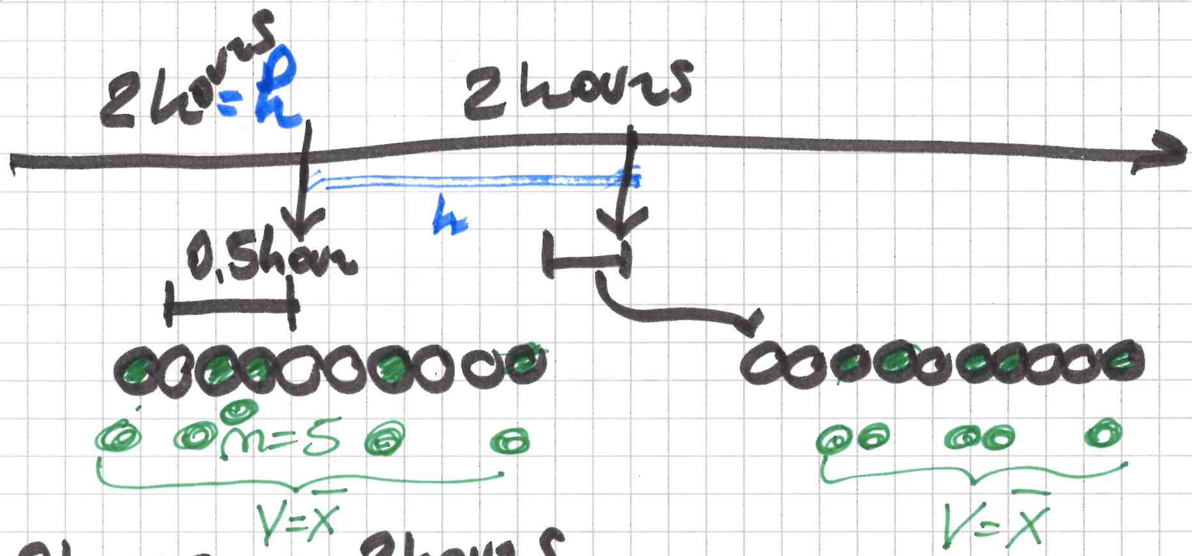
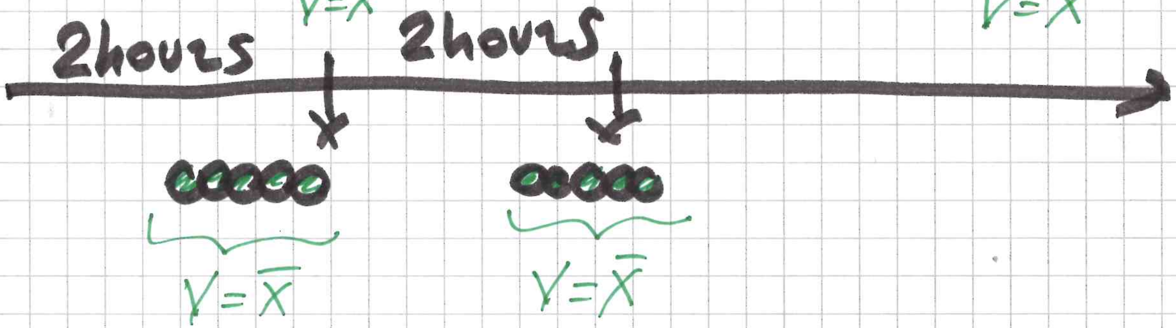
QDA 2023.06.27 (1)

SAMPLING STRATEGY

a)

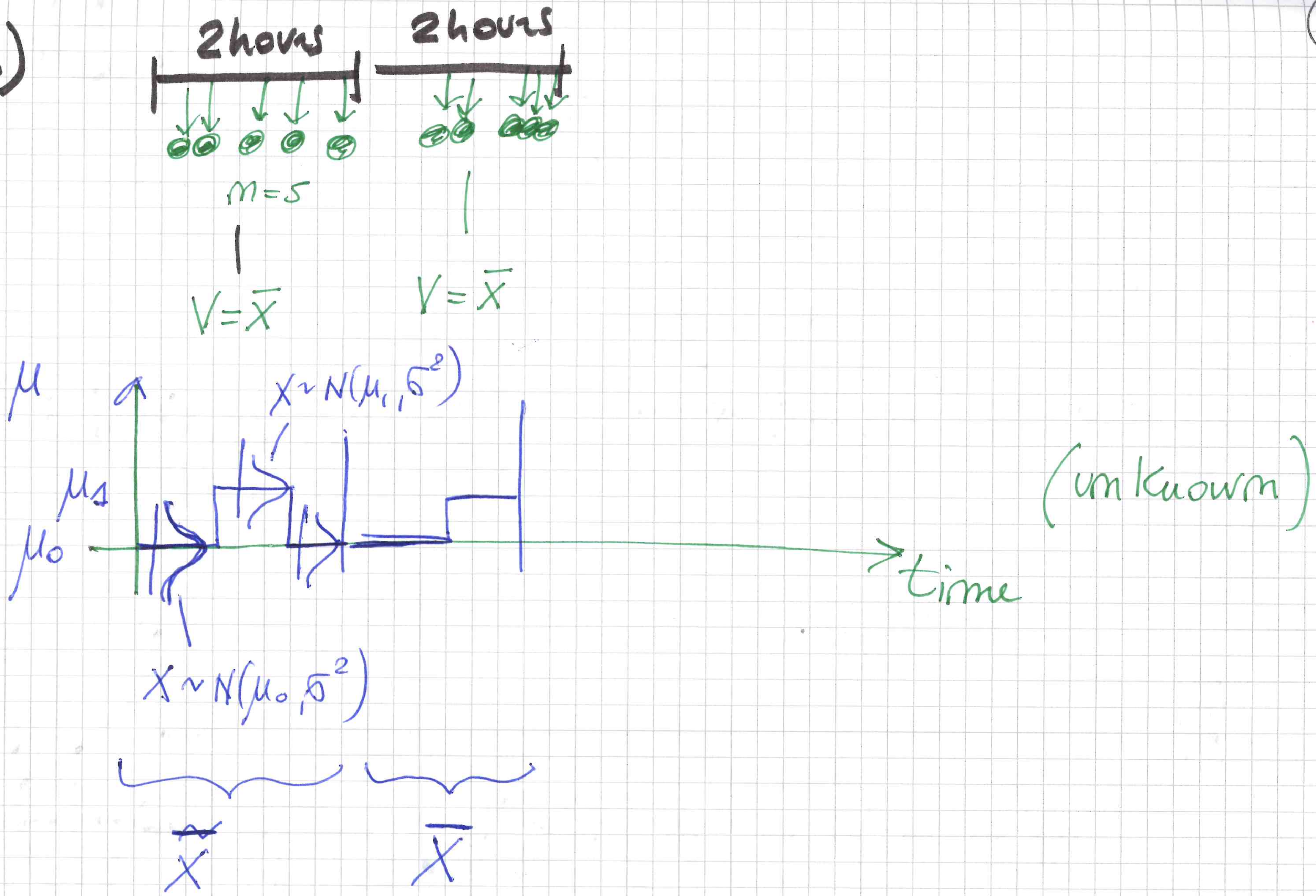


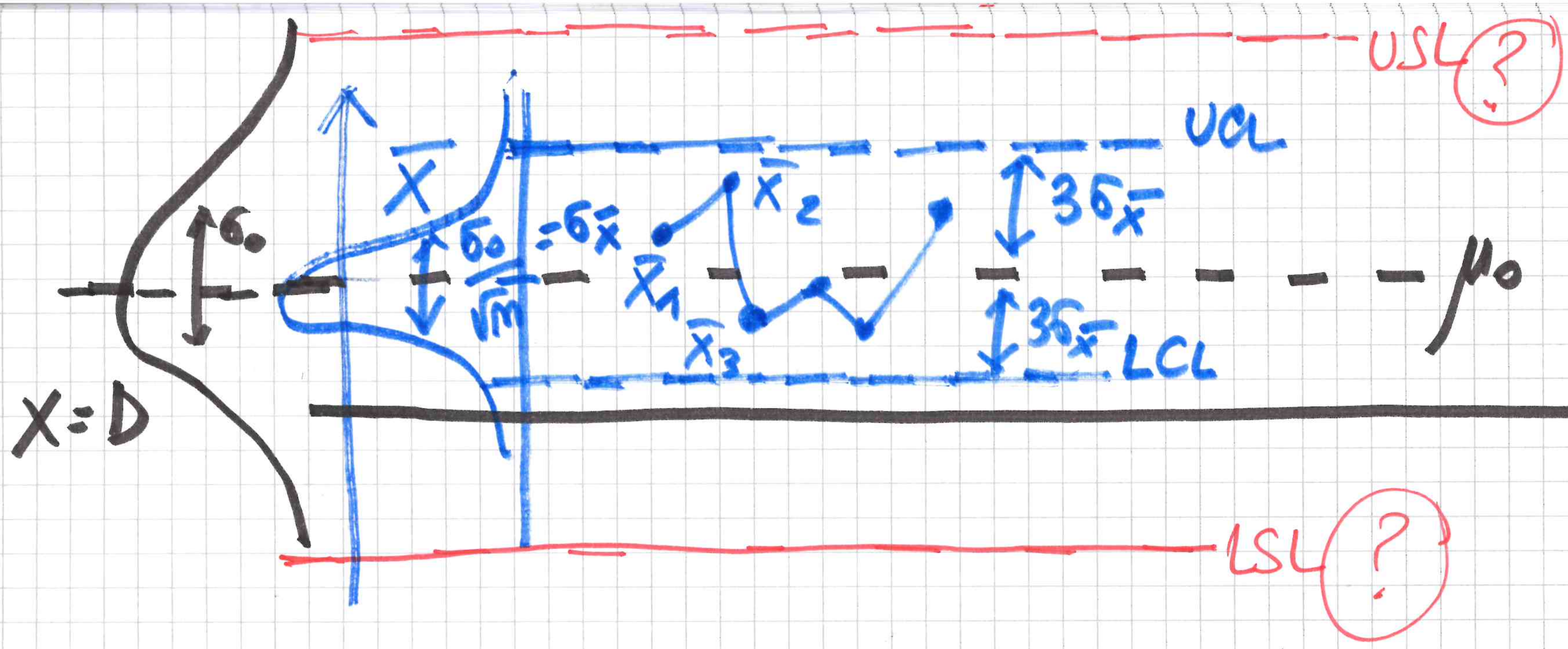
b)



SQM
SPM $\rightarrow$ $O = X \overset{iid}{\sim} N(\mu, \sigma^2)$

c)


2 hours
2 hours
$n=5$
$V=\bar{X}$
$V=\bar{X}$
μ
μ_1
μ_0
$X \sim N(\mu_1, \sigma^2)$
$X \sim N(\mu_0, \sigma^2)$
(unknown)
time
$\bar{X}$
$\bar{X}$



$\bar{X}$ control chart
$m = 20$

$$H_0 : D = X \overset{\text{iid}}{\sim} N(\mu_0, \sigma_0^2)$$

output

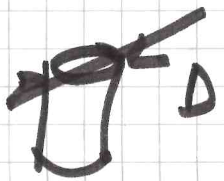
$$\longrightarrow V = \bar{X} = \frac{1}{m} \Sigma X_i \sim N\left(\mu_0, \frac{\sigma_0^2}{m}\right)$$

$$\begin{matrix} UCL \\ LCL \end{matrix} = \mu_0 \pm 3 \frac{\sigma_0}{\sqrt{m}}$$

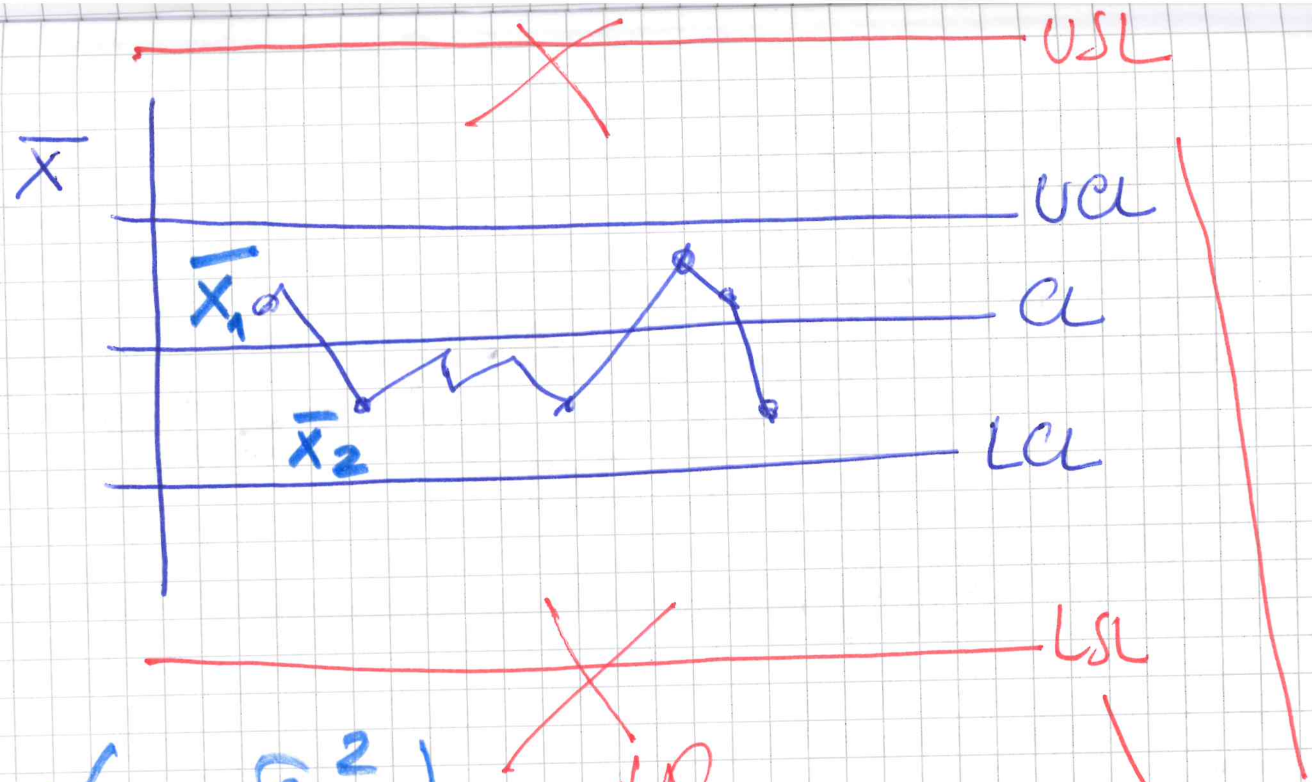
X
↑
diameter

LSL
USL

$\rightarrow X \in [LSL, USL]$
conforming
[not defected]



ERROR!



$$\bar{X} \sim N\left(\mu_0, \frac{\sigma_0^2}{n}\right)$$

$\sigma_{\bar{X}}$

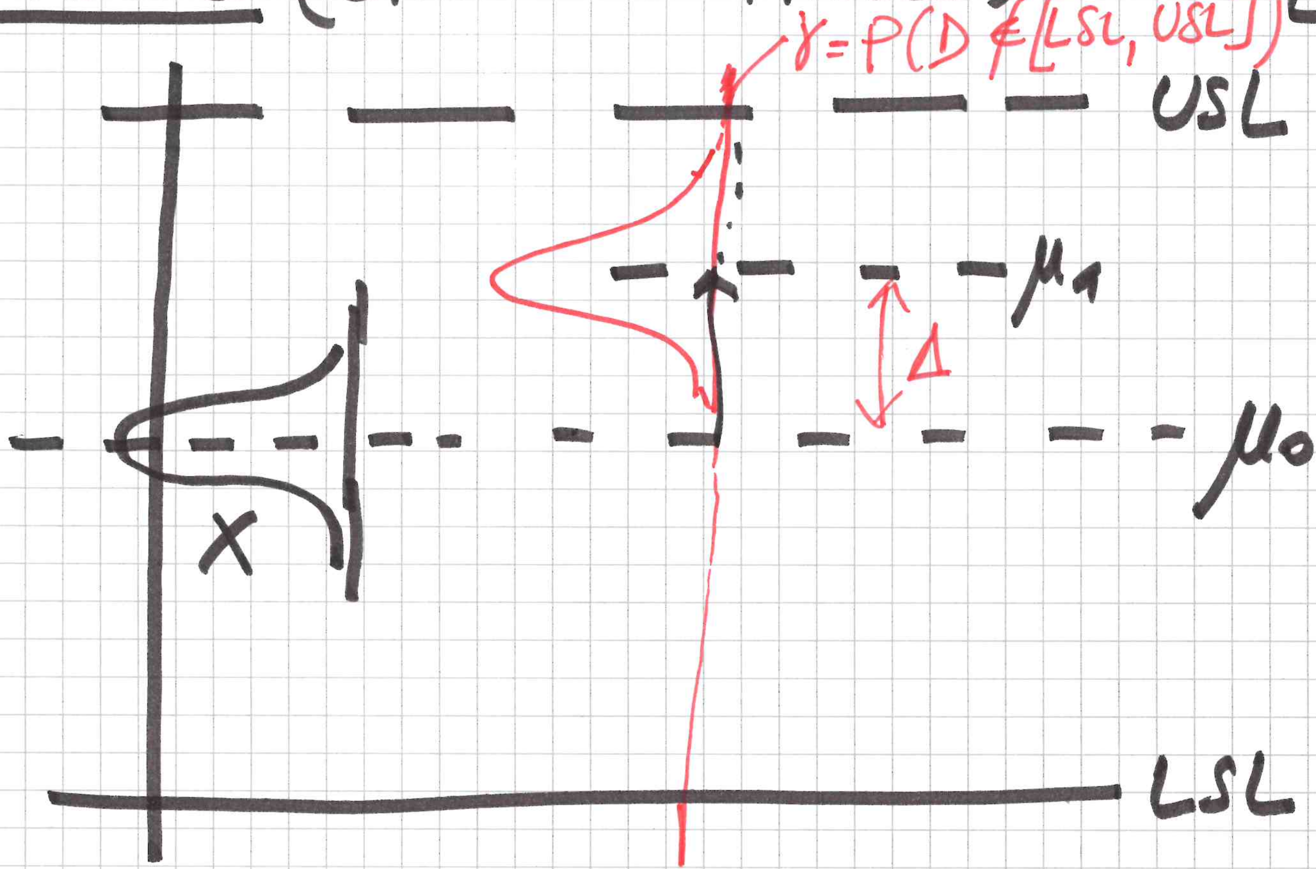
NO

UNLESS (n = 1)

$$X \sim N(\mu_0, \sigma_0^2)$$

CASE 1 (SIX SIGMA PROCESS) → C_p LARGE

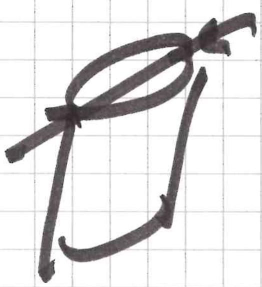
(5)



NOT THE TRADITIONAL CONTROL CHART

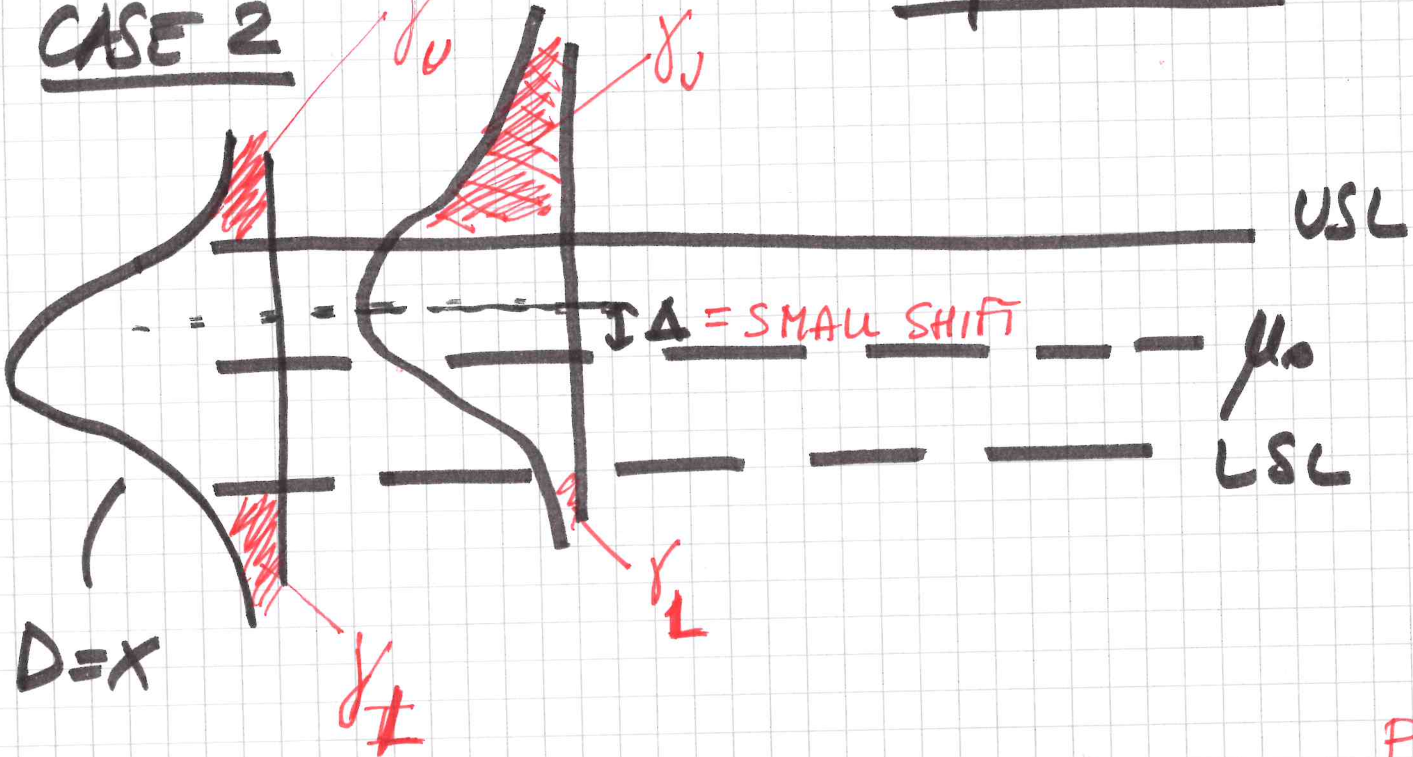
→ MODIFIED CONTROL CHART (ALARM ONLY WHEN μ_1 CLOSE TO LSL or USL)

OUT OF CONTROL MEAN



$D = X \begin{cases} \text{LSL} \\ \text{USL} \end{cases}$ OK!

CASE 2 → C_p SMALL



CONTROL CHART × SMALL SHIFT

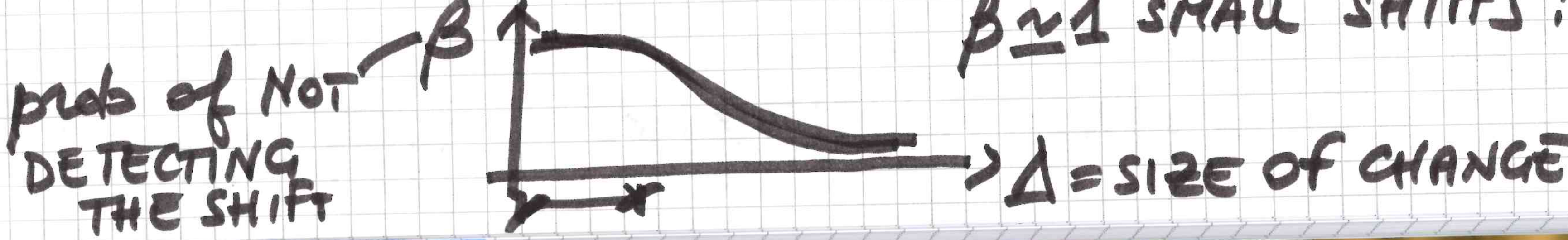
(CUSUM, EWMA, MA)

$\gamma_U + \gamma_L = \text{Prob}\{D \notin [LSL, USL]\}$ PROB of DEFECTIVE ITEM

→ SHEWHART → $\begin{matrix} UCL \\ LCL \end{matrix} = \mu_V \pm k\sigma_V$

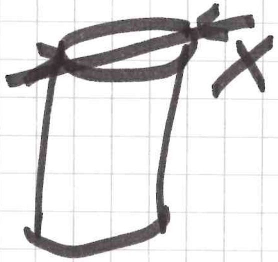
→ is the TRADITIONAL CONTROL CHART EFFECTIVE FOR SMALL SHIFTS? NO

$\beta \simeq 1$ SMALL SHIFTS!!



$$C_p = \frac{USL - LSL}{6\sigma}$$ CAPABILITY INDEX

VARIABLE CONTROL CHARTS



IN CONTROL

$$X \overset{iid}{\sim} N(\mu_0, \sigma_0^2)$$

$\mu_1 \neq \mu_0$

μ → Xbar control chart

$\sigma_1 \neq \sigma_0$

σ → R control chart or S control chart

$n > 1$

Xbar ($\bar{X}$) - R (Range)

Xbar - S (sample std deviation)

$n = 1$ → I - MR

I: individual; MR: moving range

Xbar CC

$$V = \bar{X}$$

$$H_0 : X \sim N(\mu_0, \sigma_0^2) \longrightarrow V = \bar{X} \sim N\left(\mu_0, \frac{\sigma_0^2}{m}\right)$$

$$\begin{matrix} UCL \\ CL \\ LCL \end{matrix} = \mu_V \pm K \sigma_V$$

$$\bar{X} \longrightarrow \begin{matrix} UCL \\ CL \\ LCL \end{matrix} = \mu_0 \pm K \frac{\sigma_0}{\sqrt{m}}$$

USUALLY $K = 3$

$$\bar{X} \longrightarrow \begin{matrix} UCL = \mu_0 + \overbrace{\frac{3}{\sqrt{m}}}^{A(m)} \sigma_0 \\ LCL = \mu_0 - \underbrace{\frac{3}{\sqrt{m}}}_{A(m)} \sigma_0 \end{matrix}$$

$$A(m) = \frac{3}{\sqrt{m}}$$

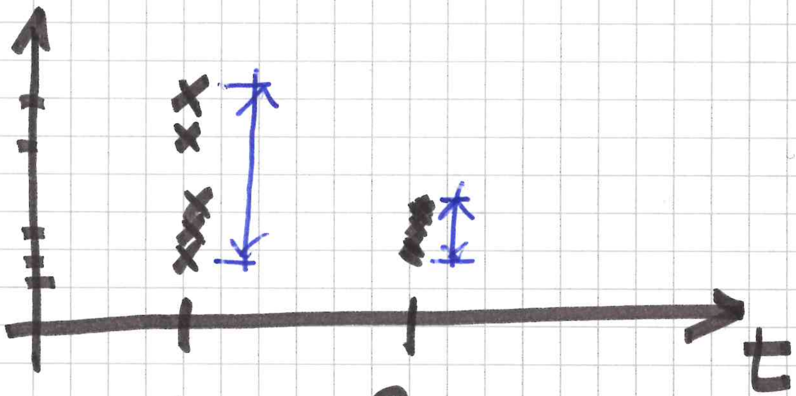
Range R control chart

STATISTIC

$$V = R \triangleq \max_i X_i - \min_i X_i \quad \text{in a sample}$$

$\mu_V \pm K\sigma_V$

D



1 (n=5)

2 (n=5)

$H_0 : X \overset{iid}{\sim} N(\mu_0, \sigma_0^2)$

$W = \frac{R}{\sigma}$

RELATIVE RANGE

$E(W) = d_2(n)$

$Var(W) = [d_3(n)]^2$

$$d_2 = E(W) = E\left(\frac{R}{\sigma}\right) = \frac{1}{\sigma} E(R) \Rightarrow E(R) = \sigma \cdot d_2 = \mu_R$$

$$d_3^2 = V\left(\frac{R}{\sigma}\right) = V\left(\frac{1}{\sigma} \cdot R\right) = \frac{1}{\sigma^2} V(R) \Rightarrow V(R) \cdot \sigma^2 \cdot d_3^2 = \sigma_R^2$$

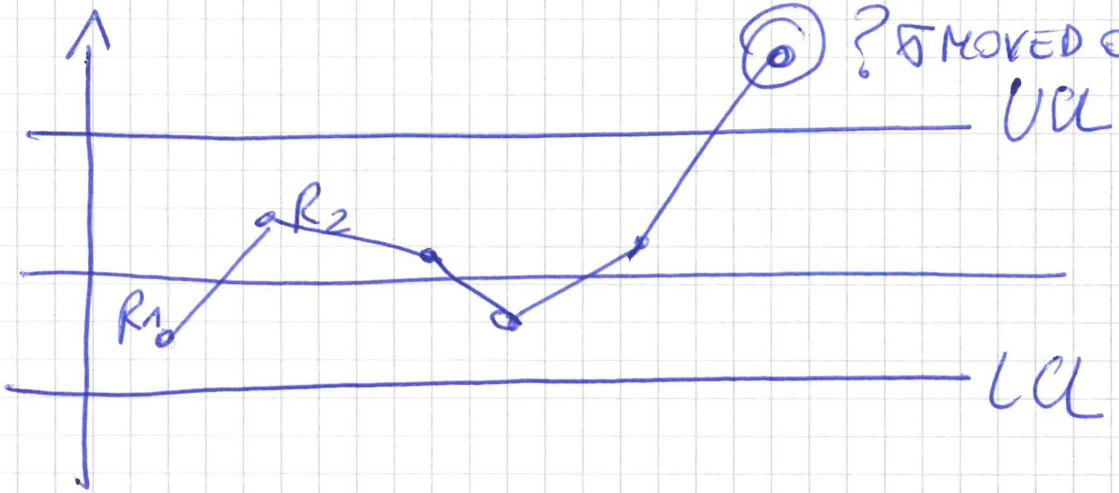
R CC

$$UCL = \mu_R + k\sigma_R = d_2 \cdot \sigma + k d_3 \cdot \sigma = \sigma(d_2 + k d_3)$$

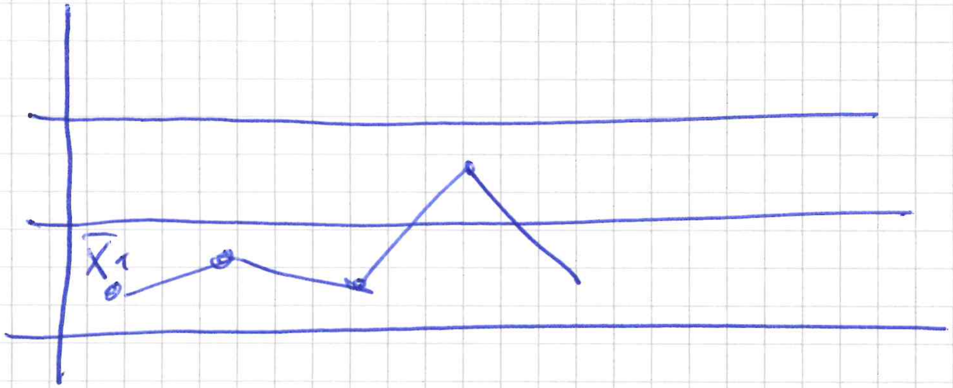
$$CL = \mu_R = d_2 \cdot \sigma$$

$$LCL = \mu_R - k\sigma_R = d_2 \cdot \sigma - k\sigma \cdot d_3 = \sigma(d_2 - k d_3)$$

$k = 3$



R cc



$\bar{X}$