# Exercise 2

The data reported in `ESE7_ex2.csv` refer to temperature measurements in a chemical process, one measure per minute (Time Series Analysis: Forecasting and Control Box, Jenkins e Reinsel 1994).

1. Design a traditional control chart for the process data.
2. Identify a suitable model for the process data.
3. By using the identified model, design both SCC and FVC charts; compare the results with those achieved at point 1.

> Let's start by importing the required libraries and loading the data.

In [ ]:
```python
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd
from scipy import stats
import qda

# Import the dataset
data = pd.read_csv('ESE7_ex2.csv')

# Inspect the dataset
data.head()
```

Out[ ]:

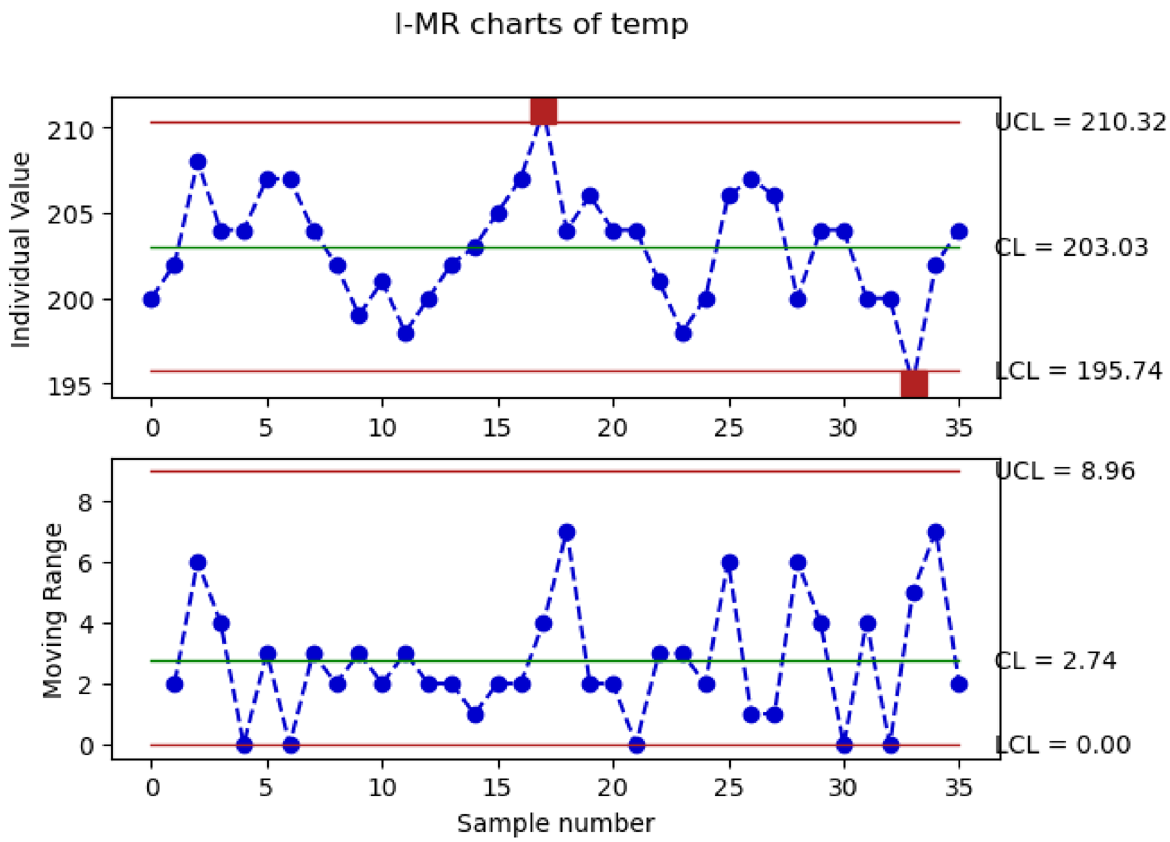
| | temp |
|---|---|
| **0** | 200 |
| **1** | 202 |
| **2** | 208 |
| **3** | 204 |
| **4** | 204 |

## Point 1

Design a traditional control chart for the process data.

> ### Solution
>
> Let’s design a control chart in a BLIND way, i.e., without checking the assumptions and without any graphical analysis of process data (**wrong approach**).

In [ ]:
```python
df_IMR = qda.ControlCharts.IMR(data, 'temp')
```




> Two observations are out of control.

> Is this result trustworthy? **NO**, unless assumptions are verified.

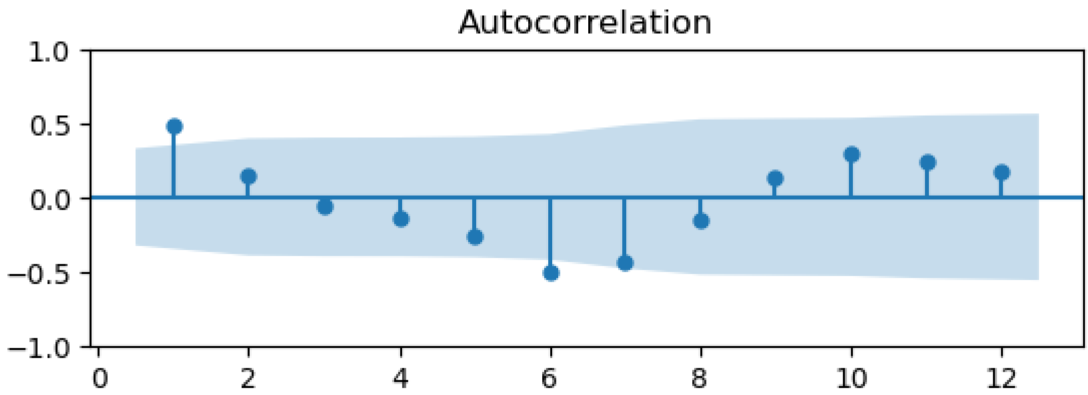
> Perform the runs test to check if the data are random. Use the `runstest_1samp` function from the `statsmodels` package.

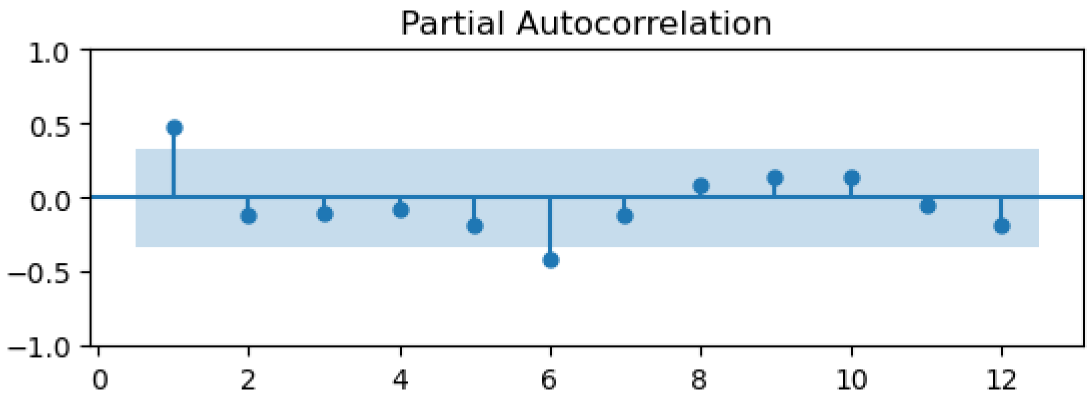
In [ ]:
```
# Import the necessary libraries for the runs test
from statsmodels.sandbox.stats.runs import runstest_1samp

_, pval_runs = runstest_1samp(data['temp'], correction=False)
print('Runs test p-value = {:.3f}'.format(pval_runs))
```

```
Runs test p-value = 0.002
```

> Plot the autocorrelation and partial autocorrelation functions of the data. Use the `plot_acf` and `plot_pacf` functions from the `statsmodels` package.

In [ ]:
```
# Plot the acf and pacf using the statsmodels library
import statsmodels.graphics.tsaplots as sgt

fig, ax = plt.subplots(2, 1)
sgt.plot_acf(data['temp'], lags = int(len(data)/3), zero=False, ax=ax[0])
fig.subplots_adjust(hspace=0.5)
sgt.plot_pacf(data['temp'], lags = int(len(data)/3), zero=False, ax=ax[1], method =
plt.show()
```




> Data are nonrandom and the autocorrelation function shows a significant autocorrelation at lag 1.

## Point 2

Identify a suitable model for the process data.

> ### Solution
>
> Let's try to fit a AR(1) model to the data.

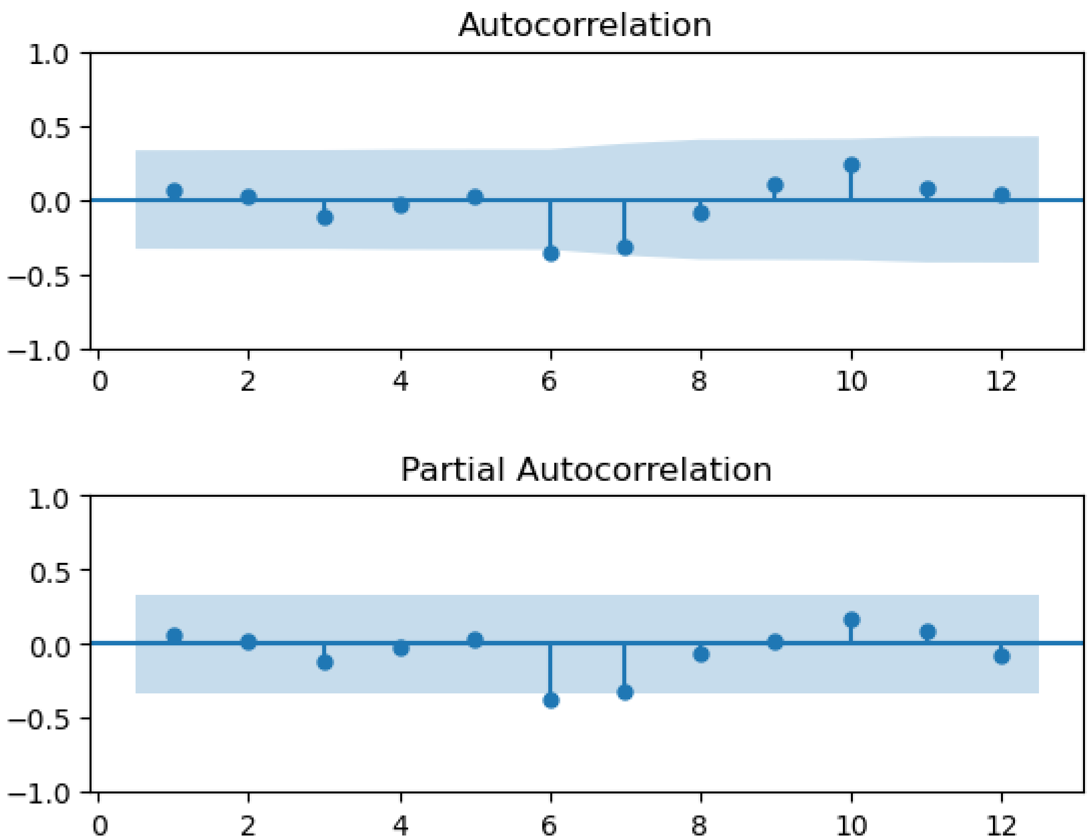
In [ ]:

```
# Add a column with the lagged temperature to use as regressor
data['temp_lag1'] = data['temp'].shift(1)

# Fit the linear regression model
import statsmodels.api as sm

x = data['temp_lag1'][1:]
x = sm.add_constant(x) # this command is used to consider a constant to the model,
y = data['temp'][1:]
model = sm.OLS(y, x).fit()
qda.summary(model)
```

```
REGRESSION EQUATION
-------------------
temp =  + 105.800 const + 0.479 temp_lag1

COEFFICIENTS
------------
     Term     Coef  SE Coef  T-Value  P-Value
    const 105.7999  30.5748   3.4604   0.0015
temp_lag1   0.4794   0.1506   3.1833   0.0032

MODEL SUMMARY
-------------
     S   R-sq  R-sq(adj)
2.9664 0.2349     0.2117

ANALYSIS OF VARIANCE
---------------------
    Source   DF    Adj SS    Adj MS  F-Value  P-Value
Regression  1.0   89.1649   89.1649  10.1332   0.0032
     const  1.0  105.3642  105.3642  11.9741   0.0015
 temp_lag1  1.0   89.1649   89.1649  10.1332   0.0032
     Error 33.0  290.3779    8.7993      NaN      NaN
     Total 34.0  379.5429       NaN      NaN      NaN
```

> Now check the residuals of the AR(1) model.

In [ ]:
```
# Check the autocorrelation of the residuals
fig, ax = plt.subplots(2, 1)
sgt.plot_acf(model.resid, lags = int(len(data)/3), zero=False, ax=ax[0])
fig.subplots_adjust(hspace=0.5)
sgt.plot_pacf(model.resid, lags = int(len(data)/3), zero=False, ax=ax[1], method =
plt.show()
```

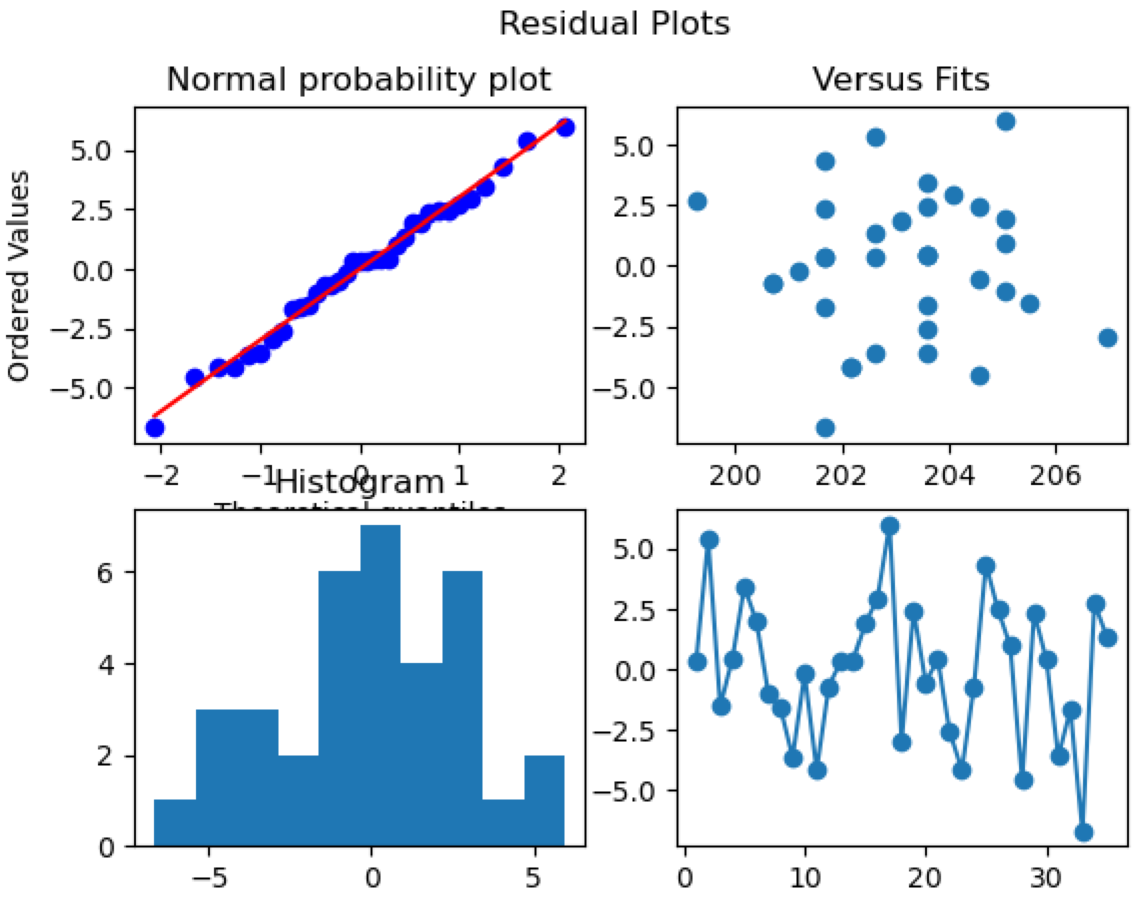



In [ ]:
```
_, pval_runs_resid = runstest_1samp(model.resid, correction=False)
print('Runs test p-value = {:.3f}'.format(pval_runs_resid))
```

```
Runs test p-value = 0.244
```

In [ ]:
```
# Plot the residuals and test for normality
fig, axs = plt.subplots(2, 2)
fig.suptitle('Residual Plots')
stats.probplot(model.resid, dist="norm", plot=axs[0,0])
axs[0,0].set_title('Normal probability plot')
axs[0,1].scatter(model.fittedvalues, model.resid)
axs[0,1].set_title('Versus Fits')
axs[1,0].hist(model.resid)
axs[1,0].set_title('Histogram')
axs[1,1].plot(np.arange(1, len(model.resid)+1), model.resid, 'o-')
_, pval_SW_res = stats.shapiro(model.resid)
print('Shapiro-Wilk test p-value on the residuals = %.3f' % pval_SW_res)
```

```
Shapiro-Wilk test p-value on the residuals = 0.954
```



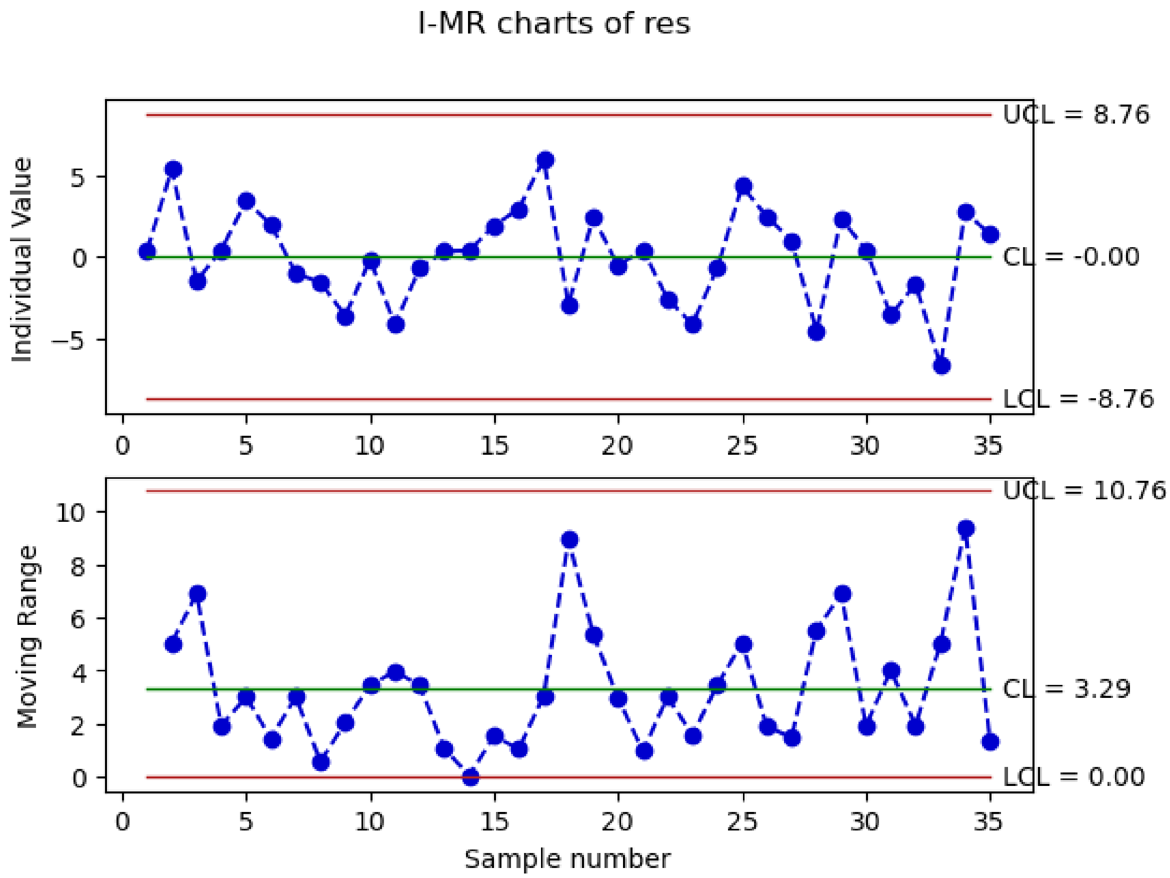

> All assumptions are verified.

# Point 3

By using the identified model, design both SCC and FVC charts; compare the results with those achieved at point 1.

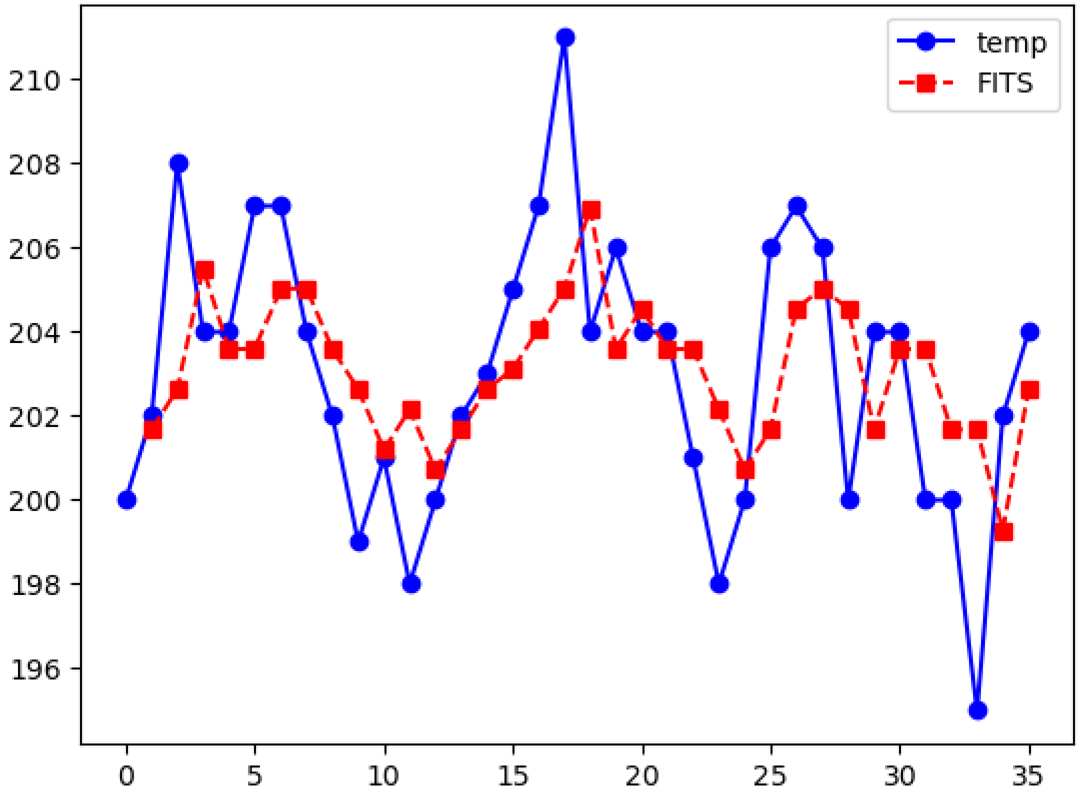
> ## Solution
>
> Let's design a SCC chart for the AR(1) model.

```python
# Put the residuals in a dataframe
df_SCC = pd.DataFrame(model.resid, columns=['res'])

# Plot the IMR control chart
df_SCC_IMR = qda.ControlCharts.IMR(df_SCC, 'res')
```




> Notice that the MR of residuals is significantly different from the MR of original observations. Thus, the approximation that is usable for TREND models is not applicable to AR(1) models.

```python
plt.plot(data['temp'], color='b', linestyle='-', marker='o', label='temp')
plt.plot(model.fittedvalues, color='r', linestyle='--', marker='s', label='FITS')
plt.legend()
plt.show()
```

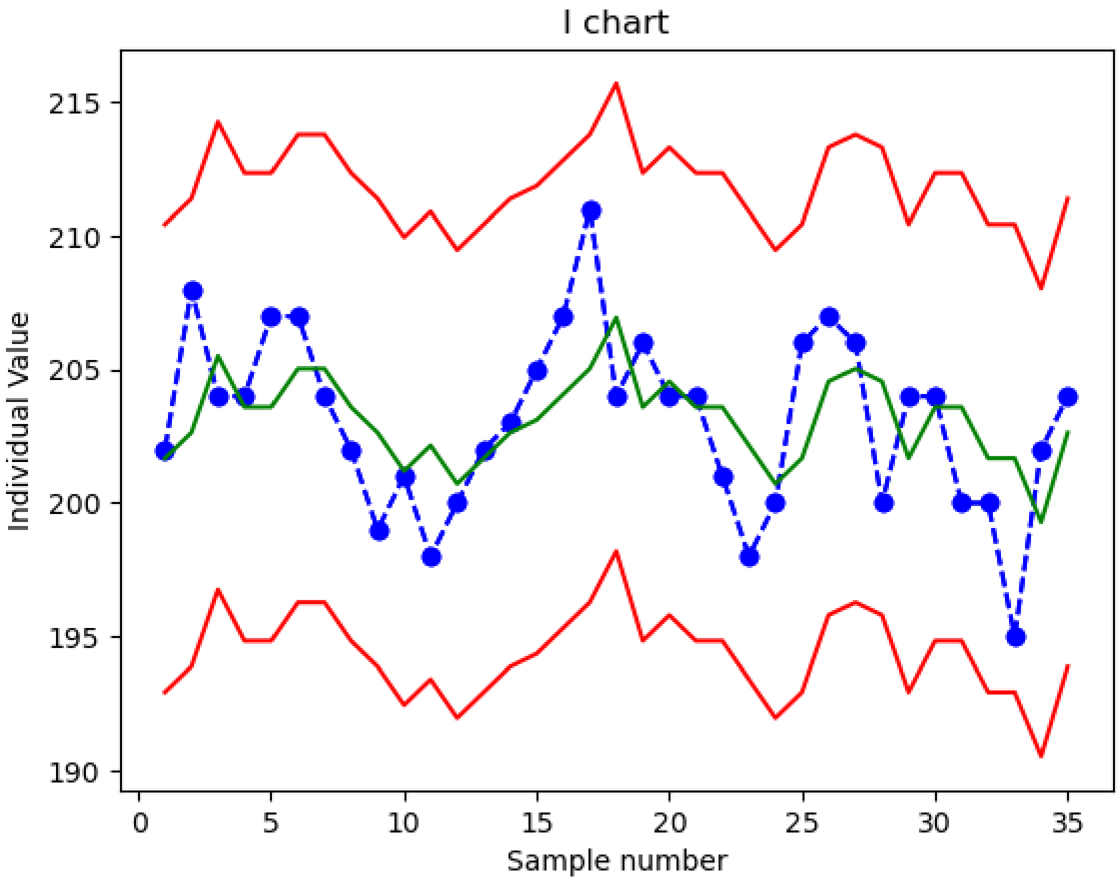



Also in this case (in analogy with the TREND control chart) we can design a model-based CC as follows.

In [ ]:
```
d2 = qda.constants.getd2(2)

MRbar_res = df_SCC_IMR['MR_CL'].iloc[0]
```

In [ ]:
```
# Create a new dataframe with the original data and the center line (the fitted va
df = pd.DataFrame({'I': data['temp'].iloc[1:], 'I_CL': model.fittedvalues}, index=

# Add the I_UCL and I_LCL columns with the upper and
# lower control limits computed from the formula
df['I_UCL'] = df['I_CL'] + 3 * MRbar_res / d2
df['I_LCL'] = df['I_CL'] - 3 * MRbar_res / d2

# Add the TEST1 column
df['I_TEST1'] = np.where((df['I'] > df['I_UCL']) | (df['I'] < df['I_LCL']), df['I'
```

In [ ]:
```
# Plot the I chart
plt.title('I chart')
plt.plot(df['I'], color='b', linestyle='--', marker='o')
plt.plot(df['I'], color='b', linestyle='--', marker='o')
plt.plot(df['I_UCL'], color='r')
plt.plot(df['I_CL'], color='g')
plt.plot(df['I_LCL'], color='r')
plt.ylabel('Individual Value')
plt.xlabel('Sample number')
# highlight the points that violate the alarm rules
plt.plot(df['I_TEST1'], linestyle='none', marker='s',
        color='r', markersize=10)

plt.show()
```




> We may conclude that no out-of-control data are present, but the trend pattern between observation 12 and 18 might be investigated more in depth.