“Einer kann sich bewußtlos stellen; aber auch bewußt?”

— L. Wittgenstein

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1 Introduction

In recent years, the rise of commission-free trading platforms has profoundly reshaped retail investor behavior and sparked growing interest among scholars in behavioral finance. One of the most prominent examples is Robinhood, a mobile-first brokerage that gained widespread popularity for eliminating trading fees and offering a highly gamified user experience. The platform attracted a large number of retail investors, particularly young and inexperienced individuals[1].

An important development in the empirical literature was the release of Robintrack, an open-source dataset that tracks the number of Robinhood users holding individual stocks over time. This data, collected via Robinhood's public API, provides a rare opportunity to directly observe the trading dynamics and portfolio shifts of real retail investors. The dataset can be downloaded from `https://robintrack.net/`.

Several recent studies, including [Fedyk, 2024] and [Welch, 2022][2], have leveraged the Robintrack dataset to examine retail investor performance. Their findings suggest that Robinhood investors, contrary to popular belief, exhibited strong market timing and outperformed passive benchmarks. In particular, these papers report significant cumulative returns and positive alpha using standard factor models.

In this paper, we revisit these claims by constructing an alternative methodology for portfolio formation based on the same dataset. Specifically, we analyze whether the returns of Robinhood users' favorite stocks exhibit stochastic dominance over benchmark indices. Our goal is to offer a more nuanced assessment of whether retail investors truly generate abnormal returns or whether previous results may be driven by sample selection or methodological choices.

2 Literature Review

The literature on retail investor performance has evolved considerably over the past two decades. Early foundational work highlighted persistent underperformance by individual investors, while more recent studies leveraging novel, high-frequency data sources have painted a more nuanced picture—one in which method choice can swing conclusions from ”retail crowd outperforms” to ”retail crowd underperforms.” This paper introduces a novel approach in studying directly investor performance, while drawing insights on the risk aversion to assess the relaibility of the collected data and possible behavioral

[1]According to Robinhood's IPO filing, the typical user on the platform is 31 years old, with an average account balance of approximately $3,500. Notably, around half of the platform's users are investing for the first time.

[2]it must be noted that the former explicitly follows the method of the latter

biases.

The canonical starting point to analyse trading and behavioral biases is the seminal work of [Barber and Odean, 2000]. Using account-level brokerage records, they show that individual investors engage in trading with excessive frequencey, incurring higher transaction costs, exhibiting poor timing. This ultimately leads to underperformance relative to buy-and-hold benchmarks. The main message of their study is in the title of the paper itself, ”Trading Is Hazardous to Your Wealth”. In their analysis of individual trading accounts from an important broker, they find that the most active accounts underperform the market by approximately 6.5% per year, net of fees. This result firmly establishes that behavioral biases such as overconfidence, turnover, and attention-driven trading harm returns of retail participants. Moreover, [Grinblatt and Keloharju, 2001] use Finnish account-level data, documenting several behavioral biases in their trading patterns. They show evidence of investors being reluctant to realize losses and that hat trading intensity rises following past returns and when stocks reach monthly highs or lows.

Building on advances in data availability, [Welch, 2022] and [Fedyk, 2024] both exploit the Robintrack dataset to construct a ”Robinhood crowd” portfolio, althoug focusing only on U.S. common stocks. [Welch, 2022] introduces two weighting schemes identical to those later used by Fedyk, and documents that, when building a portfolio with yesterday’s crowd-popularity weights to today’s returns, the resulting performance has significant positive alpha over the February 2018-August 2020 sample. The aggregate Robinhood portfolio earned an average six-factor daily alpha of 6.5 basis points. Remarkably, retail investors appear to have timed the market successfully, outperforming a U.S. stock index while achieving similar drawdowns.

[Fedyk, 2024] closely follows Welch’s methodology, aggregating hourly Robintrack counts into daily weights and applying a one-day lag to avoid look-ahead bias. She confirms Welch’s core finding of very high positive returns and documents robustness across factor models. Importantly, Fedyk also compares the ”dollar” versus ”share” weighting schemes and shows that both yield similar outperformance. The author also decomposes the returns through three behavioral channels. A ”buy-the-dip” effect is particularly pronounced in large-cap stocks, showing positive excessive returns in the very-short terms. Robinhood investors also show more activity around announcements and analyst recommendation revisions, and attention spilloveers driven by WallStreetBets sentiment.

[Ardia et al., 2023] leverage higher frequency data to reveal that Robinhood users exhibit a high-frequency, contrarian trading style: Robinhood investors disproportionally buy big losers within an hour of extreme negative returns and show higher sensitivity to overnight moves.

The literature in behavioral finance highlights the importance of limited attention and the importance of information. [Barber et al., 2021] also make use of the robintrack dataset to show that Robinhood users engage in extreme attention-induced herding episodes. Notably, these episodes are followed by large negative returns. They document that this herding behavior is driven by Robinhood's "Top Mover" interface, highlighting the importance of gameification in shaping investors preferences. [Da et al., 2009] show that increases in the Google Search Volume Index for a given ticker are correlated with investing activity of retial investors. In addition to this, increased retail attention can explain the long run underperformance of IPO stocks. Beyond raw attention effects, interface design and social media contagion further shape retail behavior. [Semenova and Winkler, 2023] use Reddit's WallStreetBets discussions to show that peer-driven cascades amplify price momentum and reversals, as traders coordinate through social network to drive short-term bubbles.

In sum, the existing literature vividly illustrates that retail-investor performance hinges critically on both measurement design and behavioral dynamics Yet, a cohesive framework that emebeds risk aversion considerations remains absent. This paper fills the gap by evaluating the performance of retail investors not only through risk-adjusted and raw returns but also through CRRA utility estimation, allowing us to compute welfare losses retail investors incur when behavioral biases misalign their portfolios. Stochastic dominance tests are also presented, supporting a less beenvolent view of Robinhood investors and raising a question of rationality. In doing so, we provide a cohesive assesment of behavioral drivers of returns and risk aversion implied by revealed preferences, offering a more structurally consistent and behaviorally meaningful evaluation of whether and how "the wisdom of the crows" truly materializes.

3 Robinhood Dataset

3.1 Description of the Dataset

The dataset records the number of Robinhood users holding at least one share of 8,619 securities, with observations taken hourly. Following [Welch, 2022] and [Fedyk, 2024], we aggregate this data on a daily basis by selecting the last observation of each trading day.

The sample spans from February 5, 2018, to August 13, 2020, covering 818 days. Note that the dataset includes non-trading days and contains some missing observations.

Since the dataset only provides the number of investors per security, we cannot track individual holdings, monetary amounts, or share quantities. Moreover, buy/sell flows are unobservable; however, we can approximate them using changes in the number of holders.

We merge this dataset with CRSP to obtain market-level information and later construct a benchmark index. The resulting dataset contains 7,613 unique securities, substantially more than in [Fedyk, 2024] and [Welch, 2022], who restrict their analysis to U.S. common stocks only. Details of the data cleaning procedure are provided in Appendix B.

In terms of security types, common stocks represent 57.3% of the dataset, while ETFs and other funds account for 26.3% and 8.7%, respectively. Structured products, REITs, and ADRs constitute the remaining share. When classifying by market capitalization, stocks dominate with 83.1%, followed by ETFs (9.3%) and other funds (3.3%).

The total number of open positions on any given day is calculated as the sum of users holding at least one share across all securities—i.e., a row-wise sum across the dataset.

Market data for each security was retrieved from CRSP[3] via WRDS. Out of the full universe, 8,099 securities were available in CRSP, as it includes only U.S.-listed assets.

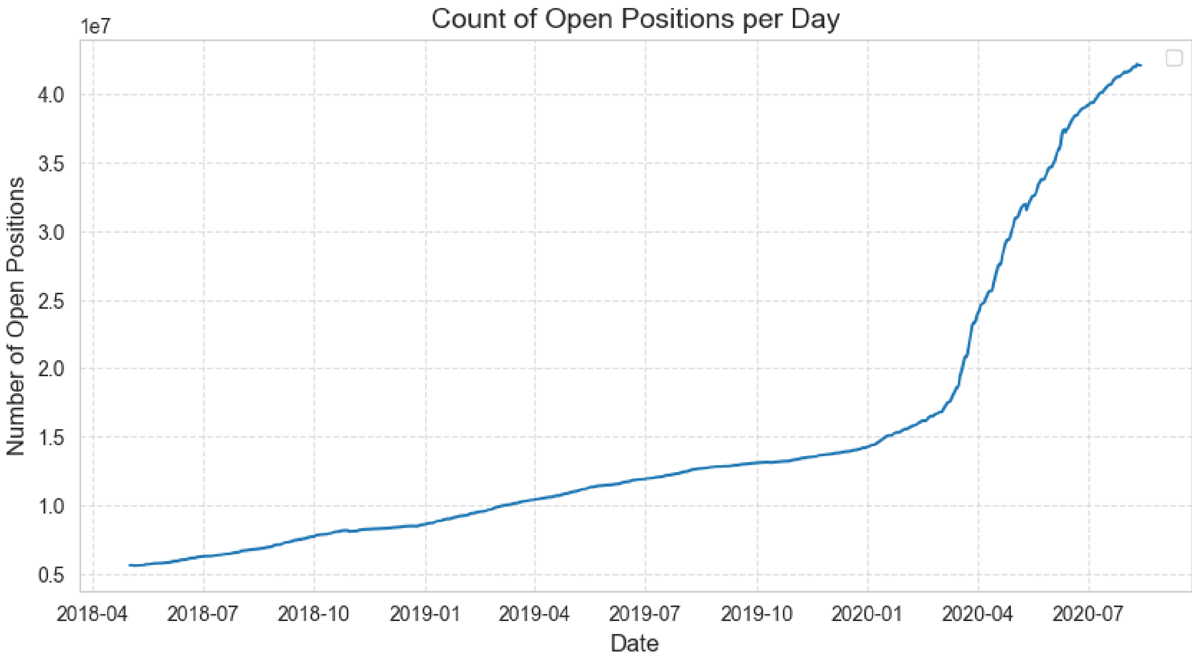

Figure 1: Daily count of open Robinhood positions, May 2018-August 2020.

The figure above shows the daily count of open positions on Robinhood from April 2018 to mid-2020. We observe a steady increase in user participation, with a sharp acceleration beginning in early 2020. This surge coincides with the onset of the COVID-19 pandemic, likely driven by a combination of heightened market volatility, increased retail interest, and fiscal stimulus payments.

[3]The Center for Research in Security Prices (CRSP), based at the University of Chicago, provides high-quality historical market data widely used in finance research and investment analysis.

4 Building the Robinhood Portfolio

As explained above, the biggest limitation of the Robintrack dataset is that it counts the number of users holding a certain security and doesn't provide any information on the amount invested in a particular security.

This section presents a comprehensive framework for constructing and evaluating such a portfolio using distinct weights methods and assumptions. We and the other authors adopt two distinct approaches to compute value and returns. By comparing these approaches, we can better understand how different assumptions about retail investor behavior influence our assessment of the Robinhood crowd's investment performance.

4.1 Weights Methods

[Fedyk, 2024] and [Welch, 2022] use the same approach to build the performance of the Robinhood crowd (or "reference index"): they build daily weights and then apply the weights from the previous day to daily stock returns, directly building portfolio returns.

First, it is necessary to define how those weights are computed. They define two different types of weights, although they yield similar findings in their analysis.

The first method is the "dollar method", which assumes that every investor represents an equal dollar amount investment in the stock.

$$w_{i,t}^{\text{dollar}} = \frac{N_{i,t}}{\sum_j N_{j,t}} \tag{1}$$

where $w_{i,t}$ is the Robinhood portfolio weight of security i at time t and $N_{i,t}$ is the number of investors in security i at time t.

Alternatively, they define the "share method", where each Robinhood investor in a stock represents a one share investment in that stock.

$$w_{i,t}^{\text{share}} = \frac{N_{i,t} \cdot P_{i,t}}{\sum_j N_{j,t} \cdot P_{j,t}} \tag{2}$$

where $P_{j,t}$ is the price of stock j at time t.

The approach we developed differs on how the weights are applied. Nonetheless, we need to define how the components of the portfolio are weighted:

$$w_{i,t}^{\text{mine}} = \frac{N_{i,t}}{\sum_j N_{j,t}} \tag{3}$$

where $w_{i,t}$ is the Robinhood portfolio weight of security i at time t and $N_{i,t}$ is the number of investors in security i at time t. This is identical to 1 but is being listed for clarity.

4.2 Alternative Methodologies for Constructing Portfolio Returns

4.2.1 Fedyk's Approach

The aggregate Robinhood Portfolio returns are derived in [Fedyk, 2024] and [Welch, 2022] by multiplying weights by their daily returns[4], assuming that the weights, however computed, represent a certain share of wealth in a stock held by Robinhood crowd.

From now on, we will define this method as the ”Fedyk” method.

Defining Returns Returns differ mathematically based on which weights are applied to returns.

Formally the returns of the Robinhood portfolio, using the dollar method, are defined as:

$$r_t^{\text{dollar}} = \sum_{i=1}^{N} w_{i,t-1}^{\text{dollar}} \cdot r_{i,t} \tag{4}$$

where $r_{i,t}$ is the realized simple return on security i at time t, and $w_{i,t-1}^{\text{dollar}}$ is the dollar weight as defined in equation 1 of security i, for the previous day to avoid look-ahead bias.

Alternatively, they derive returns for the Robinhood portfolio using the share method in the following manner:

$$r_t^{\text{share}} = \sum_{i=1}^{N} w_{i,t-1}^{\text{share}} \cdot r_{i,t} \tag{5}$$

where $r_{i,t}$ is the realized simple return on security i at time t, and $w_{i,t-1}^{\text{share}}$ is the share weight as defined in equation 2 of security i, for the previous day to avoid look-ahead bias.

Defining Value Although Fedyk and Welch never define the value of their portfolio in their papers, we can derive the value of the Robinhood Portfolio computed according to their method from compounded returns.

The value of the Robinhood portfolio using the dollar method can be defined as follows:

$$V_T^{\text{dollar}} = V_0 \prod_{t=1}^{T} (1 + r_{\text{dollar},\,t}) \tag{6}$$

where V_0 can be assumed equal to 1 without loss of generality and $r_{\text{dollar},\,t}$ is derived from equation 4.

[4] returns are computed directly by CRSP and are adjusted for dividends, e.g. if $P_0 = 10$ and $D_1 = 5$ and $P_1 = 5$ returns would be 0%

Using the share method instead, the value of the Robinhood portfolio can be defined as follows:

$$V_T^{\text{share}} = V_0 \prod_{t=1}^{T} (1 + r_{\text{share},\,t}) \tag{7}$$

where V_0 can be assumed equal to 1 without loss of generality and $r_{\text{share},\,t}$ is derived from equation 5.

4.2.2 My Approach

On the other hand, I first compute the value of the Robinhood portfolio by doing a weighted sum of the prices of the securities in the dataset. Conceptually, this represents the portfolio of an investor who decides to allocate a certain number (or percentage) of shares to each security. I will call this this method ”Mine” or simply the method built on prices.

We can therefore define the value of the Robinhood Portfolio as follows:

$$V_t^{\text{mine}} = \sum_{i=1}^{N} w_{i,t-1}^{\text{mine}} \cdot P_{i,t} \tag{8}$$

where $w_{i,t-1}^{\text{mine}}$ is the weight of security i computed according to 3 at time $t-1$ and $P_{i,t}$ is the price of security i at time t.

I then track the evolution of the value of the portfolio as defined in 8 to compute returns.

$$r_t^{\text{mine}} = \frac{V_t^{\text{mine}}}{V_{t-1}^{\text{mine}}} - 1 \tag{9}$$

4.3 Capturing the Persistence of Investor Composition

Although both approaches ultimately yield a time series of Robinhood portfolio returns, there is a fundamental difference in what these return paths represent.

In the method used by [Fedyk, 2024] and [Welch, 2022], the portfolio is effectively rebalanced every day to reflect the current composition of investor popularity. Each day's return is computed based on that day's weights and the corresponding daily stock-level returns. This provides a valid snapshot of the average return generated by the stocks held on a given day.

However, this approach does not preserve the economic exposure that investors accumulate through time. A stock that was extremely popular for several days but declines in popularity just before a price spike will have minimal influence on the portfolio's return when that spike occurs. Only the weights at time $t-1$ affect the return at time

t^5, so the model captures immediate sentiment shifts but not the cumulative effects of holding positions over time.

In contrast, the methodology I propose (9) applies weights to stock prices and computes returns from changes in total portfolio value. This implies that a stock that was heavily weighted yesterday continues to influence portfolio performance today, even if its popularity has declined. The return reflects both the dynamics of price changes and the path dependency of investor composition.

As a result, my method embeds the effects of investor flows, popularity shifts, and concentration in the actual evolution of portfolio value. The cumulative performance is not a sequence of disconnected daily snapshots, but a reflection of how crowd behavior builds, persists, and unwinds over time.

Conceptually, this distinction is important when studying behavioral dynamics. Retail investor behavior, particularly on platforms like Robinhood, is driven not only by cross-sectional preferences at a point in time but also by persistent patterns of attention, sentiment, and herding. A portfolio that evolves with these behavioral shifts provides a more realistic measure of the actual wealth path experienced by retail investors, rather than an idealized, continually rebalanced index.

In this sense, computing returns from the portfolio value offers a more structurally consistent and behaviorally meaningful representation of the Robinhood crowd's investment trajectory.

5 Comparing Returns and Distribution measures

The biggest difference does not appear when using different kinds of weights ("dollar" or "share" method) but rather when building the portfolio from prices or returns. Moreover, Fedyk and Welch build their portfolio only using common American stocks (share code 10 or 11). In my final analysis I look at all types of securities but significant differences emerge even when using the same sample. Additionally, by recreating the method employed by the other authors we can analyse its return when dealing with all kinds of securities. In section 5.2 I compare returns using only common stocks or the full sample of securities, in the other sections of this paper I will use the full sample unless explicitly stated.

As the other authors have claimed in their papers, the Portfolio built directly from returns had a significantly higher cumulative return compared to the market and positive alpha.

I will proceed to analyse in more detail the distribution of returns of different Robinhood

[5] Previous day's weights are taken to prevent look-ahead bias

portfolios, showing that my method depicts a far less rosy picture of the ”Robinhood strategy”.

Moreover, [Fedyk, 2024] has analysed extensively the differences between the portfolio obtained using the share method and the dollar method. We'll focus on the dollar method since it is the same approach I use to compute weights.

5.1 Constructing Moving Averages

Defining log returns allows us to simply compute moving averages, showing the profitability of the Robinhood portfolio at different time frames. Conceptually, the value of an n-day moving average on a given date T represents the return an investor would have earned by initiating the position at the open of day $T-n$ and holding it continuously up to close of day T. More rigorously, for a given horizon of n days, the log return over the period is:

$$r_n = \sum_{t=T-n+1}^{T} r_t = \ln\left(\prod_{t=T-n+1}^{T} R_t\right) \tag{10}$$

where r_t are daily log returns and R_t are daily gross returns.

5.2 Retail Performance Under Different Samples

5.2.1 Rolling Returns Across Investment Horizons

Common Stocks Only At short horizons (5-30 days), the two Robinhood portfolios (Fedyk and Mine) display highly similar dynamics, with both closely tracking market indices and exhibiting bursts of volatility during periods of market stress, particularly around the onset of the COVID-19 crash. This suggests that in the very short term, retail investors tend to move in tandem with broader market trends, with limited divergence in return profiles across methodologies. However, at the 120-day horizon, both Robinhood portfolios outperform the S&P 500 and a World ETF[6].

This finding is consistent with the idea that many retail investors, especially on Robinhood, engaged in ”buy-the-dip” behavior during the COVID-19 crash. Their increased exposure to beaten-down or speculative stocks during the downturn appears to have been rewarded in the subsequent rebound. Importantly, this also coincides with a period of explosive growth for the platform itself, which may have amplified attention and capital inflows into popular names.

Nonetheless, this post-crash outperformance comes after a prolonged period of clear underperformance. Prior to March 2020, both Robinhood portfolios consistently

[6] I've used Vanguard's VOO for the S&P500 and Vanguard's VT as a World Equity ETF

lag behind the benchmark indices, with my method in particular reflecting substantial drawdowns and poor stock selection.

What differentiates the two methods most clearly is the strength of the post-COVID recovery. While Fedyk's method shows a relatively steady climb, my price-based approach rebounds even more sharply after March 2020. This reflects the fact that, under my methodology, investor positions are not rebalanced away from prior favorites. As a result, stocks that surged after the crash contributed disproportionately to the portfolio's recovery.

In sum, the 120-day results provide two key insights: first, that retail traders on Robinhood did benefit from post-crisis market dynamics, and second, that the magnitude and nature of this benefit depends heavily on the modeling approach, particularly when capturing persistence in portfolio composition.

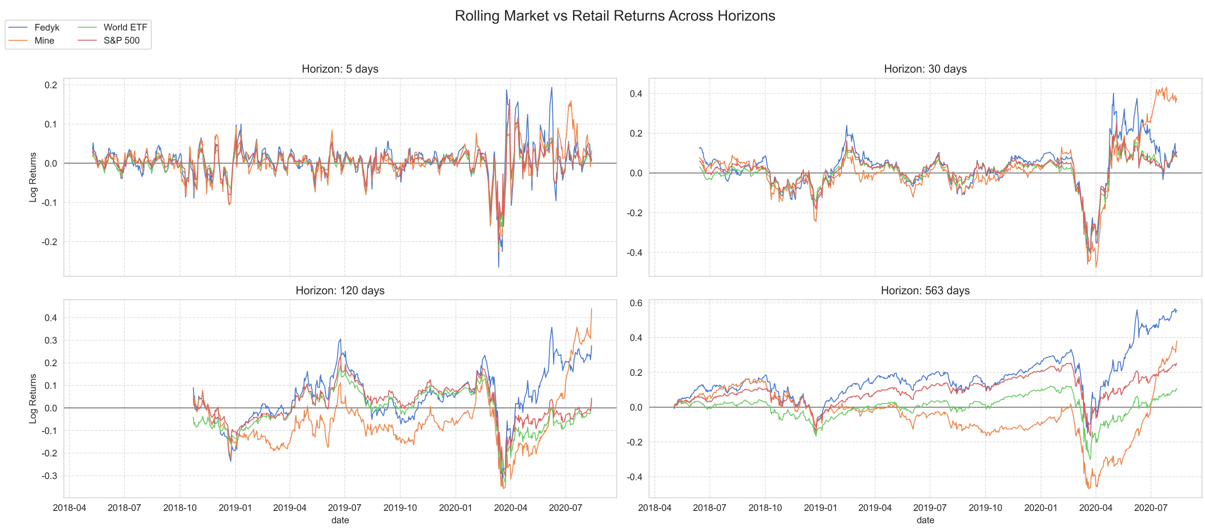

Figure 2: Rolling log-returns of Robinhood and market portfolios at four investment horizons.

Full Universe of Securities We now extend the analysis to all securities in the dataset, including ETFs, REITs, and structured products.

Expanding the sample, we still observe that short-term movements (5-30 days) remain closely correlated across all portfolios, with limited divergence in returns or volatility between methods.

However, the performance gap widens at longer horizons. In contrast to the stock-only case, where both Robinhood portfolios outperformed after the crash, here the drawdowns, particularly in the price-based portfolio, are deeper and more persistent.

Low performance spans the entire pre-COVID period: my portfolio underperforms continuously throughout 2019, while Fedyk's stays closer to the benchmarks but still lags in absolute terms.

At the 563-day horizon, both Robinhood portfolios end below the S&P 500 and the World ETF. My portfolio, while showing a stronger post-crash rebound, barely catches up to the S&P 500 by the end of the sample, and only due to its heavier exposure to post-crash winners. Fedyk's method performs slightly better but remains clearly below the benchmark, reversing the apparent outperformance seen in the stock-only sample.

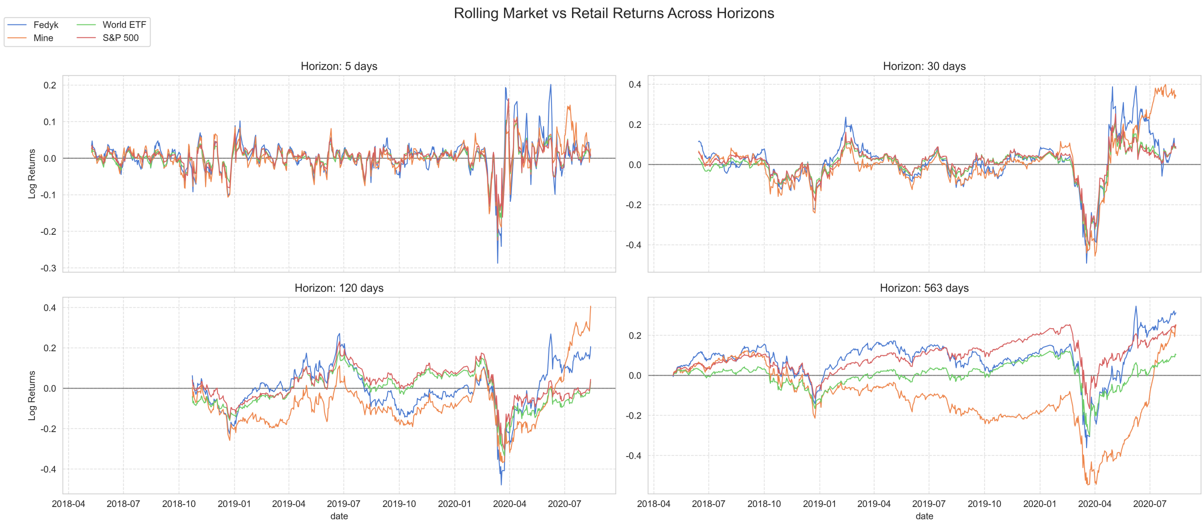

Figure 3: Rolling log-returns of Robinhood and market portfolios at four investment horizons.

A detailed description of the distributions can be found in table 2.

5.2.2 Distribuional insights

Common Stocks Only The distribution of returns across horizons provides further evidence on the differences between the Robinhood portfolios and the benchmarks.

At short horizons (5-30 days), all portfolios are tightly centered around zero, with relatively similar dispersion. Both Robinhood portfolios display slightly higher volatility than the S&P 500 and the World ETF, with standard deviations of approximately 0.045 for five-day returns, compared to about 0.03 for the benchmarks. However, mean returns remain close to zero for all portfolios, and short-term behavior shows limited divergence across methods.

At the 120-day horizon, differences become more evident. While Fedyk's portfolio maintains a positive mean log return of approximately 0.05, my price-based portfolio exhibits a negative mean return of about -0.06. The median is also much worse, at about -0.14 versus -0.02. This shift is reflected in the distribution shapes, with my portfolio showing a wider left tail and greater dispersion relative to both Fedyk's method and the benchmarks.

At the 563-day horizon, the gap further widens. The price-based portfolio remains centered around negative returns, with a mean log return of approximately -0.04, while the rebalanced portfolio achieves a positive mean return of 0.17. In contrast, the S&P 500 maintains a strong positive outcome, with a mean log return close to 0.1.

Despite the strong rebound observed after the COVID-19 crash, the cumulative performance of the price-based portfolio remains significantly weaker, often with higher standard deviation.

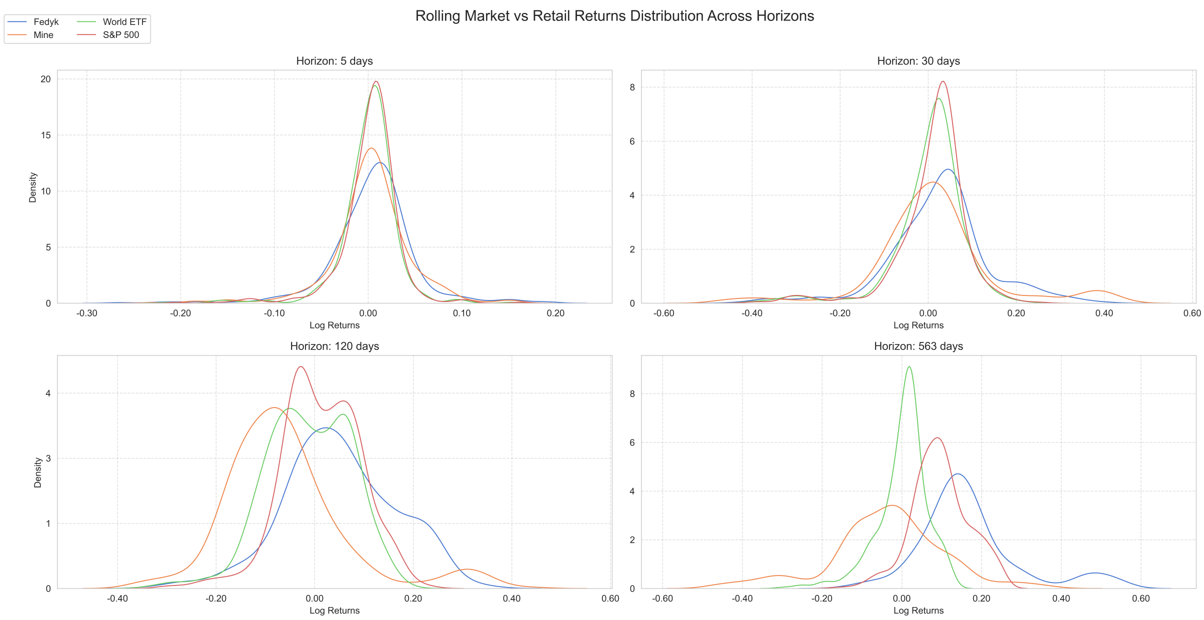

Figure 4: Distribution of rolling log-returns for retail and benchmark portfolios across investment horizons (stock-only sample)

A detailed description of the distributions can be found in table 1.

Full Universe of Securities Expanding the analysis to include all types of securities slightly alters the distributional patterns compared to the stock-only case.

At short horizons (5-30 days), return distributions remain centered around zero and maintain a similar dispersion across all portfolios. Standard deviations are comparable to the stock-only case, around 0.02 for both Robinhood portfolios at the daily level and 0.04 for five-day returns. Mean returns are very close to zero, confirming that short-term dynamics are largely unaffected by the broader sample.

At the 120-day horizon, however, differences become more evident. Both Robinhood portfolios exhibit flatter distributions with thicker left tails compared to the benchmarks. In particular, my price-based portfolio records a negative mean log return of approximately -0.07, while Fedyk's method also turns slightly negative at around -0.004, unlike in the stock-only case where it remained positive. The S&P 500 maintains a positive mean return over the same horizon, reinforcing the relative weakness of retail portfolios when more asset types are included.

At the cumulative horizon, the gap becomes substantial. My portfolio shows a strongly left-skewed distribution with a mean log return of approximately -0.10. Fedyk's portfolio, although positive at around 0.087, remains below the S&P 500, which achieves a mean return close to 0.1. The World ETF again displays lower returns, in line with its broader exposure to international markets.

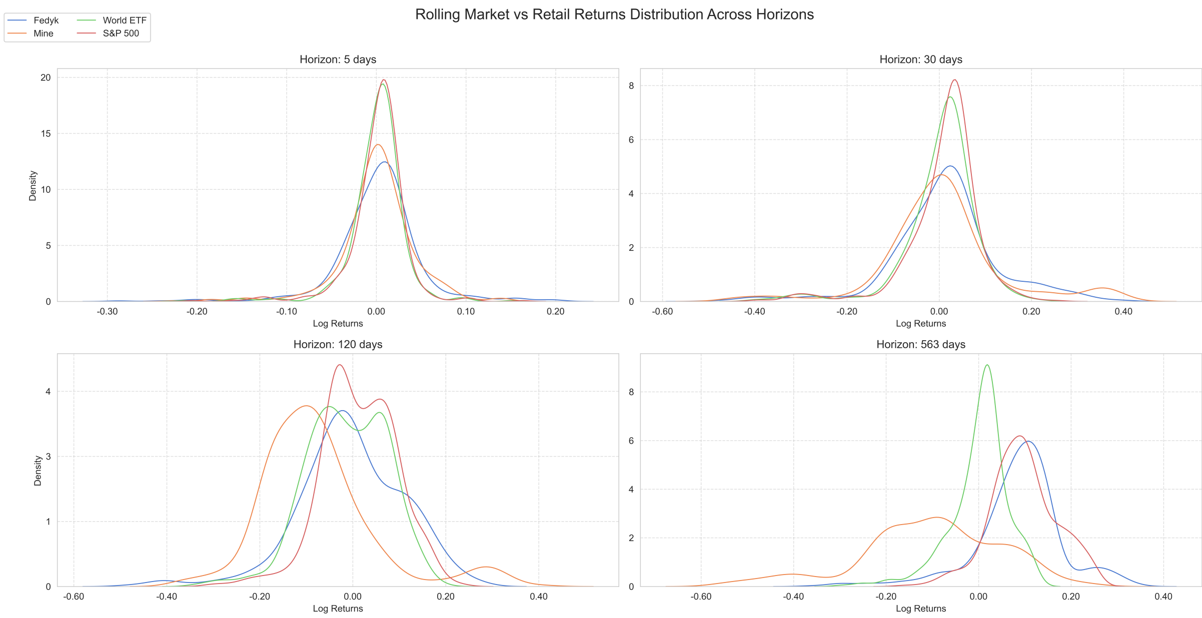

Figure 5: Distribution of rolling log-returns for retail and benchmark portfolios across investment horizons (complete sample)

Descriptive statistics supporting these findings are reported in Table 2.

5.3 Analysis of Short-Term Returns and the Impact of COVID

Comparing the daily returns for the whole period available we can already observe distinct behavior in terms of returns and distributions for the market and Robinhood indices. To test whether there are statistically significant differences between the mean daily returns of the Robinhood portfolios, the S&P 500 and the world ETF, I conduct an ANOVA test. The null hypothesis is that all portfolios have equal mean daily returns.

5.3.1 Covering the Whole Period

As expected from the results regarding the cumulative returns, mean daily returns are very similar for the Robinhood Portfolios. Fedyk's method has a mean of 0.000555, while the method based on prices has a mean of 0.000450. The ANOVA analysis conducted on these two portfolios, equal to a t-test in this case, has a p-value of 0.9273. This suggests that at the daily frequency the expected growth rate, i.e. the mean log returns, of the Robinhood portfolios are not statistically different.

In terms of standard deviation, the Robinhood portfolios have relatively larger values, around 0.02, while the market indices have values around 0.15. This is consistent with what can be seen from the distributions: retail portfolios have fatter tails while modal returns appear similar across all time series.

Conducting ANOVA tests on all possible combinations of these four timeseries, none has an acceptable p-value, results are in table 4. However, conducting a Fligner test to assess the difference in variances, the Robinhood portfolios show statistically significant different variances than the market proxies, results are in table 6.

Here below the plots of the distributions, while the returns are in table 2:

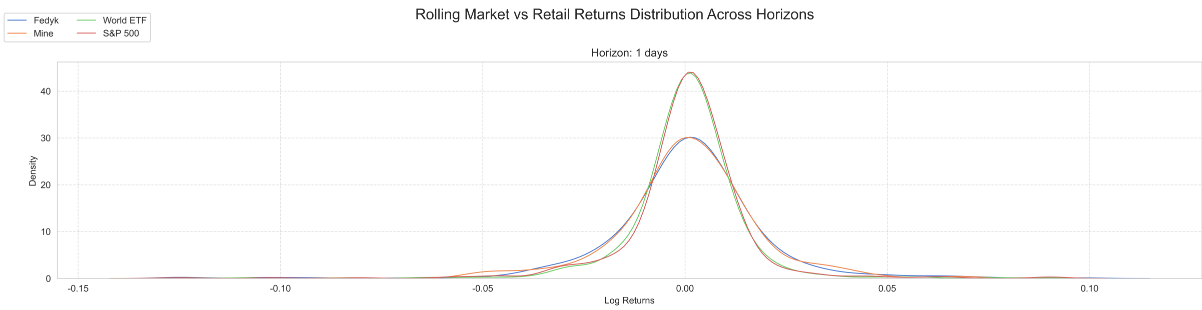

Figure 6: Distribution of daily log-returns for retail and benchmark portfolios (complete sample)

5.3.2 Excluding COVID

The results on daily returns, however, are probably impacted by the noise of the March 2020 crash. I run the same analysis filtering the dataframe up to February 3rd 2020, the date in which the pandemic was declared in the US.

In this case point estimates for means differ greatly even among Robinhood Portfolios, with the price-based approach yielding -0.000328 on average versus Fedyk's 0.000246. Nonetheless, also at this level returns are not statistically significant, results can be found in table 5.

In terms of standard deviation, the market indices show again clearly a different distribution, with variances being markably lower. This is corroborated by fligner tests, available in table 7.

Here below the plots of the distributions, while the returns are in table 3:

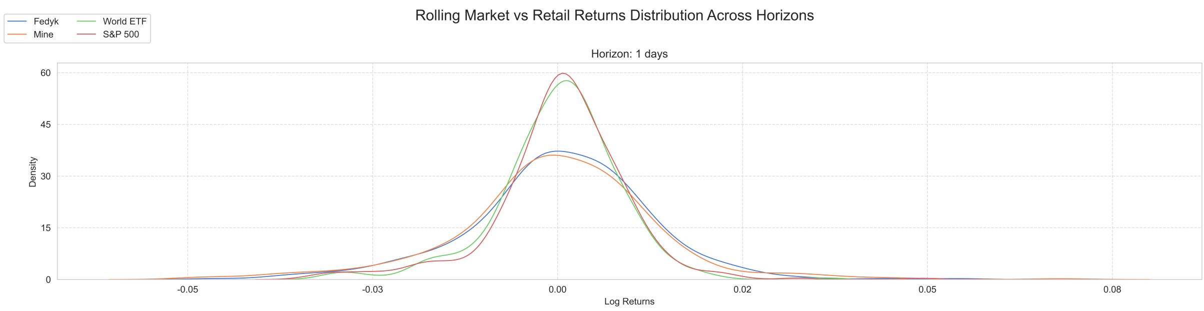

Figure 7: Distribution of daily log-returns for retail and benchmark portfolios (complete sample, excluding COVID)

5.4 Comments

Daily returns show a deceptively optimistic view, with the different series having statistically equal log-returns. However, the retail portfolios carry systematically higher day-to-day risk, and this compounds as underperformance at the cumulative level before the pandemic.

A one-way ANOVA on daily log returns fails to reject equality of means for any group of series, both including and excluding the pandemic.

However, looking at the risk dimension, standard deviations differ sharply. Fligner-Killeen tests reject homogeneity at any α level whenever a retail series is compared with a market proxy. As highlighted earlier, this can be easily seen from the shape of the distributions, with correspondingly fatter tails. In log space, larger dispersion automatically lowers long-run growth because every large swing pulls the cumulative sum away from the average trend. In other words, retail investors accept more noise than the market but earn no measurable extra drift in a single day.

Although ANOVA cannot reject equality between the Robinhood portfolios, the point estimates themselves are not identical (compare table 3). The gap in their mean returns, about -0.0006 log points, rapidly cumulates to -0.25 log points. The test lacks power because daily σ is large; the economic impact measured in compounding is evident.

Path dependence widens the difference. The price-weighted series 9 embeds a negative correlation between returns and remaining capital. When a stock falls, its contribution to the portfolio value shrinks, and therefore any subsequent rebound is applied to less capital, exactly how a normal portfolio would work. The return-weighted construction re-anchors weights every market close and therefore avoids this drag.

Losses in early 2019 thus penalise the price-weighted portfolio twice.

6 Factor Model Evaluation and Alpha Robustness

In this section I subject the representative Robinhood portfolio to a series of standard asset pricing tests in order to challenge the unusually large abnormal returns reported in [Fedyk, 2024] and [Welch, 2022], more precisely between 14.8% and 21.4% annualized alpha.

I estimate the Capital Asset Pricing Model, the Fama-French three- and four-factor specifications, and the six-factor model using exactly the same factor premia from Kenneth French's database that Fedyk and Welch employs. Unlike his analysis, which is confined to a hand-picked subset of stocks, I apply the portfolio construction procedure developed earlier in this paper to the full universe of available securities. I also extend the sample window through the COVID-19 crash to test the stability of the alpha estimates under extreme market stress. The resulting intercepts are far more muted than those in Fedyk's tables, and many lose statistical significance once pandemic volatility is incorporated. These results call into question the realism of the super-high alpha values in the existing literature and highlight the importance of broad coverage and rigorous methodology in evaluating retail-driven strategies.

6.1 Results and Interpretation

6.1.1 Robustness Check: Applying Weights to Prices on a Stock-Only Universe

Before extending Fedyk's method to all securities, we proceed by shwoing that the method proposed in this paper has considerably different effects also when restriciting the analysis to stocks only, precisely how the other authors have meade.

The results (avilable in table 8) already paint a less rosy picture. Fedyk's regressions report positive, statistically significant alphas—on the order of 0.0006 to 0.0008 per period, suggesting that his portfolio construction appears to deliver genuine excess returns above what is explained by standard factor exposures. In contrast, the "Mine" table shows alphas that are effectively zero or slightly negative (approximately -0.0001 to -0.0002) and never reach statistical significance. In practical terms, the absence of any discernible alpha implies that, once my weighting method is applied to the full universe of securities, the Robinhood strategy cannot outperform the market: at best, it simply replicates the benchmark, and any apparent outperformance vanishes.

6.1.2 Extending the Analysis to All Securities

When estimating both my ”Mine” strategy and Fedyk’s portfolio over the full universe of tradable securities, rather than restricting to U.S. common stocks, the most striking finding is the disappearance of any positive alpha. In Fedyk’s regressions, the intercepts hover just above zero (0 to 0.0004), but fail to achieve any significance. However, when I apply the same factor-model framework, including CAPM, three-, four-, and six-factor regressions, to the aggregate set of all equities, my portfolio’s intercept is slightly negative (approximately -0.0001 to -0.0004) in every specification and never approaches statistical significance. In other words, what appears to be a faint ”edge” in Fedyk’s U.S.-only sample entirely vanishes once we broaden the coverage, implying that my Robinhood-style weights simply replicate common factor exposures rather than generating true excess returns. The results are in table 9 and 10.

Beyond alpha, several factor-loading shifts arise when using the full universe. Market beta on ”Mine” (≈1.01-1.09) remains similar to Fedyk’s (≈0.99-1.11). The HML loading is far more negative in ”Mine” (-0.42 to -0.49) than in Fedyk (≈+0.09 in three-factor, -0.18 in four-factor), demonstrating a substantial tilt toward growth. SMB falls from Fedyk’s large positives (≈+0.69 in three-factor, +0.58 in four-factor) to more moderate values in ”Mine” (≈+0.20-0.23). Momentum (UMD), strongly negative for Fedyk (≈-0.32 to -0.36), becomes minor and marginal in ”Mine” (-0.08) and vanishes in the six-factor. Profitability (RMW) flips from Fedyk’s negative (≈-0.31) to a small positive (≈+0.09) in ”Mine”, while investment (CMA) becomes more strongly negative (≈-1.00 versus Fedyk’s -0.63).

6.1.3 Excluding the impact of COVID

These pre-COVID regressions paint a starkly different picture than when the pandemic period is included. By excluding observations after early February 2020, when a large fraction of the Robinhood portfolios’ return spikes occurred, the estimated alphas for both ”Mine” and ”Fedyk” become negative and highly significant. In other words, much of the apparent outperformance disappears once we remove the extreme volatility of the COVID-19 crash and subsequent rebound. This confirms our earlier return-distribution analysis showing that a disproportionate share of gains arose after February 3rd.

When observations from the COVID-19 crash are removed, the ”Mine” portfolio shows a significant negative alpha in every regression. Results are in table 11. In the CAPM, the intercept is -0.0010 (SE 0.0003, p<.01). The three-factor model yields alpha = -0.0011 (SE 0.0002, p<.01) with a market beta of 1.2400 (SE 0.0269, p<.01). Adding momentum in the four-factor model gives alpha = -0.0011 (SE 0.0002, p<.01), HML = -0.4968 (SE 0.0640, p<.01), and UMD = -0.0465 (SE 0.0530, not significant). In the six-factor model, alpha = -0.0010 (SE 0.0002, p<.01), HML = -0.2195 (SE 0.0530, p<.01), and CMA = -0.9507 (SE 0.1027, p<.01) while RMW is 0.0608 (SE 0.0749, not significant). R-squared values range from 0.8216 in CAPM to 0.8914 in the six-factor model. These results confirm that, before COVID-19, the ”Mine” portfolio underperformed all benchmarks and produced no positive alpha.

When COVID-19 observations are removed, the ”Fedyk” portfolio still produces a null or slightly negative alpha. Results are in table 12. In the CAPM regression, the intercept is -0.0004 (SE 0.0003, p < .01). The three-factor model yields alpha = -0.0002 (SE 0.0002, p < .01) with a market beta of 1.1199 (SE 0.0248, p < .01). Adding momentum in the four-factor specification gives alpha = -0.0002 (SE 0.0002, p < .01), HML = -0.3413 (SE 0.0538, p < .01), and UMD = -0.2295 (SE 0.0471, p < .01). In the six-factor model, alpha remains -0.0002 (SE 0.0002, p < .01), HML = -0.2479 (SE 0.0534, p < .01), RMW = -0.2797 (SE 0.0681, p < .01), and CMA = -0.3126 (SE 0.0875, p < .01). R-squared ranges from 0.8040 in CAPM to 0.8660 in the six-factor model. These findings show that, even before COVID-19, ”Fedyk” fails to generate any positive alpha once factor exposures are accounted for.

7 Assessing Risk Preferences: A Dual-Criterion Approach

7.1 Setup and Definitions

The results of sections 5 and 6 paint very different pictures of retail investors. Using the method set forth by [Welch, 2022] and [Fedyk, 2024], analyzed also in section 5.2.1, yields superior returns and similar drawdowns to the market. They also conclude that the Robinhood crowd has achieved positive alpha when analyzed under different factor models (table VII, IX, X in [Welch, 2022] and table 16 in [Fedyk, 2024]). These results appear to be in contrast with the existing literature on retail investing, most notably [Barber and Odean, 2000].

A more fundamental question, however, is whether those returns are attractive once investors’ attitudes toward risk are taken into account. This section evaluates the Robinhood portfolio against its market benchmarks using two complementary criteria.

First, I adopt the constant-relative-risk-aversion (CRRA) framework, in line with the majority of asset pricing work.

$$U(W) = \begin{cases} \frac{W^{1-\gamma}-1}{1-\gamma}, \gamma \neq 1 \\ \ln(W), \gamma = 1 \end{cases} \tag{11}$$

By computing the expected utility of both the Robinhood and benchmark portfolios over a grid of possible risk aversion (γ) values, I identify the cutoff γ^* such that a representative CRRA investor is indifferent between the two. This delivers a concise, parametric summary of how risk preferences may shape portfolio choice.

Then, I estimate the welfare loss derived from investing in the Robinhood portfolio rather than (i) not investing or (ii) investing in the market portfolio. By comparing the certainty-equivalent wealth under each strategy for a CRRA investor, with risk aversion calibrated inside a feasible range, I obtain dollar-denominated welfare losses that capture the risk-return profile taken by Robinhood investors.

To account for the uncertainty deriving from the limited sample size and high variance we employ also another approach to estimate risk-aversion. We directly estimate the risk aversion γ from the percentage of wealth invested in the risky asset (α), precise numbers about the wealth of Robinhood investors are not available, we therefore plot the implied risk aversion over a grid α and compare it to alternative portfolios. It must be explicitly stated that in all sections, except this last, it is assumed that investors are entirely invested in the risky asset.

7.2 Expected Utility and Cutoff

Identifying a cutoff level of risk aversion provides a clear criterion for the CRRA utility-maximizing investor: it is the minimum γ at which an alternative strategy becomes preferred to the Robinhood portfolio. However, the limited size and noise of the sample imply wide conference intervals for the estimated expected utilities at different γ levels. In practice, this remains a useful conceptual framework to understand how risk preference and beliefs may affect portfolio choice, but in limited samples, its numerical outputs are more illustrative than definitive.

Formally, we want to find γ^* defined as:

$$\gamma^* = \min \{\gamma_j : \mathbb{E}[U_p(\gamma_j)] \leq \mathbb{E}[U_m(\gamma_j)]\} \tag{12}$$

where $U_p(\cdot)$ is the utility of the Robinhood portfolio, while $U_m(\cdot)$ is the utility of the market portfolio.

In practice, we apply CRRA utility function (11) to each gross return observation and then takes the sample average. The resulting mean serves as the expected utility implied by the investor's revealed choices over the sample period. Wealth at time t is equal to gross returns, assuming the initial wealth W to be equal to one without loss of generality.

The main problem related to this approach is inherent to the sample we apply it to, having only 539 observations and inclusion of extreme events such as the COVID crash. These factors inflate the sample variability of our mean-utility estimates. Therefore, the resulting confidence intervals for the sample mean remain wide.

We don't pretend to draw absolute conclusions but a few noteworthy remarks must be stated. We compare the two Robinhood portfolios with the S&P 500 and the World ETF, resmapling returns every n days instead of taking rolling returns to avoid distortions due to autocorrelation. First, the Fedyk approach yields higher γ^* values than my approach across comparable horizons and market proxies. This can be interpreted as a relative perfomance index, telling us that if we accept the approach based directly on prices, then Robinhood investors would need to have lower risk aversion to not be convinced to hold the market.

Remarkably, investors with a risk aversion paramter even slightly lower than one would have a higher utility by investing in the S&P 500 rather than in the "Mine" portfolio. Fedyk's portfolio requires a risk aversion between 1.7 and 2; more plausible values although relatively low to empirical estimates. When comparing the two Robinhood portfolios with the World ETF, the values for γ increase in the 2-4.2 range, higlighting a less attractive performance of this ETF when compared to the Robinhood portfolios. Detailed estimates can be found in table 13.

The key takeaway from comparing the two Robinhood-based portfolios is that Fedyk's construction systematically requires a higher level of risk aversion before the market overtakes it than the "Mine" strategy does. In practical terms, this means that Fedyk's portfolio must be judged by a much more cautious investor before traditional market benchmarks become more attractive. Under Fedyk's construction, only investors with high risk aversion would prefer the S&P 500 (or the World ETF) to the Robinhood-style basket; those with moderate risk aversion could rationally stick with the Robinhood mix.

By contrast, the "Mine" portfolio flips that relationship. Its cutoff γ is much lower, which implies that even investors who are only mildly risk-averse would already find the market to deliver higher expected utility than our version of the Robinhood strategy. In other words, this portfolio construction leaves fewer "justifiable" investors: only the very least risk-averse would stick with it once they properly account for risk preferences.

7.3 Computing Welfare Loss

In this section, we want to estimate the Welfare loss from investing in the Robinhood portfolio rather than in the market or in the risk-free asset. The goal is to give a quantitative measure of the behavioral cost of choosing one strategy over another. In other words, how much would we have to pay a Robinhood investor to make them switch to another portfolio? We perform this analysis comparing the Portfolio formula set forth in this paper and Fedyk's against the world ETF, the S&P 500, and the risk free asset.

To compute the welfare loss we first need to derive the certainty equivalent for the

CRRA function. Starting from the definition of certainty equivalent and 11:

$$\begin{aligned} U(CE) &= \mathbb{E}[U(W)] \\ CE &= \left[\mathbb{E}(W^{1-\gamma})\right]^{\frac{1}{1-\gamma}} \end{aligned} \tag{13}$$

Then we define the welfare loss for a certain risk aversion parameter γ as:

$$WL_\gamma = CE_{p,\gamma} - CE_{b,\gamma} \tag{14}$$

Where $CE_{p,\gamma}$ and $CE_{b,\gamma}$ are respectively the certainty equivalent of the portfolio and the base strategy to which we compare it, both evaluated at a certain γ. In practice, the welfare loss measures the net risk-free return adjustment required to make a CRRA investor with parameter γ indifferent between the base strategy b and the risky portfolio p. If $\mathrm{WL} * \gamma > 0$, then $\mathrm{CE} * p, \gamma > \mathrm{CE} * b, \gamma$ and the investor would be willing to forgo up to $\mathrm{WL} * \gamma$ of guaranteed return to switch from b into p. Conversely, if $\mathrm{WL} * \gamma < 0$, then $\mathrm{CE} * p, \gamma < \mathrm{CE} * b, \gamma$ and the investor would demand an extra $|\mathrm{WL} * \gamma|$ of risk-free return—on top of $\mathrm{CE\_}b, \gamma$—to be indifferent between holding b and switching into p.

Welfare Loss Compared to the Risk-Free Asset When we benchmark each strategy against the the risk-free asset, every risky portfolio must bear a positive welfare loss for sufficiently high risk aversion. As can be seen in figure 8, the S&P 500 suffers the smallest penalty, the green curve stays closest to the zero line, implying that even moderately risk-averse investors would demand only a small payment to forsake cash for the index. The World ETF lies slightly below, so it too outperforms the Robinhood baskets but at a marginally higher cost in utility relative to the risk-free benchmark.

Both the ”Mine” and Fedyk portfolios incur substantially larger losses. At very short horizons, all strategies look similar, the noise of one-day returns dominates, but as we move to 30- and 60-day periods the cost of volatility in the Robinhood portfolios grows sharply. Mine has the steepest utility price, particularly for investors with γ above one or two. Fedyk lies between the market indices and my version of the Robinhood mix.

In practical terms, because the welfare-loss function WL_γ for VT or the S&P 500 is relatively flat as γ increases—so that investors need only a few basis points of extra return to switch into cash. The behavioral cost of holding those mainstream ETFs remains small over all plausible γ. In contrast, the welfare-loss curves for the Robinhood portfolios are much steeper, meaning that even modest increases in γ translate into substantially higher $|\mathrm{WL}_\gamma|$ (i.e., larger compensation demanded to leave cash). This ordering confirms that, although both Robinhood strategies can outperform in raw returns, their behavioral cost relative to cash is significantly higher than that of mainstream market ETFs. Notably, we can highlight the fact that the behavioral cost for investing in Fedyk's portfolio is lower for a wide range of γ.

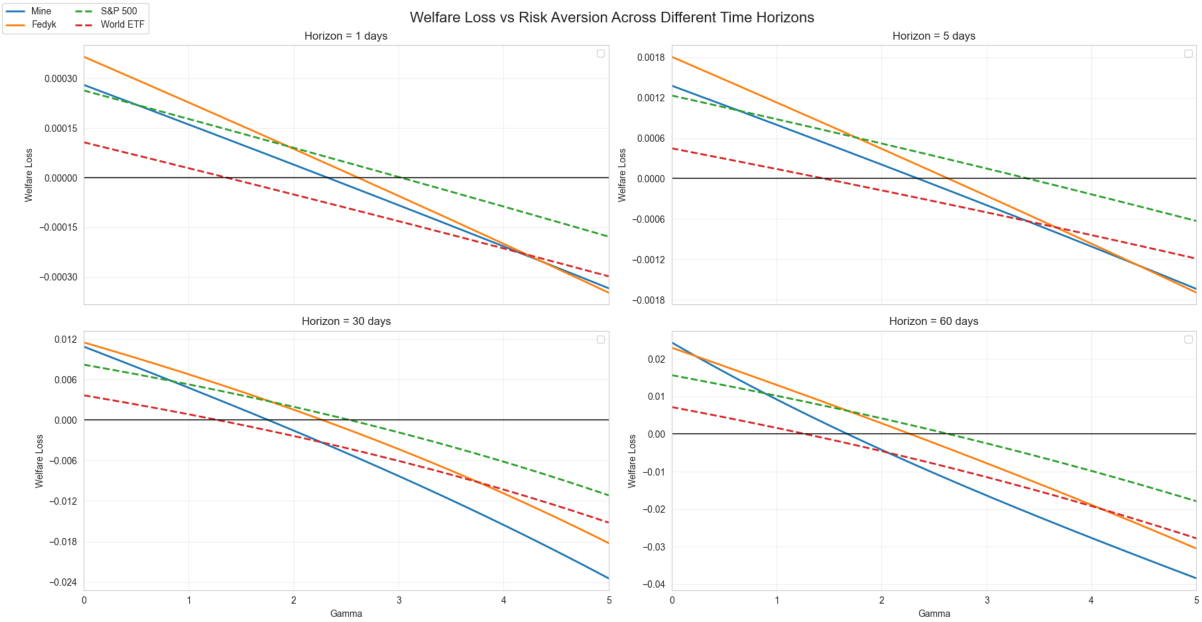

Figure 8: Welfare loss relative to a risk-free asset for four strategies plotted as a function of the CRRA risk-aversion coefficient across 1-, 5-, 30-, and 60-day return horizons.

Welfare Loss Compared to Market Indeces When we measure each Robinhood proxy against the two broad market benchmarks, striking differences emerge in how quickly risk-aversion erodes their advantage. At $\gamma = 0$ (risk neutrality), all four curves start above zero, indicating that a completely risk-neutral investor would prefer any of the Robinhood strategies to either the World ETF or the S&P 500. Figure 9 confirms what we've jsut described. First, "Mine" has a higher behavioral cost compared to Fedyk's construction. Secondly, taking the World ETF as a proxy for the market instead od the S&P 500 implies even greater behavioral costs. Lastly, as the horizon increases the value of γ for which the welfare loss is zero decreases, implying that the certainty equivalent of the Robinhood Portfolio, however computed, falls short of that of the market very quickly and for a wider array of investors.

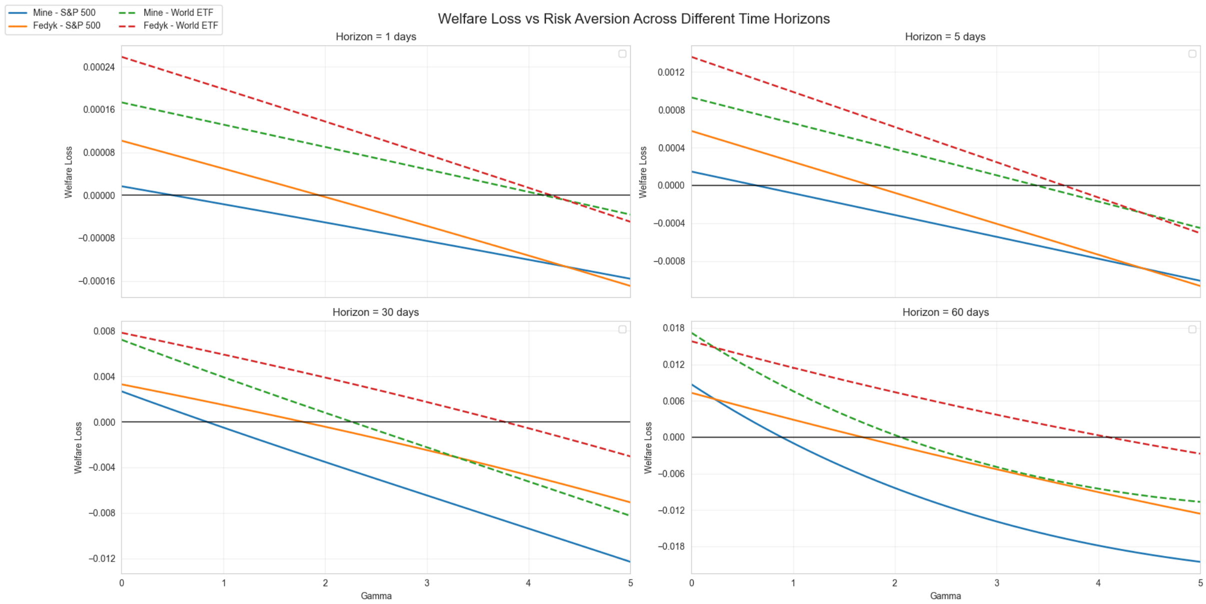

Figure 9: Welfare loss of "Mine" and Fedyk Robinhood portfolios relative to the World ETF and S&P 500 plotted as a function of the CRRA risk-aversion coefficient across 1-, 5-, 30-, and 60-day return horizons.

7.4 Implied Risk Aversion Approach

7.4.1 Deriving the Condition

To address the limitations of our cutoff γ approach, we turn to the canonical condition of maximizing expected utility by changing the share of wealth invested in the risky asset. In this framework, the agent decides to invest an amount α in the risky asset, his final wealth is therefore:

$$W_1 = (1-\alpha)W_0 R_f + \alpha W_0 \tilde{R} \tag{15}$$

where W_0 is the initial wealth, which we can assume w.l.o.g equal to 1, R_f is the gross return of the risk free asset and $\tilde{R}$ is the random variable that expresses the returns in the risky asset.

Defining $r = \tilde{R} - R_f$ we can approximate the CRRA utility of 15 with a second-order taylor expansion around R_f:

$$\mathbb{E}[U(W_1)] \approx R_f^{-\gamma}[\alpha \mathbb{E}[r] - \frac{\gamma}{2}\alpha^2 \mathbb{E}[r^2]] \tag{16}$$

The investor chooses its portfolio to maximize the expected utility:

$$\max_{0\leq\alpha\leq 1} \mathbb{E}[U(W_1)] \approx \max_{0\leq\alpha\leq 1} R_f^{-\gamma}\Big[\alpha\,\mathbb{E}[r] - \tfrac{\gamma}{2}\,\alpha^2\,\mathbb{E}[r^2]\Big] \tag{17}$$

which yields the following equation[7]:

$$\alpha^* = \frac{\mathbb{E}[r]}{\gamma\,\mathbb{E}[r^2]} \tag{18}$$

where $\mathbb{E}[r] = \mu - R_f$ and $\mathbb{E}[r^2] = \sigma^2 + (\mu - R_f)^2$. Since returns are small, $(\mu - R_f)^2 \ll \sigma^2$. We derive the following:

$$\gamma^* = \frac{\mu - R_f}{\alpha\sigma^2} \tag{19}$$

In our case, both α and γ are unknown. The variable of interest is γ, we can therefore use 19 to compute it for every α in $[0, 1]$.

7.4.2 Empirical Estimates and Interpretation

The results in this section might diverge slightly with the approach proposed above in section 7.2 for a couple of reasons. First, here we limit investors preferences to the second moment, assuming implictly that higher moments do not influence investor's utility. Secondly, the limited sample size, especially at longer horizons, limits the precision of the proposed estimates. Moreover, in section 7.2 we implicitly assumed investors to hold all their wealth either in the Robinhood portfolio or in the market i.e., using this section's notation, to have $\alpha = 1$. Notwithstanding these challenges, some general conclusions can be drawn.

A relevant point that should be discussed is the choice of not using rolling returns. Although they provide a good proxy for assessing the typical return over different horizons and across secruties, these return series suffer from autocorellation by construction, implying a very small variance especially at longer horizons. Therefore, the risk aversion estimates provided by 19 would be unreliable and orders of magnitude greater than acceptable values in asset pricing. For this reason, we focus or attention on daily, weekly, monthly and bi-monthly returns starting from the first day in the sample.

In table 14 we report the implied share invested in the risky asset given a feasible upper and lower bound for investor's risk aversion, namely $\gamma = 2$ and $\gamma = 5$. The full curves are plotted below in Figure 10. Something we should note is that since we have limited α to be in $[0, 1]$ and $\sigma^2 \geq 0$, the implied risk aversion can be negative only if the mean excess returns are negative. across all horizons. As expected, α falls in a hyperbolic fashion, since 19 implies $\alpha \propto \frac{1}{\gamma}$.

Across all four holding-period horizons and securities, lower risk aversion implies substantially higher optimal equity weights (α^*) tha higher risk aversion. Comparing across strategies, the S&P 500 series has the largest implied shares at $\gamma = 2$, rising from 0.766 at one day to 0.890 at five days before settling at 0.708 by sixty days, while

[7]Derivation: FOC: $\frac{\partial}{\partial\alpha}\Big\{R_f^{-\gamma}[\alpha\,\mathbb{E}[r] - \frac{\gamma}{2}\alpha^2\mathbb{E}[r^2]]\Big\} = R_f^{-\gamma}\big(\mathbb{E}[r] - \gamma\,\alpha\,\mathbb{E}[r^2]\big) \ = \ 0$

its $\gamma = 5$ allocations remain above both Robinhood strategies throughout. The World ETF's position at the bottom of the table highlights its lower excess-return profile: even at the longest horizon its $\gamma = 2$ weight of 0.328 barely matches the $\gamma = 5$ allocation to the Robinhood portfolios. Between the two Robinhood constructions, Fedyk is uniformly more aggressive than Mine, its $\gamma = 2$ allocations exceed Mine's by roughly 10-25 percentage points at every horizon, reflecting Fedyk's higher expected excess return per unit variance.

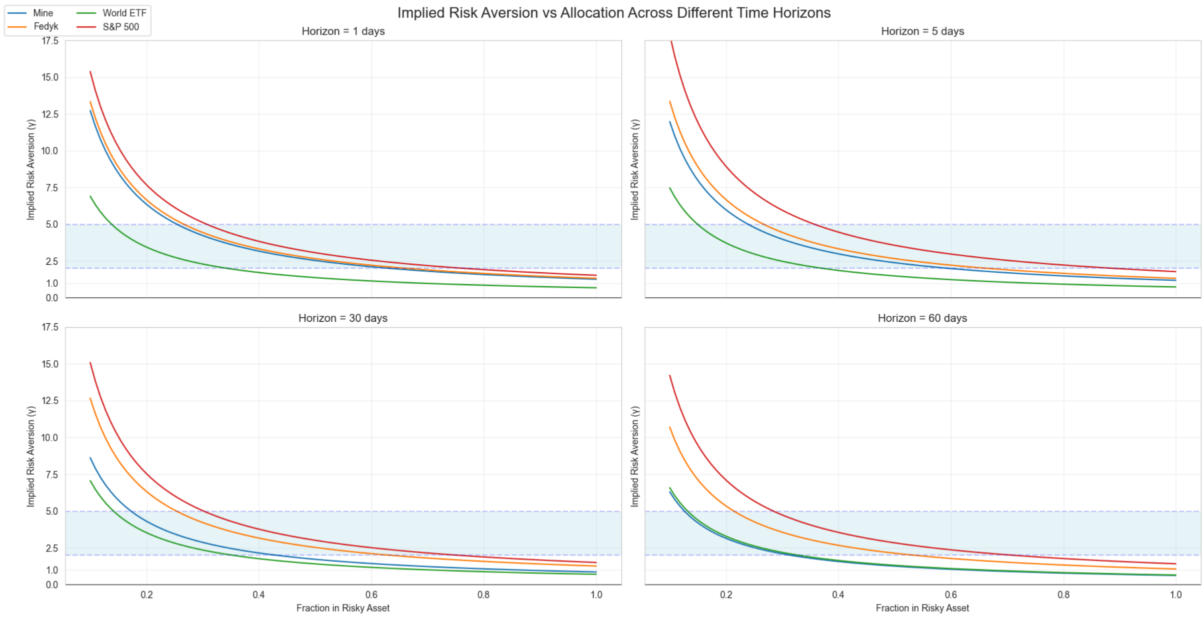

Figure 10: Implied CRRA risk aversion parameter as a function of allocation to each strategy (Mine, Fedyk, World ETF, S&P 500) across 1-, 5-, 30-, and 60-day horizons. Shaded bands mark the typical empirical range of $\gamma \in [2, 5]$.

These somewhat counterintuitive results can be interpreted by noticing that the condition we've set (19) is simply a modified version of the Sharpe ratio. It is rational for investors in this framework to allocate a greater portion of their wealth if the risky asset promises higher risk-adjusted returns. The results are in table 16.

7.5 Comparing Results

The cutoff γ estimates reported in Section 7.2 are associated with wide confidence intervals due to finite sample noise and extreme return realizations, and the implied γ values from Section 7.4.2 rely on a second-order Taylor approximation that ignores higher moments, so all inferences should be interpreted with appropriate caution.

To assess whether a fully invested CRRA investor would rationally hold the Robinhood portfolios, we compare the implied γ at $\alpha = 1$ (Table 15) to the corresponding cutoff γ^*

against the market (Table 14) for each horizon. If a portfolio's implied $\gamma(\alpha = 1)$ lies below its cutoff γ^*, then an investor with that level of risk aversion would prefer the Robinhood portfolio to the S&P 500 in expected-utility terms. This analysis is necessary since when computing the cutoff value for risk aversion we implicitly assumed investors to allocate all their resources to the risky asset.

For Fedyk, the implied γ values at $\alpha = 1$ remain below the cutoff γ^* at all horizons, indicating that an investor with risk aversion equal to the implied value would indeed rationally favor Fedyk over the S&P 500. By contrast, ”Mine” only exhibits implied $\gamma(\alpha = 1)$ below the cutoff γ^* at the sixty-day horizon, implying that shorter horizons would render a fully invested position in ”Mine” inconsistent with CRRA-rational behavior against the S&P 500. The World ETF's implied $\gamma(\alpha = 1)$ also falls below the cutoff γ^* for each horizon, but those cutoff levels exceed typical estimates of retail risk aversion, confirming that cash or a higher-certainty-equivalent benchmark would be preferred by most CRRA investors. These comparisons suggest that, under the benchmark of cutoff γ, a fully invested Fedyk position can be CRRA-justified only for investors with unusually low risk aversion, while ”Mine” fails to meet that criterion at shorter horizons.

Therefore, although the implied and cutoff measures are derived from different methodologies, their intersection highlights that full allocation to certain Robinhood portfolios cannot be reconciled with any plausible γ above the cutoff. Overall, this analysis reinforces the conclusion that retail investors' observed full allocations to these portfolios imply risk aversion levels below the cutoff thresholds, underscoring a persistent behavioral gap under standard CRRA assumptions.

8 Stochastic Dominance Analysis and Behavioral Implications

In this final empirical exercise, we compare the full return distributions of the Robinhood portfolio and the market benchmark using nonparametric stochastic dominance tests. A second-order stochastic dominance (SSD) result in favor of the market would imply that *every* risk-averse investor (including but not restricted to all CRRA preferences with $\gamma > 0$) strictly prefers the market's payoff to that of the Robinhood strategy.

8.1 Methodology

We first estimate the empirical cumulative distribution functions (CDFs) of daily returns for the two portfolios. Let

$$F_p(r) \;=\; \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}\{r_{p,t} \le r\}, \quad F_m(r) \;=\; \frac{1}{T}\sum_{t=1}^{T} \mathbf{1}\{r_{m,t} \le r\}, \tag{20}$$

where $r_{p,t}$ and $r_{m,t}$ are the portfolio and market returns on day t.

To analyse whether the portfolio is preferred over the market we employ two different tests. First-order stochastic dominance (FSD) holds if $F_p(r) \le F_m(r)$ for all r; second-order stochastic dominance (SSD) holds if

$$G_p(r) \;\equiv\; \int_{-\infty}^{r} F_p(x)\,dx \quad \text{and} \quad G_m(r) \;\equiv\; \int_{-\infty}^{r} F_m(x)\,dx \tag{21}$$

satisfy $G_p(r) \le G_m(r)$ for all r.

8.2 Results

8.2.1 Rolling-Horizon SSD

Comparing Robinhood Portfolios Across non-overlapping horizons of 1, 5, 30, and 120 trading days—as well as the full 563-day sample—we compute for each return series the empirical CDFs $F_{\text{Mine}}(r)$ and $F_{\text{Fedyk}}(r)$, their integrals $G_{\text{Mine}}(r)$ and $G_{\text{Fedyk}}(r)$, and the fraction of evaluation points satisfying

$$G_{\text{Fedyk}}(r) \;\le\; G_{\text{Mine}}(r), \tag{22}$$

which indicates Fedyk's second-order stochastic dominance over the portfolio I built. This support fraction falls monotonically with the horizon: it exceeds 98 % at the 1- and 5-day horizons, drops below 1 % at 30 days, only rebounds to about 6 % at 120 days, and reaches 0 % over the full period. These findings show that Fedyk's SSD advantage becomes increasingly stronger at longer horizons, making it preferred by every risk-averse investor. This relation even becomes a first-order stochastic dominance when accounting for cumulative returns, as can be easily seen from plot.

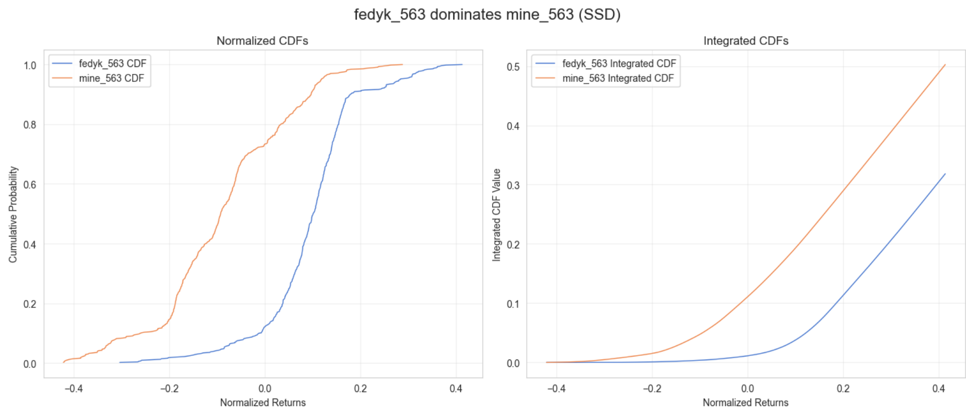

Figure 11: FSD Between the Robinhood Portfolios.

Comparing Robinhood to Broad-Market Proxies We use the Fedyk portfolio as a conservative benchmark for comparison. Its demonstrated dominance over our custom Robinhood strategy across horizons makes it an especially stringent benchmark: if our portfolio cannot overcome Fedyk's advantage, it would be even harder to outperform true market indices at longer holding periods.

Having established that the Fedyk portfolio second-order stochastically dominates our Robinhood strategy as the length of the horizon increases, we now benchmark against three widely-used market proxies: the S&P 500 ETF (VOO), the global all-world ETF (VT), and the Fama French market index (results for which are nearly identical to VOO).

At the 1- and 5-day horizons, all three market indices exhibit overwhelming SSD dominance over our Robinhood portfolio, each with over 97% of evaluation points satisfying

$$G_{\text{proxy}}(r) \leq G_{\text{RH}}(r). \tag{23}$$

By the 30-day horizon, this support fraction declines to approximately 92% for the S&P 500 and 90% for the all-world ETF, while the Fama French index mirrors the S&P 500's trajectory. Extending to 120 days, the S&P 500 retains complete dominance (100% of points), whereas the all-world ETF's dominance share falls to about 83%. Over the full 563-day sample, the S&P 500 continues to second-order dominate Robinhood, while the all-world ETF shows only weak support ($\approx$ 8%). Only the S&P 500 (VOO) and the Fama French market index preserve dominance in the long-run payoff distribution.

In other words, whether we compare the portfolio I built against the Fedyk portfolio or traditional market indices, the second-order stochastic dominance over our Robinhood strategy is nearly universal at intraday and weekly horizons. Our approach to building the Robinhood portfolio, fails SSD also against the World ETF at longer horizons.

8.2.2 A Proof of Irrationality?

Our analysis reveals a fundamental contradiction in Robinhood investors' revealed preferences. Building on the risk aversion estimates from Section 7.4.2 (Table 14), where we found strictly positive γ values in 95% confidence intervals for daily and weekly returns, we now confront these findings with stochastic dominance tests. This creates an irreconcilable tension: while Robinhood investors exhibit risk-averse preferences through positive γ estimates, their chosen portfolios fail to satisfy basic rationality requirements when compared to market alternatives.

The contradiction emerges most sharply at weekly horizons. Despite estimated risk aversion parameters ($\hat{\gamma} > 0$) that should favor market indices under second-order stochastic dominance (SSD), we find that the Fama French market index demonstrates unambiguous SSD over both Robinhood portfolios, and S&P 500 and world ETF (VT) achieve SSD over 95-99% of evaluation points. This implies that even the most conservative specification of risk-averse preferences would favor market indices over Robinhood strategies, directly contradicting the investors' actual portfolio choices.

The tension persists across all horizons: Daily returns show 95-98% SSD support for market dominance, monthly returns reveal comprehensive market dominance (90.7-100% SSD support), and only a small fraction of monthly $\hat{\gamma}$ estimates dip below zero.

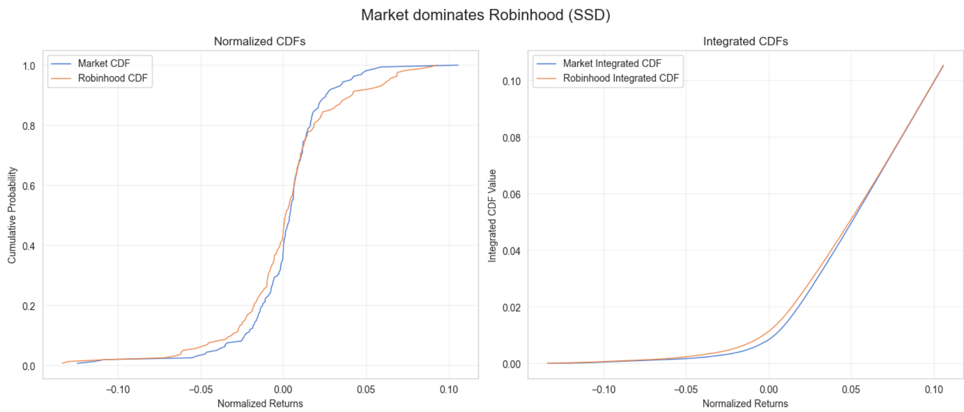

Figure 12: SSD of the Market over the Robinhood Portfolio.

These results present an inescapable conclusion: Robinhood investors' portfolio choices cannot be rationalized through standard risk-return tradeoffs. The combination of (1) positive estimated risk aversion and (2) market dominance across SSD tests creates a paradox that only resolves if we reject the premise of fully rational investors. Even under generous assumptions about return aggregation, the evidence consistently challenges the rationality hypothesis.

Our findings suggest that non-standard preferences or behavioral factors, such as lottery-seeking behavior, attention-driven trading, or misunderstanding of statistical dominance, must drive these investment decisions. The persistence of this contradiction across multiple methodologies (CRRA estimation, SSD tests, and temporal subsamples) makes it particularly robust to alternative explanations.

8.3 Implications for Investor Behavior

Our nonparametric stochastic-dominance tests deliver a clear verdict: across multiple return horizons and market proxies, the broad market consistently second-order stochastically dominates the Robinhood strategy for short-term horizons, and at monthly and longer horizons the S&P 500 (VOO) in particular preserves that dominance where other proxies fade. Because SSD implies preference by every risk-averse investor (all CRRA utilities with $\gamma > 0$), these results extend and reinforce our earlier finding—based on risk aversion estimation—that retail investors exhibit positive risk aversion yet hold a portfolio that no rational, CRRA-maximizer would choose.

Taken together, the risk aversion implied by α and stochastic-dominance evidence point to a fundamental disconnect between normative decision rules and observed retail behavior. Even though our bootstrap-corrected GMM estimates uncover strictly positive—and economically plausible—risk-aversion parameters at daily and weekly frequencies, the SSD tests show that any such risk-averse investor would uniformly prefer the market portfolio. The persistence of sub-optimal, under-diversified strategies therefore cannot be attributed to risk-aversion alone.

Instead, these findings call for behavioral explanations: investors may succumb to overconfidence in their timing ability, confirmation bias in interpreting past returns, or an illusion of control over individual trades. Such biases can sustain a self-confirming equilibrium, in which retail traders overweight episodes of success and dismiss losses as noise, reinforcing the belief that their custom portfolio offers an edge—even when the broad market outperforms on every risk-averse criterion. By combining both approaches, we provide robust evidence that, despite genuine risk aversion, retail investors systematically deviate from the optimal market portfolio—highlighting the crucial role of behavioral frictions in real-world decision-making.

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Appendix A Tables

Table 1: Descriptive Statistics for Daily and Rolling Returns

Note: Returns accounting only for investments in Stocks.

	count	mean	std	min	25%	50%	75%	max
Fedyk daily return	563	0.000990	0.019894	-0.135995	-0.005807	0.001516	0.009167	0.105441
Mine daily return	563	0.000677	0.019209	-0.125590	-0.006681	0.001363	0.009784	0.071628
VT daily return	563	0.000185	0.015106	-0.123763	-0.004592	0.000886	0.005933	0.087470
VOO daily return	563	0.000443	0.015820	-0.124870	-0.003893	0.000976	0.006653	0.091087
Fedyk 5 return	559	0.004866	0.046404	-0.265341	-0.015031	0.007872	0.025282	0.193965
Mine 5 return	559	0.002979	0.043110	-0.226991	-0.013770	0.003621	0.021740	0.159707
VT 5 return	559	0.000824	0.030805	-0.214262	-0.011045	0.003956	0.014927	0.151788
VOO 5 return	559	0.002111	0.031063	-0.204425	-0.009442	0.005716	0.016506	0.162820
Fedyk 30 return	534	0.025268	0.114955	-0.431533	-0.032448	0.035874	0.072017	0.401292
Mine 30 return	534	0.012159	0.140314	-0.474366	-0.051104	0.009054	0.054639	0.431367
VT 30 return	534	0.002735	0.081324	-0.406688	-0.024941	0.017664	0.041113	0.224464
VOO 30 return	534	0.009843	0.079924	-0.401950	-0.015450	0.027341	0.046977	0.252864
Fedyk 120 return	444	0.050804	0.114293	-0.330870	-0.021276	0.046242	0.130711	0.357556
Mine 120 return	444	-0.059004	0.127178	-0.358205	-0.139490	-0.074974	-0.015135	0.439583
VT 120 return	444	-0.012628	0.085645	-0.333294	-0.070621	-0.012804	0.059034	0.186378
VOO 120 return	444	0.013786	0.079450	-0.302877	-0.037033	0.011899	0.073431	0.231377
Fedyk 563 return	563	0.168894	0.129435	-0.148334	0.100301	0.146221	0.199060	0.565657
Mine 563 return	563	-0.038567	0.143947	-0.468443	-0.114762	-0.027966	0.041386	0.380918
VT 563 return	563	0.002293	0.063753	-0.300675	-0.023998	0.014837	0.032750	0.122378
VOO 563 return	563	0.096803	0.071714	-0.166225	0.054575	0.093080	0.136742	0.253868

Table 2: Descriptive Statistics for Daily and Rolling Returns

Note: Returns accounting for investments in all types of securities. ETF returns are identical to table 1.

	count	mean	std	min	25%	50%	75%	max
Fedyk daily returns	563	0.000555	0.020000	-0.125369	-0.006658	0.000754	0.008755	0.097995
Mine daily returns	563	0.000450	0.018710	-0.126475	-0.006472	0.000606	0.009044	0.072808
VT daily return	563	0.000185	0.015106	-0.123763	-0.004592	0.000886	0.005933	0.087470
VOO daily return	563	0.000443	0.015820	-0.124870	-0.003893	0.000976	0.006653	0.091087
Fedyk 5 return	559	0.002684	0.047496	-0.287100	-0.017592	0.005106	0.022556	0.201768
Mine 5 return	559	0.001891	0.041707	-0.224346	-0.015152	0.002165	0.020423	0.145245
VT 5 return	559	0.000824	0.030805	-0.214262	-0.011045	0.003956	0.014927	0.151788
VOO 5 return	559	0.002111	0.031063	-0.204425	-0.009442	0.005716	0.016506	0.162820
Fedyk 30 return	534	0.012231	0.118341	-0.491574	-0.044795	0.018809	0.055543	0.391200
Mine 30 return	534	0.005924	0.134409	-0.456420	-0.056865	0.006054	0.044482	0.399350
VT 30 return	534	0.002735	0.081324	-0.406688	-0.024941	0.017664	0.041113	0.224464
VOO 30 return	534	0.009843	0.079924	-0.401950	-0.015450	0.027341	0.046977	0.252864
Fedyk 120 return	444	-0.004432	0.114878	-0.478685	-0.062876	-0.010586	0.076930	0.271451
Mine 120 return	444	-0.076568	0.124668	-0.368195	-0.159984	-0.095393	-0.037126	0.406700
VT 120 return	444	-0.012628	0.085645	-0.333294	-0.070621	-0.012804	0.059034	0.186378
VOO 120 return	444	0.013786	0.079450	-0.302877	-0.037033	0.011899	0.073431	0.231377
Fedyk 563 return	563	0.086945	0.096844	-0.361597	0.050499	0.096675	0.134919	0.345822
Mine 563 return	563	-0.103880	0.152561	-0.547158	-0.193259	-0.096858	0.008830	0.253266
VT 563 return	563	0.002293	0.063753	-0.300675	-0.023998	0.014837	0.032750	0.122378
VOO 563 return	563	0.096803	0.071714	-0.166225	0.054575	0.093080	0.136742	0.253868

Table 3: Descriptive Statistics for Daily Returns, up to February 3rd 2020.

Note: Returns accounting for investments in all types of securities.

	count	mean	std	min	25%	50%	75%	max
Mine	429	-0.000328	0.013581	-0.049700	-0.006121	-0.000106	0.007070	0.065745
Fedyk	429	0.000246	0.012218	-0.047821	-0.005735	0.000549	0.007639	0.052644
VT	429	0.000200	0.008370	-0.031068	-0.003812	0.000766	0.004879	0.036545
VOO	429	0.000492	0.008938	-0.032828	-0.003072	0.000783	0.005103	0.049350

Table 4: Results for One-Way ANOVA Tests on Daily Returns

These results cover the whole period.

subset	F Statistic	p-value
(Fedyk, VT)	0.123106	0.725755
(VOO, VT)	0.078475	0.779426
(Mine, VT)	0.068498	0.793584
(Fedyk, VOO)	0.010935	0.916736
(Fedyk, Mine)	0.008335	0.927275
(Fedyk, VOO, VT)	0.069435	0.932923
(Fedyk, Mine, VT)	0.062966	0.938977
(Mine, VOO, VT)	0.046579	0.954490
(Fedyk, Mine, VOO, VT)	0.045550	0.987099
(Fedyk, Mine, VOO)	0.006693	0.993330
(Mine, VOO)	0.000046	0.994589

Table 5: Results for One-Way ANOVA Tests on Daily Returns, up to February 3rd 2020

subset	F Statistic	p-value
(Mine, VOO)	0.861444	0.353597
(Fedyk, Mine)	0.398907	0.527823
(Mine, VT)	0.321035	0.571135
(Mine, VOO, VT)	0.517251	0.596281
(VOO, VT)	0.243873	0.621550
(Fedyk, Mine, VOO)	0.451564	0.636733
(Fedyk, Mine, VT)	0.273712	0.760596
(Fedyk, Mine, VOO, VT)	0.343216	0.794081
(Fedyk, VOO)	0.055235	0.814249
(Fedyk, VT)	0.028370	0.866282
(Fedyk, VOO, VT)	0.092718	0.911456

Table 6: Results for Fligner Tests on Daily Returns

Results cover the whole period. Omitted subsets have p-value≈ 0.

subset	F Statistic	p-value
(Fedyk, Mine)	0.031996	0.858037
(VOO, VT)	0.003157	0.955192

Table 7: Results for Fligner Tests on Daily Returns, up to February 3rd 2020
Omitted subsets have p-value≈ 0.

subset	**F Statistic**	**p-value**
(Fedyk, Mine)	0.576961	0.447506
(VOO, VT)	0.002907	0.957003

Table 8: Regression results for the "Mine" portfolio using only stocks.
Note: standard errors in parentheses.

	CAPM	3 Factor	4 Factor	6 Factor
intercept	0.0002	-0.0001	-0.0002	-0.0001
	(0.0004)	(0.0004)	(0.0004)	(0.0003)
market	1.0593***	1.1082***	1.1099***	1.0252***
	(0.0427)	(0.0388)	(0.0368)	(0.0360)
hml		-0.4688***	-0.5223***	-0.2654***
		(0.0548)	(0.0662)	(0.0638)
smb		0.2215***	0.1997***	0.0498
		(0.0604)	(0.0647)	(0.0592)
umd			-0.0649	-0.0395
			(0.0489)	(0.0498)
rmw				0.1509
				(0.1011)
cma				-1.0965***
				(0.1315)
R-squared	0.7813	0.8337	0.8344	0.8631
R-squared Adj.	0.7809	0.8327	0.8331	0.8616
N	539	539	539	539

Standard errors in parentheses: * $p<.1$, ** $p<.05$, ***$p<.01$

Table 9: Regression results for the ”Mine” portfolio the full universe of securities.
Note: standard errors in parentheses.

	CAPM	3 Factor	4 Factor	6 Factor
intercept	-0.0001	-0.0003	-0.0004	-0.0003
	(0.0004)	(0.0003)	(0.0003)	(0.0003)
market	1.0447***	1.0847***	1.0870***	1.0103***
	(0.0380)	(0.0346)	(0.0322)	(0.0318)
hml		-0.4165***	-0.4859***	-0.2488***
		(0.0500)	(0.0602)	(0.0591)
smb		0.2292***	0.2009***	0.0580
		(0.0535)	(0.0582)	(0.0533)
umd			-0.0842*	-0.0670
			(0.0444)	(0.0442)
rmw				0.0931
				(0.0912)
cma				-1.0018***
				(0.1253)
R-squared	0.8057	0.8492	0.8505	0.8760
R-squared Adj.	0.8053	0.8484	0.8494	0.8746
N	539	539	539	539

Standard errors in parentheses: * p$<$.1, ** p$<$.05, ***p$<$.01

Table 10: Regression results for the ”Fedyk” portfolio the full universe of securities.
Note: standard errors in parentheses.

	CAPM	3 Factor	4 Factor	6 Factor
intercept	0.0000	0.0004	0.0002	0.0003
	(0.0004)	(0.0003)	(0.0003)	(0.0003)
market	1.1111***	1.0260***	1.0346***	0.9923***
	(0.0485)	(0.0358)	(0.0298)	(0.0352)
hml		0.0895	-0.1759**	-0.0076
		(0.0566)	(0.0715)	(0.0789)
smb		0.6928***	0.5845***	0.4463***
		(0.0556)	(0.0858)	(0.0648)
umd			-0.3222***	-0.3611***
			(0.0786)	(0.0823)
rmw				-0.3092***
				(0.1101)
cma				-0.6263***
				(0.1394)
R-squared	0.7899	0.8671	0.8829	0.8968
R-squared Adj.	0.7895	0.8663	0.8820	0.8956
N	539	539	539	539

Standard errors in parentheses: * $p<.1$, ** $p<.05$, *** $p<.01$

Table 11: Regression results for the ”Mine” portfolio the full universe of securities and excluding COVID.

Note: standard errors in parentheses.

	CAPM	3 Factor	4 Factor	6 Factor
intercept	-0.0010***	-0.0011***	-0.0011***	-0.0010***
	(0.0003)	(0.0002)	(0.0002)	(0.0002)
market	1.3287***	1.2400***	1.2325***	1.1063***
	(0.0306)	(0.0269)	(0.0293)	(0.0251)
hml		-0.4596***	-0.4968***	-0.2195***
		(0.0524)	(0.0640)	(0.0530)
smb		0.0720	0.0493	0.0092
		(0.0533)	(0.0617)	(0.0567)
umd			-0.0465	-0.0456
			(0.0530)	(0.0436)
rmw				0.0608
				(0.0749)
cma				-0.9507***
				(0.1027)
R-squared	0.8216	0.8619	0.8622	0.8914
R-squared Adj.	0.8212	0.8609	0.8608	0.8897
N	405	405	405	405

Standard errors in parentheses: * $p<.1$, ** $p<.05$, *** $p<.01$

Table 12: Regression results for the "Fedyk" portfolio the full universe of securities and excluding COVID.
Note: standard errors in parentheses.

	CAPM	3 Factor	4 Factor	6 Factor
intercept	-0.0004	-0.0002	-0.0002	-0.0002
	(0.0003)	(0.0002)	(0.0002)	(0.0002)
market	1.1957***	1.1199***	1.0832***	1.0381***
	(0.0304)	(0.0248)	(0.0247)	(0.0257)
hml		-0.1575***	-0.3413***	-0.2479***
		(0.0468)	(0.0538)	(0.0534)
smb		0.4918***	0.3797***	0.3272***
		(0.0526)	(0.0549)	(0.0527)
umd			-0.2295***	-0.2642***
			(0.0471)	(0.0460)
rmw				-0.2797***
				(0.0681)
cma				-0.3126***
				(0.0875)
R-squared	0.8040	0.8459	0.8554	0.8660
R-squared Adj.	0.8035	0.8448	0.8539	0.8640
N	405	405	405	405

Standard errors in parentheses: * $p<.1$, ** $p<.05$, *** $p<.01$

Table 13: Cutoff γ^* for the two Robinhood Portfolio Proxies and market Indeces.

Horizon	**Fedyk - S&P 500**	**Mine - S&P 500**	**Fedyk - World ETF**	**Mine - World ETF**
1 Days	1.940	0.503	4.221	4.147
5 Days	1.759	0.637	3.658	3.387
30 Days	1.782	0.839	3.762	2.262
60 Days	1.682	0.888	4.099	2.057

Table 14: Implied risky-asset share α by holding-period horizon (days) and CRRA coefficient across portfolios

Portfolio	**1 Day**	**5 Days**	**30 Days**	**60 Days**
Fedyk, $\gamma = 2$	0.664	0.665	0.631	0.532
Fedyk, $\gamma = 5$	0.266	0.267	0.253	0.214
Mine, $\gamma = 2$	0.588	0.597	0.429	0.314
Mine, $\gamma = 5$	0.236	0.239	0.172	0.126
S&P 500, $\gamma = 2$	0.766	0.890	0.751	0.707
S&P 500, $\gamma = 5$	0.307	0.357	0.301	0.284
World ETF, $\gamma = 2$	0.344	0.372	0.352	0.373
World ETF, $\gamma = 5$	0.138	0.149	0.141	0.132

Table 15: Implied risk aversion for full allocation in the risky asset by holding-period horizon (days) across portfolios

Portfolio	**1 Day**	**5 Days**	**30 Days**	**60 Days**
Fedyk	1.333	1.336	1.267	1.070
Mine	1.272	1.197	0.862	0.630
S&P 500	1.537	1.788	1.509	1.421
World ETF	0.691	0.747	0.707	0.658

Table 16: Sharpe ratios by holding-period horizon (days) across portfolios

	1 Days	**5 Days**	**30 Days**	**60 Days**
Fedyk	0.027	0.049	0.121	0.157
Mine	0.024	0.041	0.097	0.124
S&P 500	0.025	0.047	0.111	0.149
World ETF	0.010	0.018	0.051	0.069

Appendix B Handling Missing Data

The original Robinhood dataset contains missing values for 3,331 securities, primarily in the earlier periods. This means that these securities don't have information for a certain date.

To ensure consistency we adopt a similar method as [Fedyk, 2024]. Their Robinhood portfolio is constructed using the available securities on a daily basis, hence securities with missing values are simply not taken into account for the day. Moreover we drop all securities that they have defined as problematic in the appendix.

Since our CRSP dataset is also a bit different from the one they use, we drop entirely securities that have more than one entry per day.