Exercise 2

In a process for the production of metal laminates we collected 100 sequential measurements of laminate width (time series 'A' "Statistical Control by monitoring and feedback adjustment" Box Luceño – J. Wiley)

Identify and fit a model for the data.

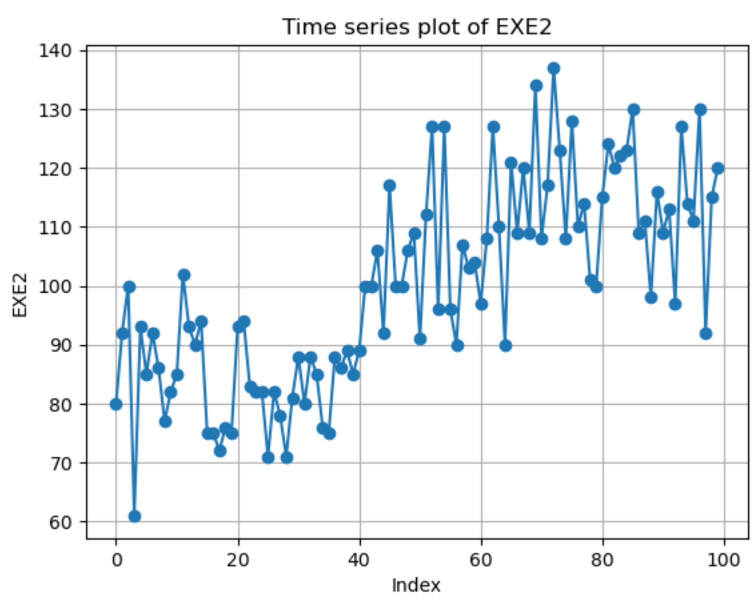
In a future class: Design a SCC control chart and a FVC control chart

In [ ]:
```python
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats
import seaborn as sns

# Import the dataset
data = pd.read_csv('ESE4_ex2.csv')

# Inspect the dataset
data.head()
```

Out[ ]:

	EXE2
0	80
1	92
2	100
3	61
4	93

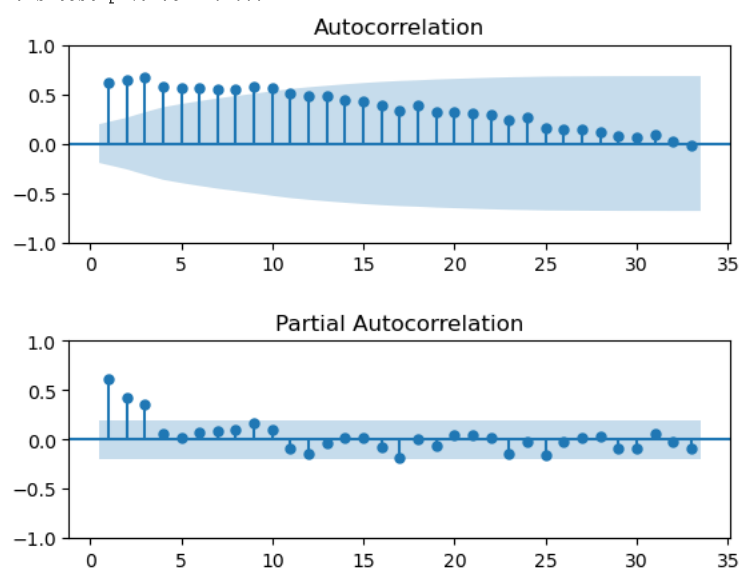
In [ ]:
```python
# Plot the data
plt.plot(data['EXE2'], 'o-')
plt.xlabel('Index')
plt.ylabel('EXE2')
plt.title('Time series plot of EXE2')
plt.grid()
plt.show()
```



> let's verify the time dependence assumption with runs test and ACF/PACF

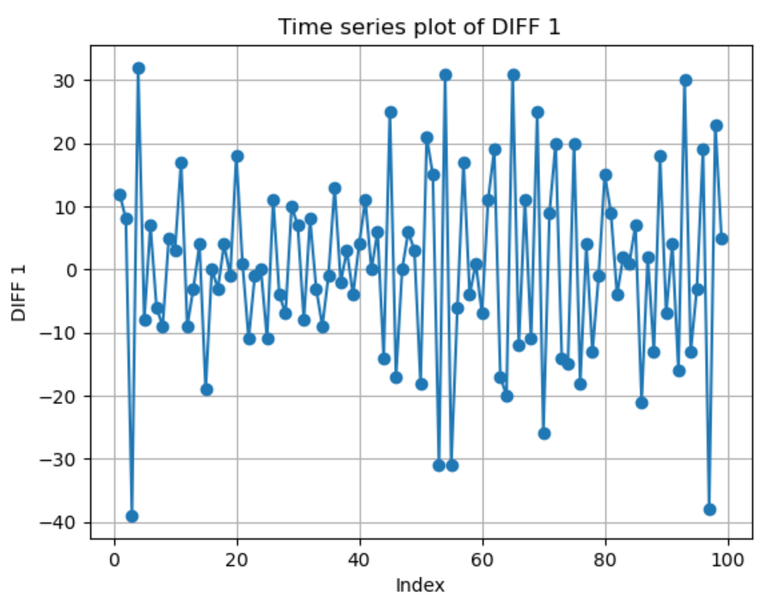
In [ ]:
```python
# Import the necessary libraries for the runs test
from statsmodels.sandbox.stats.runs import runstest_1samp

_, pval_runs = runstest_1samp(data['EXE2'], correction=False)
print('Runs test p-value = {:.3f}'.format(pval_runs))

# Plot the acf and pacf using the statsmodels library
import statsmodels.graphics.tsaplots as sgt

fig, ax = plt.subplots(2, 1)
sgt.plot_acf(data['EXE2'], lags = int(len(data)/3), zero=False, ax=ax[0])
fig.subplots_adjust(hspace=0.5)
sgt.plot_pacf(data['EXE2'], lags = int(len(data)/3), zero=False, ax=ax[1], method = 'ywm')
plt.show()
```

```
Runs test p-value = 0.000
```



> The process is NON-STATIONARY.
>
> Let's try to apply the difference operator.

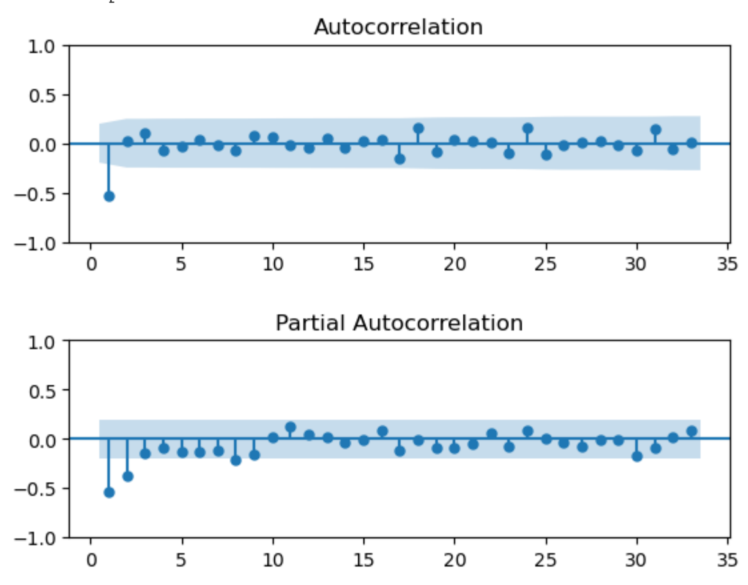
In [ ]:
```python
data['diff1'] = data['EXE2'].diff(1)

plt.plot(data['diff1'], 'o-')
plt.xlabel('Index')
plt.ylabel('DIFF 1')
plt.title('Time series plot of DIFF 1')
plt.grid()
plt.show()
```



> Let's verify again the time dependence assumption with runs test and ACF/PACF on the DIFF1 data

In [ ]:
```python
_, pval_runs = runstest_1samp(data['diff1'][1:], correction=False)
print('Runs test p-value = {:.3f}'.format(pval_runs))

fig, ax = plt.subplots(2, 1)
sgt.plot_acf(data['diff1'][1:], lags = int(len(data)/3), zero=False, ax=ax[0])
fig.subplots_adjust(hspace=0.5)
sgt.plot_pacf(data['diff1'][1:], lags = int(len(data)/3), zero=False, ax=ax[1], method = 'ywm')
plt.show()
```

```
Runs test p-value = 0.000
```



> After the differencing operation, the most suitable model seems to be an MA(1). Thus the investigated model is ARIMA(0,1,1)

In [ ]:
```python
# calculate an ARIMA model: import the necessary library
import qda
```

> The function `qda.ARIMA()` requires as inputs:
>
> 1. The dataframe with the data.
> 2. The `order` parameter, i.e., the (p, d, q) of the model: $AR(p), I(d), MA(q)$.
> 3. The `add_constant` parameter, i.e. the presence of a constant term in the model:
>   - `False`, for no constant term.
>   - `True`, for a constant term.

In [ ]:
```python
# fit model ARIMA with constant term
x = data['EXE2']
model = qda.ARIMA(x, order=(0,1,1), add_constant = True)

qda.ARIMAsummary(model)
```

```
---------------------
ARIMA MODEL RESULTS
---------------------
ARIMA model order: p=0, d=1, q=1

FINAL ESTIMATES OF PARAMETERS
-------------------------------
 Term    Coef  SE Coef  T-Value    P-Value
const  0.3111    0.226   1.3765 1.6867e-01
ma.L1 -0.8143    0.064 -12.7215 4.4928e-37

RESIDUAL SUM OF SQUARES
-------------------------
  DF          SS       MS
97.0 12318.5893 126.9958

Ljung-Box Chi-Square Statistics
----------------------------------
 Lag  Chi-Square  P-Value
  12      8.0673   0.7799
  24     14.9539   0.9221
  36     27.3625   0.8491
  48     39.2028   0.8133
```

> The calculated ARIMA model is in the form:
>
> $Y_t - Y_{t-1} = \nabla Y_t = \mu - \theta_1 \epsilon_{t-1} + \epsilon_t$
>
> The constant term ha a p-value of 0.169. Let's remove the constant value by omitting the `trend` parameter.

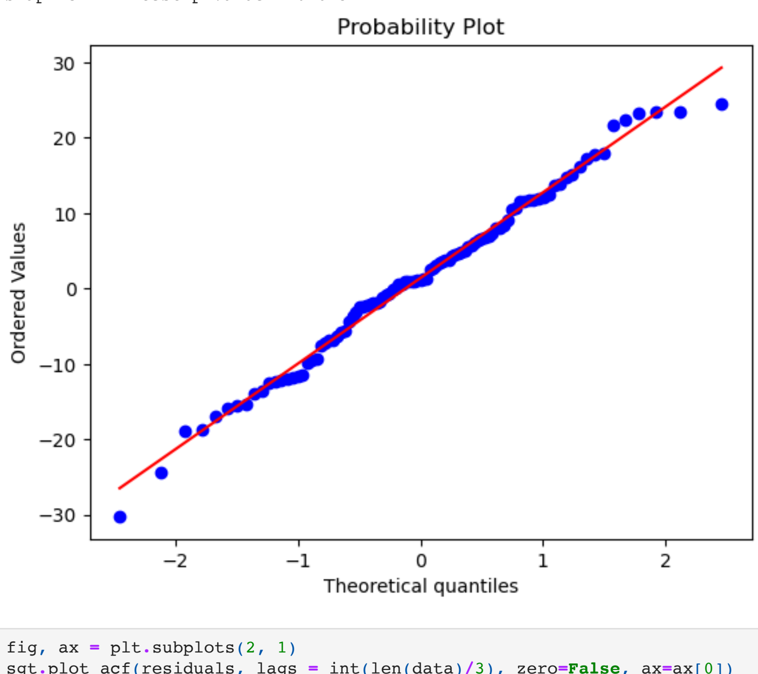
In [ ]:
```python
# fit model ARIMA with constant term
x = data['EXE2']
model = qda.ARIMA(x, order=(0,1,1), add_constant=False) # ARIMA(p,d,q), no constant term

qda.ARIMAsummary(model)
```

```
---------------------
ARIMA MODEL RESULTS
---------------------
ARIMA model order: p=0, d=1, q=1

FINAL ESTIMATES OF PARAMETERS
-------------------------------
 Term    Coef  SE Coef  T-Value    P-Value
ma.L1 -0.7854   0.0626 -12.5422 4.3837e-36

RESIDUAL SUM OF SQUARES
-------------------------
  DF         SS       MS
98.0 12528.408 127.8409

Ljung-Box Chi-Square Statistics
----------------------------------
 Lag  Chi-Square  P-Value
  12      8.5337   0.7422
  24     15.6625   0.8999
  36     27.8377   0.8329
  48     39.5483   0.8023
```

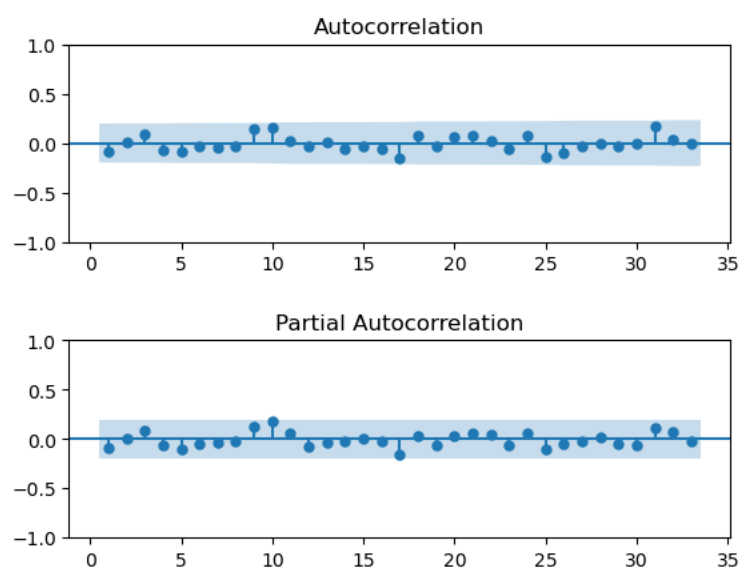
> The calculated ARIMA model is in the form:
>
> $Y_t - Y_{t-1} = \nabla Y_t = \theta_1 \epsilon_{t-1} + \epsilon_t$
>
> Let's check the assumptions on the residuals

In [ ]:
```python
#extract the residuals
residuals = model.resid[1:]

# Perform the Shapiro-Wilk test
_, pval_SW = stats.shapiro(residuals)
print('Shapiro-Wilk test p-value = %.3f' % pval_SW)

# Plot the qqplot
stats.probplot(residuals, dist="norm", plot=plt)
plt.show()
```

```
Shapiro-Wilk test p-value = 0.643
```

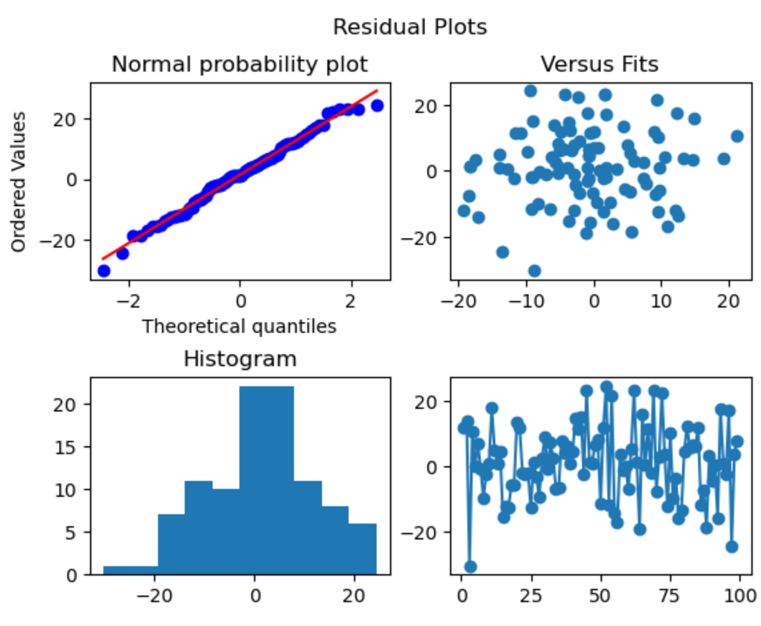


In [ ]:
```python
fig, ax = plt.subplots(2, 1)
sgt.plot_acf(residuals, lags = int(len(data)/3), zero=False, ax=ax[0])
fig.subplots_adjust(hspace=0.5)
sgt.plot_pacf(residuals, lags = int(len(data)/3), zero=False, ax=ax[1], method = 'ywm')
plt.show()
```



Try at home: Bartlett test and LBQ test on ARIMA model residuals

In [ ]:
```python
fig, axs = plt.subplots(2, 2)
fig.suptitle('Residual Plots')
stats.probplot(residuals, dist="norm", plot=axs[0,0])
axs[0,0].set_title('Normal probability plot')
axs[0,1].scatter(model.fittedvalues[1:], residuals)
axs[0,1].set_title('Versus Fits')
fig.subplots_adjust(hspace=0.5)
axs[1,0].hist(residuals)
axs[1,0].set_title('Histogram')
axs[1,1].plot(np.arange(1, len(residuals)+1), residuals, 'o-')
plt.show()
```



> The model is adequate.