UNIVERSITÀ DEGLI STUDI DI TORINO
MultiAgent Systems Course A.A. 2017/2018 Prof. Marco Maggiora

Influence in social network

Master's Students in Physics of Complex System

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2.1 Users and Companies

The initial world's configuration is given by the instantiation of N Users. The Users live in a world with C companies (which are indexed from 0 to $C-1$) and are provided with a set of opinions, one for each company.

An opinion is a positive integer which ranges from 0 to $R-1$, let R denote the opinion range. For each User the opinions are stored within a vector $\mathbf{v} \in \mathbb{N}^C$ which we denote as the opinion vector, the entry $\mathbf{v}_c$ is the opinion on the c-th company.

Each User has an inclination $I \in \{-1, 0, 1\}$ whose possible values denote respectively a "bad", "neutral" and "good" average opinion across the companies, its computation is described with further detail in the following pages.

The simulation is made up by a series of D temporal steps which we call days. On first day the Users' opinions are randomly intialized. On each day the Users influence each other with one-to-one interactions in which opinions can be exchanged.

2.2 Interaction

Each User has a degree k which is the number of Users to whom he can send messages during the day.

Let's consider an interacting couple and call the two Users A and B having respectively degree k_A and k_B, the interaction consists in three steps :

1 A sends a message to B

   The message contains information on A's opinion vector $\mathbf{v}_A$ and degree k_A

2 B receives A's message

   B compares its opinion vector $\mathbf{v}_B$ with $\mathbf{v}_A$, given the subset of companies on which the two Users' opinion are different, a company's index is extracted according to a uniform distribution. If $\mathbf{v}_A$ and $\mathbf{v}_B$ are equal the interaction stops.

3 Influence

   Let c denote the extracted index, two possible events may occur:

   - The opinion on c-th company remains unchanged for B while A changes it to B's opinion. This event occurs with probability $k_B/(k_A + k_B)$.
   - The opinion on c-th company remains unchanged for A while B changes it to A's opinion. This event occurs with probability $k_A/(k_A + k_B)$.

     Basically each User can influence the other on a differing opinion with a probability proportional to its degree.

2.3 Inclination Computation

The Inclination for each User is computed by considering the average of the opinions across the companies.

Let's consider an opinion vector $\mathbf{v}$ with entries $\mathbf{v}_c$ ($c = 1, .., C$). Each entry $\mathbf{v}_c$ is drawn uniformly from the discrete set $\{0, 1, .., R-1\}$. Recalling that the uniform discrete distribution $\mathcal{U}\{a, b\}$ has mean $\frac{a+b}{2}$ and variance $\frac{(b-a+1)^2-1}{12}$ we have that $\mathcal{U}\{0, R-1\}$ has mean $\frac{R-1}{2}$ and variance $\frac{R^2-1}{12}$

Let S be the average of $\mathbf{v}$ entries:

$$S = \sum_{c=1}^{C} \frac{\mathbf{v}_c}{C}$$

For the central limit theorem, for $C \to \infty$ the random variable S will follow a normal distribution with mean $\frac{R-1}{2}$ and variance $\frac{R^2-1}{12C}$.

In order to assess wether the inclination $I = -1$ we check if the value of S lies under the quantile of order 1/3 of the normal distribution $\mathcal{N}(\frac{R-1}{2}, \frac{R^2-1}{12C})$. If $I = 0$ then S is between the 1/3 and 2/3 quantile, and $I = 1$ if S is over the 2/3 quantile.

The 1/3 quantile of the standard normal $\mathcal{N}(0, 1)$ is $q^{(S)}_{1/3} = -0.4399132$ and by simmetry $q^{(S)}_{2/3} = -q^{(S)}_{1/3}$. The 1/3 quantile of $\mathcal{N}(\mu, \sigma^2)$ is $q_{1/3} = \mu + \sigma q^{(S)}_{1/3}$ and the same applies for the 2/3 quantile.

So, in summary, given the User's opinion vector $\mathbf{v}$ we compute its inclination according to the average of components S:

$$I := \begin{cases} -1 & \text{if } S < \frac{R-1}{2} + q^{(S)}_{1/3} \frac{R^2-1}{12C}, \\ 1 & \text{if } S > \frac{R-1}{2} + q^{(S)}_{2/3} \frac{R^2-1}{12C}, \\ 0 & \text{if else} . \end{cases}$$

2.4 The User Class

3 Graphs and results

4 Error estimation

5 Conclusions

6 Altre cose che non so dove mettere