(1)

QDA 2023.03.16

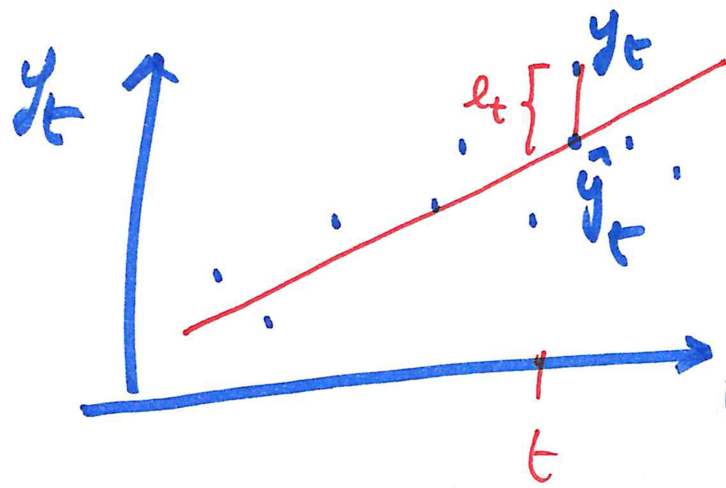
$$Y_t = \beta_0 + \beta_1 x_t$$

$$\hat{y}_t = b_0 + b_1 x_t \longrightarrow (y_t - \hat{y}_t) = e_t$$

error $\uparrow e_t$

$y_t \uparrow$ OBSERVED

$\hat{y}_t \uparrow$ "PREDICTION" $t \neq 1 \dots T$ / ESTIMATE t



$\hat{\mu}_t = b_0 + b_1 t$

$t \rightarrow x_t = t$

2

$$\begin{cases} \left.\dfrac{\partial SSE}{\partial \beta_0}\right|_{\beta_0 = b_0} = 0 & (1) \rightarrow b_0 \\ \left.\dfrac{\partial SSE}{\partial \beta_1}\right|_{\beta_1 = b_1} = 0 & (2) \rightarrow b_1 \end{cases}$$

NORMAL EQUATIONS

(1) $$\sum_t (y_t \underbrace{- b_0 - b_1 x_t}_{\hat{\mu}_t = \hat{y}_t}) = \sum_t (y_t - \hat{y}_t) = \boxed{\sum_t e_t = 0}$$

(2) $$\sum_t x_t (y_t - b_0 - b_1 x_t) = \boxed{\sum_t x_t \cdot e_t = 0}$$

③

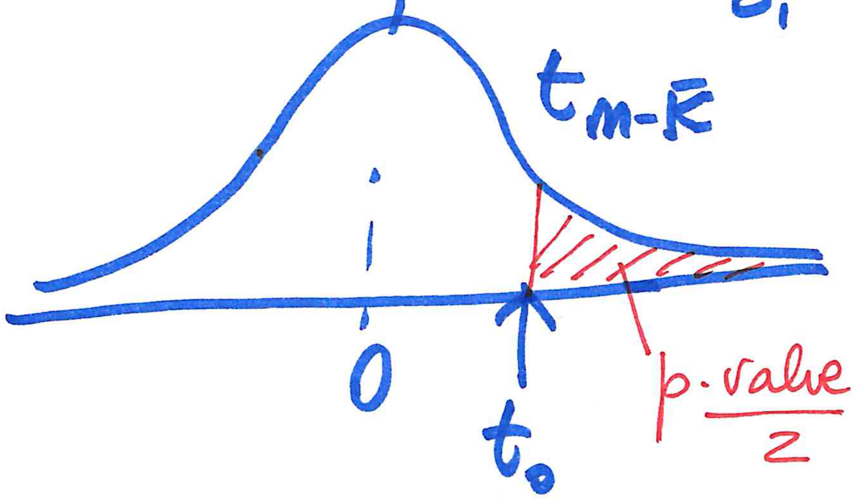
<u>**TEST 1**</u> $H_0: \beta_i = 0$ $H_1: \beta_i \neq 0$ for a given i

TEST STATISTIC

$$t_0 = \frac{b_i - 0}{\hat{\sigma}_{b_i}} \sim t_{m-K}$$

$\leftarrow$ $H_0: \beta_i = 0$

$m-K$: $p+1$ $\rightarrow$ intercept β_0; p $\rightarrow$ # regressors



$\Rightarrow$ p-value $> \alpha$ we cannot reject $H_0: \beta_i = 0$

<u>ex</u>: $Y_t = \beta_0 + \underbrace{\beta_1}_{=0} x_t \rightarrow \underline{Y_t = \beta_0} + 0 x_t$

4

$\Rightarrow x_t$ is not supposed to influence y_t's

$H_0: \beta_1 = 0$ (ACCEPTED.)

R^2 ← consider regression model important

$R^2 \approx 80\%$ → let's care

$R^2 = 30\%$ → linear model NOT RELEVANT

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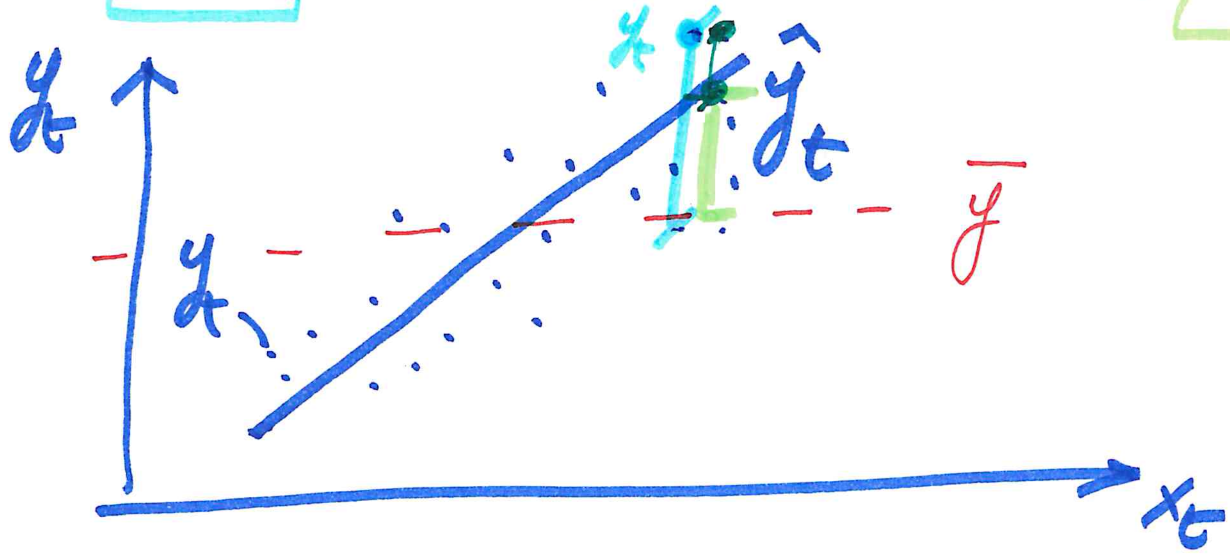
$H_0: \beta_1 = \beta_2 = \ldots \beta_k = \ldots \beta_{\underbrace{K-1}_{p}} = 0$

$H_1: \beta_j \neq 0$ for at least one j in $1 \ldots p$

$\rightarrow \; Y_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \ldots + \beta_{\underbrace{K-1}_{p = K-1}} x_{pt}$

$K = p+1$

$$\sum_t (y_t - \bar{y})^2 = \sum_t (y_t - \hat{y}_t)^2 + \sum_t (\hat{y}_t - \bar{y})^2$$



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$$\sum_t (y_t - \bar{y})^2 = \sum_t ((y_t - \hat{y}_t) + (\hat{y}_t - \bar{y}))^2 =$$

$$= \sum_t \Big[\overset{a}{(y_t - \hat{y}_t)} + \overset{b}{(\hat{y}_t - \bar{y})}\Big]^2 =$$

$$= \sum_t \underbrace{(y_t - \hat{y}_t)^2}_{a^2} + \sum_t \underbrace{(\hat{y}_t - \bar{y})^2}_{b^2} + \overbrace{2\sum_t \underbrace{(y_t - \hat{y}_t)}_{e_t}(\hat{y}_t - \bar{y})}^{*}$$

$\cancel{2}ab$

e_t

$$* \quad \sum_t e_t \cdot (\hat{y}_t - \bar{y}) =$$

$$= \sum_t e_t \hat{y}_t - \sum_t e_t \cdot \bar{y} = \sum_t e_t \hat{y}_t - \underbrace{\bar{y} \overbrace{\sum e_t}^{0}}$$

$$= \sum_t e_t (b_0 + b_1 x_t) = \underbrace{\sum_t e_t \cdot b_0}_{0} + \underbrace{b_1 \sum_t e_t x_t}_{0} \; 0$$

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$$\sum_t (y_t - \bar{y})^2 = \sum_t (y_t - \hat{y}_t)^2 + \sum_t (\hat{y}_t - \bar{y})^2$$

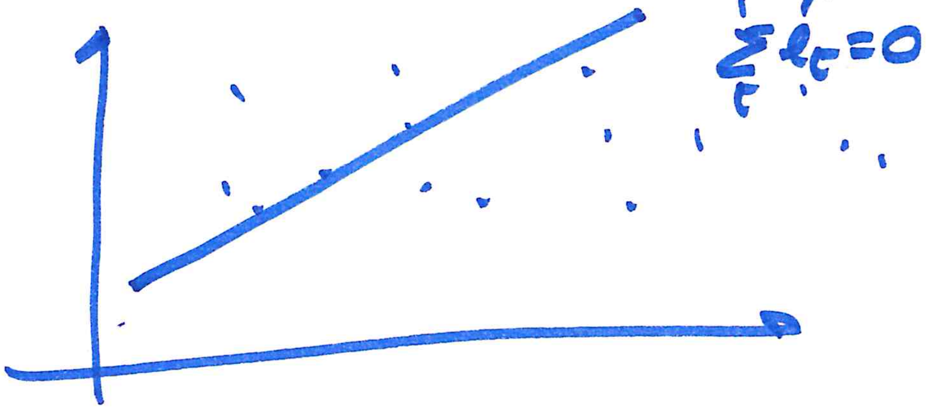
$$SS_T = SS_E + SS_R$$

degrees of freedom

$$n - 1 = n - \underline{K} + \underline{K} - 1$$

$\hat{\beta}_0, \hat{\beta}_1 \ldots \hat{\beta}_p$

$\sum_t e_t = 0 \quad \sum x_t e_t = 0$



$$\bar{y} = \frac{\sum y_t}{n}$$

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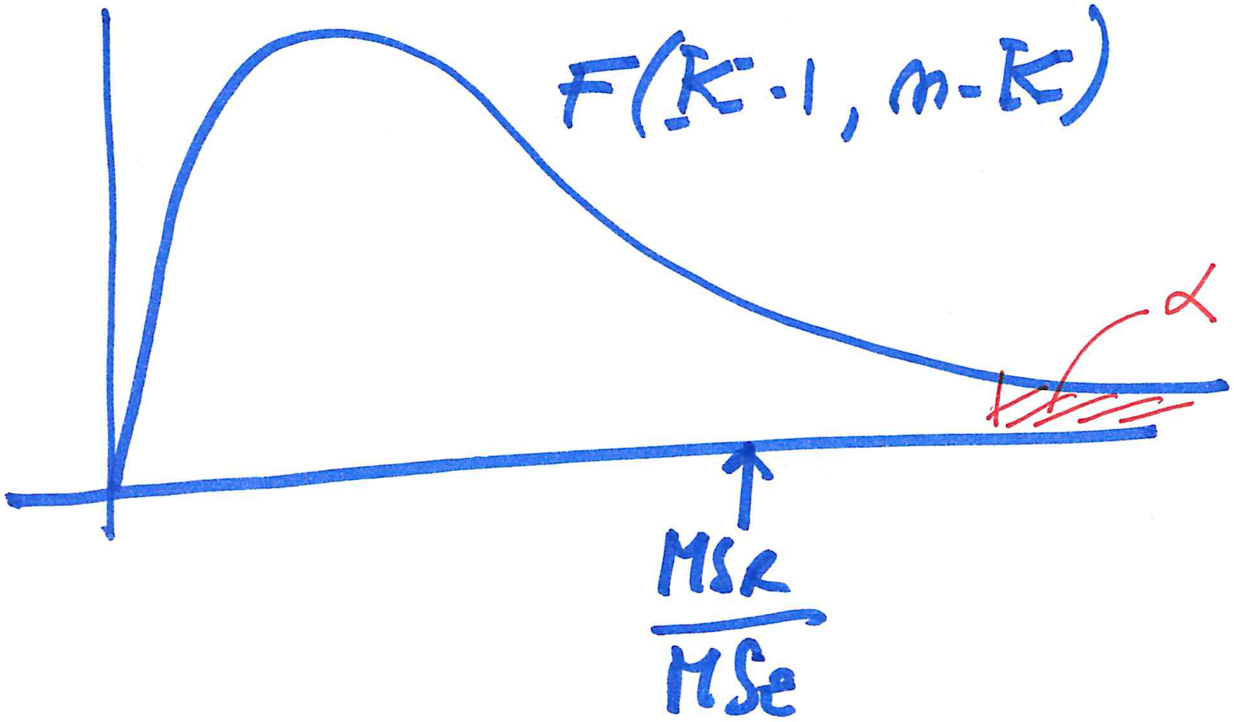
$H_0 : \beta_1 = \dots = \beta_p = 0$

$H_1 : \neq$ at least one

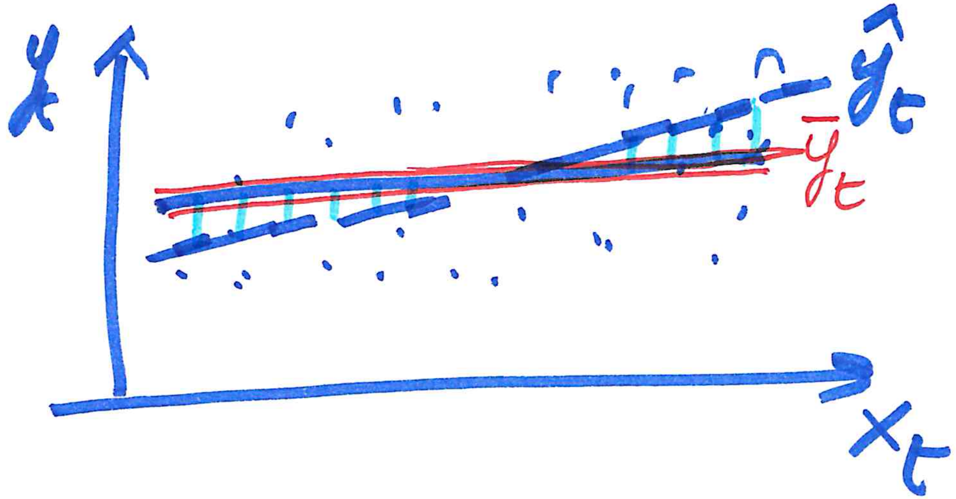
$\boxed{\text{If } H_0 \text{ is true}} \rightarrow \boxed{\dfrac{MS_R}{MS_E} \sim F(K-1, n-K)}$

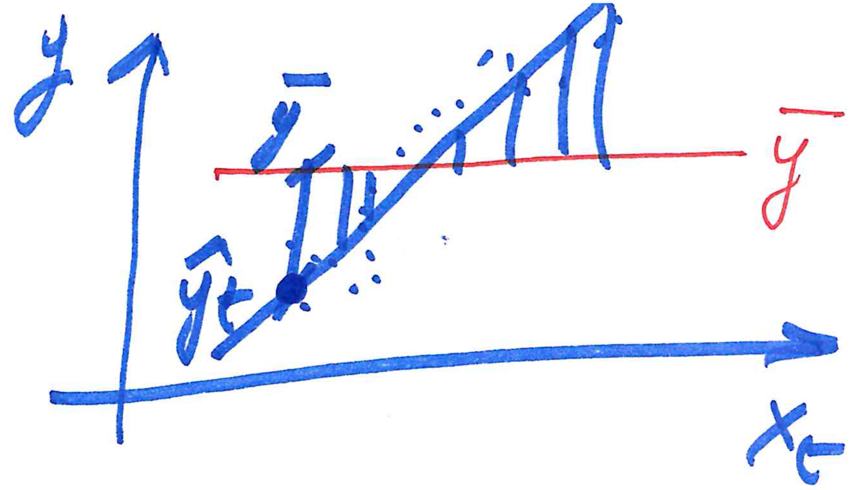
$\dfrac{SS_R}{df_R} = \dfrac{SS_R}{K-1}$

$\dfrac{SS_E}{df_E} = \dfrac{SS_E}{n-K}$



⑨





$$\underbrace{\sum_t e_t}_{SS_E} + \underbrace{\sum (\hat{y}_t - \bar{y})^2}_{SS_R} = SS_T$$

"small" → "small" MS_R

$\beta_1 \cong 0$

"ACCEPT" H_0

$$SS_R = \sum_t (\hat{y}_t - \bar{y})$$

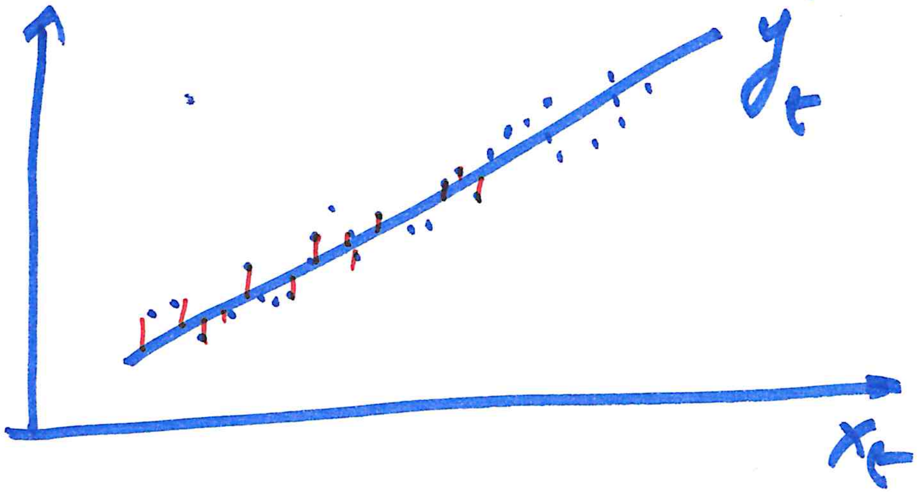
? "large" → "large" MS_R

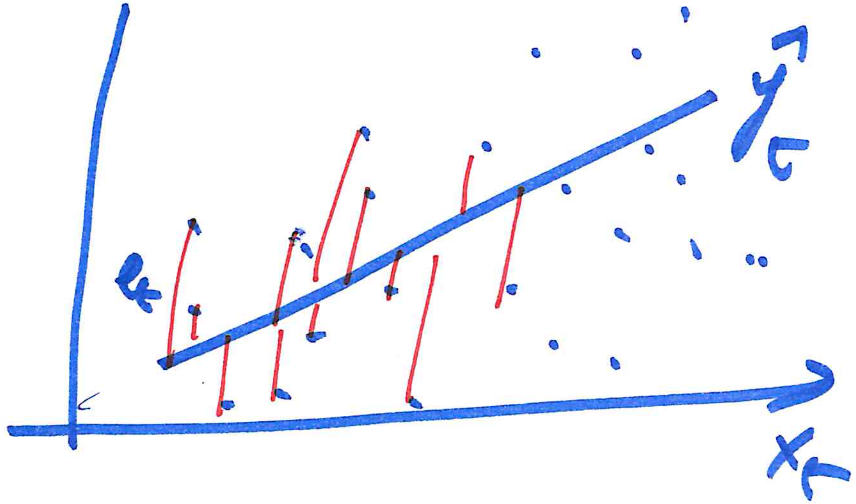
$\beta_1 \neq 0$

REJECT H_0

$$\frac{MS_R}{MS_E} \sim F$$

10

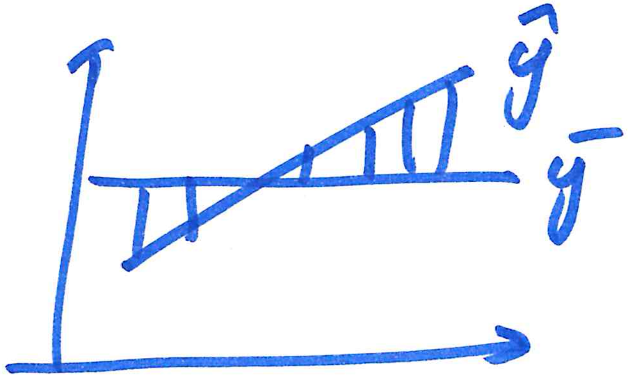




Small MSE

large MSE

$$\hat{\sigma}_\varepsilon = MS_E = \frac{SS_E}{m-K} \qquad SS_E - \sum_t e_t^2$$



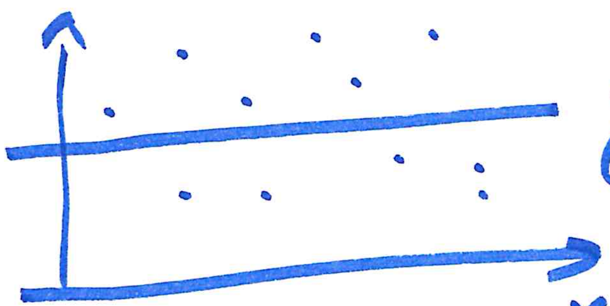
$$SS_R = \sum(\hat{y}_t - \bar{y})^2$$

(11)

$$R^2 \triangleq \frac{SS_R}{SS_T} = \frac{SS_T - SS_E}{SS_T} = 1 - \frac{SS_E}{SS_T}$$

$$SS_T = SS_E + SS_R$$

$$0 \leq R^2 \leq 1$$



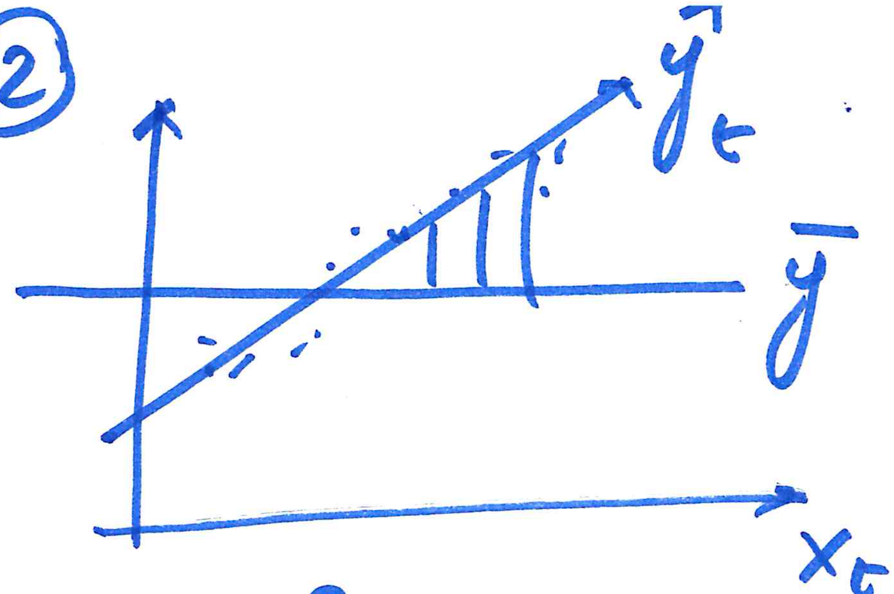
x_t

$\hat{y}_t \simeq \bar{y}$ $\quad SS_R = \Sigma(\hat{y}_t - \bar{y})^2 \to$ SMALL $\to 0$

$R^2 \to 0$ $\quad x_t$ the regressor is meaningless

→ we can assume that the regression model is not to be considered

(12)



$R^2 \rightarrow 1$

$SS_R = \Sigma(\hat{y}_t - \bar{y})^2$ large

→ regression is meaningful as x_t is influencing data