Probability and Statistics > Random Numbers >
Interactive Entries > Interactive Demonstrations >

Random Number

CONTRIBUTE To this Entry

A random number is a number chosen as if by chance from some specified distribution such that selection of a large set of these numbers reproduces the underlying distribution. Almost always, such numbers are also required to be independent, so that there are no correlations between successive numbers. Computer-generated random numbers are sometimes called pseudorandom numbers, while the term "random" is reserved for the output of unpredictable physical processes. When used without qualification, the word "random" usually means "random with a uniform distribution." Other distributions are of course possible. For example, the Box-Muller transformation allows pairs of uniform random numbers to be transformed to corresponding random numbers having a two-dimensional normal distribution.

It is impossible to produce an arbitrarily long string of random digits and prove it is random. Strangely, it is also very difficult for humans to produce a string of random digits, and computer programs can be written which, on average, actually predict some of the digits humans will write down based on previous ones.

There are a number of common methods used for generating pseudorandom numbers, the simplest of which is the linear congruence method. Another simple and elegant method is elementary cellular automaton rule 30, whose central column is given by 1, 1, 0, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, ... (OEIS A051023), and which provides the random number generator used for large integers in the Wolfram Language. Most random number generators require specification of an initial number used as the starting point, which is known as a "seed." The goodness of random numbers generated by a given algorithm can be analyzed by examining its noise sphere.

When generating random numbers over some specified boundary, it is often necessary to normalize the distributions so that each differential area is equally populated. For example, picking θ and ϕ from uniform distributions *does not* give a uniform distribution for sphere point picking.

In order to generate a power-law distribution $P(x)$ from a uniform distribution $P(y)$, write $P(x) = C x^n$ for $x \in [x_0, x_1]$. Then normalization gives

$$\int_{x_0}^{x_1} P(x)\,dx = C\,\frac{\left[x^{n+1}\right]_{x_0}^{x_1}}{n+1} = 1, \qquad (1)$$

so

$$C = \frac{n+1}{x_1^{n+1} - x_0^{n+1}}. \qquad (2)$$

Let Y be a uniformly distributed variate on $[0, 1]$. Then

$$\begin{aligned} D(x) &= \int_{x_0}^{x} P(x')\,dx' && (3)\\ &= C\int_{x_0}^{x} x'^n\,dx' && (4)\\ &= \frac{C}{n+1}\left(x^{n+1} - x_0^{n+1}\right) && (5)\\ &\equiv y, && (6)\end{aligned}$$

and the variate given by

$$\begin{aligned} X &= \left(\frac{n+1}{C}\,y + x_0^{n+1}\right)^{1/(n+1)} && (7)\\ &= \left[\left(x_1^{n+1} - x_0^{n+1}\right)y + x_0^{n+1}\right]^{1/(n+1)} && (8)\end{aligned}$$

is distributed as $P(x)$.

SEE ALSO:
Bays' Shuffle, Box-Muller Transformation, Cliff Random Number Generator, Quasirandom Sequence, Random Variable, Schrage's Algorithm, Stochastic, Uniform Distribution

REFERENCES:

Bassein, S. "A Sampler of Randomness." *Amer. Math. Monthly* **103**, 483-490, 1996.

Bennett, D. J. *Randomness.* Cambridge, MA: Harvard University Press, 1998.

Bratley, P.; Fox, B. L.; and Schrage, E. L. *A Guide to Simulation, 2nd ed.* New York: Springer-Verlag, 1996.

Dahlquist, G. and Bjorck, A. Ch. 11 in *Numerical Methods.* Englewood Cliffs, NJ: Prentice-Hall, 1974.

Deak, I. *Random Number Generators and Simulation.* New York: State Mutual Book & Periodical Service, 1990.

Evans, M.; Hastings, N.; and Peacock, B. *Statistical Distributions, 3rd ed.* New York: Wiley, p. 5, 2000.

Forsythe, G. E.; Malcolm, M. A.; and Moler, C. B. Ch. 10 in *Computer Methods for Mathematical Computations.* Englewood Cliffs, NJ: Prentice-Hall, 1977.

Gardner, M. "Random Numbers." Ch. 13 in *Mathematical Carnival: A New Round-Up of Tantalizers and Puzzles from Scientific American.* New York: Vintage, pp. 161-172, 1977.

James, F. "A Review of Pseudorandom Number Generators." *Computer Physics Comm.* **60**, 329-344, 1990.

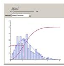
Kac, M. "What is Random?" *Amer. Sci.* **71**, 405-406, 1983.

Kenney, J. F. and Keeping, E. S. *Mathematics of Statistics, Pt. 1, 3rd ed.* Princeton, NJ: Van Nostrand, pp. 200-201 and 205-207, 1962.

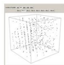
Kenney, J. F. and Keeping, E. S. *Mathematics of Statistics, Pt. 2, 2nd ed.* Princeton, NJ: Van Nostrand, pp. 151-154, 1951.

Knuth, D. E. Ch. 3 in *The Art of Computer Programming, Vol. 2: Seminumerical Algorithms, 3rd ed.* Reading, MA: Addison-Wesley, 1998.

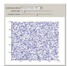
Marsaglia, G. "A Current View of Random Number Generators." In *Computer Science and Statistics: Proceedings of the Symposium on the Interface, 16th, Atlanta, Georgia, March 1984* (Ed. L. Billard). New York: Elsevier, 1985.



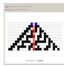
The Method of Inverse Transforms
Ryan Carroll



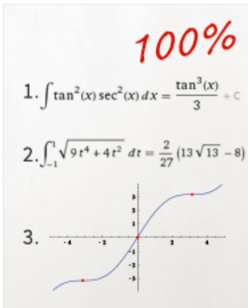
Poor Statistical Qualities for the RANDU Random Number Generator
Paul Savory



Mersenne Twister and Friends
Ed Pegg Jr



Using Rule 30 to Generate Pseudorandom Real Numbers
Chris Boucher



Marsaglia, G. "DIEHARD: A Battery of Tests for Random Number Generators." http://stat.fsu.edu/~geo/diehard.html.

Mascagni, M. "Random Numbers on the Web." http://archive.ncsa.uiuc.edu/Apps/CMP/RNG/mascagni/www-rng.html.

Niederreiter, H. *Random Number Generation and Quasi-Monte Carlo Methods.* Philadelphia, PA: SIAM, 1992.

Nijenhuis, A. and Wilf, H. *Combinatorial Algorithms for Computers and Calculators, 2nd ed.* New York: Academic Press, 1978.

Park, S. and Miller, K. "Random Number Generators: Good Ones are Hard to Find." *Comm. ACM* **31**, 1192-1201, 1988.

Peterson, I. *The Jungles of Randomness: A Mathematical Safari.* New York: Wiley, 1997.

Pickover, C. A. "Computers, Randomness, Mind, and Infinity." Ch. 31 in *Keys to Infinity.* New York: W. H. Freeman, pp. 233-247, 1995.

Press, W. H.; Flannery, B. P.; Teukolsky, S. A.; and Vetterling, W. T. "Random Numbers." Ch. 7 in *Numerical Recipes in FORTRAN: The Art of Scientific Computing, 2nd ed.* Cambridge, England: Cambridge University Press, pp. 266-306, 1992.

Schrage, L. "A More Portable Fortran Random Number Generator." *ACM Trans. Math. Software* **5**, 132-138, 1979.

Schroeder, M. "Random Number Generators." In *Number Theory in Science and Communication, with Applications in Cryptography, Physics, Digital Information, Computing and Self-Similarity, 3rd ed.* New York: Springer-Verlag, pp. 289-295, 1990.

Sloane, N. J. A. Sequence A051023 in "The On-Line Encyclopedia of Integer Sequences."

Weisstein, E. W. "Books about Randomness." http://www.ericweisstein.com/encyclopedias/books/Randomness.html.

Wilf, H. S. *Combinatorial Algorithms: An Update.* Philadelphia, PA: SIAM, 1989.

Referenced on Wolfram|Alpha: Random Number

CITE THIS AS:

Weisstein, Eric W. "Random Number." From *MathWorld*--A Wolfram Web Resource. http://mathworld.wolfram.com/RandomNumber.html