EXERCISE CLASS 1 (Part 2/3)

EXERCISE 1

A courier service guarantees high priority items to be delivered by 10.30 a.m. of next working day. A manager analyzed an historical set of data and he found out that the delivery time follows a normal distribution with mean equal to 10.00 a.m. and standard deviation equal to 9 minutes.

- a. Compute the expected nonconformity of the delivery process, where a nonconforming delivery is a late delivery (express the result in ppm, i.e., parts per million);
- b. Compute the probability that an item is delivered between 9.40 a.m. and 10.10 a.m.
- c. Compute the time when we can expect that 90% of items will be delivered.

In [ ]:
```
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Import the library for hypothesis testing scipy
import scipy.stats as stats

# Increase the plot and font size
plt.rcParams['figure.figsize'] = [8, 6]
plt.rcParams['font.size'] = 14
```

Express the time variable in minutes; assume 10.00 a.m. as time 0 Delivery time is:

$$X \sim N(0, 9^2)$$

Point a)

$$Pr(X \leqslant 30) = \phi\left(\frac{30-0}{9}\right) = \phi(3.3333)$$

In [ ]:
```
# Input the data of the problem
sigma = 9  # variance
mu = 0   # mean

#calculate the cumulative probability of z1 (standard normal distribution)
z1 = (30 - mu)/sigma
p1 = stats.norm.cdf(z1)

print("z1 = %.6f" % z1)
print("Cumulative normal of z1 = %.6f" % p1)

P_late = 1-p1
print("Probability of late delivery = %.6f" % P_late)
```

```
z1 = 3.333333
Cumulative normal of z1 = 0.999571
Probability of late delivery = 0.000429
```

In [ ]:
```
#or, as an alternative, compute the cumulative probability of x1 = 30 (with mean 0
x1 = 30
p1 = stats.norm.cdf(x1,0,9)

print("x1 = %.6f" % x1)
print("Cumulative normal of x1 = %.6f" % p1)

P_late = 1-p1
print("Probability of late delivery = %.6f" % P_late)
```

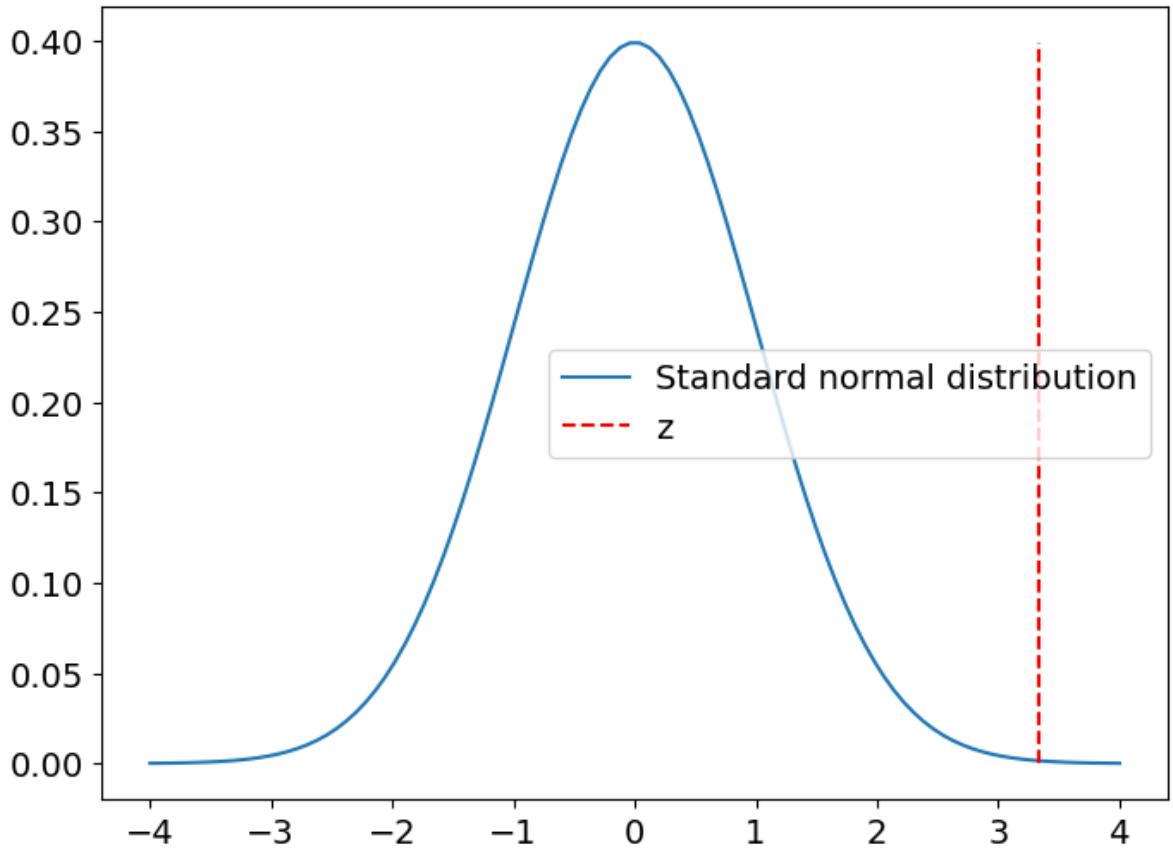
```
x1 = 30.000000
Cumulative normal of x1 = 0.999571
Probability of late delivery = 0.000429
```

In [ ]:
```
# Plot the normal distribution
x = np.linspace(-4, 4, 100)
y = stats.norm.pdf(x, 0, 1)
plt.plot(x, y, label='Standard normal distribution')


# Plot the z value
plt.vlines(z1, 0, np.max(y), color='red', linestyles='dashed', label='z')

plt.legend()

plt.show()
```



The probability of late delivery is:

$$1 - P(X \leqslant 30) = 1 - 0.999571 = 0.000429 = 0.0429\% = 429ppm$$

In [ ]:
```
#probability of late delivery in ppm

Pppm = P_late * 1000000
print("Probability of late delivery in ppm = %.0f ppm" % Pppm)
```

```
Probability of late delivery in ppm = 429 ppm
```

Point b)

Compute the probability that an item is delivered between 9.40 a.m. and 10.10 a.m.

$$Pr(-20 \leqslant X \leqslant 10) = Pr(X \leqslant 10) - Pr(X \leqslant -20)$$

In [ ]:
```
#calculate the cumulative normal of z1
z_a = (-20 - mu)/sigma
z_b = (10 - mu)/sigma
p_a = stats.norm.cdf(z_a)
p_b = stats.norm.cdf(z_b)

Prob_b = p_b - p_a

print("Probability = %.6f" % Prob_b)
```

```
Probability = 0.853606
```

> The probability that an item is delivered between 9.40 a.m. and 10.10 a.m. is:
>
> $$Pr(X \leqslant 10) - Pr(X \leqslant -20) = 85.36\%$$

Point c)

Compute the time when we can expect that 90% of items will be delivered.

In [ ]:
```
# compute the inverse cumulative function of the standard normal distribution @ P =
z_90 = stats.norm.ppf(0.90)

print("The lower 90%% percentile of the standard normal distribution is z_90 = %.6f

# then go back to the normal distribution with zero mean and standard deviation 9
x_90 = z_90*sigma + mu

print("The answer is x_90 = %.6f" % x_90)
```

```
The lower 90% percentile of the standard normal distribution is z_90 = 1.281552
The answer is x_90 = 11.533964
```

In [ ]:
```
# Plot the normal distribution
x = np.linspace(-4, 4, 100)
y = stats.norm.pdf(x, 0, 1)
plt.plot(x, y, label='Standard normal distribution')


# Plot the z value such that the its cumulative probability is P = 0.9
plt.vlines(z_90, 0, np.max(y), color='red', linestyles='dashed', label='z_90')

x_fill = np.linspace(-4, z_90, 100)
y_fill = stats.norm.pdf(x_fill, 0, 1)
plt.fill_between(x_fill, y_fill, color='red', alpha=0.5)
```

```
plt.legend()

plt.show()
```