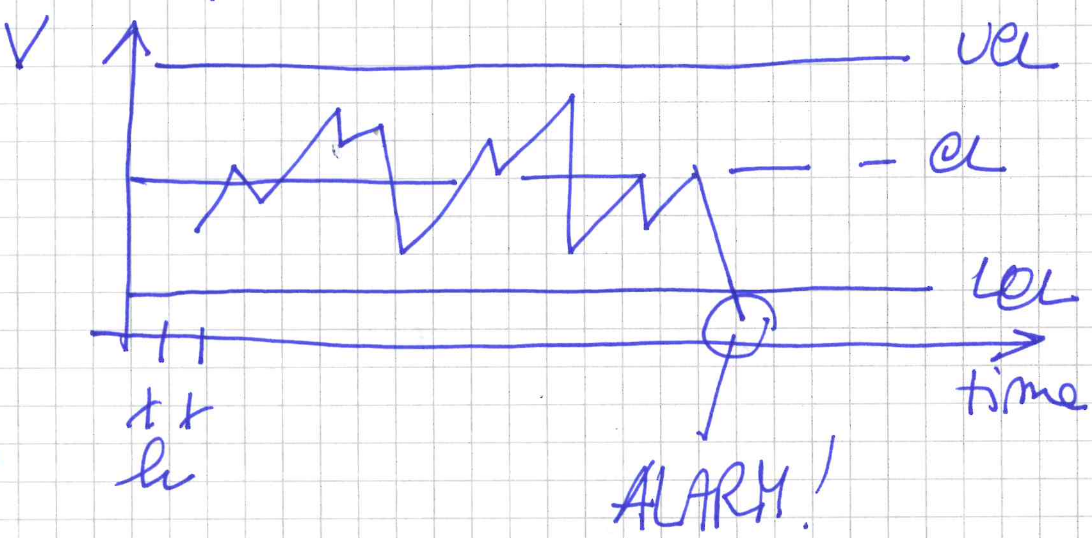
QDA 2023·05·03

sample n → V statistic

- $\bar{X}$ monitoring stability of μ
- R monitoring the stability of σ

$$\boxed{H_0: X \sim N(\mu_0, \sigma_0^2)}$$

Shewhart c.c. $\boxed{\begin{matrix} UCL \\ LCL \end{matrix} = \mu_V \pm K\sigma_V}$ $\quad \underline{K=3}$



$V=\bar{X}$: GIVEN H_0
$$\nearrow \mu_V = \mu_0 = \mu_{\bar{X}}$$
$$\searrow \sigma_V = \sigma_{\bar{X}} = \frac{\sigma_0}{\sqrt{n}}$$

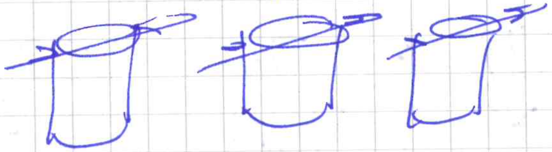
$V=R$: GIVEN H_0
$$\nearrow \mu_V = \mu_R = d_2(n)\,\sigma_0$$
$$\searrow \sigma_V = \sigma_R = d_3(n)\,\sigma_0$$

What if μ_0 and σ_0 are <u>UNKNOWN</u> ②

IN-CONTROL STATE (*)

$$X = D \sim N(\mu_0, \sigma_0^2)$$

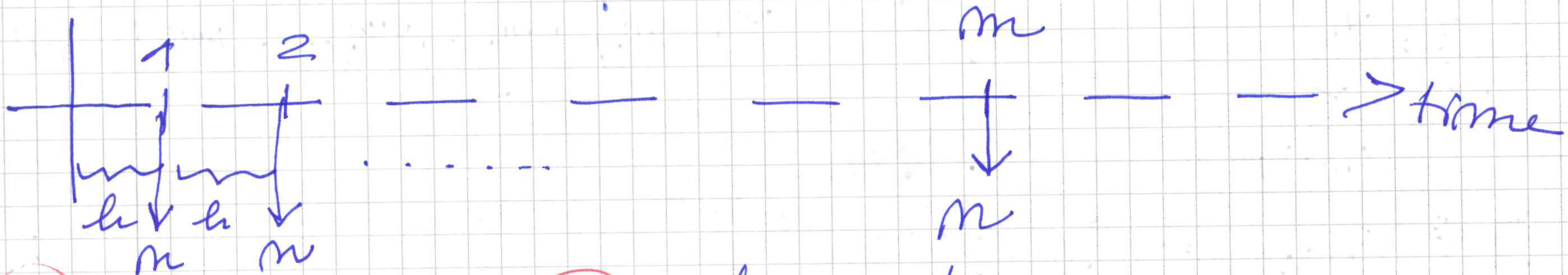
? ?



$m=3$

$\Rightarrow$ WE HAVE TO ESTIMATE $\hat{\mu}_0$ $\hat{\sigma}_0$

<u>Phase 1</u> or <u>Design phase</u> of the CC



m samples of size n from the process

$m = 20, 30$ → observing the process enough to have a clear <u>picture</u> of the in-control state*

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i	X_{ij}	$\bar{X}_i$	R_i
1	$X_{11} \dots X_{1n}$	$\bar{X}_1$	R_1
2	$X_{21} \dots X_{2n}$	$\bar{X}_2$	R_2
$\vdots$			
j^*		$\bar{X}_{j^*}$	R_{j^*}
m	$X_{m1} \dots X_{mn}$	$\bar{X}_m$	R_m

CAUSE: MACHINE TOOL PBM

$$E(R) = \mu_R = d_2(n) \cdot \sigma_0$$

?

$$\sigma_0 = \frac{E(R)}{d_2(n)} \Rightarrow \boxed{\hat{\sigma}_0 = \frac{\bar{R}}{d_2(n)}}$$

$$\hat{\mu}_R = \hat{E(R)} = \bar{R} = \frac{1}{m}\sum_i R_i$$

$$\hat{\sigma}_0 = \frac{\bar{R}}{d_2(n)}$$

$$H_0: \boxed{X \overset{iid}{\sim} N(\mu_0, \sigma_0^2)}$$

$$\hat{\mu}_0 = \sum_{i=1}^{m} \frac{\bar{X}_i}{m} = \bar{\bar{X}} = \frac{\sum_i \sum_j X_{ij}}{n\,m}$$

GRAND MEAN

$\bar{X}$ cc

$$\begin{matrix} UCL \\ CL \\ LCL \end{matrix} = \hat{\mu}_0 \pm K \frac{\hat{\sigma}_0}{\sqrt{n}} = \bar{\bar{X}} \pm K \frac{\bar{R}}{d_2 \sqrt{n}}$$

$K = 3$

R CC

$$\left.\begin{matrix}UCL\\ CL\\ LCL\end{matrix}\right. = d_2\hat{\sigma}_0 \pm k\, d_3\hat{\sigma}_0 = \cancel{d_2}\cdot\frac{\bar{R}}{\cancel{d_2}} \pm k\, d_3\frac{\bar{R}}{d_2} =$$

$$= \bar{R} \pm k\frac{d_3}{d_2}\cdot\bar{R} \qquad k=3$$

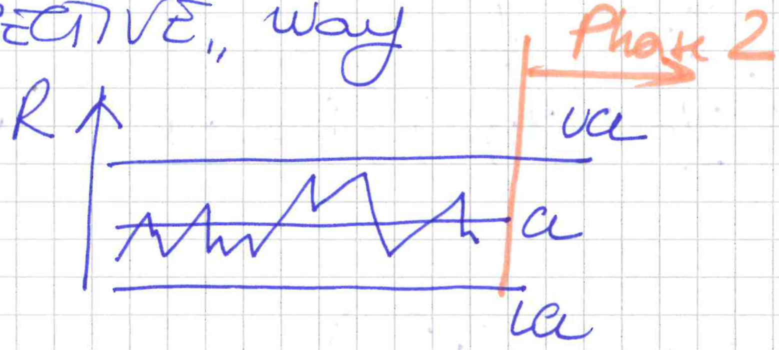
$$= \bar{R}\left(1 \pm k\frac{d_3}{d_2}\right)$$

Comments:

① $\boxed{LCL_{Range} \geq 0}$

$$R_i = \max_j x_{ij} - \min_j x_{ij}$$

② use control limits in "RETROSPECTIVE" way



j^* sample is OOC → process → looking for a "cause" behind this OOC

ASSIGNABLE CAUSE

$\hat{\sigma}_0$ $\hat{\mu}_0$

$X \overset{iid}{\sim} N(\mu_0, \sigma_0^2)$

$\bar{X} = V$ $\bar{X} \sim N(\mu_{\bar{X}}, \sigma_{\bar{X}}^2)$

$R = V$ $R \sim ?(\mu_R, \sigma_R^2)$

$\mu_R = d_2 \sigma$ $\sigma_R^2 = d_3^2 \sigma_0^2$

$\mu_V \pm K \sigma_V$

$K = 3$

$\underline{\underline{n \cdot m \text{ data}}} \rightarrow 5 \times 30 = 150$ $\hat{\mu}$, $\hat{\sigma}$

$n = 5$, $m = 30$

$ARL_0 = \dfrac{1}{\alpha}$?

α ?

FALSE ALARM PROB

$ARL_1 = \dfrac{1}{1-\beta}$?

β ?

PROB OF NOT DETECTING AN OOC STATE

$Y \sim ?\,(\mu_Y, \sigma_Y^2)$

Phase 1 – Phase 2

Phase 1 (DESIGN) | Phase 2 (USAGE)

$\underbrace{\qquad\qquad\qquad}_{m \text{ samples}}$ | $m+1$ $\quad$ $m+2$

m samples

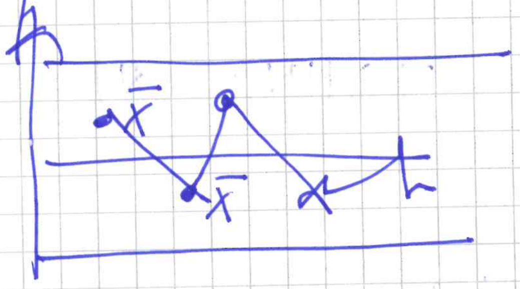
$\downarrow$

$\hat{\mu}$ $\quad$ $\hat{\sigma}$ $\rightarrow$ UCL_s, LCL_s

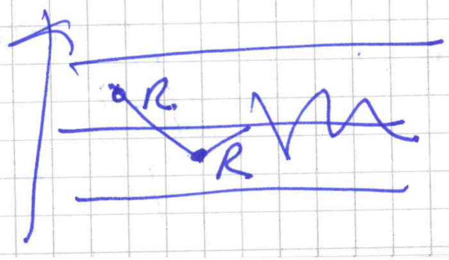
μ_0 known

σ_0 known

CONTROL LIMITS known



$\bar{X}$



R

$ARL_0 = \frac{1}{\alpha}$ $\quad ARL_1 = \frac{1}{1-\beta}$

average # samples for a false alarm

average # of samples for a justified alarm

$\boxed{\bar{X} \text{ control chart}}$

$H_0 : X \sim N(\mu_0, \sigma_0^2)$ IN CONTROL $\rightarrow \bar{X} \sim N\left(\mu_0, \frac{\sigma_0^2}{n}\right)$

$H_1 : X \sim N(\mu_1, \sigma_0^2)$ OUT of CONTROL $\rightarrow \bar{X} \sim N\left(\mu_1, \frac{\sigma_0^2}{n}\right)$

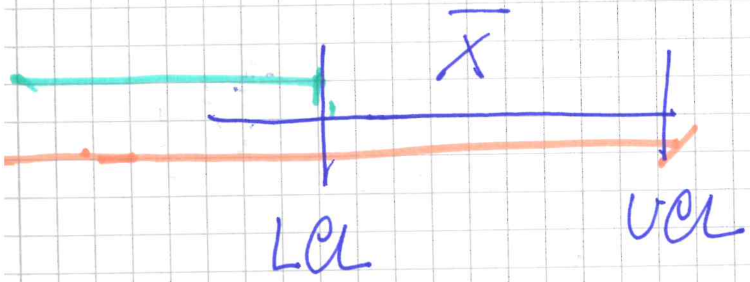
$\Delta = \mu_1 - \mu_0$

μ_1: NEW OOC MEAN

μ_0: OLD IC MEAN

$\beta(\Delta) = P\left\{\left[\bar{X} \in [LCL, UCL]\right] \mid H_1\right\} =$

$$= P\{\bar{X} \le UCL \mid H_1\} - P\{\bar{x} \le LCL \mid H_1\} =$$



$$= P\left\{Z \le \frac{UCL-\mu_1}{\sigma_{\bar{x}}}\right\} - P\left\{Z \le \frac{LCL-\mu_1}{\sigma_{\bar{x}}}\right\}$$

$$Z = \frac{\bar{X} - \overbrace{\mu_1}^{\mu_{\bar{x}}}}{\sigma_{\bar{x}}}$$

$$= \Phi\left(\frac{UCL-\mu_1}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{LCL-\mu_1}{\sigma_{\bar{x}}}\right) \overset{*}{=}$$

$$\overset{*}{=} \Phi\left(\frac{\mu_0 + K\overbrace{\sigma_{\bar{x}}}^{\sigma_0/\sqrt{n}} - (\mu_0+\Delta)}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{\mu_0 - K\sigma_{\bar{x}} - (\mu_0+\Delta)}{\sigma_{\bar{x}}}\right)$$

$$= \Phi\left(\frac{\cancel{\mu_0} + K\sigma_{\bar{x}} - \cancel{\mu_0} - \Delta}{\sigma_{\bar{x}}}\right) - \Phi\left(\frac{\cancel{\mu_0} - K\sigma_{\bar{x}} - \cancel{\mu_0} - \Delta}{\sigma_{\bar{x}}}\right)$$

$$\sigma_{\bar{x}} \leftarrow \sigma_0/\sqrt{n}$$

$$\Delta = \mu_1 - \mu_0 \longrightarrow \delta \cdot \sigma = \Delta \qquad \delta = \frac{\Delta}{\sigma} = \frac{\mu_1 - \mu_0}{\sigma}$$

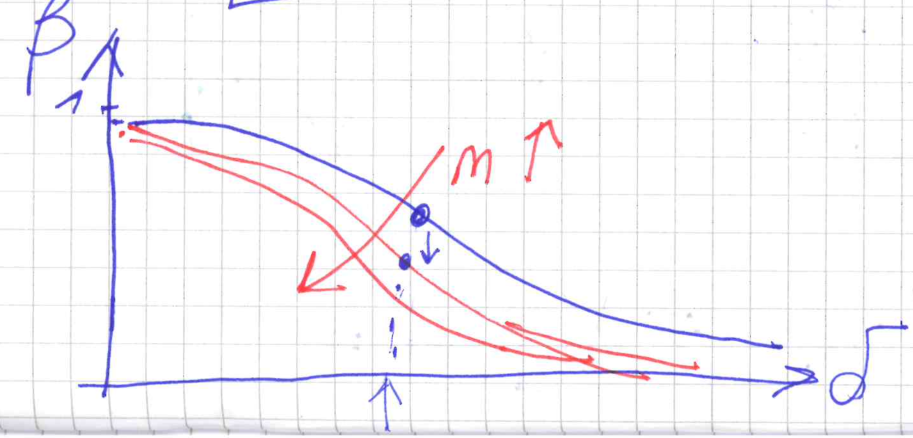
$$\Delta = \mu_1 - \mu_0$$
absolute shift

$$\delta = \frac{\Delta}{\sigma}$$
shift in standard deviation units

$$\Delta = \delta \cdot \sigma$$

$$\beta(\delta) = \Phi\left(\frac{k\frac{\cancel{\sigma}}{\sqrt{n}} - \delta\cancel{\sigma}}{\cancel{\sigma}/\sqrt{n}}\right) - \Phi\left(\frac{-k\frac{\cancel{\sigma}}{\sqrt{n}} - \delta\cancel{\sigma}}{\cancel{\sigma}/\sqrt{n}}\right) =$$

$$= \underbrace{\Phi\left(k - \delta\sqrt{n}\right) - \Phi\left(-k - \delta\sqrt{n}\right)}$$



OC curve

(10)

δ^*

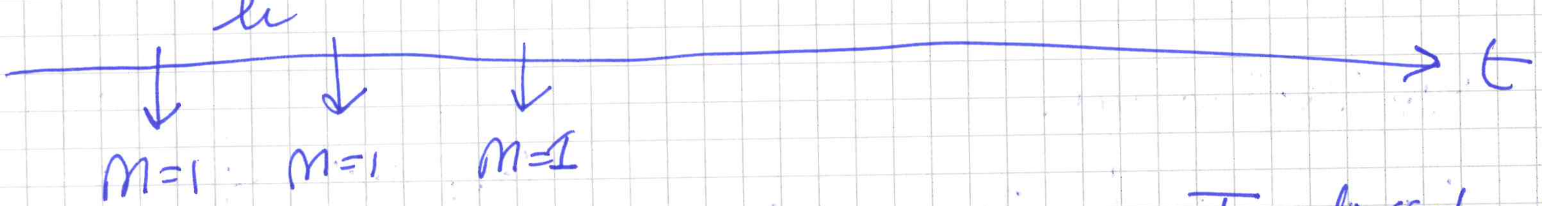
$\beta(\delta, n, k)$ $\qquad$ $ARL = \dfrac{1}{1-\beta(\delta, n, k)}$ $\qquad$ $ATS = ARL \cdot h$

$\uparrow$ time between two samples

always for $\bar{X}$ cc : α

$\delta = 0$ (NO SHIFT) — $\beta = 1-\alpha$

$$1-\alpha = \beta(\delta=0) = \Phi\big(k - \underbrace{\delta\sqrt{n}}_{0\,(\delta=0)}\big) - \Phi\big(-k - \underbrace{\delta\sqrt{n}}_{0\,(\delta=0)}\big)$$



$\bar{X}(n=1)$ $\wr$ Individual X cc

$X \overset{iid}{\sim} N(\mu_0, \sigma_0^2)$

$n=1$ $\;<\;$ Individual X cc / MR = Moving Range cc $\;\wr\; R$

I-MR

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t	X_t	MR_t
1	x_1	
2	x_2	$\lvert x_2 - x_1 \rvert$
3	x_3	$\lvert x_3 - x_2 \rvert$
$\vdots$	$\vdots$	
	x_i	
$\vdots$	$\vdots$	
m	x_m	

$$V = MR_i = \lvert x_i - x_{i-1} \rvert \qquad i = 2 \ldots\ldots m$$

$$= \max(x_i, x_{i-1}) - \min(x_i, x_{i-1})$$

$$\left.\begin{aligned} \mu_V = \mu_{MR} &= d_2(\underset{\substack{\uparrow \\ m=2}}{2}) \cdot \sigma_0 \\ \sigma_V = \sigma_{MR} &= d_3(2) \cdot \sigma_0 \end{aligned}\right\} \text{R control } (m=2)$$

$X \sim N(\mu_0, \sigma_0^2)$

$\bar{X} = X = V$ $\quad \mu_V \pm K\sigma_V = \boxed{\mu_0 \pm K\sigma_0 = \begin{matrix} UCL \\ LCL \end{matrix} \quad \text{I cc}}$

$\bar{X}(m=1) \quad \frac{\sigma_0}{\sqrt{m}} = \frac{\sigma_0}{\sqrt{1}}$

$MR = V$ $\quad \mu_V \pm K\sigma_V = \boxed{d_2(2)\,\sigma_0 \pm K\, d_3(2)\,\sigma_0 \quad \text{MR cc}}$

Phase 1

$$\hat{\mu}_0 = \bar{X} = \bar{\bar{X}} = \frac{\sum_m \cancel{\sum_n} x_{ij}}{m \cdot n}$$

$n = 1$

$$\hat{\sigma}_0 = \frac{\overline{MR}}{d_2(2)}$$

$n = 2$

$$\boxed{\hat{\sigma}_0 = \frac{\bar{R}}{d_2(n)}}$$