QDA 2023.05.11 ①

Xbar-S control chart

NON IID CONTROL CHARTS
- $n=1$
- $n>1$

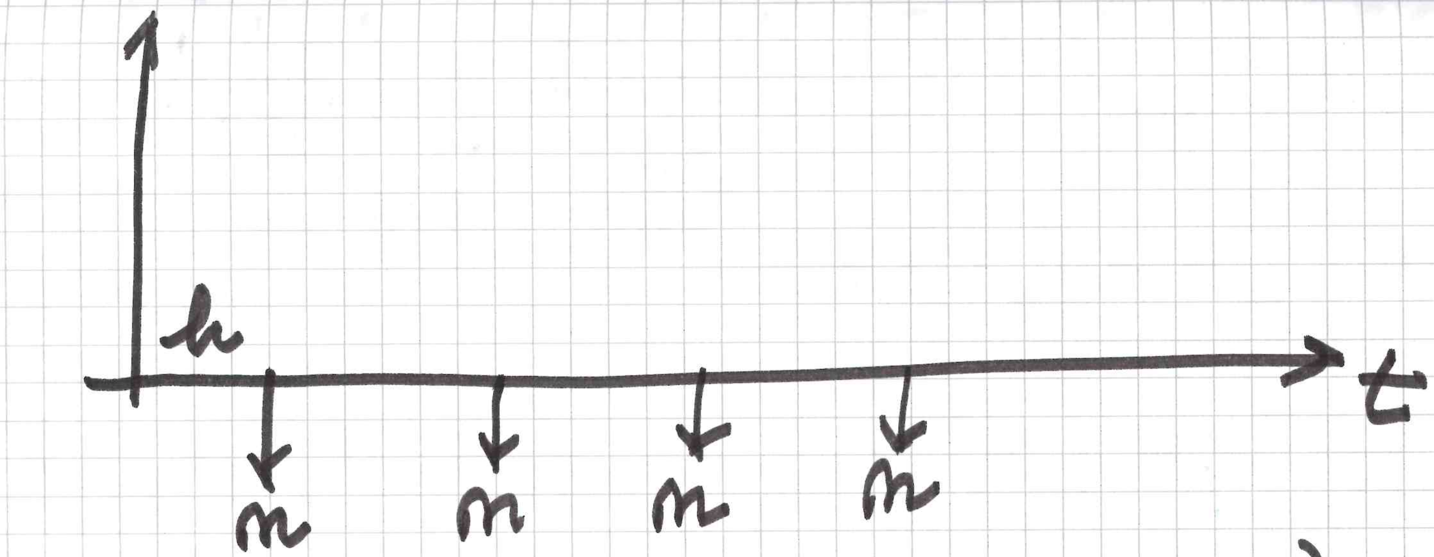
MECH ENG

MNG & FOOD ENG

MULTIVARIATE CC

FINAL CLASS : TOPICS × THESIS & PROJECTS (what's next)

- FINAL SIMULATION of the EXAM (to be scheduled)



- Is the mean stable (in control) $V = \bar{X}$
- Is the variability of the process stable (in control)

$V = S$

S is the sample standard deviation

All data in a sample of size n $(j = 1 \ldots n)$

$$S = \sqrt{\frac{\sum_j (X_j - \bar{X})^2}{n-1}} \qquad S^2 = \frac{\sum_j (X_j - \bar{X})^2}{n-1}$$

Shewhart control chart

$$\underset{\substack{\uparrow \\ \text{SAMPLE STATISTIC}}}{V}$$

$$\begin{matrix} UCL \\ CL \\ LCL \end{matrix} = \mu_V \pm \underset{\substack{\uparrow \\ K=3}}{K}\sigma_V$$

$V = S$

$\mu_V = \mu_S = E(S)$?

$\sigma_V^2 = \sigma_S^2 = V(S)$?

S control chart

$$\underset{\uparrow}{X} \overset{iid}{\sim} N(\mu, \sigma^2)$$

$$E(S) = \boxed{c_4(n)} \cdot \sigma$$

$E(S^2) = \sigma^2$ UNBIASED ESTIMATOR

$$V(S) \overset{\substack{\text{BASIC COMP} \\ \text{IN STATS} \\ \uparrow}}{=} E(S^2) - [E(S)]^2 = \sigma^2 - c_4^2 \cdot \sigma^2 =$$

$$= \sigma^2 (1 - c_4^2)$$

$$\boxed{V(X) = E(X^2) - [E(X)]^2}$$

S control chart

$$\begin{matrix} UCL \\ CL \\ LCL \end{matrix} = \mu_s \pm \overset{k}{3}\,\sigma_s =$$

$$= c_4 \cdot \sigma \pm 3\sqrt{1 - c_4^2} \cdot \sigma$$

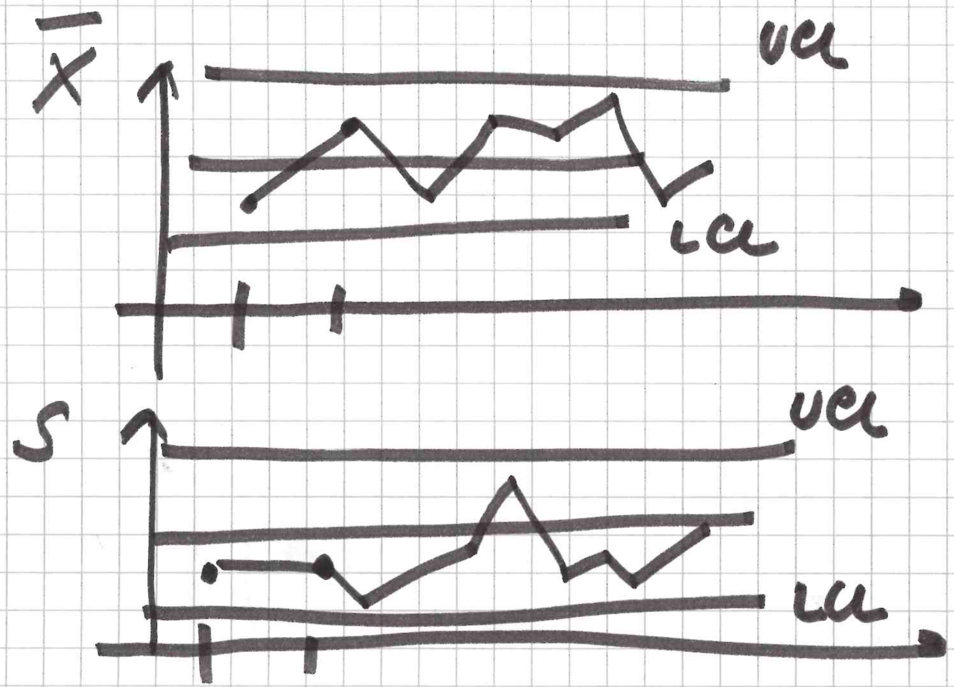
$$= \sigma \left(c_4 \pm 3\sqrt{1 - c_4^2} \right)$$

UNKNOWN ?

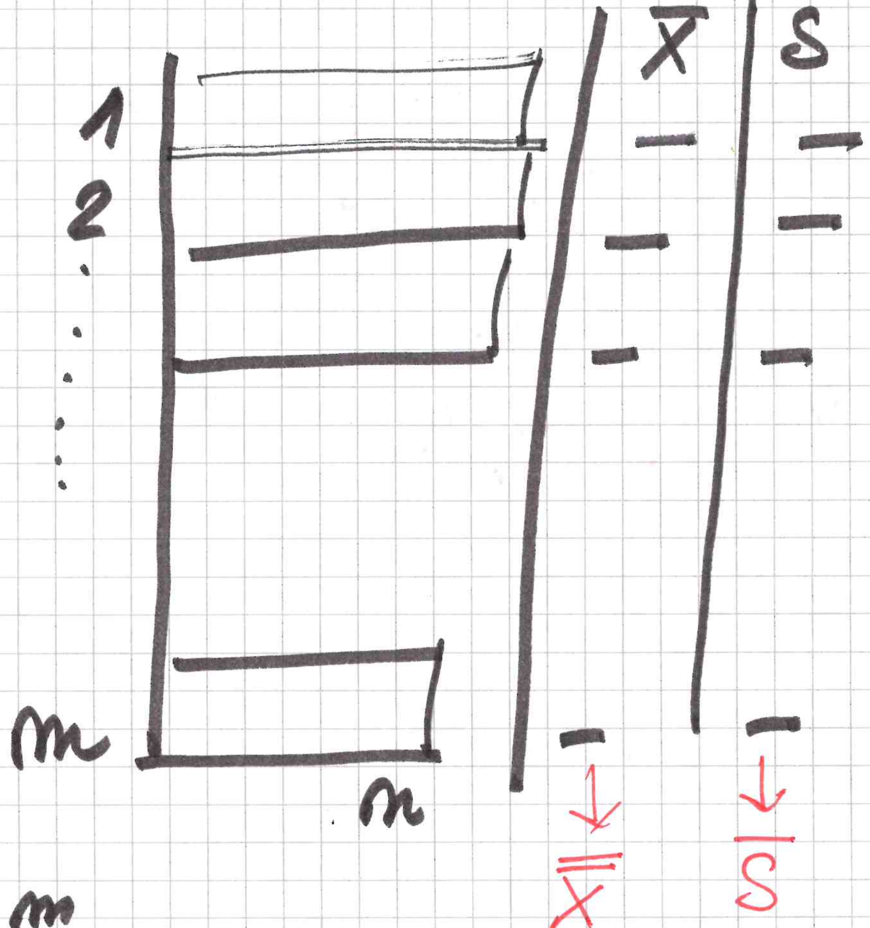
Phase 1

$$\begin{cases} c_4 + 3\sqrt{1 - c_4^2} = B_6 \\ c_4 - 3\sqrt{1 - c_4^2} = B_5 \end{cases}$$

(answer to the ex on the slides)



Phase I?



$$X_i \overset{iid}{\sim} N(\mu, \sigma)$$

? ?

$(\bar{x})$ $$\hat{\mu} = \bar{\bar{x}} = \sum_{i=1}^{m} \frac{1}{m} \bar{x}_i$$

$(\bar{x}$ and $s)$ $$\hat{\sigma} = \frac{\bar{s}}{c_4(n)}$$

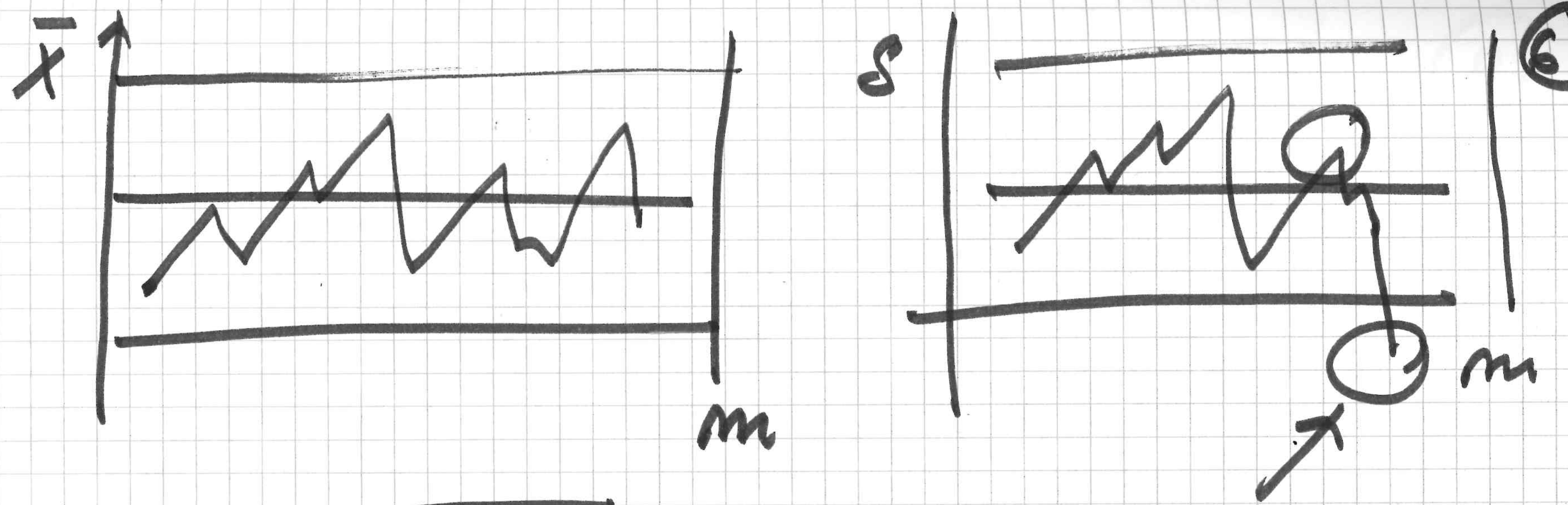
if n constant

$$E(s) = c_4(n) \cdot \sigma$$

$$\hat{\sigma} = \frac{\widehat{E(s)}}{c_4(n)} = \frac{\bar{s}}{c_4}$$

$c_4(n)$ c_4

6



m_i is not constant : at each sample m_i can be different

why ? → I can produce a ≠ lots of parts A in h time units → following this variable production speed for that part type

→ implement variable sampling policy

Variable $n \longrightarrow \bar{X}-S$ (no $\bar{X}$-R)

④

$\bar{X}$

$$\begin{matrix} UCL_i \\ CL \\ LCL_i \end{matrix} = \hat{\mu} \pm K \frac{\hat{\sigma}}{\sqrt{n_i}}$$

S

$$\begin{matrix} UCL_i \\ CL_i \\ LCL_i \end{matrix} = c_4(n_i) \cdot \hat{\sigma} \pm K \sqrt{1 - c_4(n_i)^2} \cdot \hat{\sigma}$$

$$\bar{X}_i = \frac{1}{n_i} \sum_j X_{ij}$$

$$\hat{\mu} = \bar{\bar{X}} = \frac{\sum_{i=1}^{m} n_i \bar{X}_i}{\sum_{i=1}^{m} n_i} = \frac{\sum_i \sum_j X_{ij}}{\sum_i n_i}$$

$$\hat{\sigma} = \frac{S_p}{c_4(d)} \quad S_p \text{ pooled}$$

$$S_p = \sqrt{\frac{\sum_{i=1}^{m} (n_i - 1) S_i}{\sum_i (n_i - 1)}}$$

$$\downarrow$$

$$d = \underline{\sum_{i=1}^{m} n_i - m + 1}$$

Homework → Think how to design a control chart "probabilistic" (based on the real distribution of V statistic) for the S^2 control chart assuming $X \overset{iid}{\sim} N(\mu, \sigma^2)$

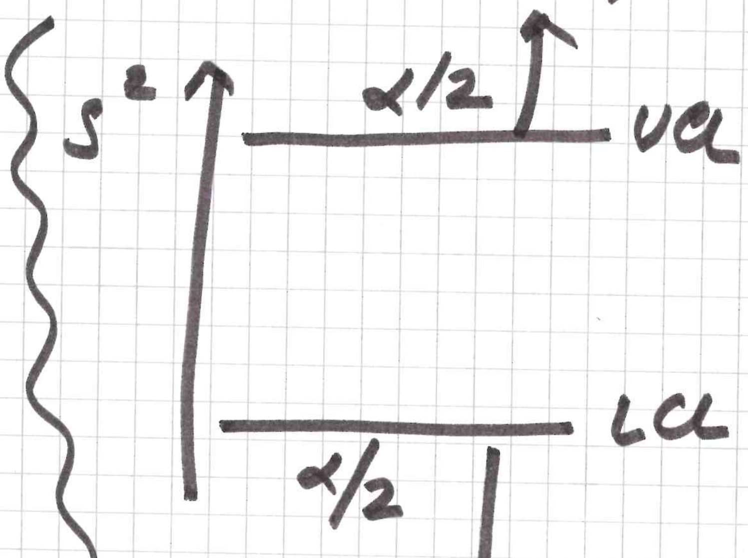
without Shewhart $\mu_V \pm k\sigma_V$

- H_0: $\boxed{X \sim N(\mu, \sigma^2)}$

$$\text{Prob}\{ s^2 \leq la \} = \alpha/2$$

$$\text{Prob}\{ s^2 \geq ua \} = \alpha/2$$

α FIRST TYPE ERROR (FALSE ALARM PROB)



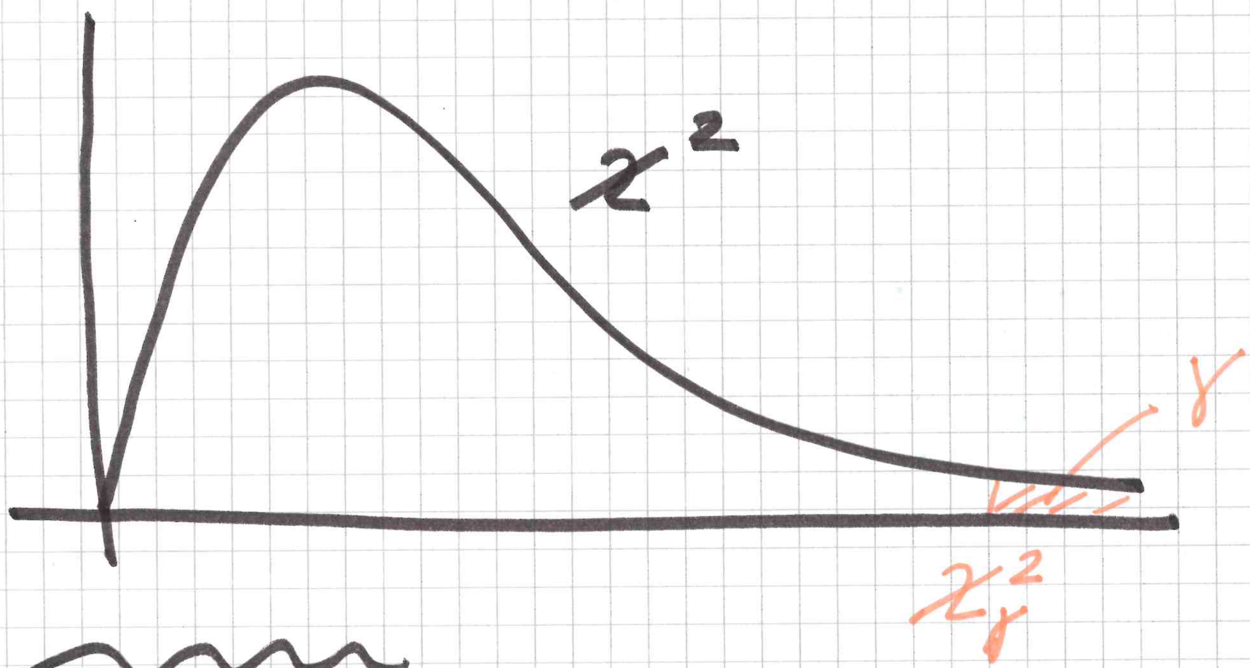
s^2 ? (,)

la ?

ua ?

$$\boxed{\frac{(n-1)s^2}{\sigma^2} \sim \chi^2(n-1)}$$

Himt



H_0 PROCESS is im CONTROL

$V \sim D(\quad) \longrightarrow P(V > UCL) = \alpha$

ex $V: \bar{X}$

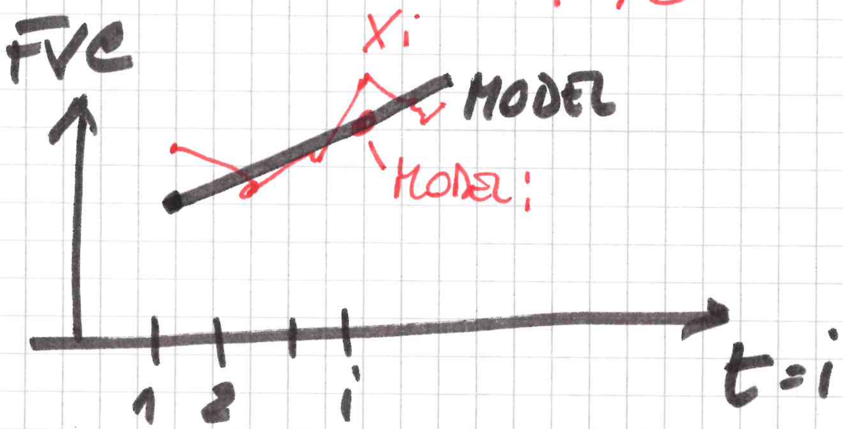
SQM - NON IID data

m=1 $\quad X \overset{iid}{\sim} \cancel{N}(\mu, \sigma^2)$

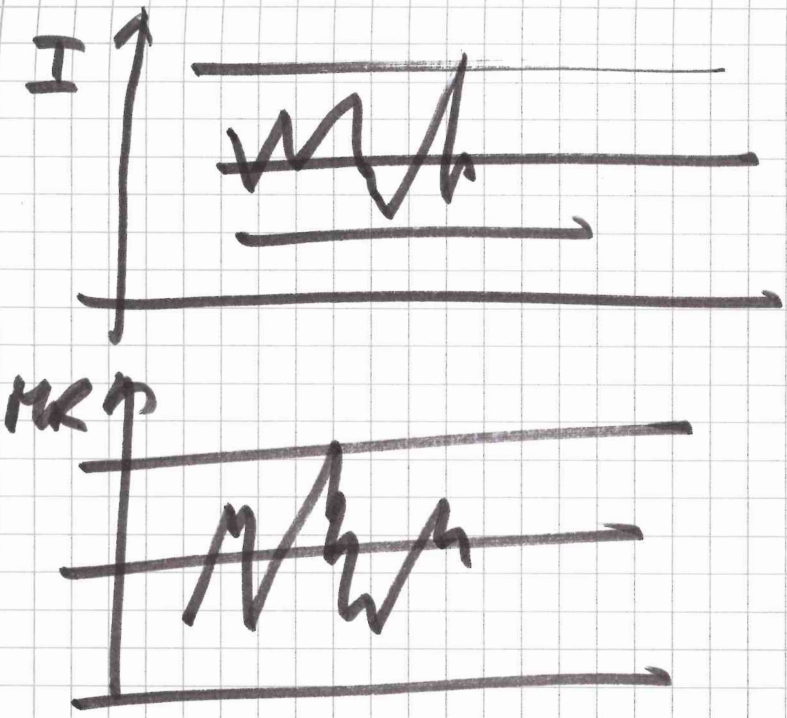
↳ MODEL the DATA $\quad X_i = MODEL_i + \varepsilon_i$

$\varepsilon_i \overset{iid}{\sim} N$

FVC Fitted-Value Chart



SCC Special Cause (Control) Chart $\Big\{$ CONTROL CHART (I-MR)



on $e_i = \hat{\varepsilon}_i$

If the MODEL is a simple TREND $MODEL_i = \beta_0 + \beta_1 t_i$

TREND CONTROL CHART

$CL_i = MODEL_i = \beta_0 + \beta_1 t_i$

$3\,\hat{\sigma}_{\varepsilon} = 3\,\dfrac{\overline{MR}_{res}}{d_2}$