QDA 2023 03 09

→ AUTOCORRELATION - TESTS (BARTLETT'S LBQ TEST)

→ MULTIPLE TESTS ON THE SAME DATA → BONFERRONI'S INEQUALITY

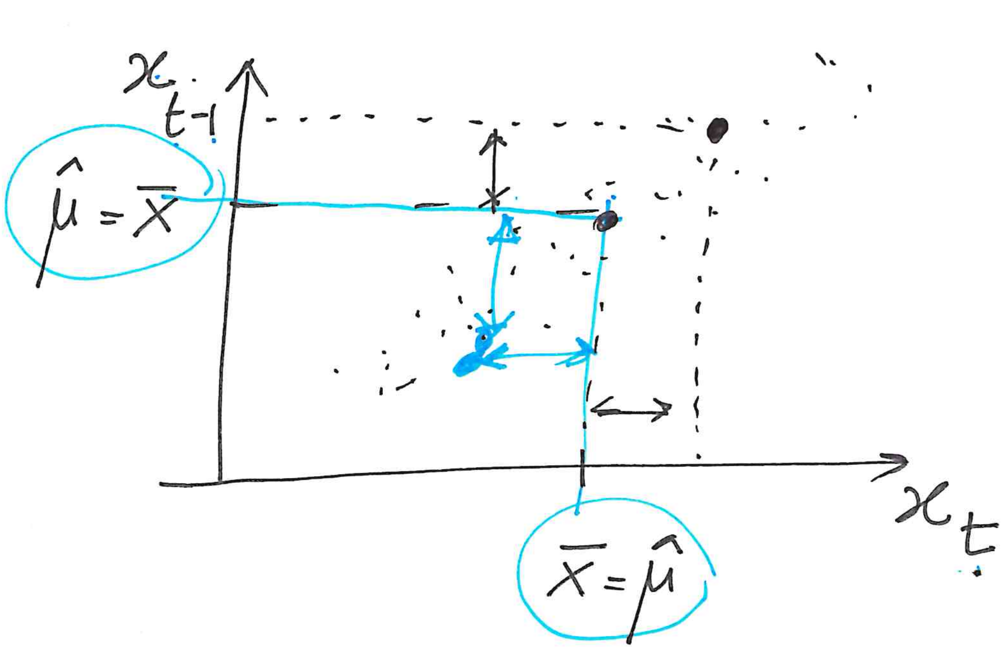
lagged time series ($K=1$)

t	X_t	X_{t-1}
1.	x_1	–
2	x_2	x_1
3	x_3	x_2
⋮	⋮	⋮
t	x_t	x_{t-1}

$K=2$

$x_1 \; x_2 \; x_3 \; \dots \; x_t, x_{t+1}, \dots$

$K = 1$ time lags



positive autocorrelation at lag $k=1$

AUTOCOVARIANCE (FUNCTION k)
AT LAG K

$$\gamma_{k,t} = E\left[(X_t - \underline{\underline{\mu_t}})(X_{t-k} - \underline{\underline{\mu_{t-k}}})\right]$$

STATIONARY PROCESS $\longrightarrow \mu_t = \mu$

$$\gamma_{k,t} = \gamma_k \qquad \gamma_0 = ? \qquad \gamma_0 = E\left[(X_t-\mu)^2\right] = \sigma^2$$

TIME LAG

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$x_1, x_2, x_3 \ldots\ldots x_n$

→ ESTIMATE $\hat{\gamma}_k = c_k$ "sample" autocovariance

[Remind $\mu \rightarrow x_1 \ldots x_n \rightarrow$ ESTIMATE $\hat{\mu} = \bar{x}$ sample mean]

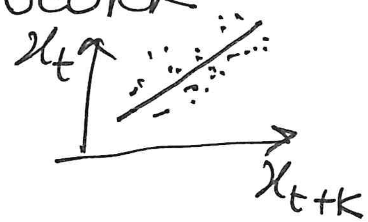
$$\boxed{\hat{\gamma}_k = c_k = \frac{1}{n} \sum_{t=1}^{n-k} (\underbrace{x_t}_{\text{data}} - \underbrace{\bar{x}}_{\hat{\mu}})(x_{t+k} - \bar{x})}$$

AUTOCORRELATION

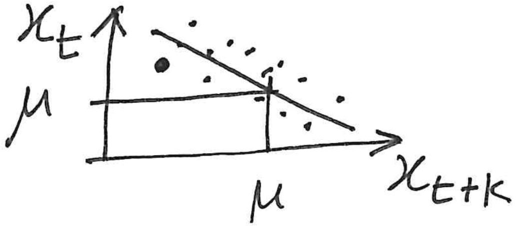
$$\rho_k = \frac{\gamma_k}{\gamma_0}.$$

$-1 \leq \rho_k \leq 1$

POSITIVE AUTOCORR



NEGATIVE AUTOCORR



$\rho_k = 0$ NO AUTOCORR

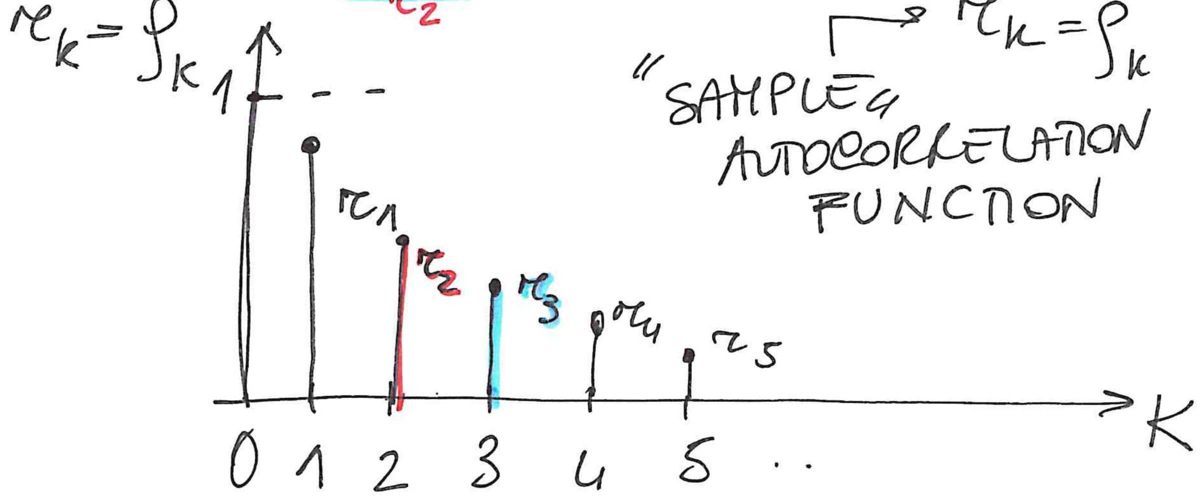
④

$$\hat{\rho}_k = r_k = \frac{\sum_{t=1}^{n-k} (x_t - \bar{x})(x_{t+k} - \bar{x})}{\sum_{t=1}^{n} (x_t - \bar{x})^2}$$

↑ SAMPLE AUTOCORRELATION

$x_1, x_2, x_3, x_4 \ldots\ldots x_t, x_{t+1}, \ldots\ldots x_n$

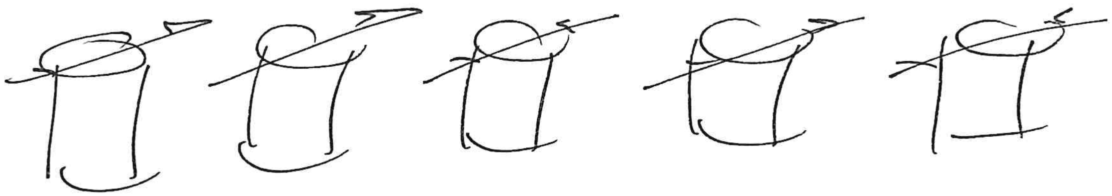
r_2



"SAMPLE" AUTOCORRELATION FUNCTION → $r_k = \hat{\rho}_k$

POSITIVELY AUTOCORRELATED PROCESS

⑤



$X_t \overset{iid}{\sim} N($

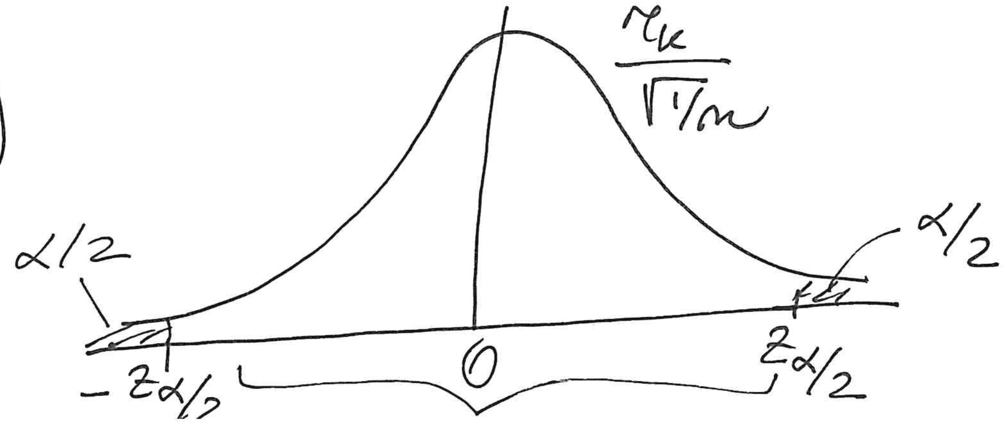
$r_1 = \hat{\rho}_1 = 0.53$

TEST BARTLETT

$H_0 : \rho_k = 0 \qquad H_1 : \rho_k \neq 0$

GIVEN H_0 $\longrightarrow$ TEST STATISTIC : $\boxed{r_k \overset{approx}{\sim} N\left(0, \frac{1}{m}\right)}$

$$\frac{r_k - 0}{\sqrt{1/m}} \sim N(0,1)$$



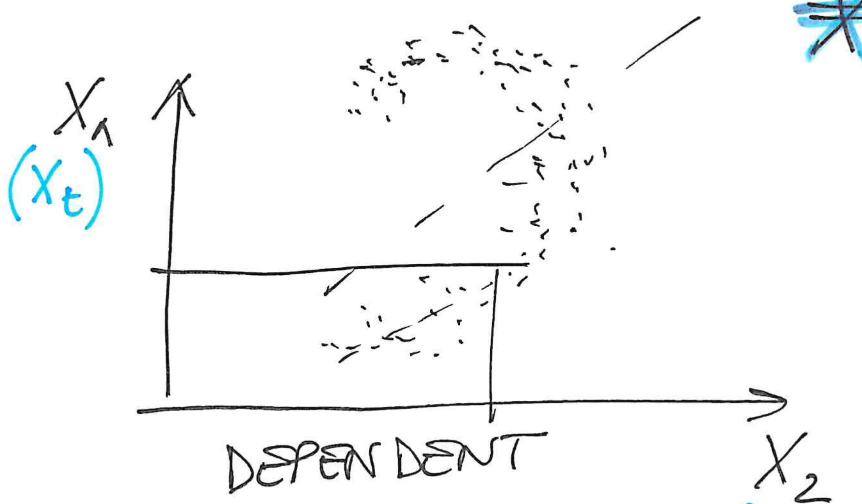
⑥

$$|r_k| > \frac{z_{\alpha/2}}{\sqrt{n}}$$ REJECT H_0

(REJECT THE ASSUMPTION OF NO AUTOCORRELATION AT LAG k)

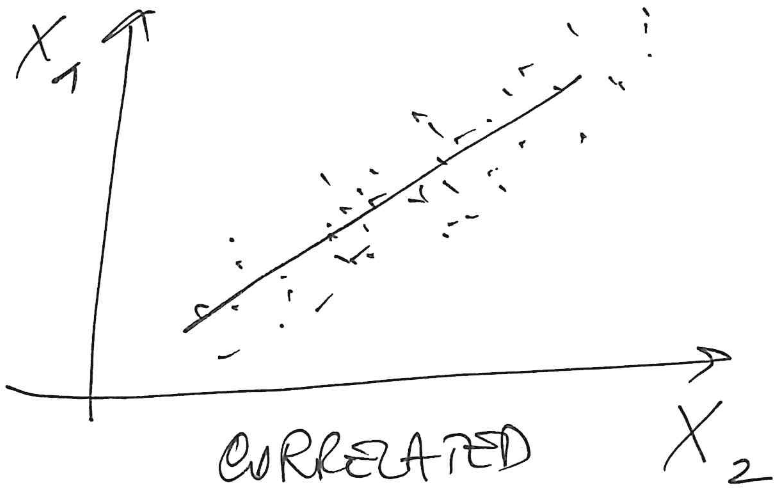
CORRELATION $\not\Leftarrow$ $\Rightarrow$ DEPENDENCE

UNCORRELATION $\Leftarrow$ $\not\Rightarrow$ INDEPENDENCE



DEPENDENT

~~NOT~~ UNCORRELATED (X_{t+1})



CORRELATED & DEPENDENT

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$K = 1, 2, 3, 4$ LAGS $\longrightarrow x_1 \ldots\ldots x_n$

1) $H_0 : \rho_1 = 0$ vs $H_1 : \rho_1 \neq 0$ α_1 = FIRST TYPE ERROR

2) $H_0 : \rho_2 = 0$ vs $H_1 : \rho_2 \neq 0$ α_2

3) $H_0 : \rho_3 = 0$ vs $H_1 : \rho_3 \neq 0$ α_3

4) $H_0 : \rho_4 = 0$ vs $H_1 : \rho_4 \neq 0$ α_4

α' = probability of rejecting at least one null hypothesis | they are all true

α' → OVERALL FIRST TYPE ERROR

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Bonferroni inequality

$$\boxed{\alpha' \leq \sum_{i=1}^{N} \alpha_i}$$

N → # TESTS

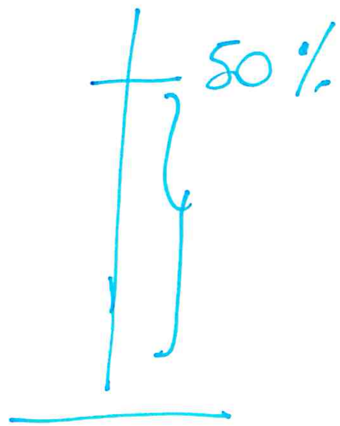
Ex : N = 10 TESTS (WINDOWS)

$\alpha_i = 5\% = 0.05$

↑ FALSE ALARM PROBABILITY

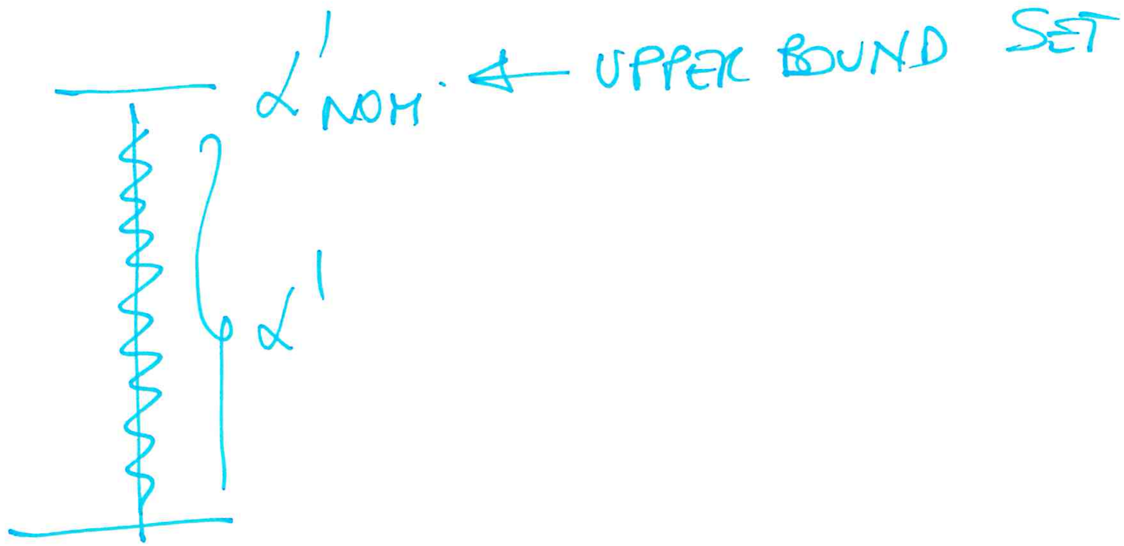
$$\alpha' \leq \sum \alpha_i = 10 \cdot 0.05 = 0.5 = 50\%$$

↑ OVERALL FALSE ALARM



Use Bonferroni's inequality to "control" α'

N TESTS $\Rightarrow \underset{\substack{\uparrow \\ \text{TARGET VALUE}}}{\alpha'_{NOM}} \Rightarrow \underset{\substack{\uparrow \\ \text{i-th single test}}}{\alpha_i} = \boxed{\dfrac{\alpha'_{NOM}}{N}} \Rightarrow \alpha' \leq \sum_{i=1}^{N} \dfrac{\alpha'_{NOM}}{N} = \alpha'_{NOM}$



Ex $N = 10$ TESTS $\alpha'_{NOM} = 10\% \Rightarrow \alpha_i = \dfrac{10\%}{10} = 0.01 = 1\%$

$\alpha' \leq 10\% = \sum \alpha_i$

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N TESTS ARE INDEPENDENT

$$1-\alpha' = \prod_{i=1}^{N} (1-\alpha_i)$$

α': OVERALL FIRST TYPE ERROR

α_i: for the i-th test

LBQ TEST

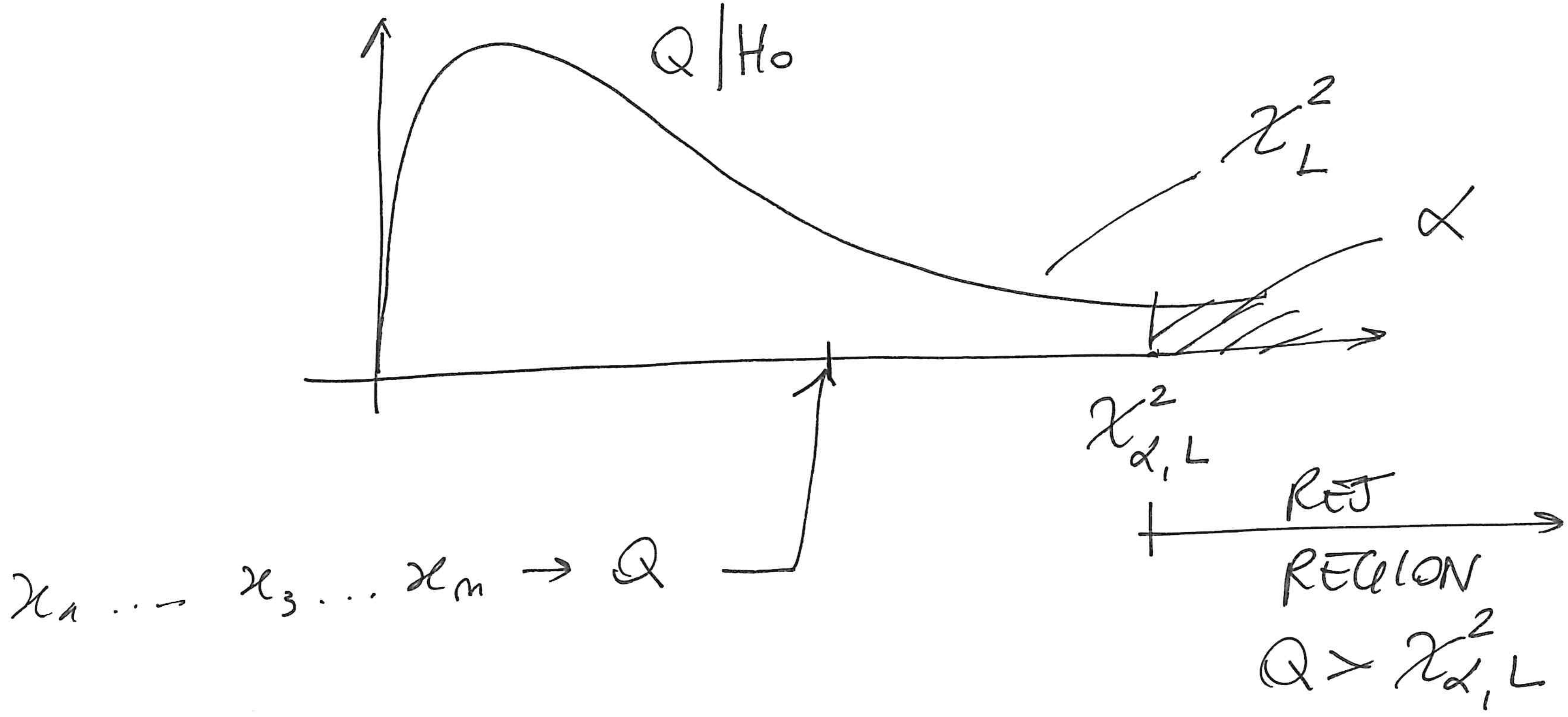
Ljung Box Pierce

$H_0 : \rho_i = 0 \quad i = 1, 2, \dots L$ vs $H_1 : \exists i \in [1, \dots L] \ni \rho_i \neq 0$

$\ni$: SUCH THAT

$$Q = m(m+2) \sum_{k=1}^{L} \frac{r_k^2}{m-k}$$

H_0 is true $\Rightarrow Q \sim \chi^2_L$


$Q | H_0$
χ^2_L
α
$\chi^2_{\alpha, L}$
REJ
REGION
$Q > \chi^2_{\alpha, L}$
$x_1 \dots x_3 \dots x_n \to Q$