# Determination of optimal price for revenue and profit maximization in retail products using generalized linear models

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## OBJECTIVE

The aim of this project was to determine the optimal prices that maximize revenue and profit for several retail products. For this purpose, demand models were obtained by fitting linear models (Ordinary Least Squares and Poisson) that predict demand at different historical prices. The models were then used to obtain revenue and profit functions which were differentiated to obtain the optimal price that maximizes both metrics. Prices were considered for the maximization of both revenue and profit because both can be business objectives depending on the situation (e.g., revenue is more important than profit for initial market penetration and growth).

## METHODOLOGY

### Tools

- R

### Dataset

The price dataset was downloaded from Kaggle. It contains historical monthly data on prices and sold quantities (demand) for several retail products between January and December 2017.

An R script was programmed to automatically extract the most suitable products for the chosen price optimization methodology, obtain the price values and generate plots for each product. The purpose was to automate the script so that it is suitable for any future dataset containing historical price data.

### Demand modelling

First, an exploratory data analysis was conducted, and coefficients of variation were computed for all products to see which ones had the highest price variability across time. Then, 2 types of linear models were used for describing price elasticity of demand: Ordinary Least Squares (OLS) and Poisson. Poisson models are a type of generalized linear model (GLM) that are well suited for count data and use the natural logarithm as the link function. OLS models are also a type of generalized linear models where the residual distribution is gaussian and the link function is the identity function.

### Product selection

The products that had OLS models with a determination coefficient (R2) > 0.50 and a negative Alpha coefficient were selected for the price optimization process. In the case of the Poisson models, the criteria for selection were having negative Alpha and a McFadden Pseudo R-squared value higher than 0.50. In the case were a product had both an OLS and Poisson model that met these requirements, the optimal price from the model with the smallest Akaike

Information Criterion (AIC) was chosen. In the case were the optimal price obtained from the process was less than -90% smaller than the average price, the product was discarded from the results. This happened mostly with GLM models.

The process to obtain optimal prices from the demand models will be explained next.

**Optimal pricing for OLS Models**

The formula for the fitted OLS demand models is as follows:

$$d(p) = \alpha p + \beta$$

Where d(p) is the demand and p the price, with α and β being the regression coefficients.

From this, we can obtain the revenue function by multiplying the demand (sold quantity) by the price as follows:

$$R(p) = p * d(p) = \alpha p^2 + p * \beta$$

And the profit function as:

$$L(p) = (p - c) * d(p) = \alpha p * (p - c) + \beta * (p - c)$$

Where c is the variable cost.

So, to find the price that maximizes both the revenue and profit functions, the derivatives of these functions must be obtained and equated to 0.

For the revenue function the process is as follows:

$$R'(p) = 2\alpha p + \beta$$

$$2\alpha p + \beta = 0$$

$$p = \frac{-\beta}{2\alpha}$$

And for the profit function:

$$L'(p) = 2\alpha p - \alpha c + \beta$$

$$2\alpha p - \alpha c + \beta = 0$$

$$p = \frac{-\beta + \alpha c}{2\alpha}$$

**Optimal pricing for Poisson (GLM) models**

The formula fitted for the Poisson models is as follows:

$$d(p) = e^{\alpha p + \beta}$$

Thus, the revenue function can be obtained as:

$$R(p) = d(p) * p = p * e^{\alpha p + \beta}$$

And the profit function as:

$$L(p) = d(p) * (p - c) = (\mathrm{p} - \mathrm{c}) * p * e^{\alpha p + \beta}$$

As stated above, the optimal prices can be obtained by differentiating the functions and equating them to 0. In the case of the revenue function the process is as follows:

$$R'^{(p)} = e^{\alpha p + \beta}(1 + \alpha p)$$

$$e^{\alpha p + \beta}(1 + \alpha p) = 0$$

$$p = \frac{-1}{\alpha}$$

And for the profit function:

$$L'(p) = e^{\alpha p + \beta}(p + (\mathrm{p} - \mathrm{c}) + \alpha(\mathrm{p} - \mathrm{c}) * \mathrm{p})$$

$$e^{\alpha p + \beta}(p + (\mathrm{p} - \mathrm{c}) + \alpha(\mathrm{p} - \mathrm{c}) * \mathrm{p}) = 0$$

$$p = \frac{c}{2 + \alpha(p - c)}$$

**RESULTS**

**Table 1.** *Optimal prices obtained for the selected products*

| Product | Optimal revenue price | Optimal profit price | Demand model | Average price | Average price vs Optimal revenue price |
|---|---|---|---|---|---|
| Bed 1 | 24.05 | 25.56 | OLS | 42.21 | -43% |
| Bed 4 | 24.26 | 26.76 | OLS | 46.73 | -48% |
| Consoles 1 | 19.65 | 23.65 | OLS | 28.24 | -30% |
| Watches 6 | 19.08 | 29.08 | GLM | 132.53 | -86% |
| Watches 7 | 182.85 | 185.85 | OLS | 305.69 | -40% |
| Watches 8 | 124.20 | 128.70 | OLS | 184.50 | -33% |

The optimal prices for all products consistently exhibit values below the corresponding average prices, implying that the products are currently marketed at prices that reduce their market demand. This observation suggests the possibility that the company has a proclivity for overpricing its products. Therefore, a strategic reduction in prices could hold the potential to stimulate demand, resulting in greater levels of both revenue and profit.

**Figure 1.** *Demand curves obtained for each product through OLS and Poisson models*

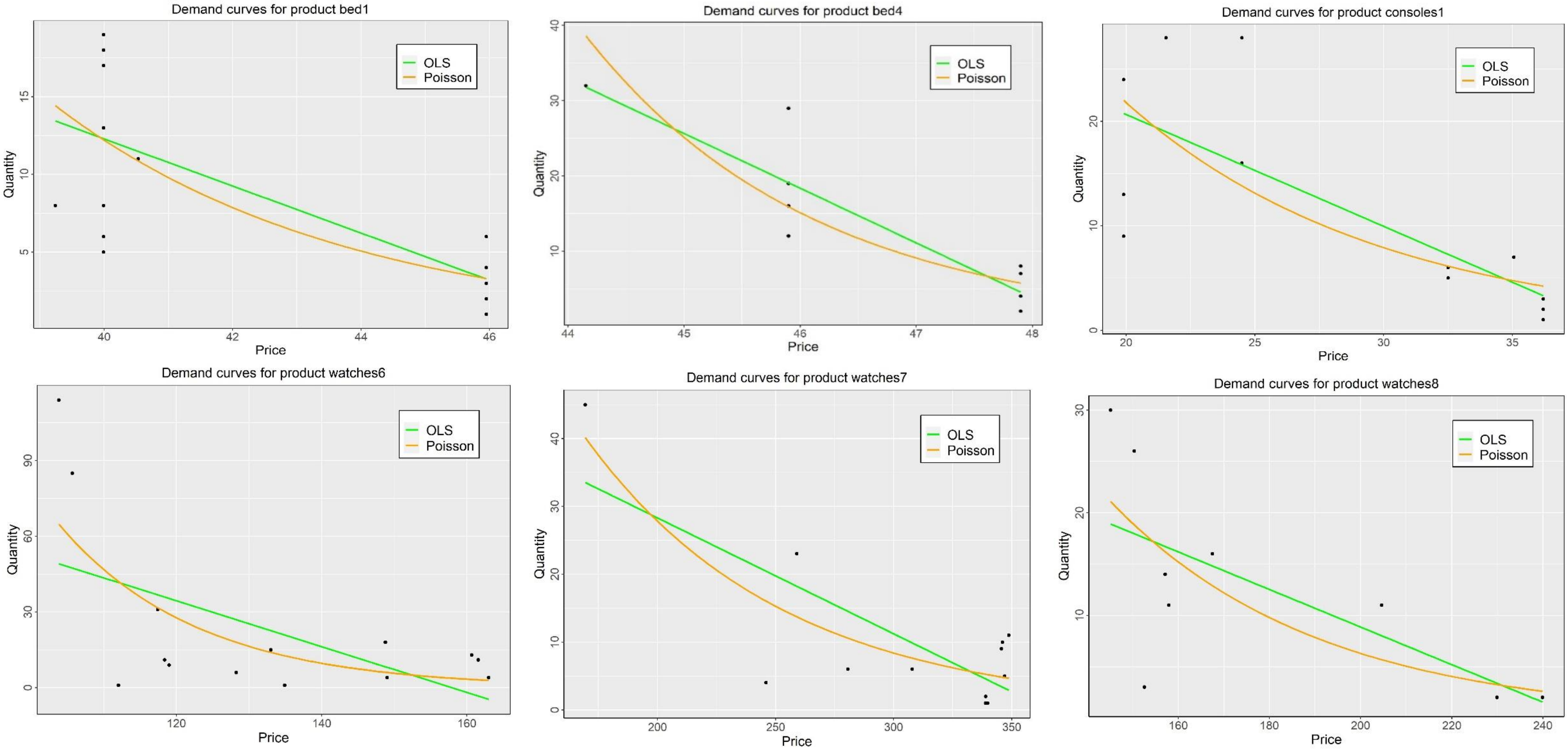
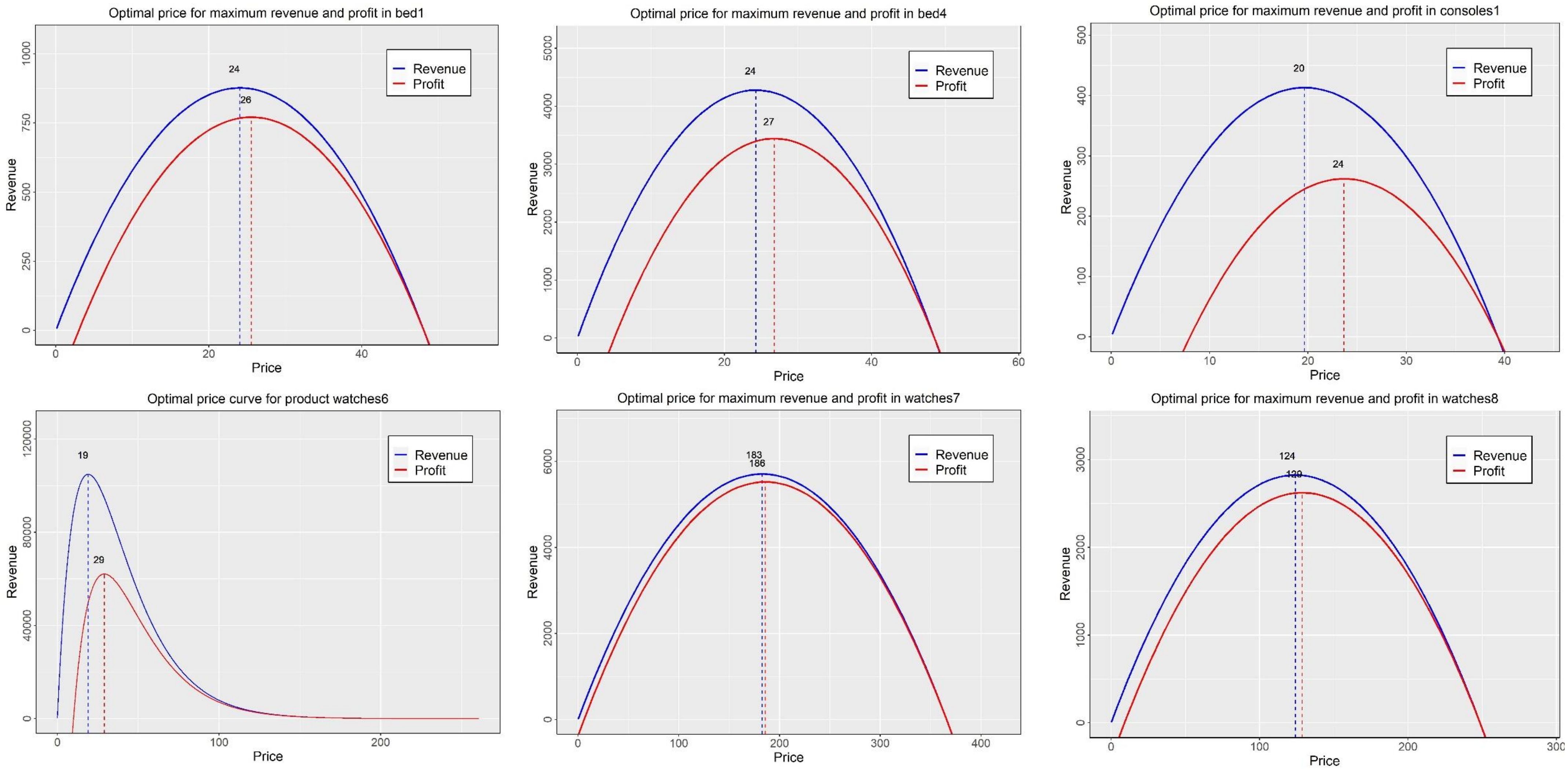


**Figure 2.** *Optimal price plots*



**CONCLUSIONS**

- The analysis revealed that optimal prices for all products consistently fell below their respective average prices, suggesting a potential situation wherein the company may be overpricing its products. This overpricing could lead to reduced demand, subsequently impacting both revenue and profitability adversely.

- It is advisable to conduct the optimal pricing strategy for distinct customer segments, which could result in better revenue and profit results. The implementation of clustering algorithms, such as k-means or fuzzy c-means, could aid in this endeavor.

- Optimal prices were successfully determined for those products that met the prerequisites of the price optimization methodology employed in this study. For products failing to meet these criteria, alternative techniques, including controlled regression or customer-segmentation-based optimal pricing, may be explored. However, the discussion of such methodologies falls beyond the scope of this project.

- Both Ordinary Least Squares (OLS) and Generalized Linear Models (GLM) demonstrated competence in describing price elasticity of demand, as evidenced by the goodness-of-fit metrics obtained in this investigation. Nevertheless, it is noteworthy that GLM-based outcomes consistently yielded optimal price estimations significantly below the mean price, thus warranting a more comprehensive inquiry.

- To ensure that the determined optimal price estimations align with business objectives, it is recommended that experts with domain-specific knowledge assess the suitability of pricing products at the designated values.