1

QDA 2023/03/01

Quality feature $\rightarrow Y$

DETERMINISTIC — RANDOM VARIABLE (R.Y.)

$$Y_t \overset{(*)}{=} \underbrace{\mu_t + \varepsilon_t}$$

independently and identically distributed

Hp: $\mu_t = \mu$ $\quad \varepsilon_t \overset{iid}{\sim} N(0, \sigma_\varepsilon^2)$

equivalently: $\underset{\text{R.V.}}{Y_t} \overset{iid}{\sim} \boxed{N(\mu_t, \sigma_\varepsilon^2)}$

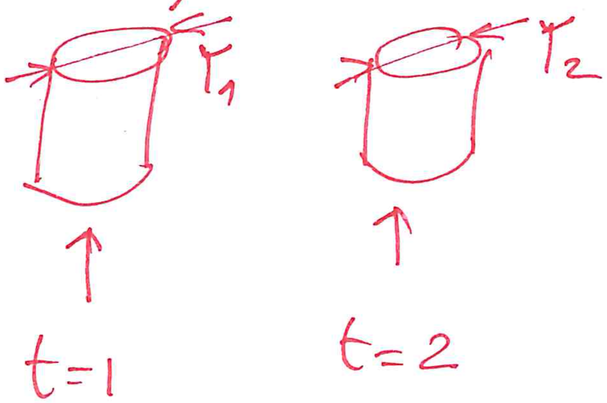
$E(\varepsilon_t) = 0$

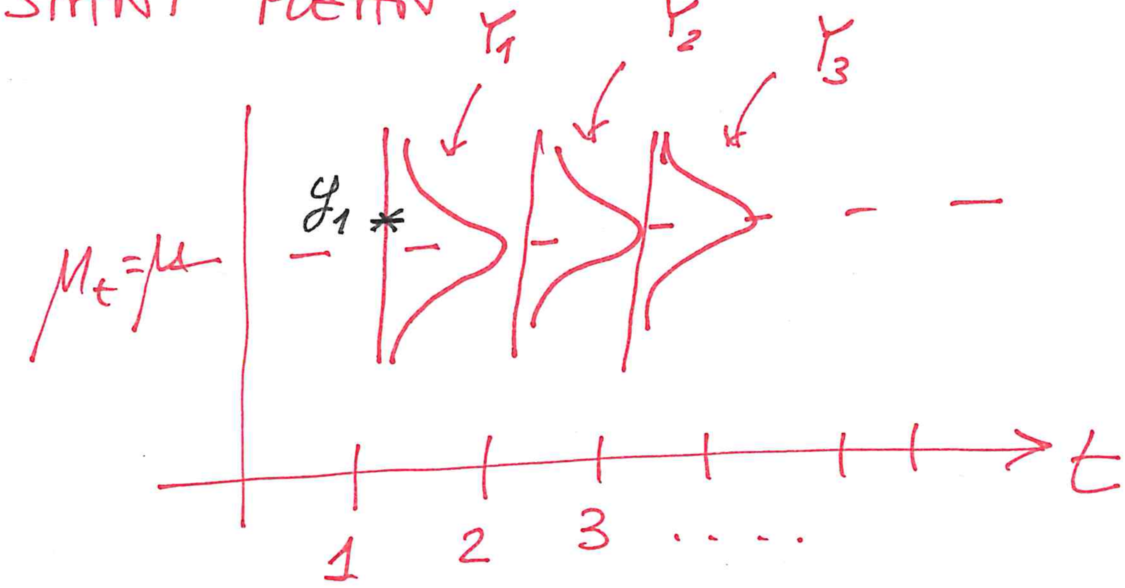
$$E(Y_t) \overset{(*)}{=} E(\mu_t + \varepsilon_t) = \mu_t$$

$$V(Y_t) \overset{(*)}{=} V(\mu_t + \varepsilon_t) = \sigma_\varepsilon^2$$

②

* $\mu_t = \mu \Rightarrow$ CONSTANT MEAN





Y → DISTRIBUTION R.V.

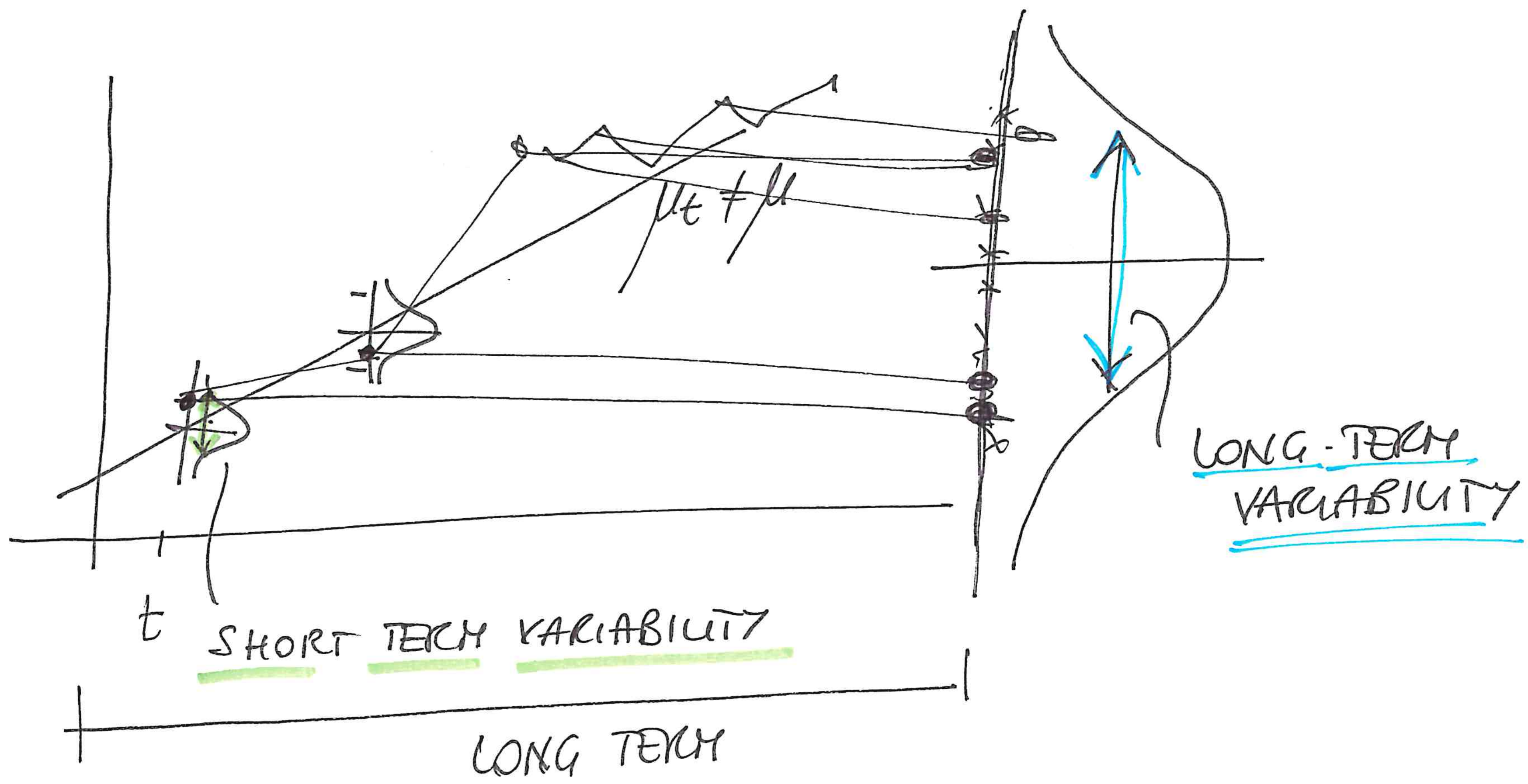
y → OCCURRENCE DATA OBSERVED NUMBER

* NORMALITY

* CONSTANT VARIANCE

$$\sigma_\varepsilon^2 = \sigma_t^2$$

* INDEPENDENCE (UNCORRELATION = special type of independence)



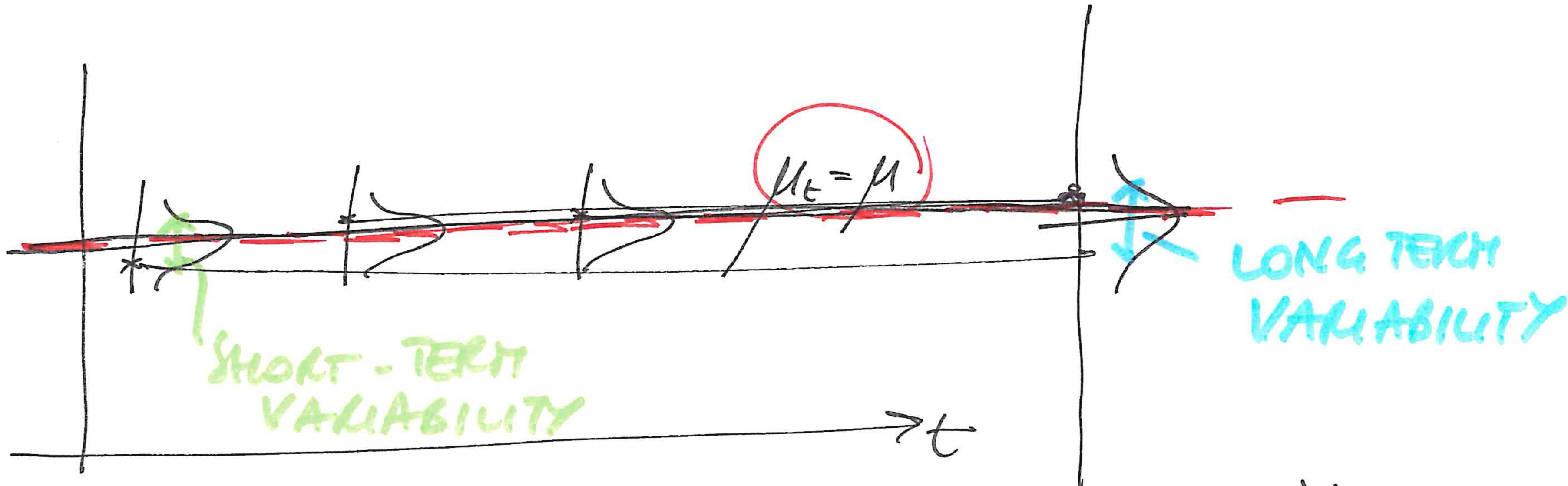
TREND $\mu_t \neq \mu$ $\Longrightarrow$ LONG-TERM VARIABILITY > SHORT-TERM VARIABILITY

as the LONG-TERM VARIABILITY is including the SHORT-TERM one + DRIFT of the MEAN μ_t $\nearrow_{\text{TREND}}$

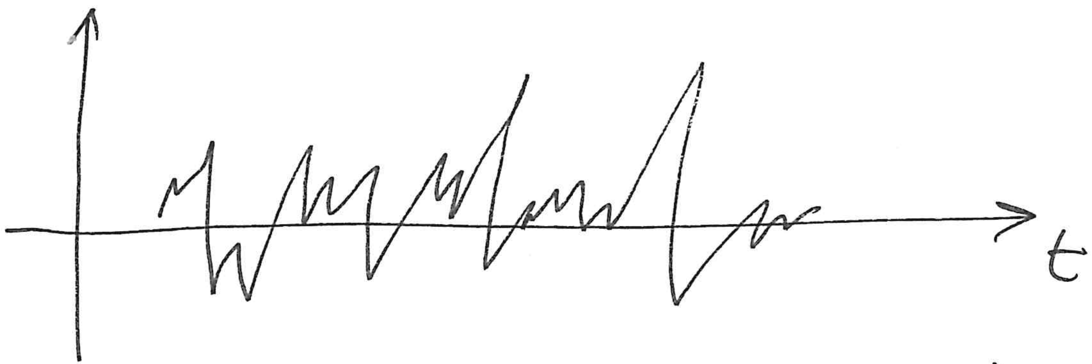
4

NO TREND

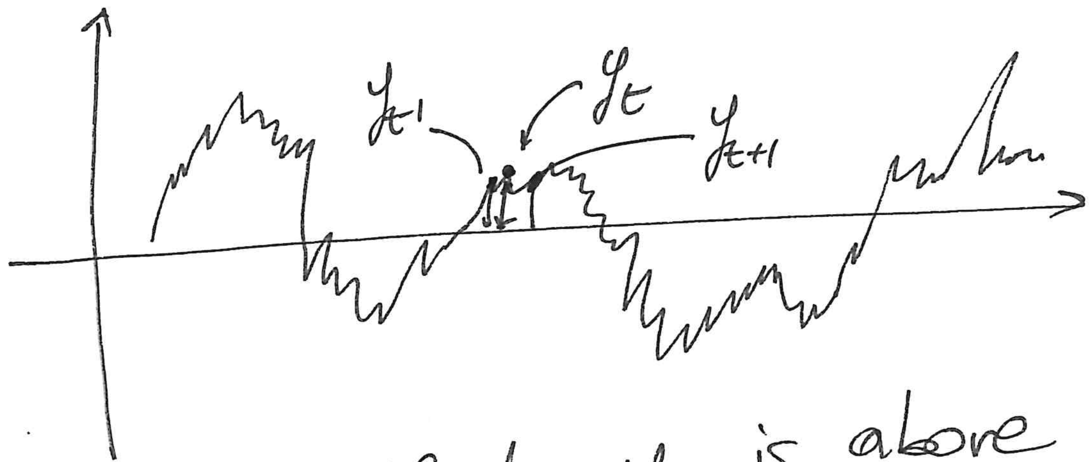
$\mu_t = \mu$



SHORT - TERM = LONG - TERM VARIABILITY as $\mu_t = \mu$ → SECOND SOURCE OF VARIABILITY DISAPPEARED



AUTOCORRELATION (MEANDERING)



probability that y_t is above the mean depends on the " " y_{t-1} " " "

6 (~~STATIS~~TICAL) TEST of HYPOTHESIS $\Rightarrow$ $\boxed{\text{RUNS TEST}}$

H_0 : W_t are coming out from a <u>RANDOM</u> SEQUENCE

↑ NULL

RANDOM

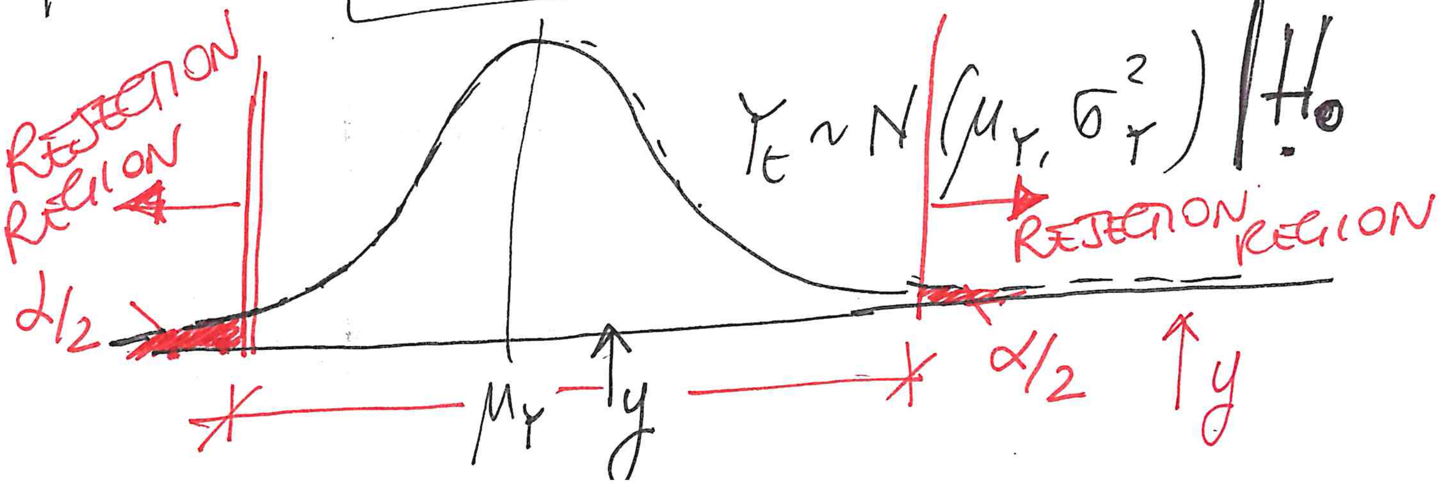
H_1 : W_t are coming out from a <u>NON</u> RANDOM SEQUENCE

↑ ALTERNATIVE

$\boxed{\text{ASSUME } H_0 \text{ IS TRUE}}$ → TEST STATISTIC $Y_t \sim D(\ ,\ )$

$Y_t = \#$ RUNS

$Y_t \sim N(\mu_Y, \sigma_Y^2) \mid H_0$



7 α = first-type error = probability of REJECTING H_0 even if H_0 is TRUE

I set $\alpha = 1\%,\ 5\%,\ 10\%$

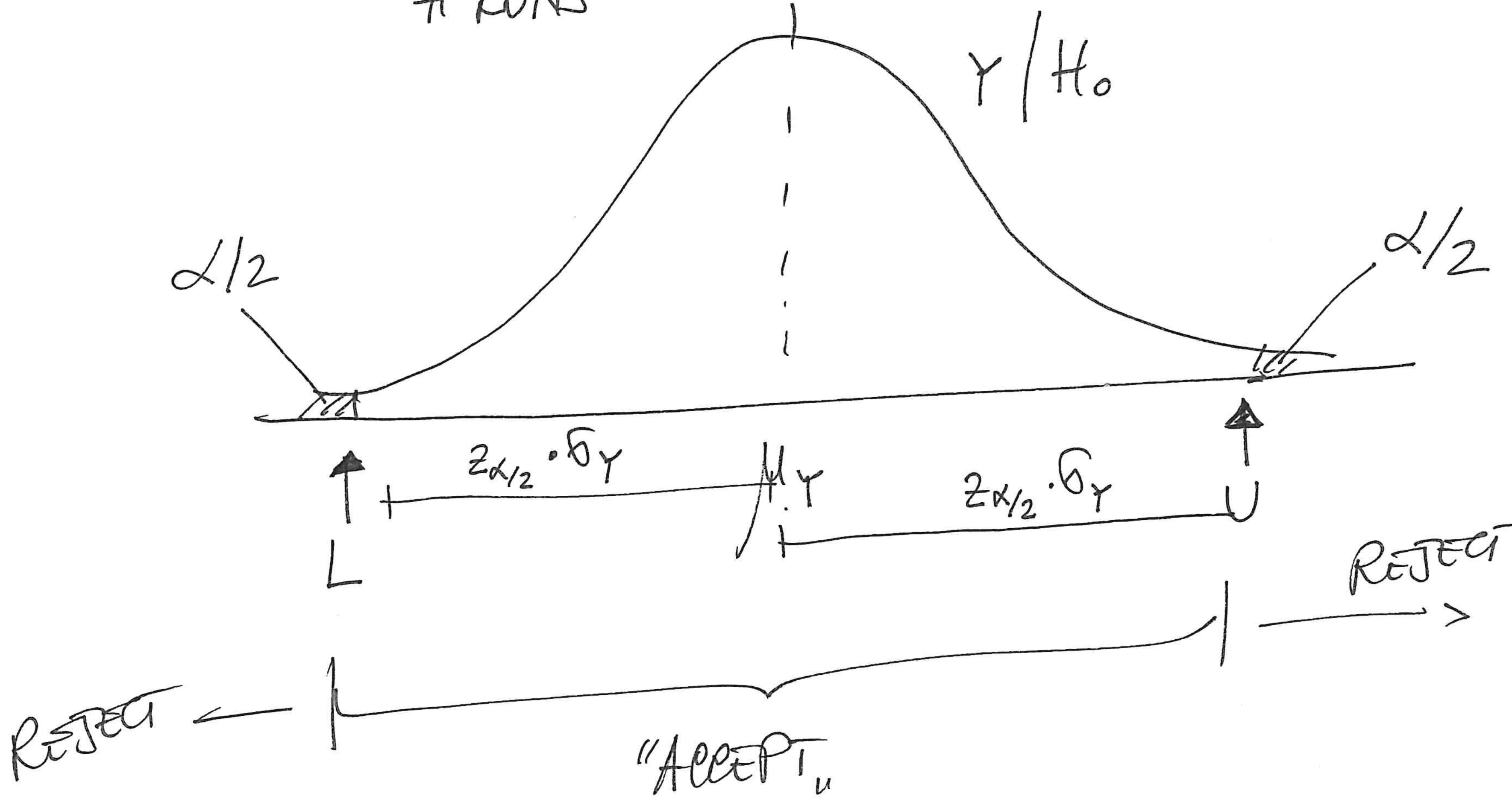
DECISION	REALITY: H_0 IS TRUE	(Ho IS NOT TRUE) H_1 IS TRUE
"ACCEPT" H_0	✗ $1-\alpha$	β
REJECT H_0	α	✗ ~~$1-\alpha$~~ $1-\beta$
	1	1

β = second-type error = probability of "accepting" H_0 even if H_0 IS NOT TRUE

H_0 is TRUE

$Y \sim N(\mu_Y, \sigma_Y^2)$

RUNS



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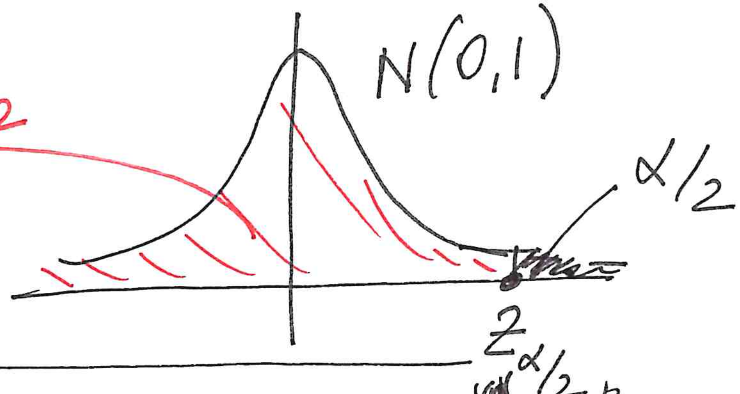
$$L = \mu_Y - K \cdot \sigma_Y$$

$$U = \mu_Y + K\,\sigma_Y$$

$$K = z_{\alpha/2}$$



$$z_{\alpha/2} = \Phi^{-1}(1-\alpha/2) \Longleftarrow \boxed{\Phi(z_{\alpha/2}) = 1-\alpha/2}$$

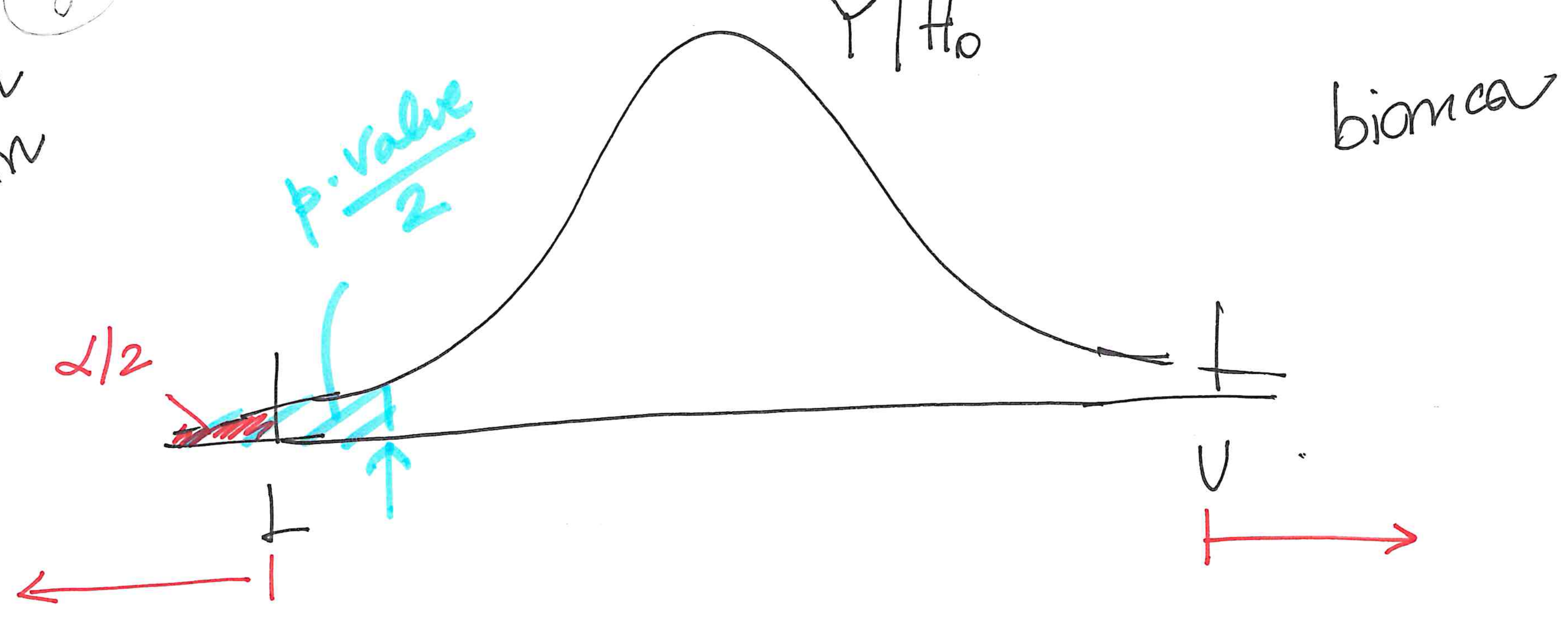
$$P(Z \leq z_{\alpha/2}) = 1-\alpha/2$$

Z: $N(0,1)$

$z_{\alpha/2} \simeq 2$ $\quad \alpha = 5\%$

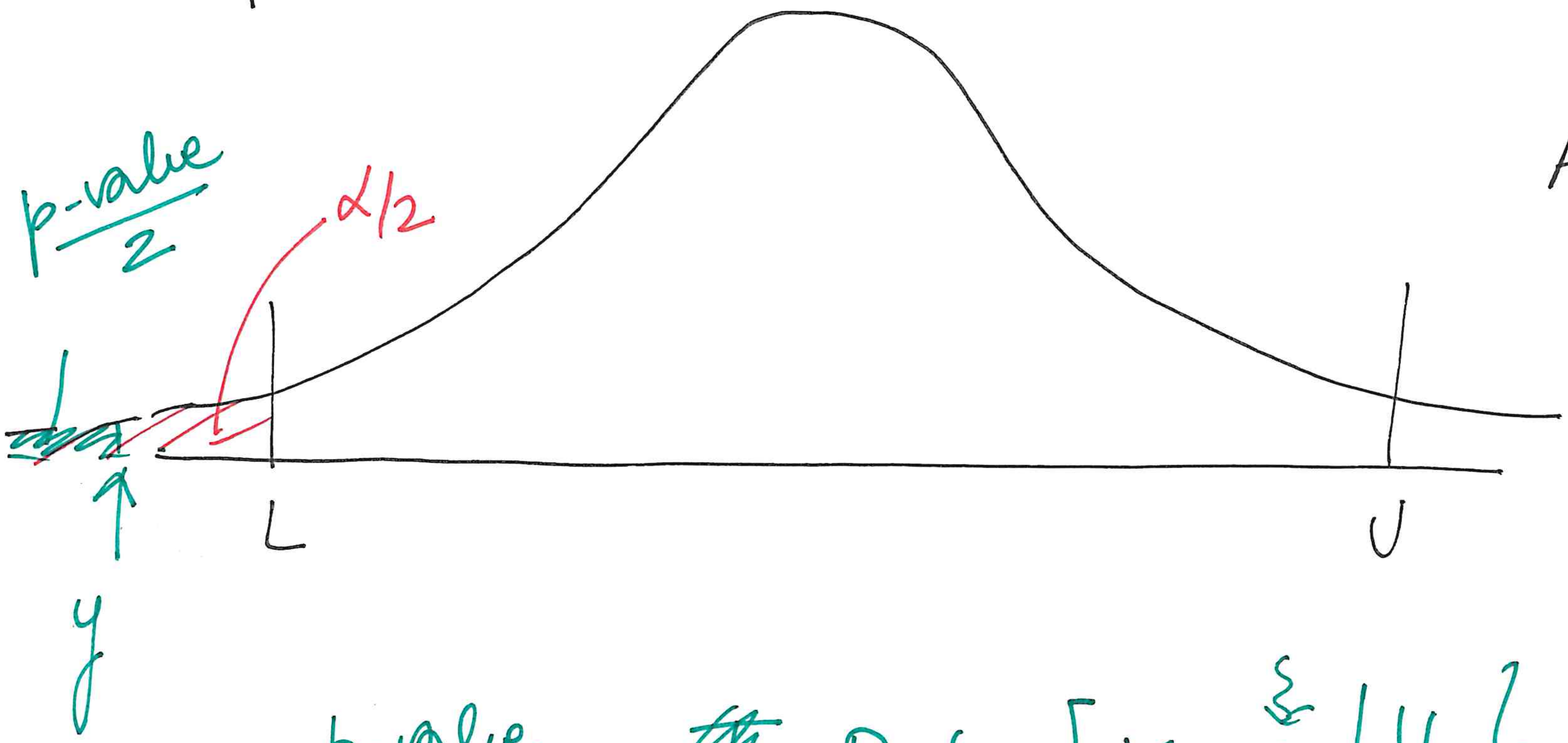
9

m
m



(10) p-value > α $\Rightarrow$ "ACCEPT" H_0
(NON REJECT)

p-value $\leq \alpha$ $\Rightarrow$ REJECT H_0



Alice

$$\frac{\text{p-value}}{2} = \text{Prob}\left\{ Y \leq y \mid H_0 \right\}$$

$Y \sim N(\mu_Y, \sigma_Y^2)$