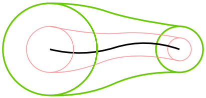
Robust Direct Trajectory Optimization

Using Approximate Invariant Funnels

Journal Club, Team 3



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Introduction

An algorithm that reason about robustness

Reference

- Z. Manchester and S. Kuindersma, "DIRTREL: Robust Trajectory Optimization with Ellipsoidal Disturbances and LQR Feedback"

Figure 1: Zachary Manchester

Figure 2: Scott Kuindersma

An algorithm that reason about robustness

Contribute

- In the case of ellipsoidal disturbance sets, fast evaluations of robust cost and constraint functions.
- Algorithm that improves tracking performance over non-robust formulations while incurring only a modest increase in computational cost.
- Evaluation of the algorithm in several simulated robot control tasks.

Background

Trajectory optimization via DIRTRAN

Characteristics

- NLP that can be solved using SQP packages such as SNOPT.
- Straight forward inclusion of state constraints and avoid numerical pitfalls such as the "tail wagging the dog" effect.
- Usually the problem size is large

$$
\begin{aligned}
\underset{x_{1:N},\, u_{1:N-1},\, h}{\text{minimize}} \quad & g_N(x_N) + \sum_{i=1}^{N-1} g(x_i, u_i) \\
\text{subject to} \quad & x_{i+1} = x_i + f(x_i, u_i) \cdot h \qquad \forall i = 1 : N-1 \\
& u_i \in \mathcal{U} \qquad \forall i = 1 : N-1 \\
& x_i \in \mathcal{X} \qquad \forall i = 1 : N \\
& h_{\min} \leq h \leq h_{\max}
\end{aligned}
$$

Figure 3: DIRTRAN optimization problem

Direct Transcription With Ellipsoidal Disturbances

Assume well defined $w_i \in W$ disturbances that enter into the dynamics
Hence, we can write the disturbed dynamics as

$$x_{i+1} = f_h(x_i, u_i, w_i)$$

Given a disturbance sequence $w_{1:N-1}$ we can calulate the state and input deviations from the nominal values.

Deviations formulation

$$\delta x_{i+1} = f_h(x_i + \delta x_i, u_i + \delta u_i, w_i) - x_{i+1}$$

$$\delta u_i = -K_i \delta x_i$$

Characteristics

- Penalize deviations of the closed-loop system from the nominal trajectory in the presence of disturbances, w_i.
- Quadratic cost of the form $\delta x_i^T Q^\ell \delta x_i + \delta u_i^T R^\ell \delta u_i$, where $Q^\ell \geq 0$ and $R^\ell \geq 0$ are positive semidefinite cost matrices.
- Need of a well-defined disturbance sequence, $w_{1:N-1}$.

Robust cost averaged over the entire disturbance set and summed along the trajectory:

$$\ell_W(x_{1:N}, u_{1:N-1}) \approx \frac{1}{Vol(W)} \int_W (\delta x_N^T Q_N^\ell \delta x_N + \sum_{i=1}^{N-1} (\delta x_i^T Q^\ell \delta x_i + \delta u_i^T R^\ell \delta u_i))\, dW$$

but this integral cannot be easily computed.

Robust Cost Function

Assumptions

- Parametrization of the ellipsoidal set W by a symmetric positive-definite matrix D, such that

$$w^T D^{-1} w \leq 1$$

- Parametrization of the ellipsoidal bounds on the state deviations δx_i by a symmetric positive-definite matrix E_i, such that

$$\delta x_i^T E_i^{-1} \delta x_i \leq 1$$

- Linearization of the disturbed dynamics around the nominal trajectory:

$$\delta x_{i+1} \approx A_i \delta x_i + B_i \delta u_i + G_i w$$

Thanks to the previous assumptions we can write the robust cost as:

$$\ell_W(x_{1:N}, u_{1:N-1}) = Tr(Q_N^\ell E_N) + \sum_{i=1}^{N-1} Tr((Q^\ell + K_i^T R^\ell K_i) E_i)$$

In addition to augmenting the DIRTRAN optimization problem with $\ell_W(x_{1:N}, u_{1:N-1})$, we must also ensure that the closed-loop system obeys state and input constraints.

Robust state constraints

$x_i^W = x_i \pm col(E_i^{1/2})$

Robust input constraints

$u_i^W = u_i \pm col((K_i E_i K_i^T)^{1/2})$

$$
\begin{aligned}
\underset{x_{1:N},\, u_{1:N-1},\, h}{\text{minimize}} \quad & \ell_{\mathcal{W}}(x_{1:N}, u_{1:N-1}) + g_N(x_N) + \sum_{i=1}^{N-1} g(x_i, u_i) \\
\text{subject to} \quad & x_{i+1} = f_h(x_i, u_i) && \forall i = 1 : N-1 \\
& u_i \in \mathcal{U} && \forall i = 1 : N-1 \\
& u_i^{\mathcal{W}} \in \mathcal{U} && \forall i = 1 : N-1 \\
& x_i \in \mathcal{X} && \forall i = 1 : N \\
& x_i^{\mathcal{W}} \in \mathcal{X} && \forall i = 1 : N \\
& h_{\min} \leq h \leq h_{\max}
\end{aligned}
$$

Figure 4: DIRTREL optimization problem

Examples

Pendulum with Uncertain Mass

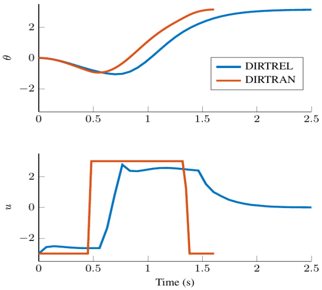

Figure 5: Direct Transcription vs DIRTREL nominal trajectories from the reference paper

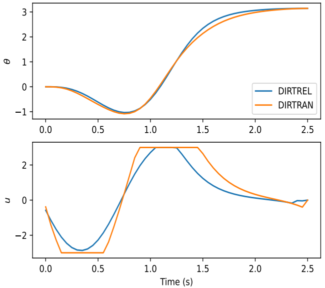

Figure 6: Direct Transcription vs DIRTREL nominal trajectories from my implementation

Pendulum with Uncertain Mass

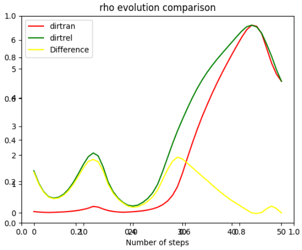

Figure 7: Comparison between the RoA dimension rho

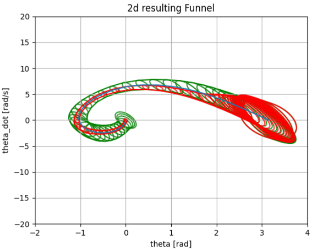

Figure 8: Comparison between the RoA representation via funnels

Cart Pole with Unmodeled Friction

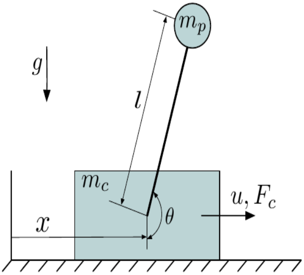

Figure 9: Schematic of the cart pole system

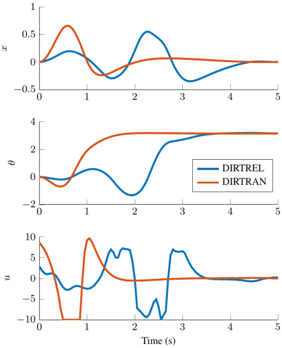

Figure 10: Resulting nominal trajectories from DIRTRAN and DIRTREL

Quadrotor with Wind Gusts

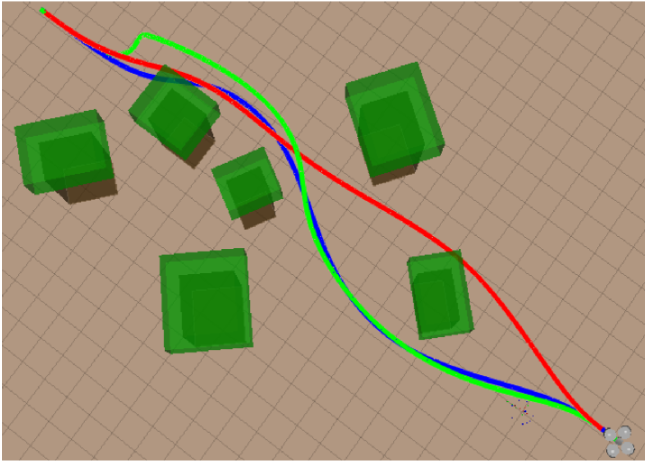

Figure 11: Resulting nominal trajectories from DIRTRAN and DIRTREL

Robot Arm with Fluid-Filled Container

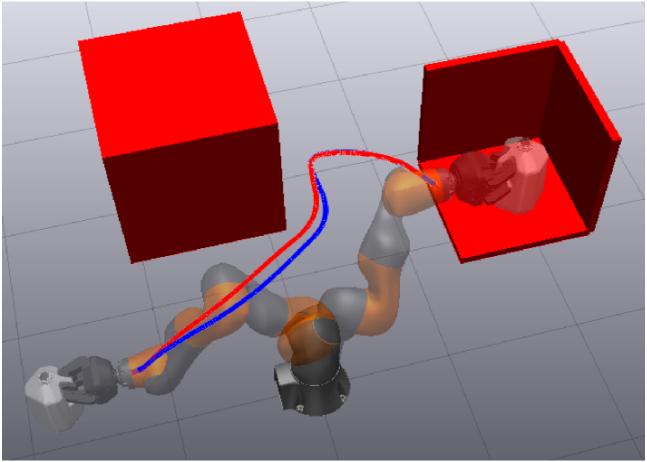

Figure 12: Resulting nominal trajectories from DIRTRAN and DIRTREL

Thanks for your attention

Appendix

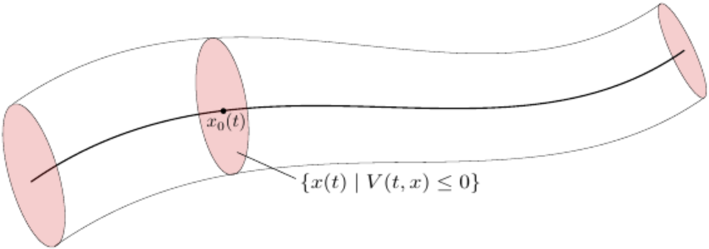

Figure A1: Conceptual depiction of an invariant funnel around a nominal trajectory

By defining

$$S(w) = w^T D^{-1} w - 1 \leq 0 \quad \textit{and} \quad V(t, \delta x) = \delta x^T E(t) \delta x - 1 \leq 0$$

we can define a robust invariant funnel by imposing

$$\dot{V}(t, \delta x, w) \leq 0$$

when $V(t, \delta x) = 0$ and for all disturbances $w \in W = \left\{ w | S(w) \leq 0 \right\}$

It can be shown that imposing the SOS formulation with this notation will lead to obtain the propagation rule for E that is used to compute the robust cost in DIRTREL.