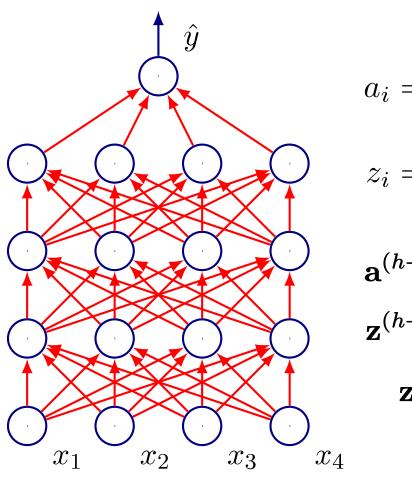




### Fully Connected Networks



$$a_i = \sum_{j \prec i} w_{i,j} z_j$$

$$z_i = f(a_i)$$

$$\mathbf{a}^{(h+1)} = \mathbf{W}^{(h)} \mathbf{z}^{(h)}$$

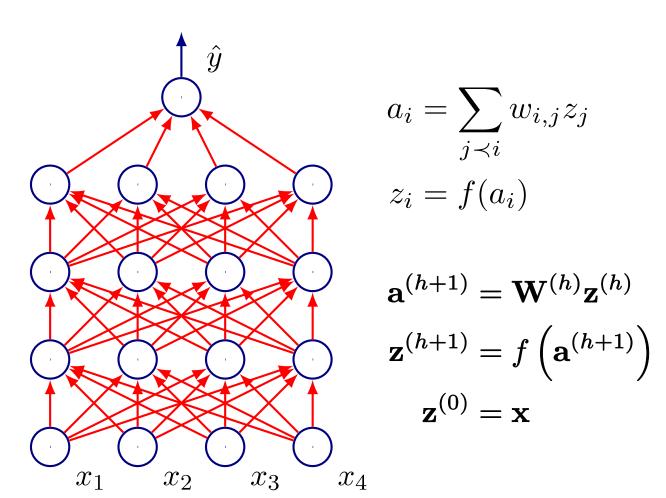
$$\mathbf{z}^{(h+1)} = f\left(\mathbf{a}^{(h+1)}\right)$$

$$\mathbf{z}^{(0)} = \mathbf{x}$$

2023/2024



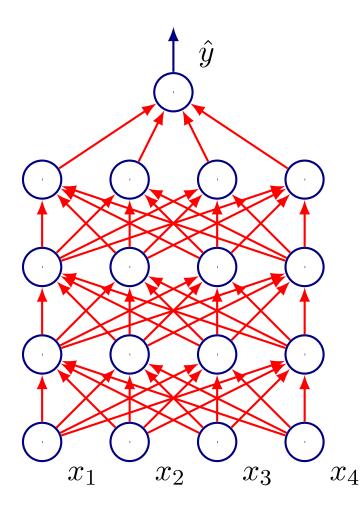
### Fully Connected Networks



- ▶ Each element is connected to the other, so we have
  - $\blacktriangleright$  (5\*4) + (5\*4) + (5\*4) + (5\*4) + 5 connections



#### Fully Connected Networks



$$a_i = \sum_{j \prec i} w_{i,j} z_j$$

$$z_i = f(a_i)$$

$$\mathbf{a}^{(h+1)} = \mathbf{W}^{(h)} \mathbf{z}^{(h)}$$
 $\mathbf{z}^{(h+1)} = f\left(\mathbf{a}^{(h+1)}\right)$ 
 $\mathbf{z}^{(0)} = \mathbf{x}$ 

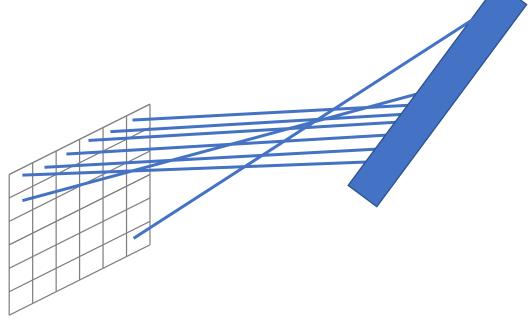
- ▶ Each element is connected to the other, so we have
  - $\blacktriangleright$  (5\*4) + (5\*4) + (5\*4) + (5\*4) + 5 connections
- For a generic network with *k* layers and size  $d_h$  for each layer h, we have

$$\sum_{h=1}^{k} d_h \cdot d_{h-1}$$



#### Further Issues with Images

- ▶ The «flat» approach is not suited to learn
  - Spatial patterns
  - Spatial hierarchies

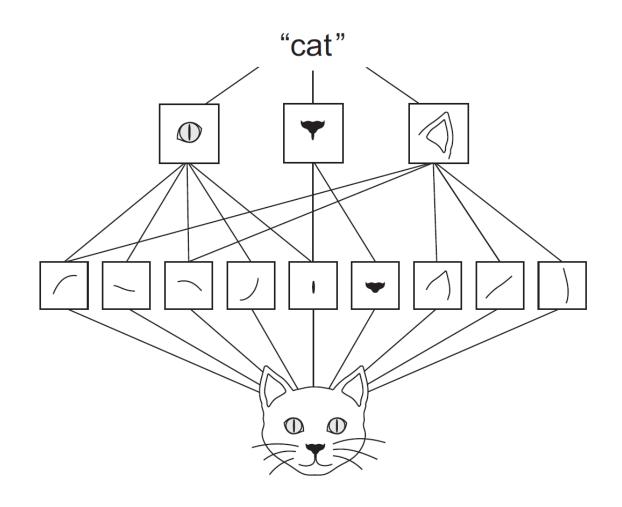


2023/2024

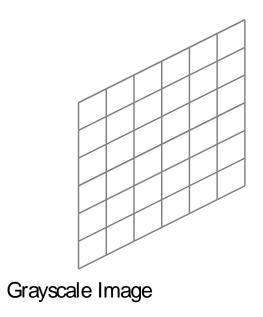


### Example of patterns and hierarchies

- ▶ How to identify such patterns?
- ▶ How to generalize them?

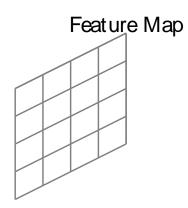




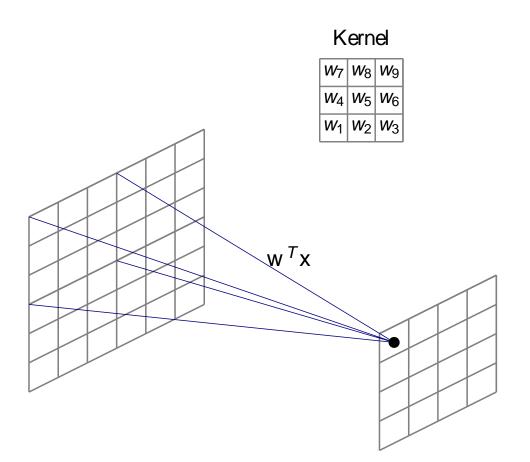


#### Kernel

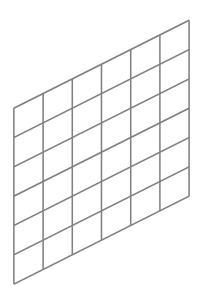
<i>W</i> <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
<i>W</i> <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
<i>W</i> <sub>1</sub>	<i>W</i> <sub>2</sub>	<b>W</b> 3





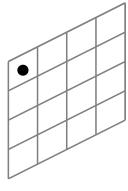




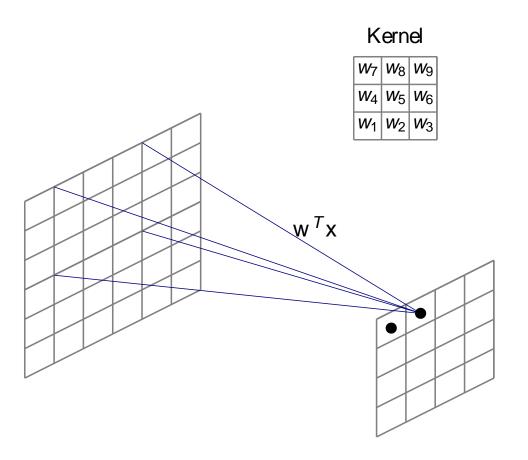


#### Kernel

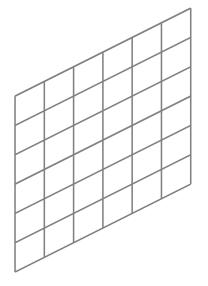
W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
W <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
W <sub>1</sub>	<b>W</b> <sub>2</sub>	<b>W</b> 3





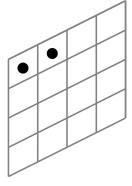




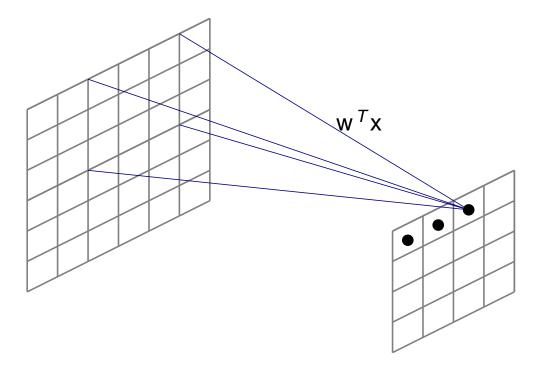


#### Kernel

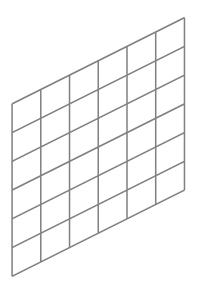
W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
<i>W</i> <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
<i>W</i> <sub>1</sub>	<i>W</i> <sub>2</sub>	<i>W</i> <sub>3</sub>





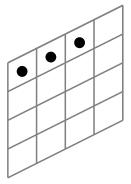




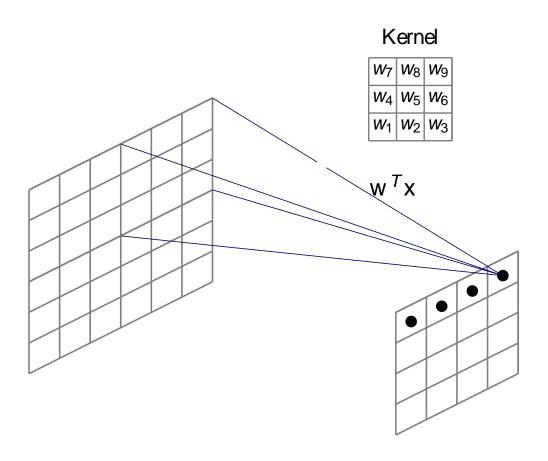


#### Kernel

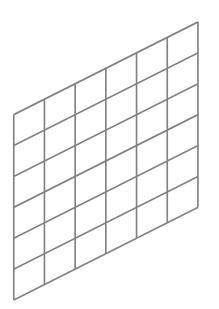
W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
<i>W</i> <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
<i>W</i> <sub>1</sub>	<i>W</i> <sub>2</sub>	<i>W</i> <sub>3</sub>





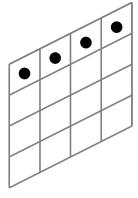




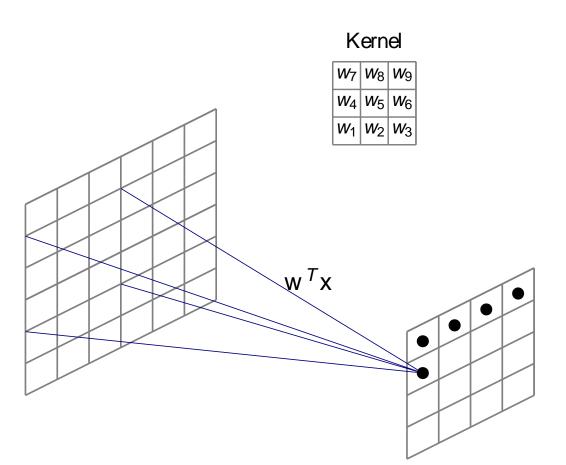


#### Kernel

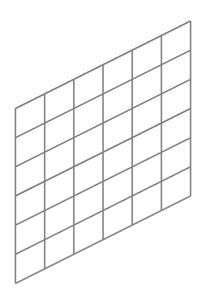
W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
<i>W</i> <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
<i>W</i> <sub>1</sub>	<b>W</b> <sub>2</sub>	<i>W</i> <sub>3</sub>





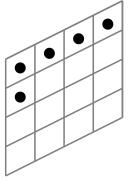




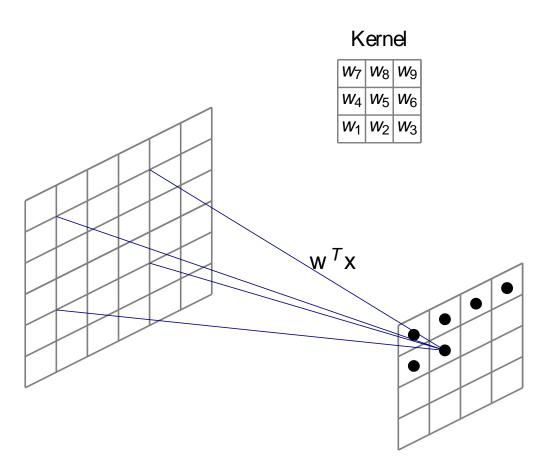


#### Kernel

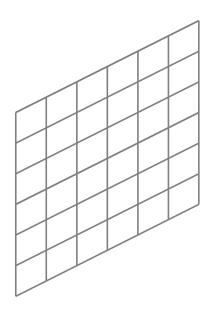
W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
W <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
W <sub>1</sub>	<i>W</i> <sub>2</sub>	<b>W</b> 3





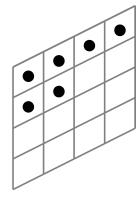




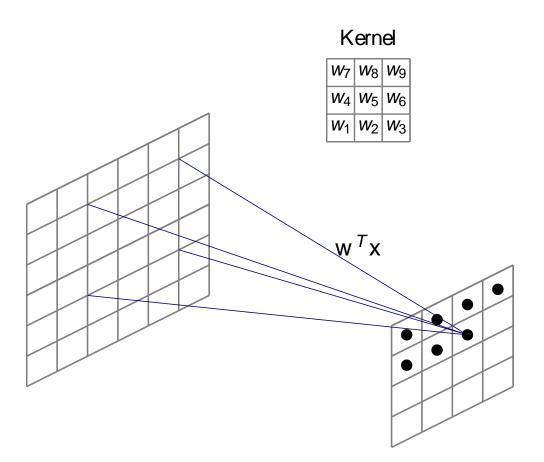


#### Kernel

W <sub>7</sub>	<i>W</i> <sub>8</sub>	<b>W</b> 9
<i>W</i> <sub>4</sub>	<i>W</i> <sub>5</sub>	<i>W</i> <sub>6</sub>
<i>W</i> <sub>1</sub>	<b>W</b> <sub>2</sub>	<i>W</i> <sub>3</sub>

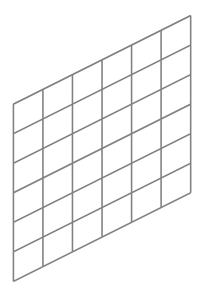


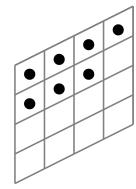




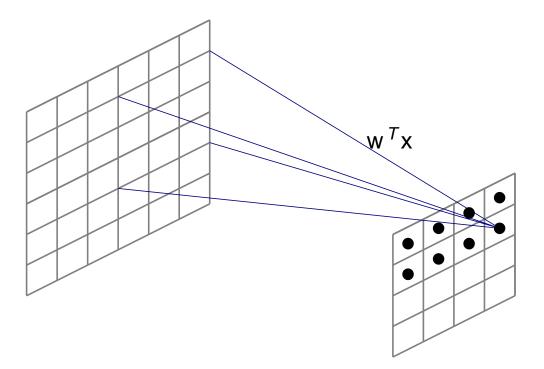
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## Convolution

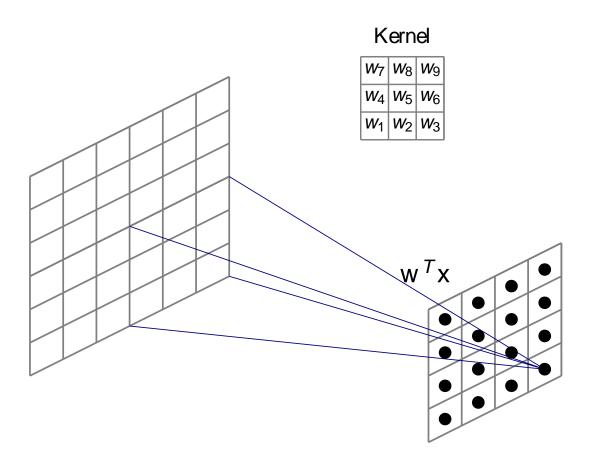






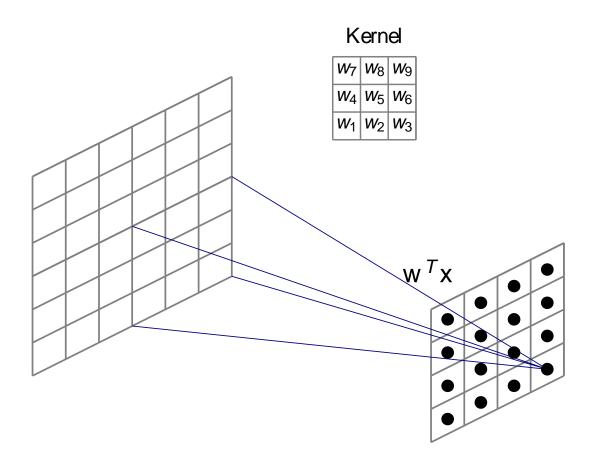








▶ What is the number of parameters?

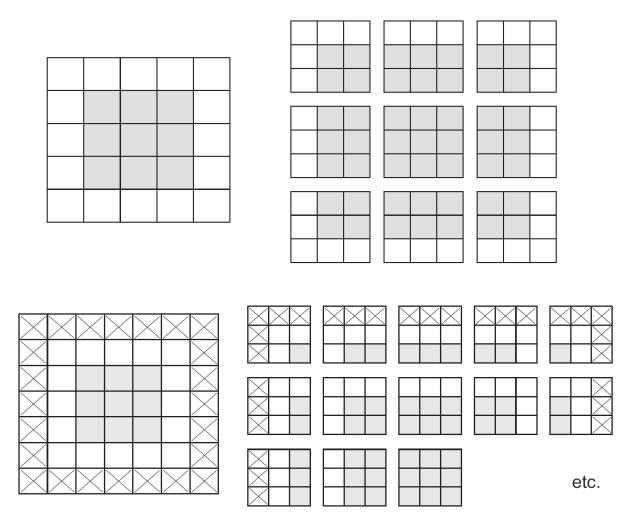


## ■ Why CNNs?

- Convolution leverages four ideas:
  - Sparse interactions
    - need to store fewer parameters, computing output needs fewer operations  $(O(m \times n) \text{ versus } O(k \times n))$
  - Parameter sharing
    - ▶ Same kernel is used throughout the input, so instead learning a parameter for each location, only a set of parameters is learnt
  - ▶ Equivariant representations
  - ▶ Ability to work with inputs of variable size



- The result of the convolution with a 3x3 feature maps shriks the images by 2 pixels along each dimension
- ▶ To avoid this effect, one might use padding, that is, add one external box of appropriate width and height.



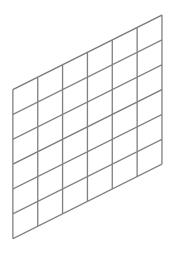
2023/2024



- ▶ The center of the convolution are not necessarily contiguous
- ▶ The distance between two consecutive windows is the stride

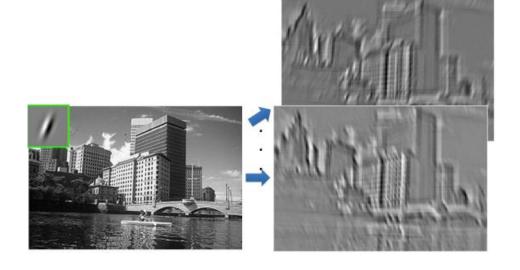
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		-					Γ
3	4						L
		1	3			4	
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		•					

## Multiple Filters



## Kernel 8

# Kernel | W<sub>7</sub> | W<sub>8</sub> | W<sub>9</sub> | | W<sub>4</sub> | W<sub>5</sub> | W<sub>6</sub> | | W<sub>1</sub> | W<sub>2</sub> | W<sub>3</sub> |

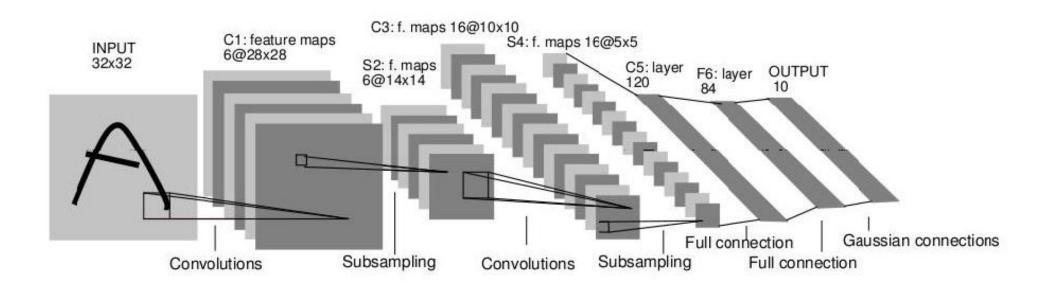


- ▶ We can use multiple kernels...
- ... and each kernel identifies a feature map



#### Convolutional Networks

▶ Neural Networks that use convolution in place of general matrix multiplication in at least one layer

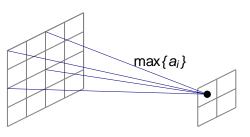




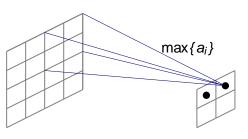




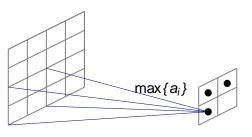




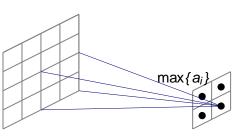






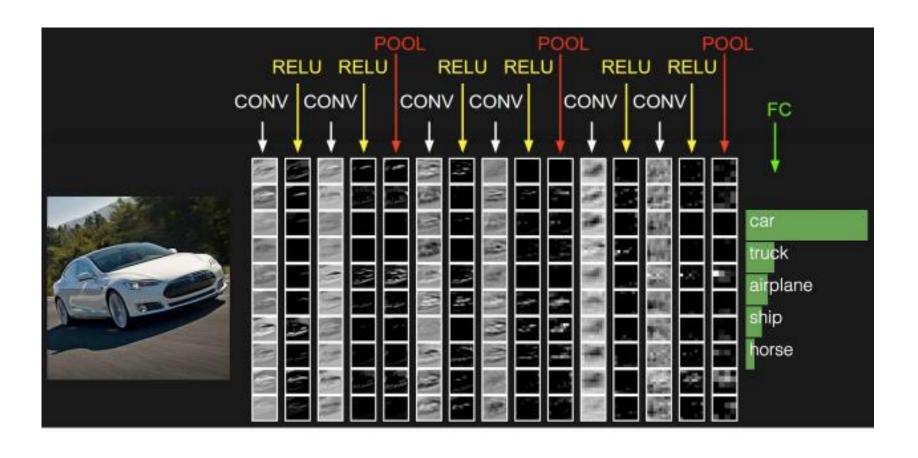






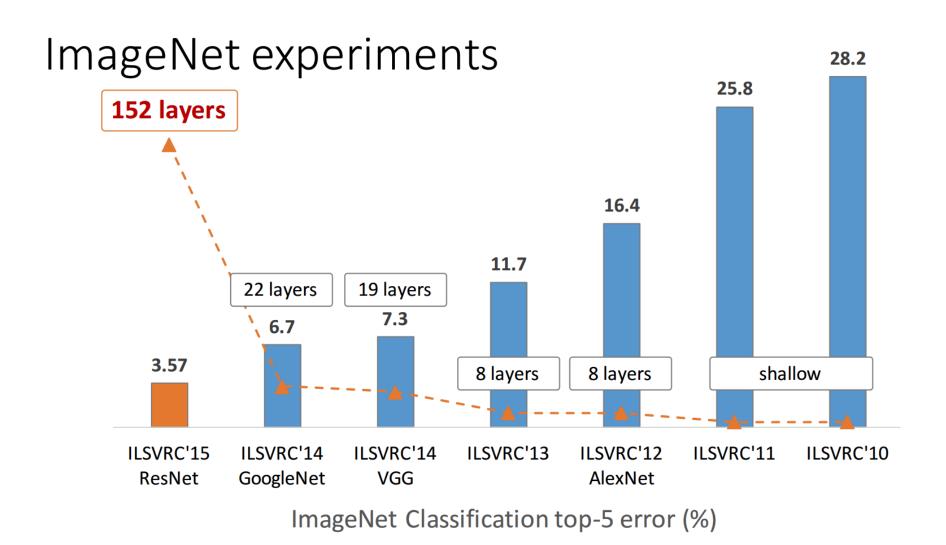


#### Convolutional neural networks





#### Deep Learning and Convolution

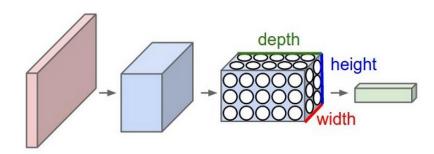




### Convolution in general

- ▶ Not just 2-D image as a running example
  - Operates on volumes
    - ▶ E.g., RGB Images would be depth 3 input

▶ Operates on 1-D vector



### Example with RGB Images (1/4)

```
import tensorflow as tf
from tensorflow.keras import datasets, layers, models, optimizers
# CIFAR 10 is a set of 60K images 32x32 pixels on 3 channels
IMG CHANNELS = 3
IMG ROWS = 32
IMG COLS = 32
#constant
                           Multiclass classification
BATCH SIZE = 128
EPOCHS = 20
CLASSES = 10
VALIDATION SPLIT = 0.2
OPTIM = tf.keras.optimizers.RMSprop()
```





## Example with RGB Images (2/4)

```
Number of filters
#define the convnet
                                                         Kernel size
                                                                    No padding (otherwise, use 'same')
def build(input_shape, classes):
 model = models.Sequential()
 model.add(layers.Convolution2D(32, (3, 3), activation='relu', padding='valid'
                         input shape=input shape))
 model.add(layers.MaxPooling2D(pool size=(2, 2)))
 model.add(layers.Dropout(0.25))
 model.add(layers.Flatten())
 model.add(layers.Dense(512, activation='relu'))
 model.add(layers.Dropout(0.5))
 model.add(layers.Dense(classes, activation='softmax'))
  return model
```

2023/2024

## Example with RGB Images (3/4)

```
# data: shuffled and split between train and test sets
(X train, y train), (X test, y test) = datasets.cifar10.load data()
# normalize
X train, X test = X train / 255.0, X test / 255.0
# convert to categorical
# convert class vectors to binary class matrices
y train = tf.keras.utils.to categorical(y train, CLASSES)
y test = tf.keras.utils.to categorical(y test, CLASSES)
model=build((IMG ROWS, IMG COLS, IMG CHANNELS), CLASSES)
model.summary()
```

2023/2024

## Example with RGB Images (4/4)

```
# use TensorBoard,
callbacks = [
  # Write TensorBoard logs to `./logs` directory
  tf.keras.callbacks.TensorBoard(log dir='./logs')
# train
model.compile(loss='categorical crossentropy', optimizer=OPTIM, metrics=['accuracy'])
model.fit(X train, y train, batch size=BATCH SIZE,
  epochs=EPOCHS, validation split=VALIDATION SPLIT,
  verbose=VERBOSE, callbacks=callbacks)
score = model.evaluate(X test, y test,
                     batch size=BATCH SIZE, verbose=VERBOSE)
print("\nTest score:", score[0])
print('Test accuracy:', score[1])
```