QDA 2023 04 06

(1)

ARIMA(p, d, q)

Remember AR process that is non stationary

eg Random Walk

$$X_t = \xi + 1 \cdot X_{t-1} + \varepsilon_t$$

BX_t $\phi_1 = 1$

$$\underbrace{X_t - X_{t-1}}_{} = \xi + \varepsilon_t$$

$$\boxed{(1-B) X_t = \xi + \varepsilon_t}$$

ARIMA(0, 1, 0)

p=0 q=0

$$\nabla X_t = \xi + \varepsilon_t$$

NABLA

INTEGRATED

NON STATIONARY AR process

→ I process

$$A(B)X_t = \boxed{\underbrace{A'_p(B)}_{\substack{\text{STATIONARY}\\ \text{AR}}}\ \underbrace{\underbrace{(1-B)}_{\nabla}{}^{d}}_{\text{NONSTATIONARY PART} \to \underline{I}} X_t = C_q(B)\varepsilon_t}\ ARIMA(p,d,q)$$

order $A(B) \to$ AR including both the stationary and nonstationary parts

$$\sum_{i=-\infty}^{t} X_t \ \text{(INTEGRATED)} = X_t + X_{t-1} + X_{t-2} \ldots = 1 + BX_t + B^2X_t + \ldots$$

$$= (1+B+B^2+B^3+\ldots)X_t = \underbrace{\sum_{i=0}^{\infty}(1+B^i)}\, X_t = \frac{X_t}{1-B}$$

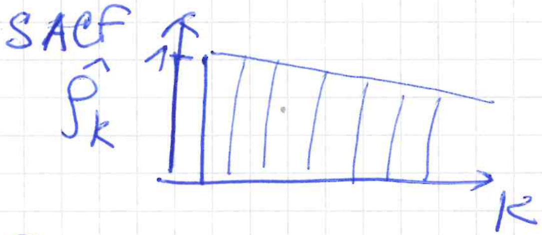
$$= \frac{X_t}{\nabla} = \underbrace{\nabla^{-1}}_{\text{INVERSE } \nabla} X_t$$

t	X_t	$\nabla X_t = X_t - X_{t-1}$	$\nabla^2 X_t$ $\nabla(\nabla X_t)$
1	X_1	–	
2	X_2	$\nabla X_2 = X_2 - X_1$	
⋮	X_3	$\nabla X_3 = X_3 - X_2$	
⋮	⋮		
t	X_t		

$\to$ is this stationary? NO

$\to$ is this stationary? ...

NON STATIONARY?



$\Rightarrow$ I

After applying the ∇ operator d times $\to$ STATIONARITY

$\Rightarrow$ I have been able to identify $\underline{\underline{d}}$

Pay attention to OVERDIFFERENCING $\to$ applying too many times the ∇ operator $\Rightarrow$ OVERESTIMATING d

Box suggests to compare the variance of the different time series (4)

$$(X_t) \qquad \nabla X_t \qquad \nabla^2 X_t \qquad \nabla^3 X_t \quad \cdots \quad \nabla^d X_t$$

$$\text{Var} \quad \text{Var}(X_t) \quad \text{Var}(\nabla X_t) \quad \text{Var}(\nabla^2 X_t) \quad \cdots \quad \underbrace{\text{Var}(\nabla^{d^*} X_t)}_{\text{MIN}}$$

→ Keep d^* ∋ the variance in min

such that

ARIMA $(?, d^*, ?)$

→ Start from $X_t' = \nabla^{d^*} X_t$ → identify p, q

by looking to the SACF

and the SPACF ⟹ p, q

of X_t'

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$$\nabla X_t = \varepsilon_t - \theta_1 \varepsilon_{t-1}$$

$$X_t - X_{t-1} = \varepsilon_t - \theta_1 \varepsilon_{t-1} \qquad IMA(1,1)$$

$$A(B) = 1 - \sum_{i=1}^{p} \phi_i B^i \quad (*)$$

$$C(B) = 1 - \sum_{i=1}^{q} \theta_i B^i \quad (**)$$

$$\underbrace{A_p(B)}_{(*)} \underbrace{\nabla^d X_t} = \underbrace{C_q(B)}_{(**)} \varepsilon_t$$

$$\nabla(\nabla(\nabla \cdot \underbrace{(\nabla X_{t}}_{X_t - X_{t-1}})))$$