [machine code] JUMP instructions.



From Recursive to Iterative Functions (cf. [18])

$$f(x) = \begin{cases} b(x_0) & \text{if } x = x_0 \\ a(x, f(d(x))) & \text{otherwise} \end{cases}$$

s.t. a, b , and so on may denote any pieces of code.

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To transform recursive function f into iterative form, we need to:

1. Identifies an increment \oplus to the argument of f , i.e., $x' = x \oplus y$ such that $x = \text{prev}(x')$, where prev is based on the arguments of the recursive call. In this case, $\text{prev}(x) = d(x)$ and, if d^{-1} exists, $x \oplus y = d^{-1}(x)$, can be plugged in for y .
2. Derives an incremental program $f'(x, r)$ that computes $f(x)$ using an accumulator r of $f(\text{prev}(x))$.
3. Forms an iterative version that initializes using the base case of f and iteratively applies f' until reaching the desired argument.

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$$f(x) = \left\{ \begin{array}{l} x_1 = x_0; r = b(x_0); \\ \text{while } (x_1 \neq x) \{ \\ \quad x_1 = d^{-1}(x_1); \\ \quad r = a(x_1, r); \\ \} \\ \text{return } r; \end{array} \right\}$$

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Note that, when a is in the form $a(a_1(x), y)$ and a is associative, we do not need d^{-1} and x_1 .

Tail-recursive Factorial Function

```
int fact(int n) {  
    if (n == 0) {  
        return 1;  
    }  
    return n * fact(n - 1);  
}
```

The replacement of $n * ((n - 1) * (n - 2))$ by $(n * (n - 1)) * (n - 2)$ is valid due to the **associativity** of multiplication.

$$f(x) = \{$$
$$r = b(x_0);$$
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Note that, (i) when dealing with IEEE754 numbers, multiplication is **not** strictly associative, and (ii) the latter *might be* slower due to multiply bigger numbers.

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```
define i32 @fact(int)(i32 %n) {  
entry:  
    %cmp3 = icmp eq i32 %n, 0  
    br i1 %cmp3, label %return, label %if.else ; label %if.else.preheader  
; if.else.preheader -> vector.ph -> vector.body -> middle.block -> if.else.preheader7  
if.else:  
    %n.tr5 = phi i32 [ %sub, %if.else ], [ %n.tr5.ph, %if.else.preheader7 ]  
    %acc.tr4 = phi i32 [ %mul, %if.else ], [ %acc.tr4.ph, %if.else.preheader7 ]  
    %sub = add nsw i32 %n.tr5, -1  
    %mul = mul nsw i32 %n.tr5, %acc.tr4  
    %cmp = icmp eq i32 %sub, 0  
    br i1 %cmp, label %return, label %if.else  
return:  
    %acc.tr.lcssa = phi i32 [ 1, %entry ], [ %4, %middle.block ], [ %mul, %if.else ]  
    ret i32 %acc.tr.lcssa  
}
```

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