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QDA 2023 06 26

SPM

Statistical < Process / Quality Monitoring (SQM)

Statistical < Quality / Process Control → SPC (SQC)

Monitoring = observing the quality of a process (or product) with time to detect CHANGES

Control = acting on some inputs (or process parameters) to keep the quality of a process stable with time

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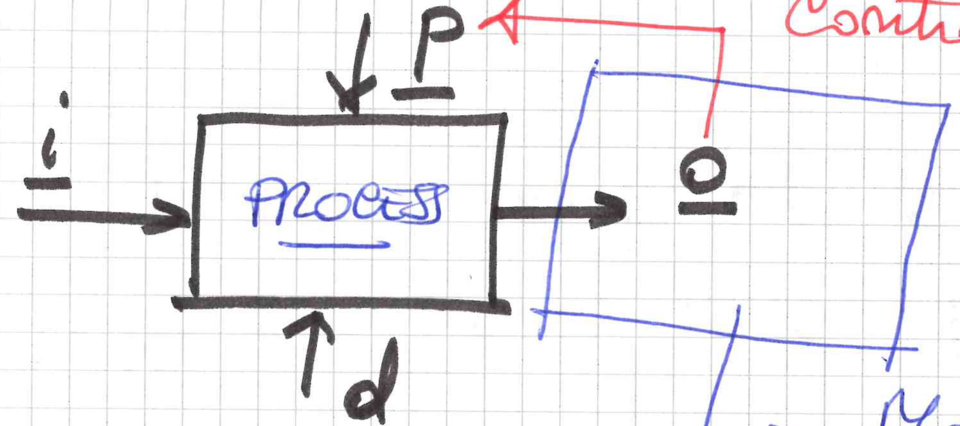


Control – acting on the p to keep the o stable

Monitoring – observe the output to detect if the process is stable

① SAMPLING POLICY
→ observe the process output with time

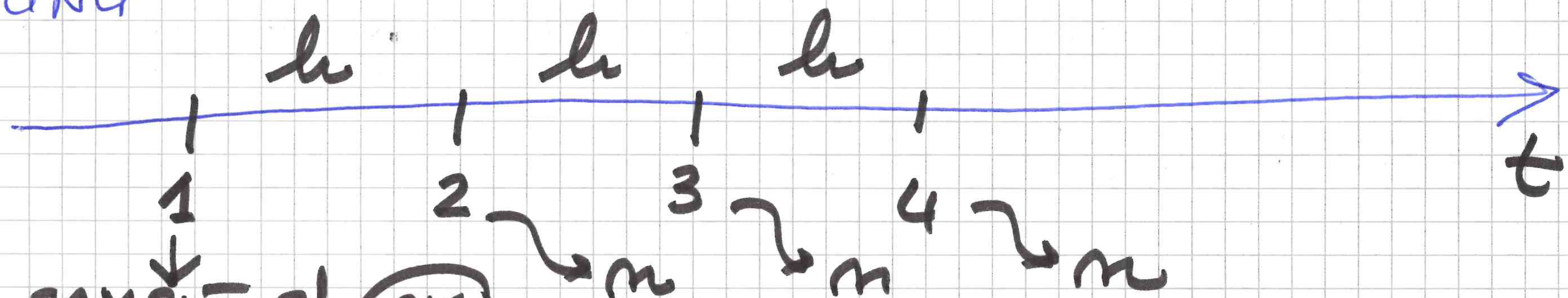
② → each time interval → is the process in control?

③ → if it is in control – proceed
→ if it is supposed to be not in control, try to figure out what is wrong * and remove the issue to keep the process back to the in-control state

DIAGNOSIS

3 * pay attention to FALSE ALARMS
(the process appears to be out of control even if nothing happened)

1 SAMPLING

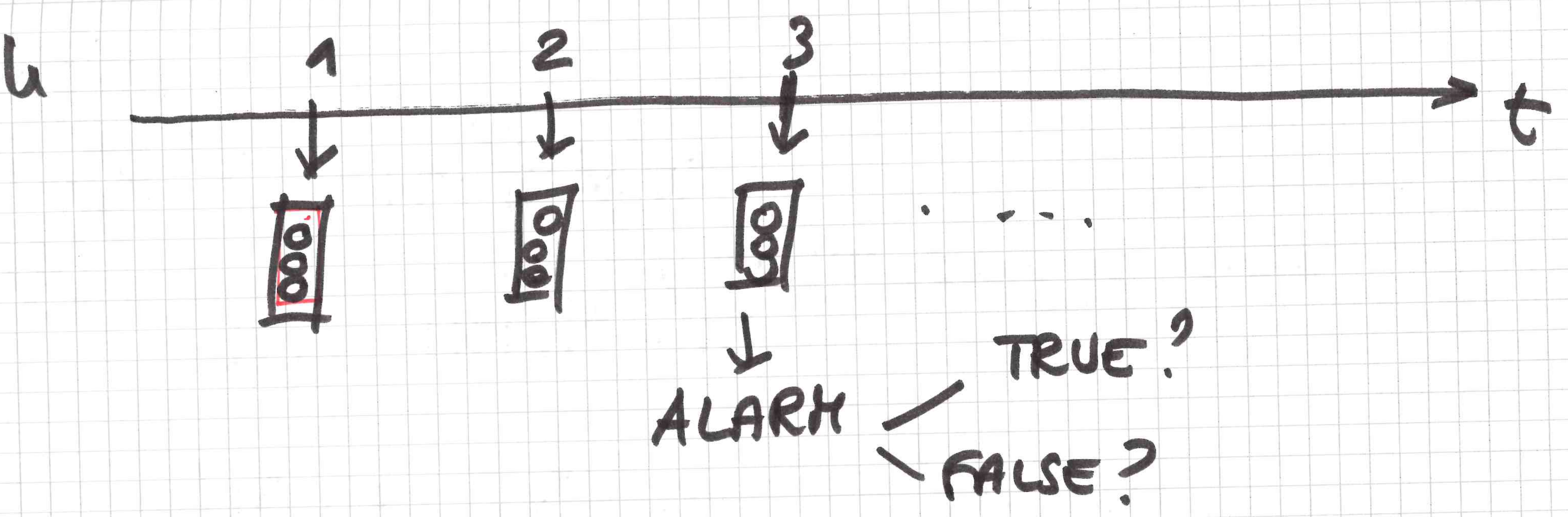


SAMPLE of DATA (ex) m pins → <u>diameter</u> of the sample

h = 1 hour, 1 minute, 1 day, 3 days ...

At each i-th sample : is the process <u>in-control</u> ? → OK

or is the process <u>out</u> of <u>control</u> ? → ALARM



ex Quality feature = diameter of pin

output = O = diameter

When the process is in control $\boxed{O \sim N(\mu_0, \sigma^2)}$

$X = O \sim N(\mu_x, \sigma_x^2)$ IN CONTROL STATE

μ_0

quality feature

STABLE?

STABLE?

Is the process mean μ_x stable with time?

$\leftarrow h \rightarrow$

t: 1, 2, 3, …, i

1 $\downarrow$ sample n $\downarrow$ $\hat{\mu}_{x_1} = \bar{X}_1$

2 $\downarrow$ sample n $\downarrow$ $\hat{\mu}_{x_2} = \bar{X}_2$

i $\downarrow$ sample n $\downarrow$ $\hat{\mu}_{x_i} = \bar{X}_i$

TEST : $H_0: \mu_x = \mu_0$ ↳ in control value

$H_1: \mu_x \neq \mu_0$

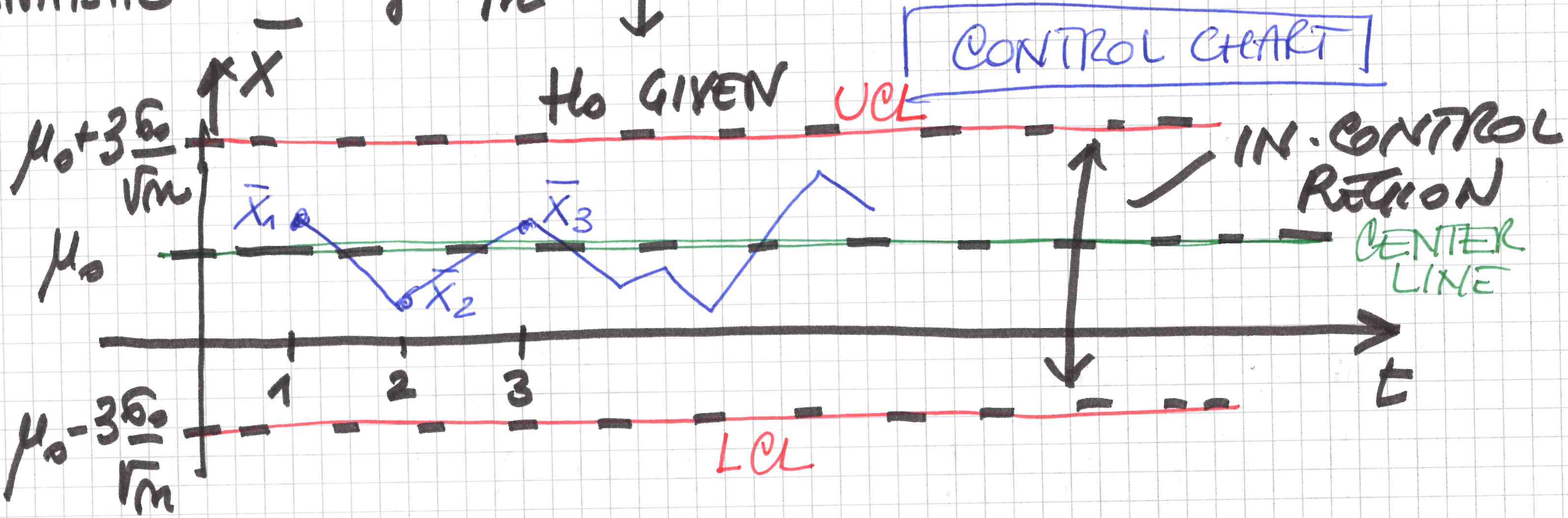
2 IS THE PROCESS STABLE?

[IN-CONTROL STATE
$\cancel{X} = 0 \sim N(\mu_0, \sigma_0^2)$]

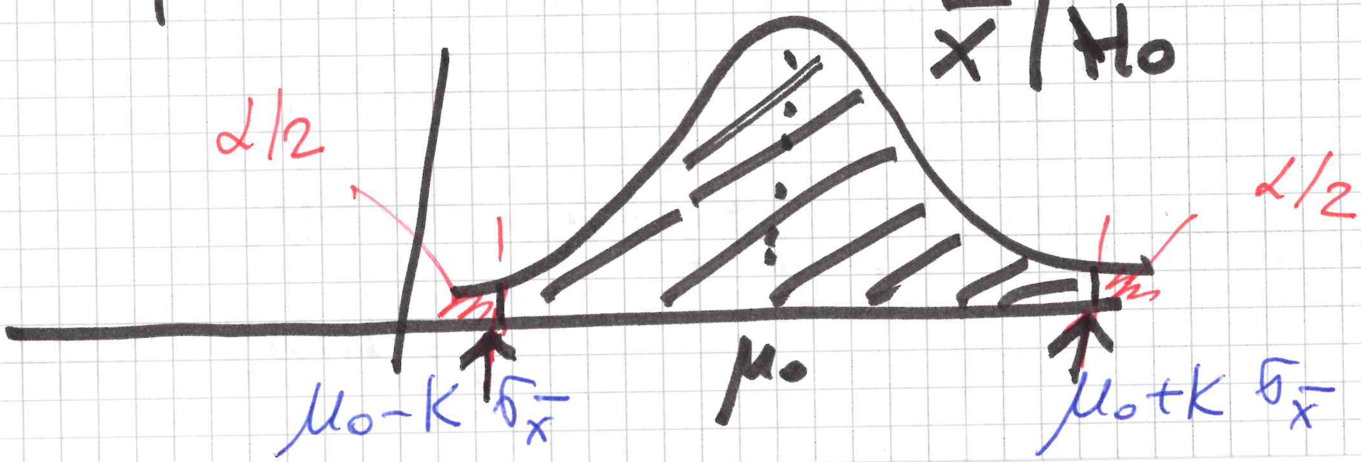
TEST STATISTIC $\bar{X} = \sum_{j=1}^{m} \frac{X_j}{m} \sim N\left(\mu_0, \frac{\sigma_0^2}{m}\right)$

$\downarrow$

H_0 GIVEN



if the process is in-control



USUALLY
$k = 3$

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CONTROL CHART

Upper Control Limit = UCL

Center Line = CL

Lower Control Limit = LCL

sample → $V = \bar{X}$ (sample statistic)

$V \sim N(\mu_V, \sigma_V^2)$ IN CONTROL

ex $\underbrace{\bar{X}}_{V} \sim N(\underbrace{\mu_0}_{\mu_V}, \underbrace{\frac{\sigma_0^2}{n}}_{\sigma_V^2})$

$UCL = \mu_V + K\sigma_V$

$CL = \mu_V$

$LCL = \mu_V - K\sigma_V$

usually $K = 3$

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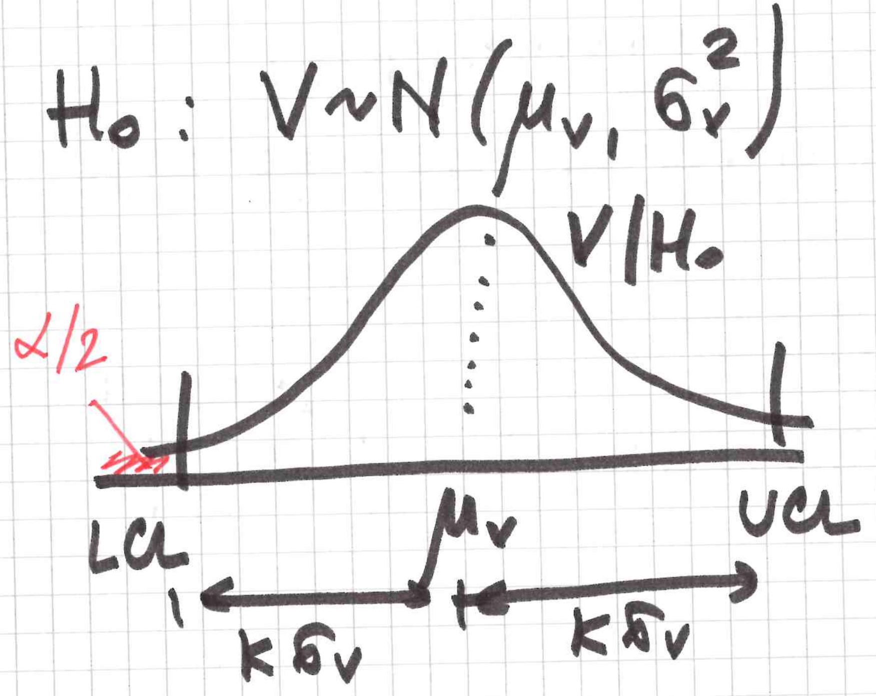
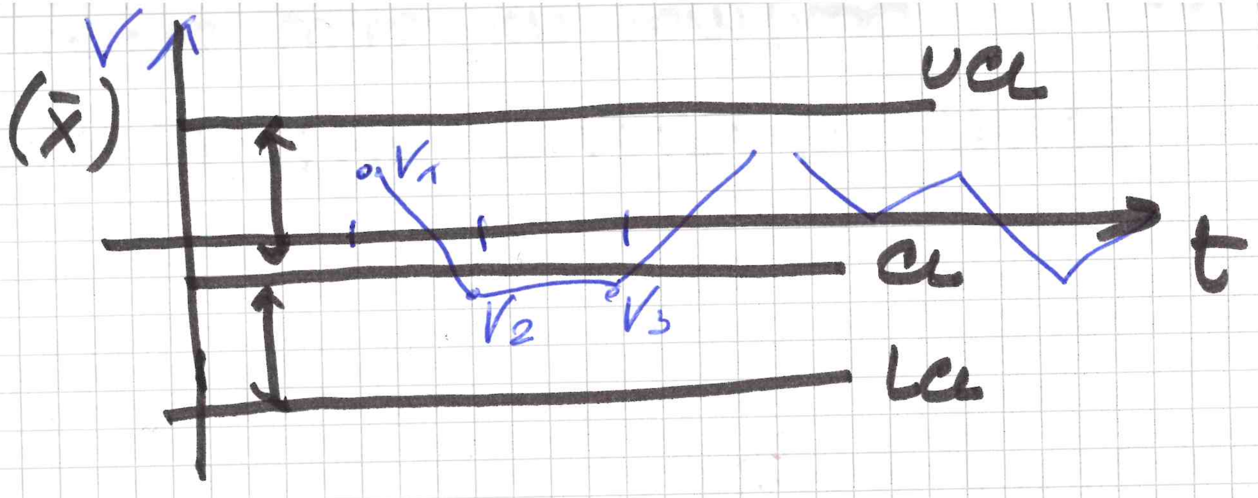
$K - \alpha$?

$H_0 : V \sim N(\mu_v, \sigma_v^2)$

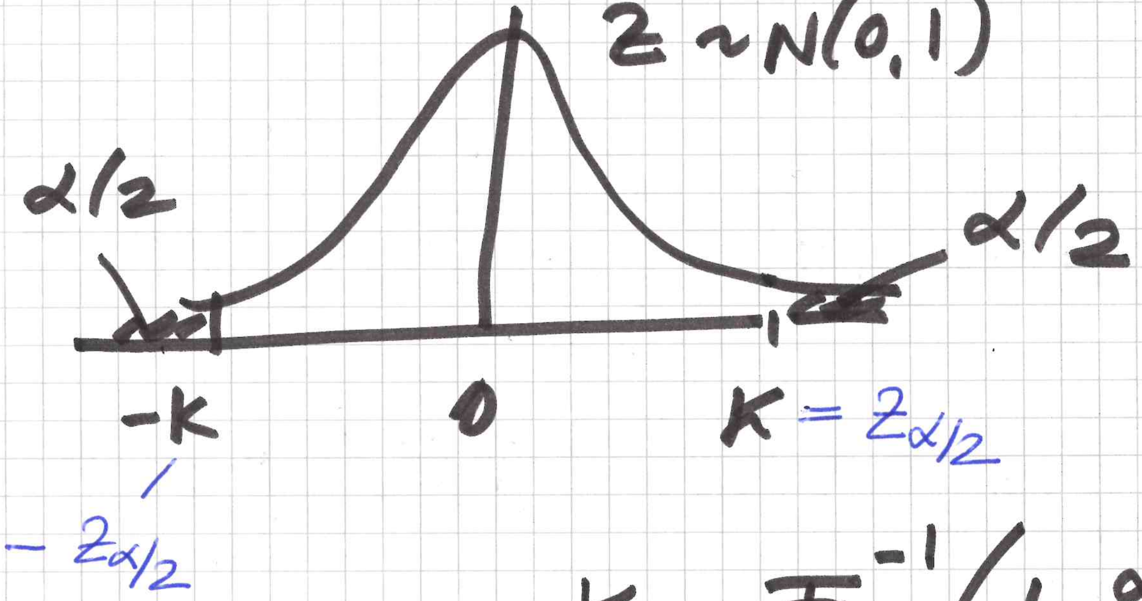




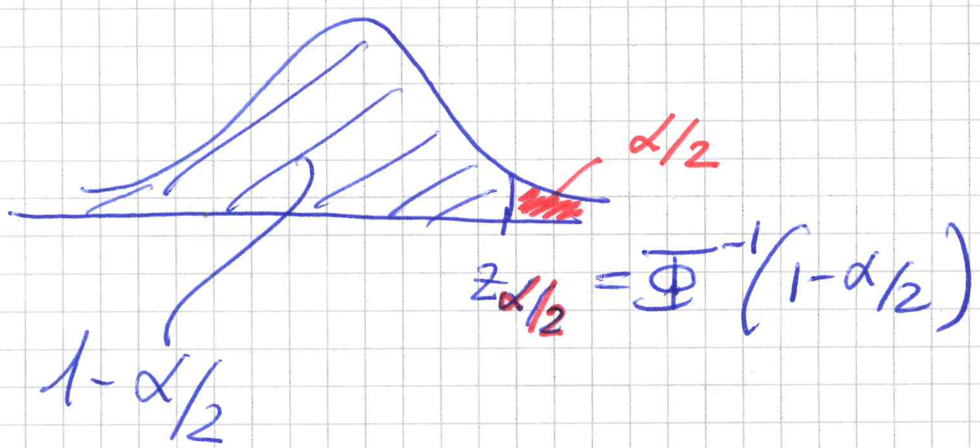
CONTROL CHART

$$\alpha/2 = P\{V \le LCL \mid H_0\} = P\Big\{ \underbrace{\frac{V-\mu_v}{\sigma_v}}_{Z \sim N(0,1)} \le \frac{LCL-\mu_v}{\sigma_v} \Big| H_0 \Big\}$$

$$= \Phi\left(\frac{LCL-\mu_v}{\sigma_v}\right) = \Phi\left(\frac{\cancel{\mu_v} - k\cancel{\sigma_v} - \cancel{\mu_v}}{\cancel{\sigma_v}}\right) = \Phi(-k)$$



$$K = \Phi^{-1}\left(1 - \alpha/2\right) = z_{\alpha/2}$$



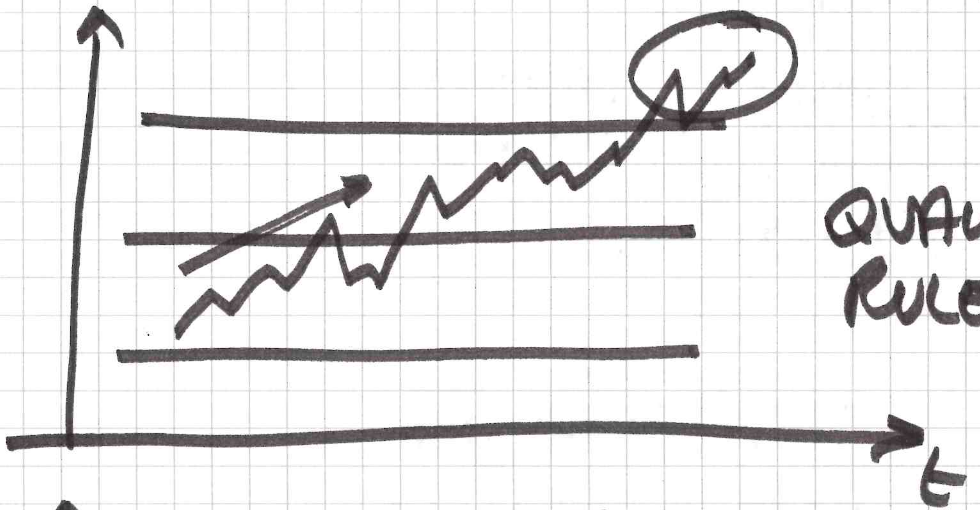
$\underline{\underline{K=3}}$? $\Rightarrow \alpha = 0.0027$
$= 2.7‰$

α = FALSE ALARM PROBABILITY

→ Y is falling OUT OF THE CONTROL LIMITS while the process was in control even if H_0 is TRUE → I'm rejecting H_0

⤳ H_0 was true

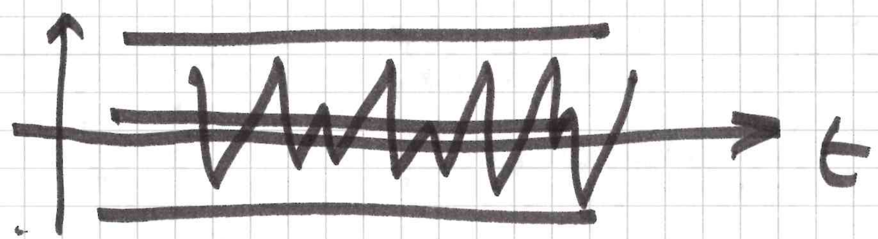
QUANTITATIVE ALARM RULE $V \notin [LCL, UCL]$



QUALITATIVE RULES: TREND plot of the control chart

↓

ALARM!



OSCILLATION

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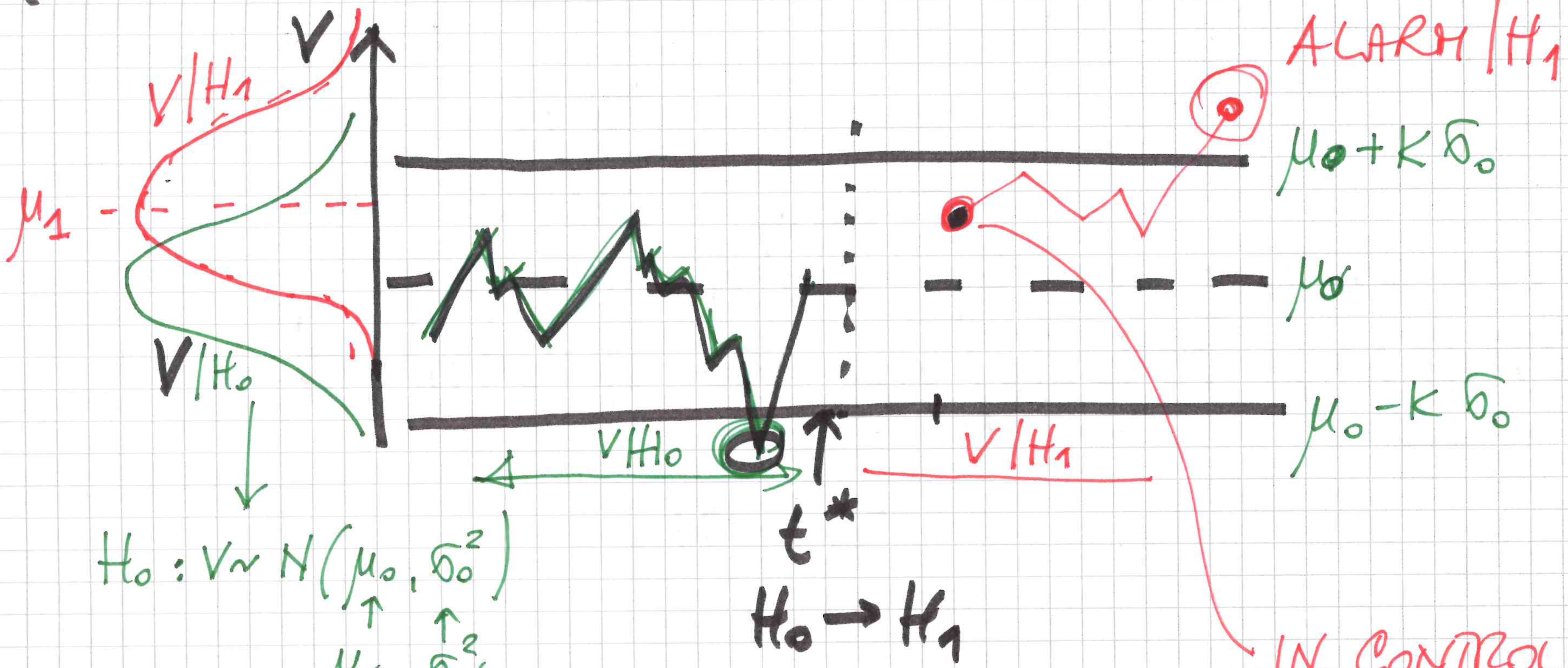
β ? probability of not having an ALARM while the process is out of CONTROL

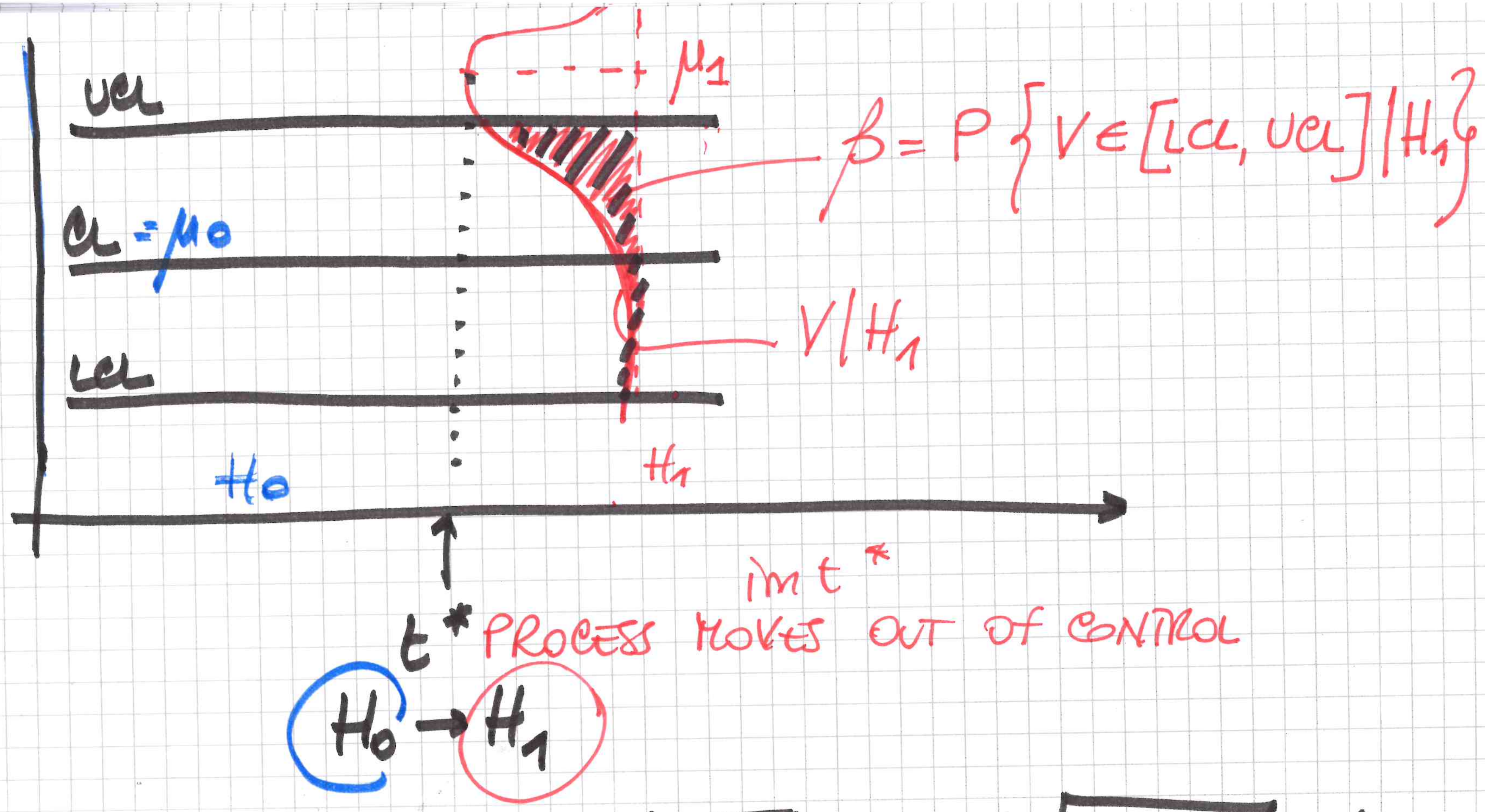
CONTROL CHART \ REAL STATE PROCESS	H_0: IN-CONTROL	H_1: OUT OF CONTROL
ALARM	α	$1-\beta$
NOT ALARM	$1-\alpha$	β



$H_0 : V \sim N(\underset{\mu_V}{\underset{\uparrow}{\mu_0}}, \underset{\sigma_V^2}{\underset{\uparrow}{\sigma_0^2}})$

$H_1 : V \sim N(\mu_1, \sigma_0^2) \quad \mu_1 \neq \mu_0$

$\beta = \text{Prob}(\text{IN-CONTROL} | H_1)$

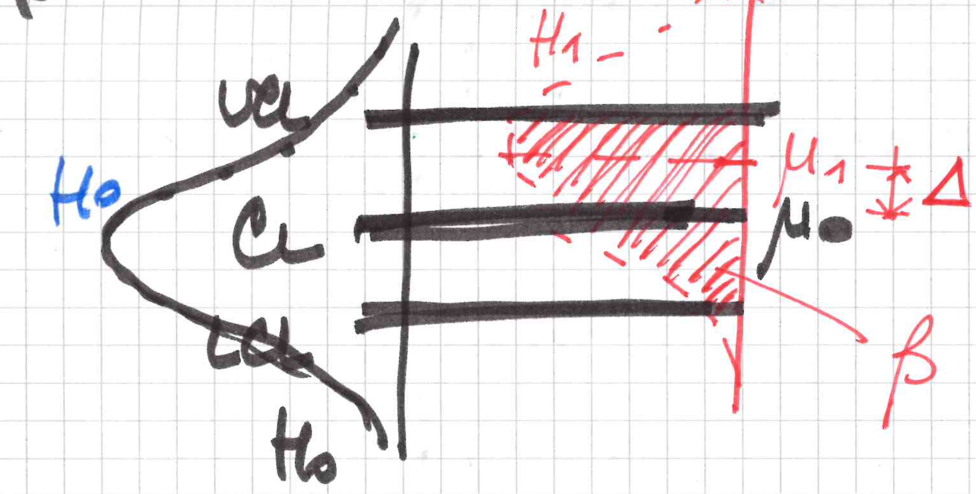


β depends on the $\boxed{\text{size}}$ of the $\boxed{\text{SHIFT}}$ from the IN-CONTROL to the OUT-OF-CONTROL STATE

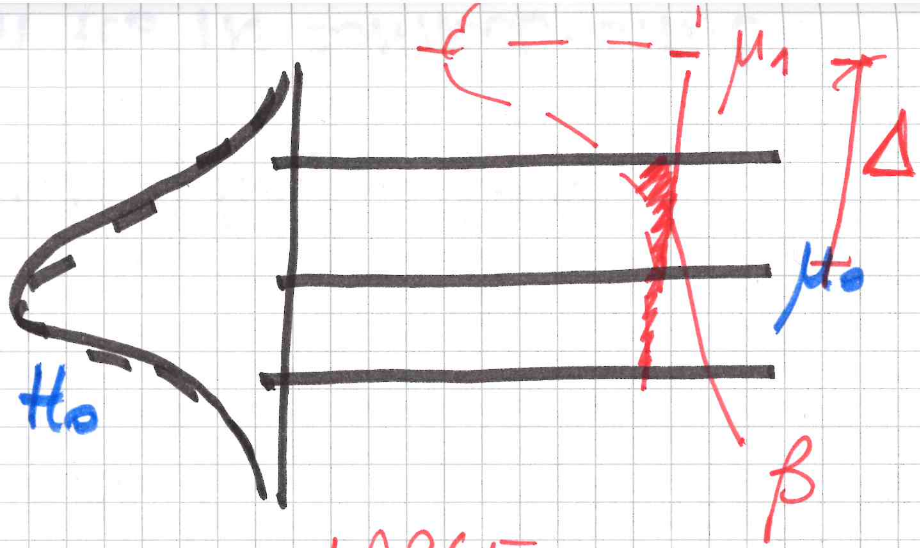
↳ H_0

↳ H_1

μ



SMALL SHIFT

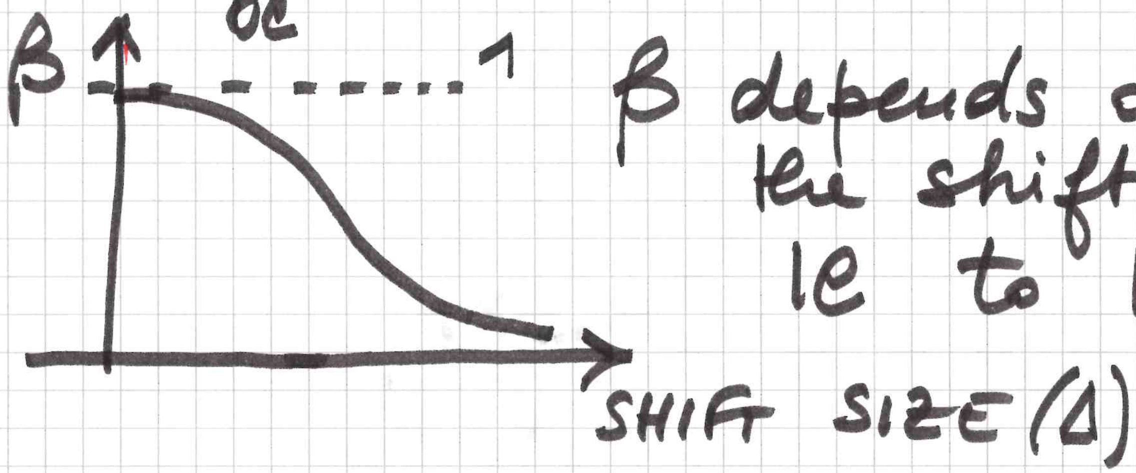


LARGE SHIFT

$\mu_0 \rightarrow \mu_1$

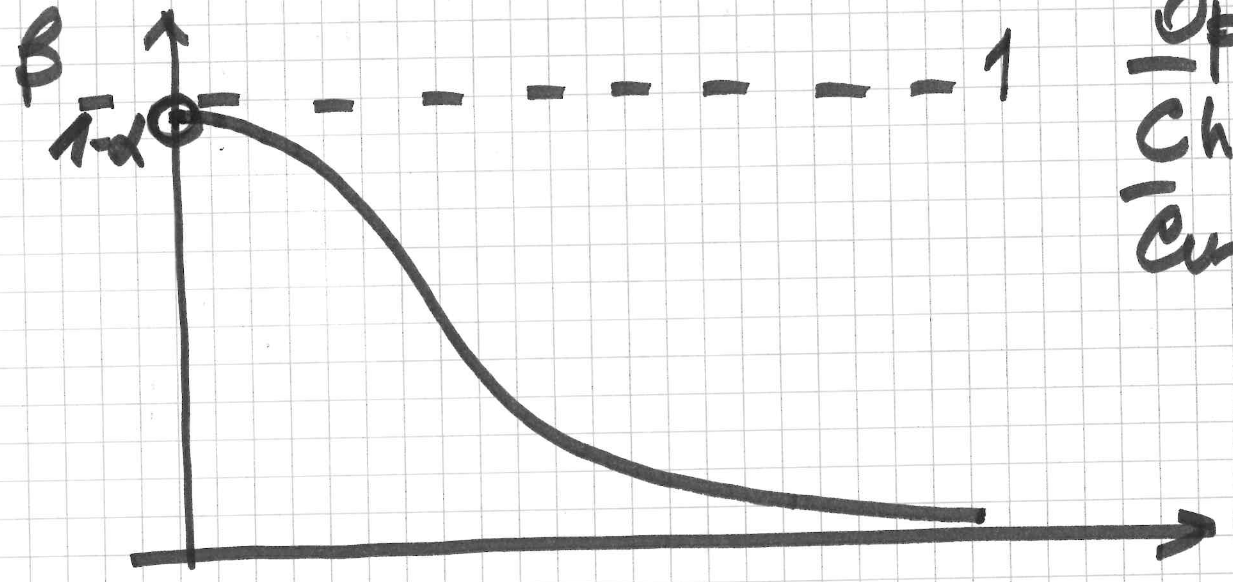
IC — IN CONTROL

OOC — OUT OF CONTROL



β depends on the $\boxed{\text{size}}$ of the shift from the IC to the OOC state

size → shift SIZE Δ



Operating **C**haracteristic **C**urve (OC)

$\beta = P(\text{NO ALARM} \mid H_1)$

$\beta(\Delta) = \beta(\Delta = \text{SIZE OF THE SHIFT } H_1 - H_0)$

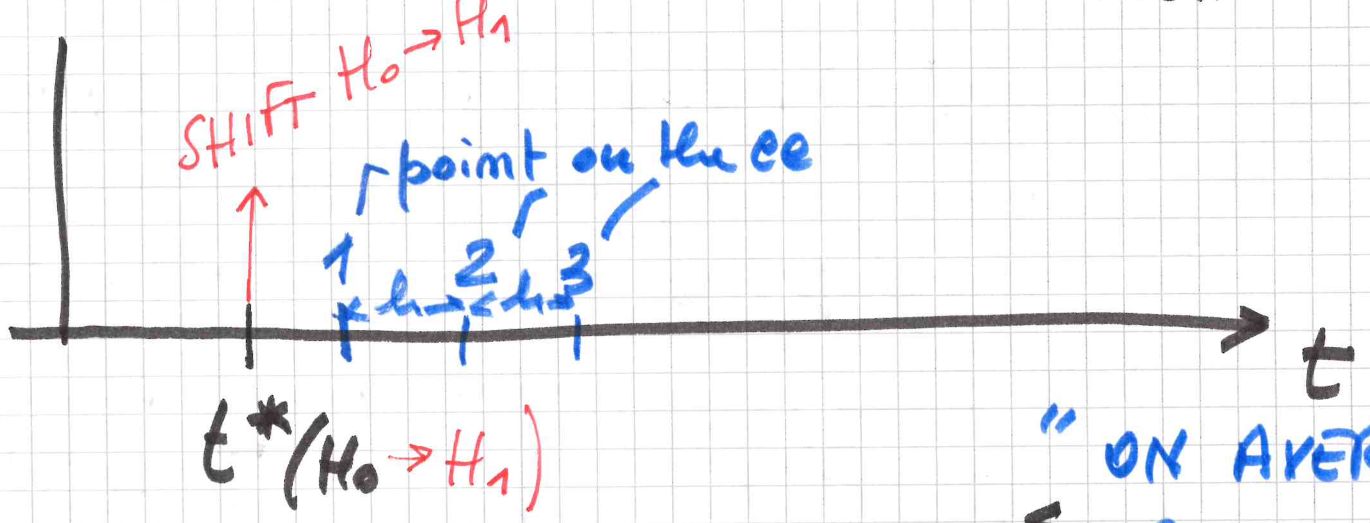
H_1 vs H_0

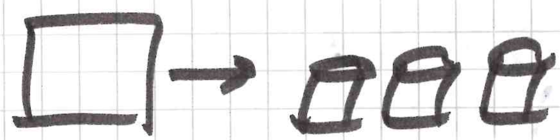
$\beta(\Delta = 0) = P(\text{NO ALARM} \mid H_0) = 1 - \alpha$

NO SHIFT → STILL im H_0 IN CONTROL-STATE

ARL = Average Run length =

$= \text{average } \underbrace{\#\text{ of samples for an ALARM}}_{\text{Run length}}$

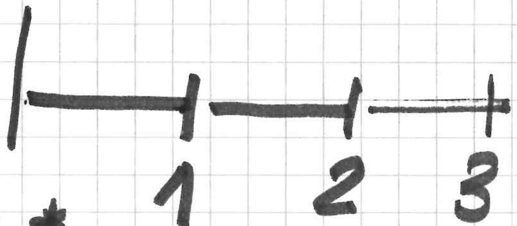




" ON AVERAGE " how many samples do I need to wait after t^* to detect the change with an alarm

ATS = Average Time for Signal

$= ARL \cdot h$



t^*

$H_0 \to H_1$

ALARM

$P = (V \notin [lcl, ucl] \mid H_1)$

$P(\text{detecting } H_1 \mid RL = 1)$

$RL = 1$	$1-\beta$
$RL = 2$	$\beta(1-\beta)$
$RL = 3$	$\beta \cdot \beta \cdot (1-\beta) = \beta^2(1-\beta)$
$\vdots$	$\vdots$
$RL = j$	$\boxed{\beta^{j-1}(1-\beta)}$

$$\underbrace{ARL}_{\text{Average}} = \sum_{j=1}^{\infty} j \cdot \underbrace{P(RL=j)} = \sum_{j=1}^{\infty} j \cdot \beta^{j-1} \cdot (1-\beta)$$

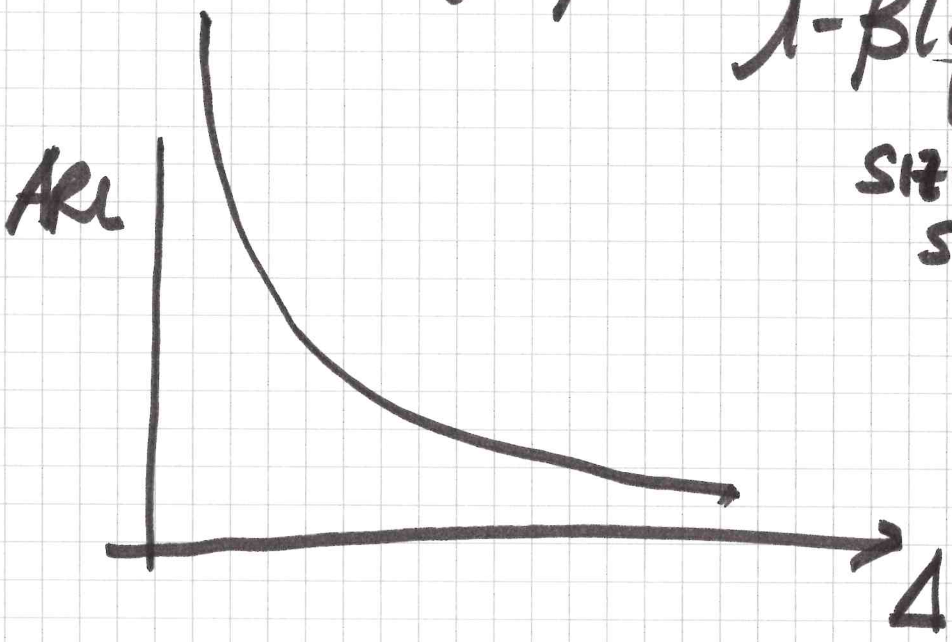
$$= \underbrace{\ldots\ldots}_{\text{HW}} = \frac{1}{1-\beta}$$

Average # of SAMPLES BEFORE A FALSE ALARM

$$ARL(0) = \frac{1}{1-\beta(0)} = \frac{1}{\alpha}$$

$\Delta = 0$

$$ARL(\Delta) = \frac{1}{1-\beta(\underbrace{\Delta}_{\text{SIZE OF THE SHIFT}})}$$



K=3
(NORMALITY of V)
$\alpha = 0.0027$
$ARL_0 = 370$