EXERCISE 3

Using the data of Exercise 2, estimate the scores for all the observations by projecting the input variables onto the space spanned by the first 2 PCs estimated on the first m = 40 data.

In [ ]:
```
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

# Import the dataset
data = pd.read_csv('ESE05_ex2.csv')

# Print the first 5 rows of the dataset
data.head()
```

Out[ ]:

	Ph1	Ph2	Ph3
0	0.801765	1.224439	0.460368
1	1.529265	2.354552	0.512780
2	-0.767775	0.185170	-0.669498
3	0.983894	1.103092	0.237085
4	0.678907	1.879489	0.279479

In [ ]:
```
# Extract a subset of the data with only the first 40 rows
data40 = data.iloc[:40, :]

# Print the size of the new dataset
print(data40.describe())
```

```
             Ph1        Ph2        Ph3
count  40.000000  40.000000  40.000000
mean    0.670381   1.893018   0.197875
std     0.776173   0.944301   0.557657
min    -1.152475   0.185170  -0.887846
25%     0.071758   1.275474  -0.088188
50%     0.733344   1.824113   0.314454
75%     1.127810   2.596343   0.527662
max     2.508370   4.136243   1.710663
```

In [ ]:
```
# Standardize the data by subtracting the mean and dividing by the standard deviat
data40_std = (data40 - data40.mean()) / data40.std()

data40_std.describe()
```

Out[ ]:

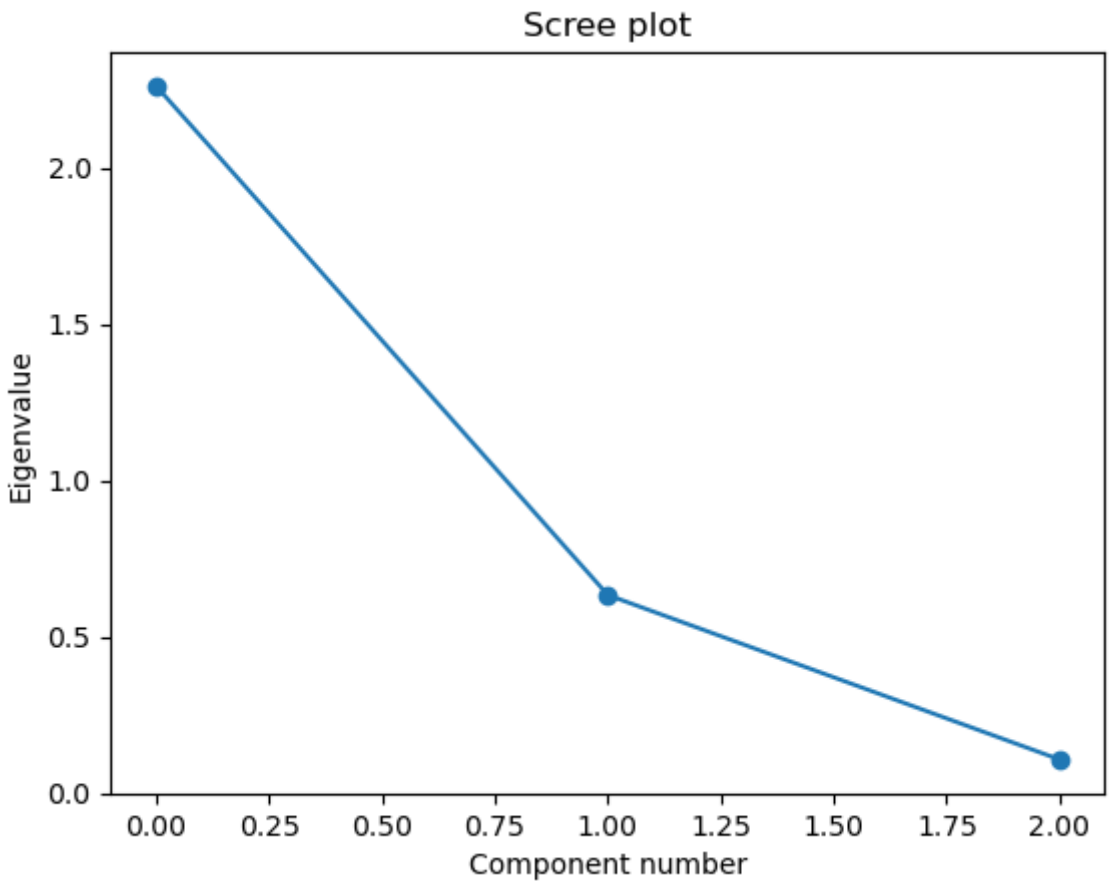
	Ph1	Ph2	Ph3
count	4.000000e+01	4.000000e+01	4.000000e+01
mean	-2.164935e-16	-3.497203e-16	5.551115e-18
std	1.000000e+00	1.000000e+00	1.000000e+00
min	-2.348518e+00	-1.808585e+00	-1.946935e+00
25%	-7.712494e-01	-6.539699e-01	-5.129733e-01
50%	8.112069e-02	-7.296887e-02	2.090500e-01
75%	5.893392e-01	7.448102e-01	5.913797e-01
max	2.368016e+00	2.375541e+00	2.712758e+00

In [ ]:
```
# import the libraries for PCA
from sklearn.decomposition import PCA

# Create the PCA object
pca_std = PCA()
# Fit the PCA object to the data
pca_std.fit(data40_std)
# Print the eigenvalues
print("Eigenvalues \n", pca_std.explained_variance_)
# Print the eigenvectors
print("\nEigenvectors \n", pca_std.components_)
# Print the explained variance ratio
print("\nExplained variance ratio \n", pca_std.explained_variance_ratio_)
# Print the cumulative explained variance ratio
print("\nCumulative explained variance ratio \n", np.cumsum(pca_std.explained_varia
```

```
Eigenvalues
 [2.26040708 0.63278177 0.10681116]

Eigenvectors
 [[ 0.63109812  0.47862724  0.6104352 ]
 [-0.25959246  0.87188528 -0.41524428]
 [ 0.73097669 -0.10359551 -0.67449318]]

Explained variance ratio
 [0.75346903 0.21092726 0.03560372]

Cumulative explained variance ratio
 [0.75346903 0.96439628 1.        ]
```

> Plot the scree plot to compare the eigenvalues.

In [ ]:
```
# Plot the eigenvalues (scree plot)
plt.plot(pca_std.explained_variance_, 'o-')
plt.xlabel('Component number')
plt.ylabel('Eigenvalue')
plt.title('Scree plot')
plt.show()
```



> Now compute the scores for all the observations by projecting the input variables onto the space spanned by the first 2 PCs estimated on the first m = 40 data.
>
> In case of PCA on the non-standardized data:
>
> $$\mathbf{z_i} = (\mathbf{X} - \bar{\mathbf{x}})\mathbf{u_i},\ i = 1, 2$$
>
> In case of PCA on the standardized data:
>
> $$\mathbf{z_i} = \mathbf{X}^*\mathbf{u_i},\ i = 1, 2$$
>
> where $\mathbf{X}^*$ is the standardized data matrix.

In [ ]:
```
# Standardize the data by subtracting the mean and dividing by the standard deviat
data_std = (data - data40.mean()) / data40.std()

data_std.describe()
```

Out[ ]:

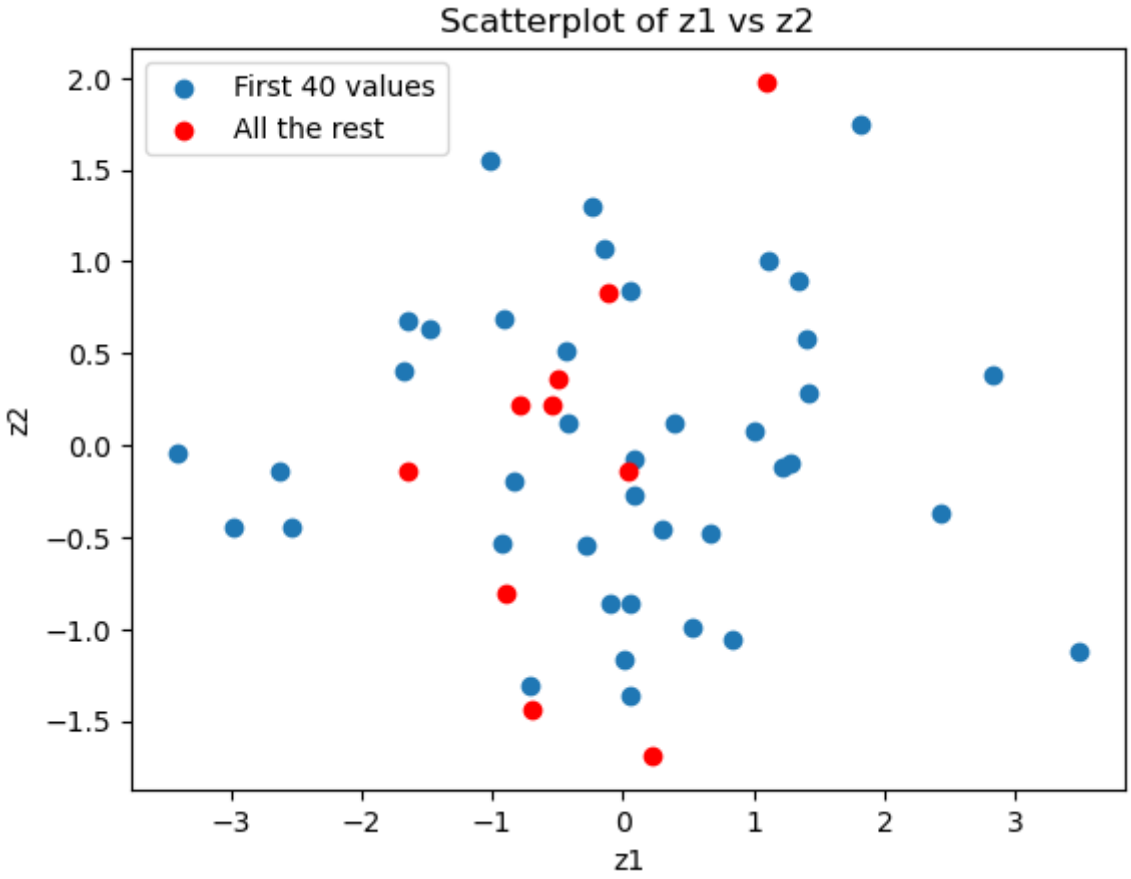
	Ph1	Ph2	Ph3
count	50.000000	50.000000	50.000000
mean	-0.014128	-0.051510	-0.069929
std	0.912933	1.022038	0.930891
min	-2.348518	-1.808585	-1.946935
25%	-0.509515	-0.804393	-0.582474
50%	0.013467	-0.123363	-0.105645
75%	0.502223	0.693697	0.531263
max	2.368016	2.375541	2.712758

In [ ]:
```
# Compute the scores
scores = pca_std.transform(data_std)
# create a dataframe with the scores
scores_df = pd.DataFrame(scores, columns = ['z1', 'z2', 'z3'])
# Print the first rows of the scores dataframe
scores_df.head()
```

Out[ ]:

	z1	z2	z3
0	0.055288	-0.856707	-0.120407
1	1.276992	-0.095601	0.377358
2	-2.984452	-0.450018	-0.117951
3	-0.102546	-0.863401	0.334493
4	0.089403	-0.076107	-0.089187

In [ ]:
```
# Create a scatterplot with the first 40 values of the two scores
plt.scatter(scores_df['z1'][:40], scores_df['z2'][:40], label='First 40 values')
# Add the rest of the values to the scatterplot with a different color
plt.scatter(scores_df['z1'][40:], scores_df['z2'][40:], color='r', label='All the
plt.xlabel('z1')
plt.ylabel('z2')
plt.title('Scatterplot of z1 vs z2')
plt.legend()
plt.show()
```


Scatterplot of z1 vs z2
First 40 values
All the rest
z1
z2
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−3
−2
−1
0
1
2
3