EXERCISE 2

Design an X-bar and R control chart for the data in `ESE06_ex2.csv`. Which conclusions can be drawn about the process?

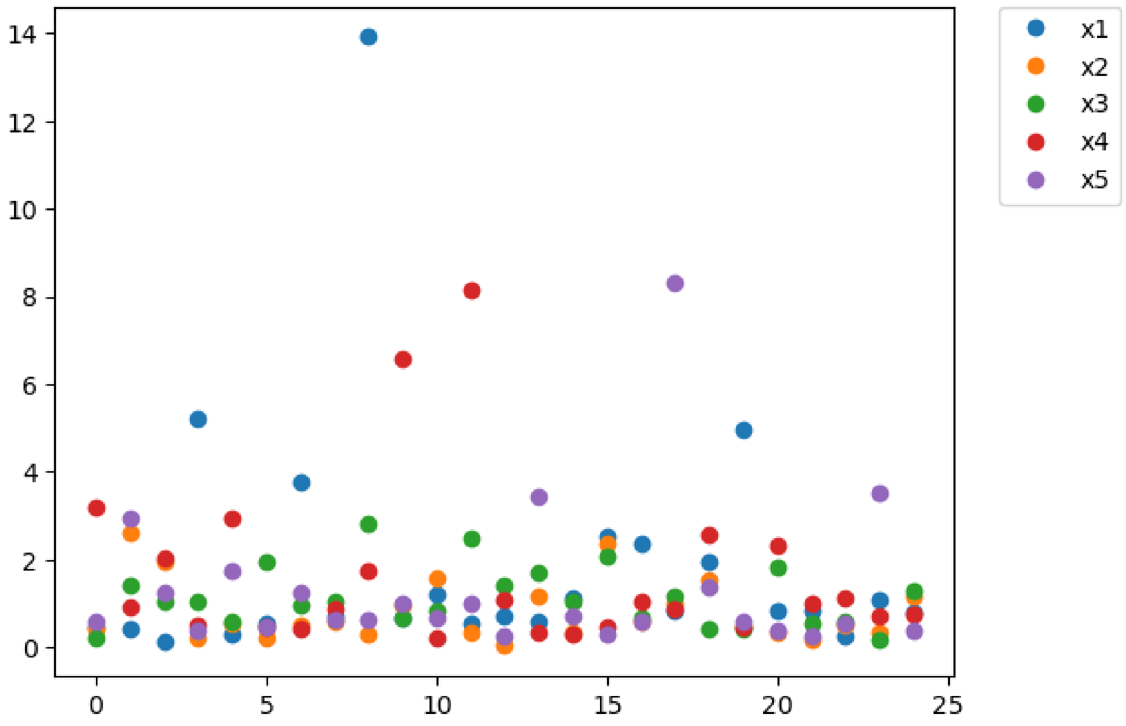
In [ ]:
```
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats
import qda

# Import the dataset
data = pd.read_csv('ESE06_ex2.csv')

# Inspect the dataset
data.head()
```

Out[ ]:

	x1	x2	x3	x4	x5
0	0.473	0.405	0.213	3.187	0.572
1	0.430	2.623	1.415	0.915	2.933
2	0.148	1.938	1.057	2.019	1.256
3	5.209	0.211	1.047	0.492	0.388
4	0.308	0.536	0.570	2.951	1.741

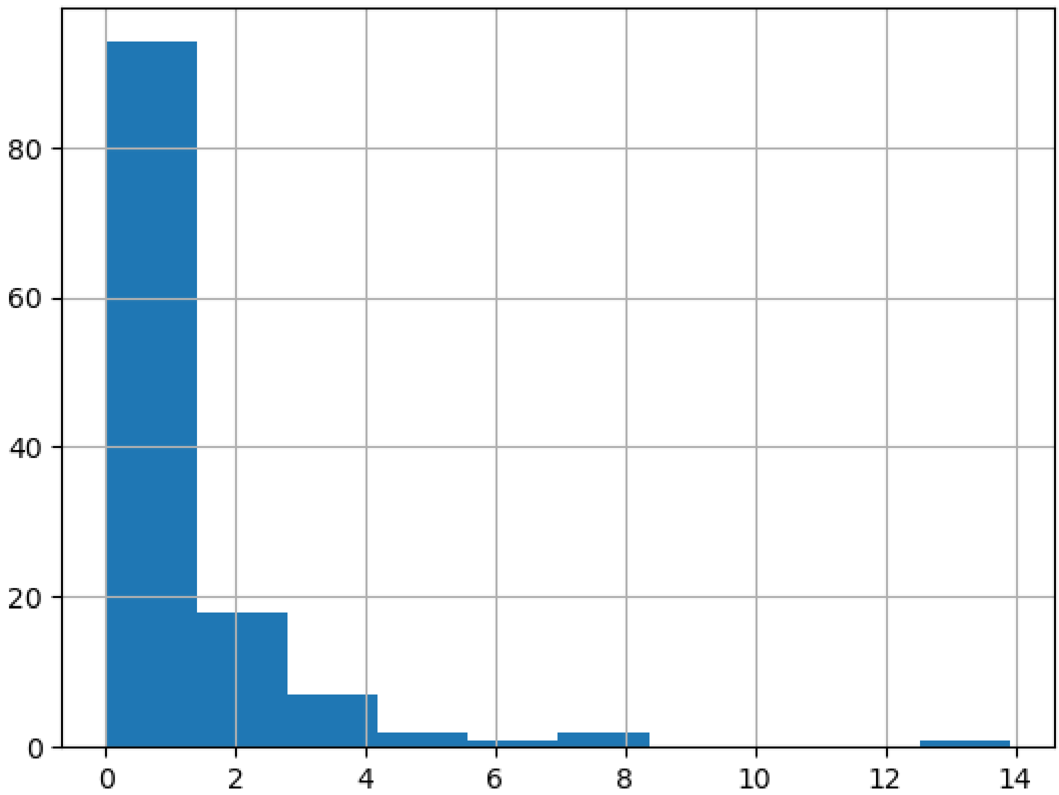
> ## Solution
>
> Inspect the data by plotting the individual datapoints.

In [ ]:
```
# Make a scatter plot of all the columns against the index
plt.plot(data['x1'], linestyle='none', marker='o', label = 'x1')
plt.plot(data['x2'], linestyle='none', marker='o', label = 'x2')
plt.plot(data['x3'], linestyle='none', marker='o', label = 'x3')
plt.plot(data['x4'], linestyle='none', marker='o', label = 'x4')
plt.plot(data['x5'], linestyle='none', marker='o', label = 'x5')
# place the legend outside the plot
plt.legend(bbox_to_anchor=(1.05, 1), loc=2, borderaxespad=0.)
plt.show()
```



> Looks like outliers are present, or - more likely - the distribution is skewed.

In [ ]:
```
# Stack the data into a single column
data_stack = data.stack()

# Plot a histogram of the data_stack
data_stack.hist()
plt.show()
```

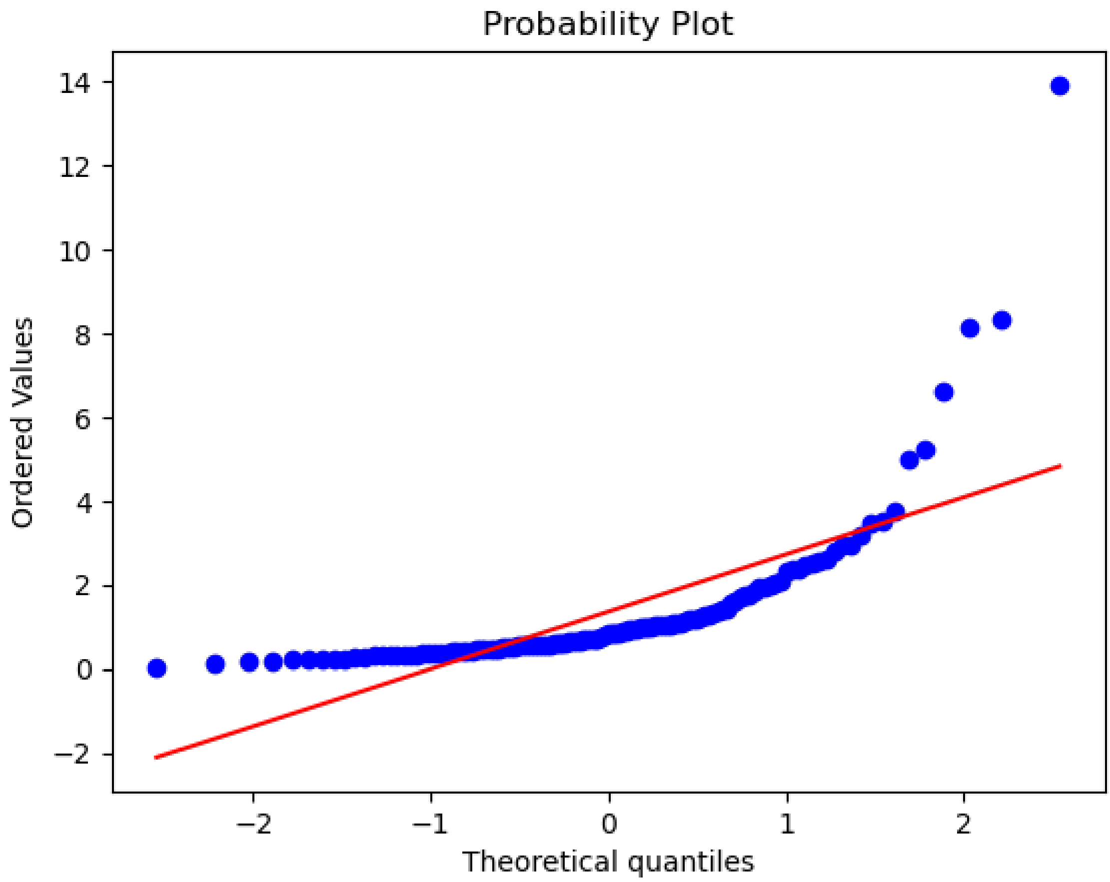


> The distribution is highly skewed.

> Verify the assumption of normality.

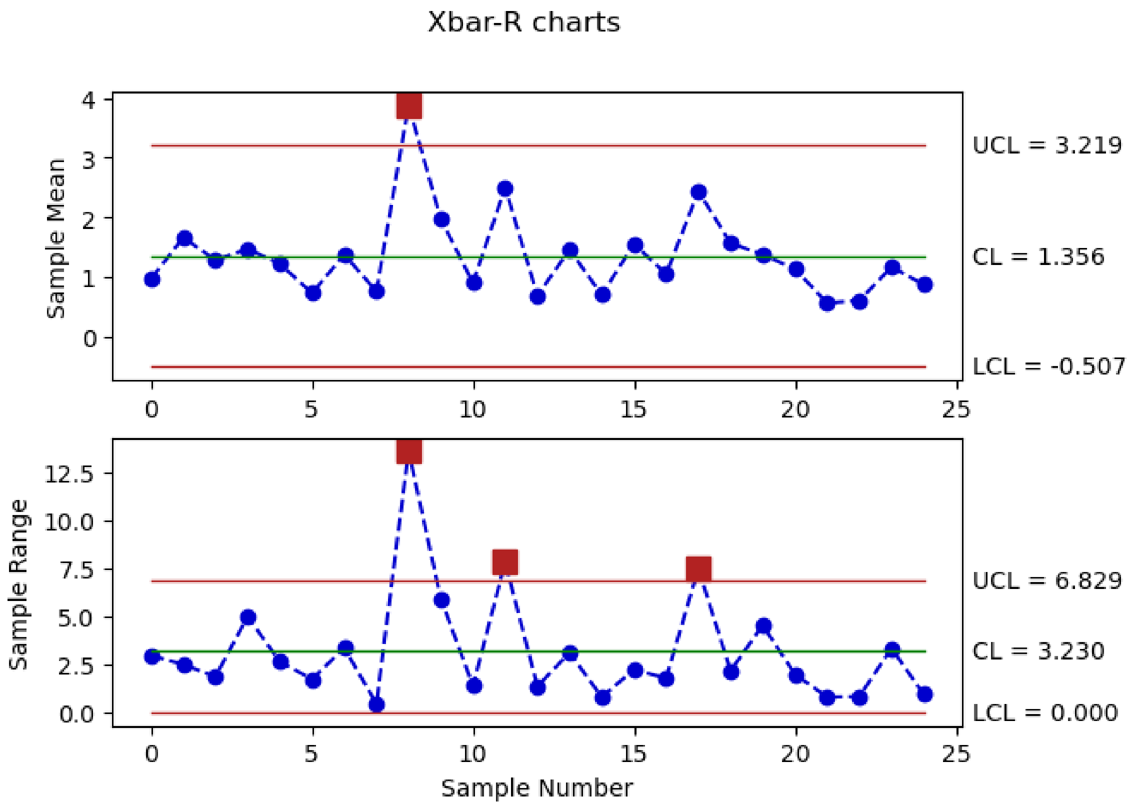
In [ ]:
```
# We can use the Shapiro-Wilk test
_, p_value_SW = stats.shapiro(data_stack)
print('p-value of the Shapiro-Wilk test: %.3f' % p_value_SW)

# QQ-plot
stats.probplot(data_stack, dist="norm", plot=plt)
plt.show()
```

```
p-value of the Shapiro-Wilk test: 0.000
```



> The data are non-normal. Therefore, we cannot use the X-bar and R chart on the raw data. We need to transform the data.
>
> But what happens if we neglect the normality violation and use the X-bar and R chart on the raw data?

In [ ]:
```
# X-bar and R charts
data_XR = qda.ControlCharts.XbarR(data)
```

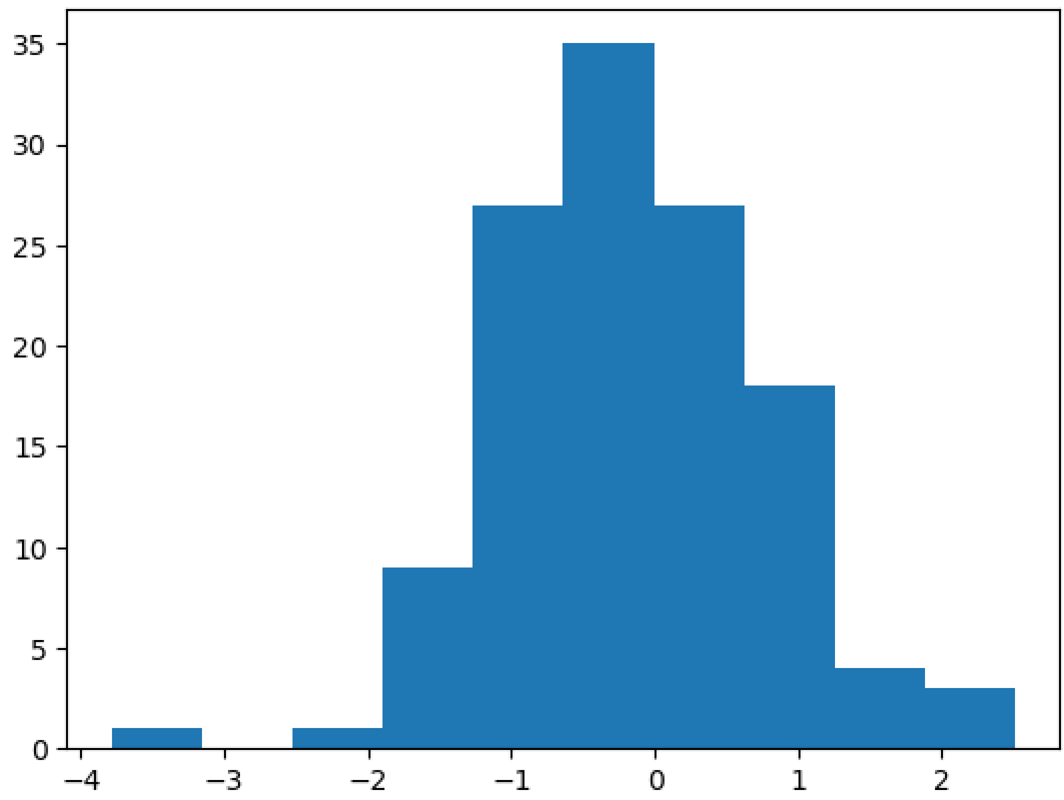


> OOC observations may be due to a violation of control chart assumptions...

> Let's transform the data to make it more normal using the Box-Cox transformation.
>
> Remember the Box-Cox transformation is defined as:
>
> $$x_{BC,i} = \begin{cases} \frac{x_i^\lambda - 1}{\lambda} & \text{if } \lambda \neq 0 \\ \ln x_i & \text{if } \lambda = 0 \end{cases}$$

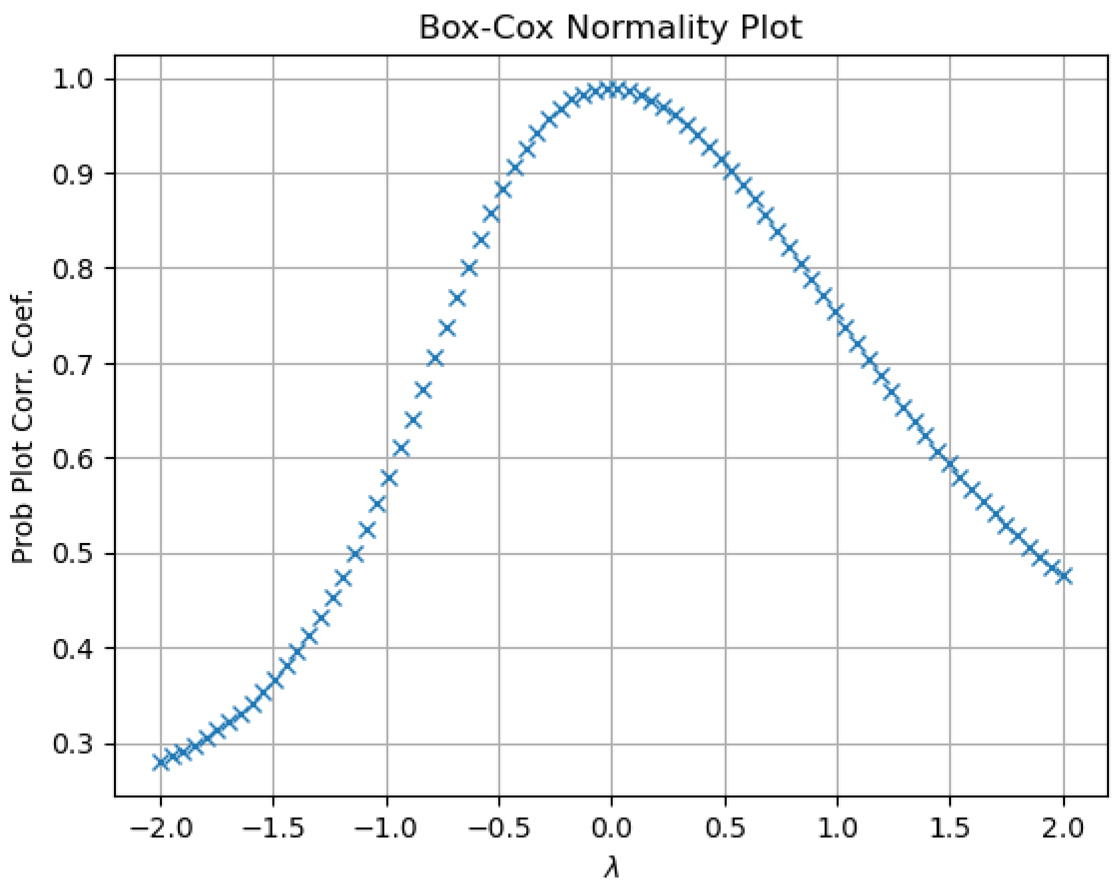
In [ ]:
```
# Box-Cox transformation and return the transformed data
[data_BC, lmbda] = stats.boxcox(data_stack)

print('Lambda = %.3f' % lmbda)

# Plot a histogram of the transformed data
plt.hist(data_BC)
plt.show()
```

```
Lambda = -0.037
```



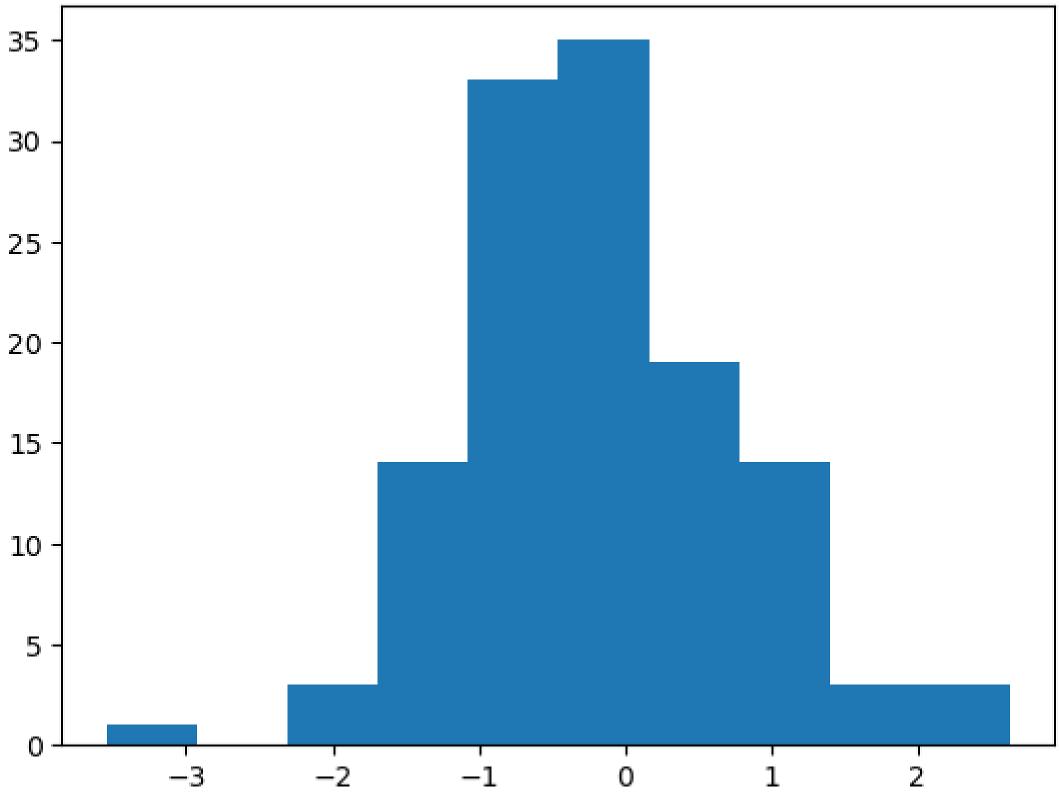
In [ ]:

```
# It is also possible to find the best value of lambda for the transformation
fig = plt.figure()
ax = fig.add_subplot(111)
stats.boxcox_normplot(data_stack, -2, 2, plot=ax)
# add grid
ax.grid(True)
```



By default, the Box-Cox function used Lambda = -0.037. A more interpretable (and very close to optimum) value is Lambda = 0.

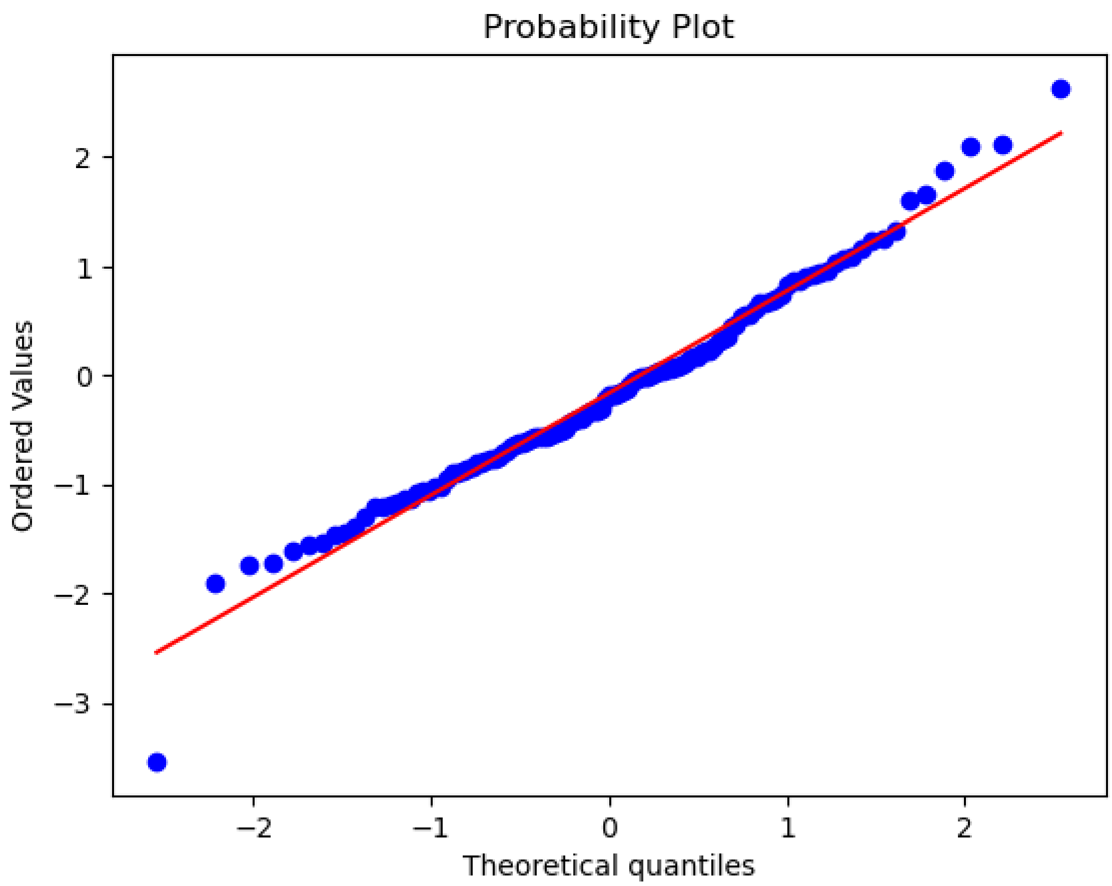
In [ ]:
```
# Use lambda = 0 for Box-Cox transformation and return the transformed data
data_BC = stats.boxcox(data_stack, lmbda=0)

# Plot a histogram of the transformed data
plt.hist(data_BC)
plt.show()
```



> Now the data seem to follow a normal distribution. Let's verify this by testing the normality.

In [ ]:
```
# We can use the Shapiro-Wilk test
_, p_value_SW = stats.shapiro(data_BC)
print('p-value of the Shapiro-Wilk test: %.3f' % p_value_SW)

# QQ-plot
stats.probplot(data_BC, dist="norm", plot=plt)
plt.show()
```

```
p-value of the Shapiro-Wilk test: 0.107
```



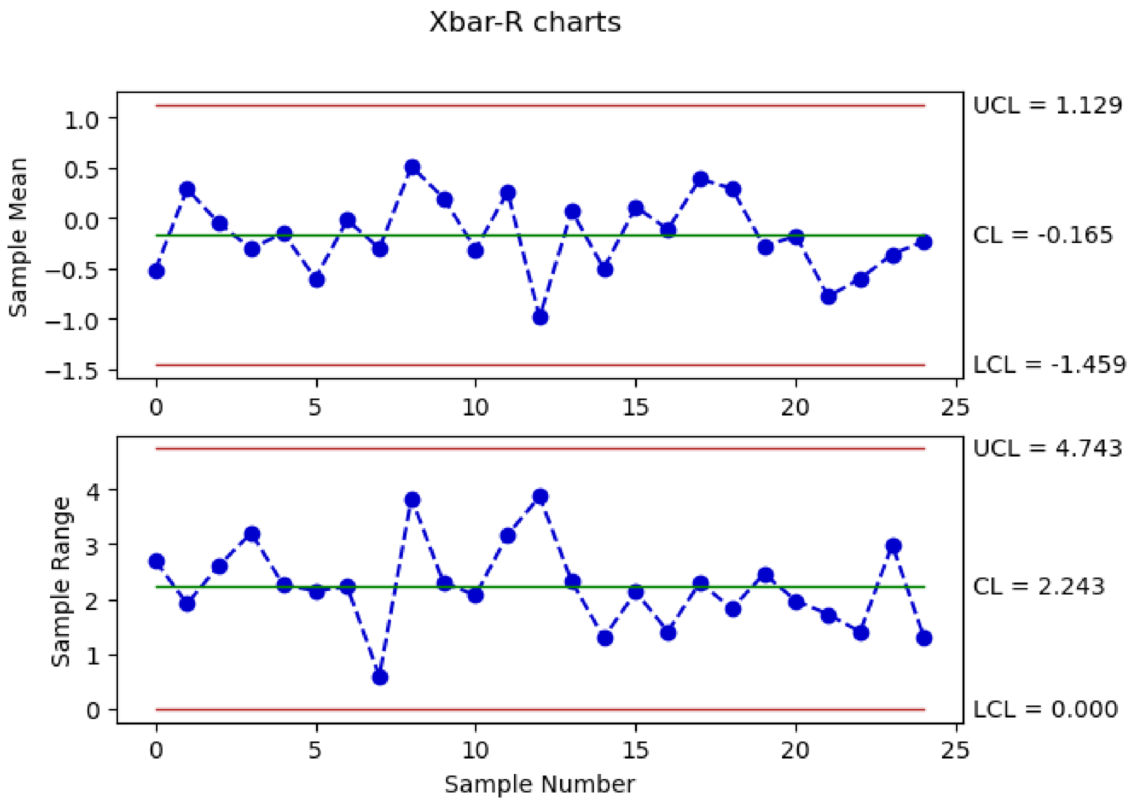
> Normality is verified. We can now use the X-bar and R chart on the transformed data.

In [ ]:
```
# First we need to unstack the data
data_BC_unstack = data_BC.reshape(data.shape)
# and convert it to a DataFrame
data_BC_unstack = pd.DataFrame(data_BC_unstack, columns = data.columns)

# Print out the transformed data
data_BC_unstack.head()
```

Out[ ]:

	x1	x2	x3	x4	x5
0	-0.748660	-0.903868	-1.546463	1.159080	-0.558616
1	-0.843970	0.964319	0.347130	-0.088831	1.076026
2	-1.910543	0.661657	0.055435	0.702602	0.227932
3	1.650388	-1.555897	0.045929	-0.709277	-0.946750
4	-1.177655	-0.623621	-0.562119	1.082144	0.554460

In [ ]:
```
# X-bar and R charts
data_BC_XR = qda.ControlCharts.XbarR(data_BC_unstack)
```



> The X-bar and R chart show that the process is in control.