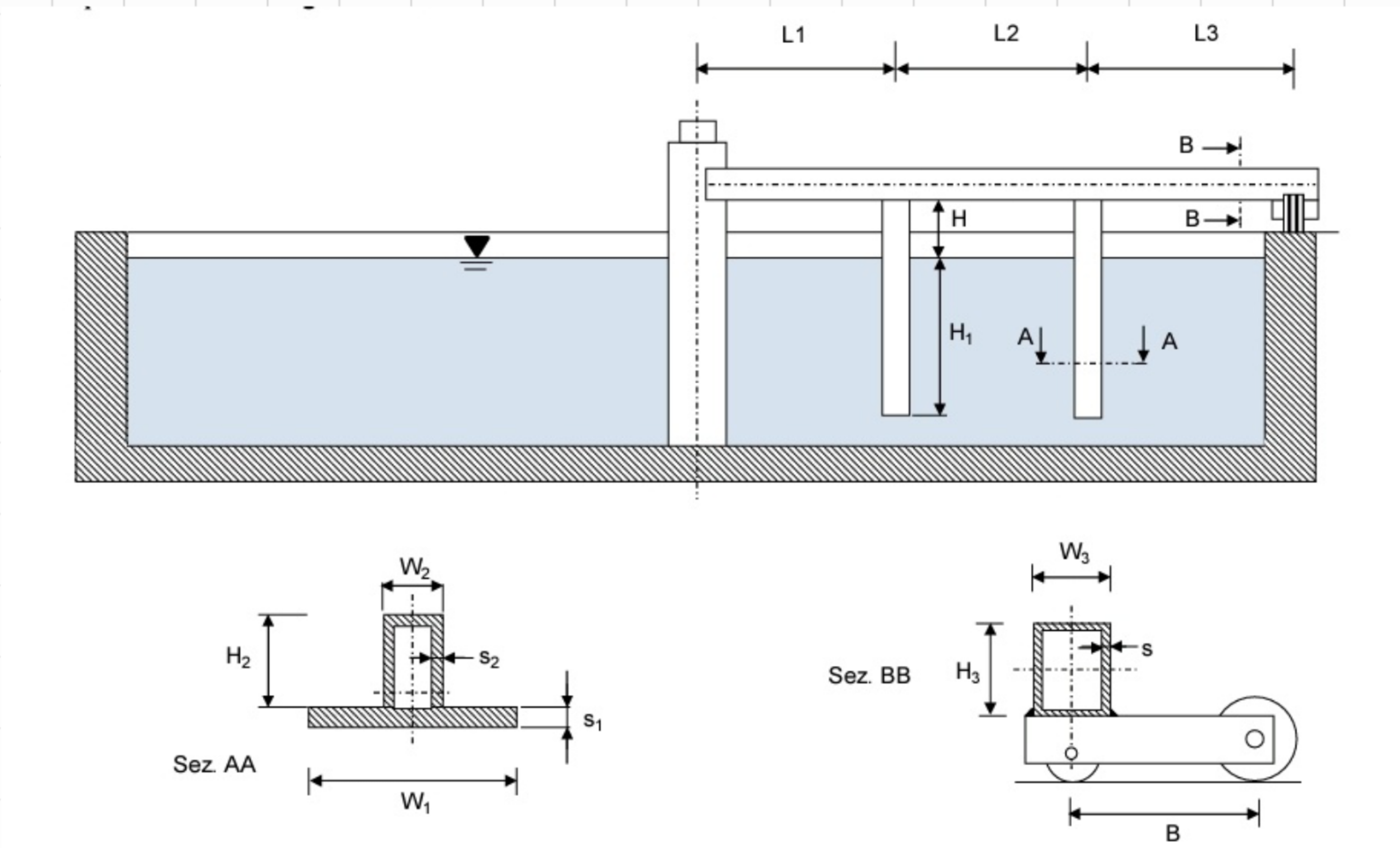
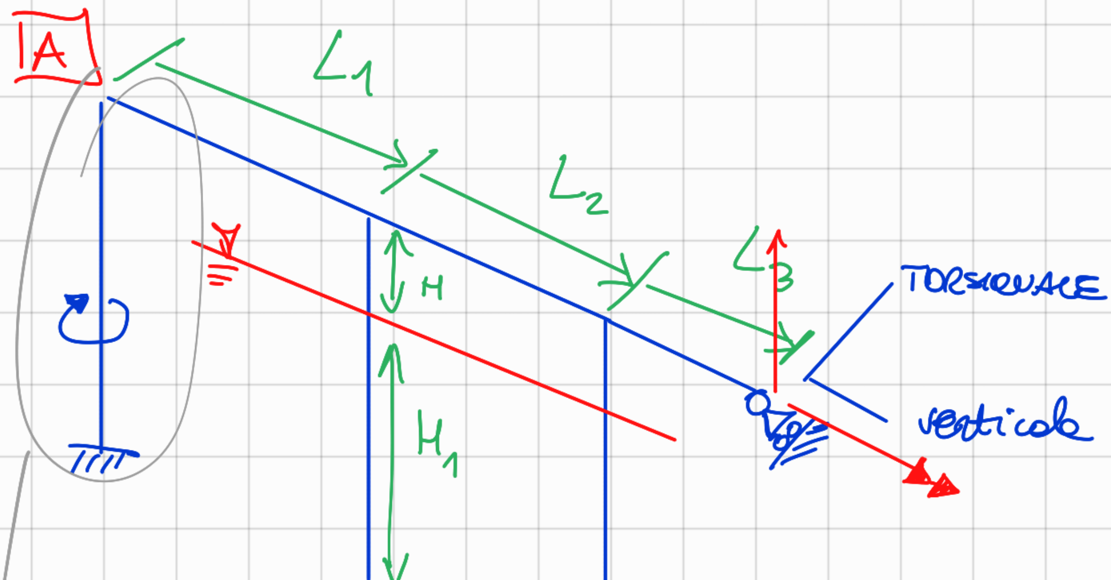
$$\overline{\partial z} \qquad GI_p$$

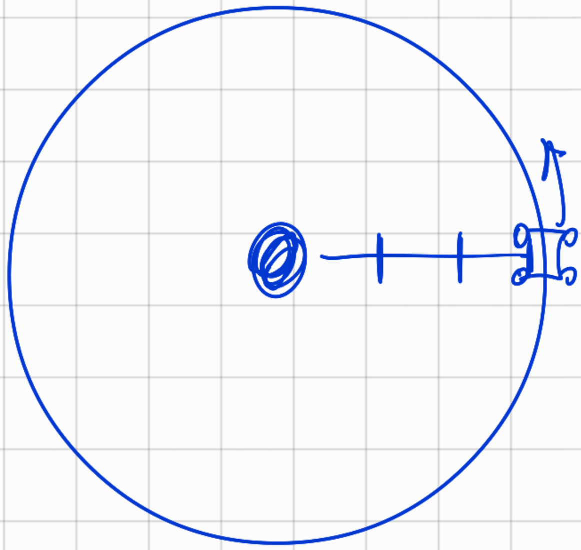
$$\Delta\theta_{sx} = \frac{M_{z1}}{GI_p} l_3 + \frac{M_{z1}}{GI_p} l_2 + \frac{M_{z2}}{GI_p} l_1$$

$$\Delta\theta_{dx} = \frac{M_{z1}}{GI_p} (l_4 + l_5) + \frac{M_{z2}}{GI_p} l_6$$





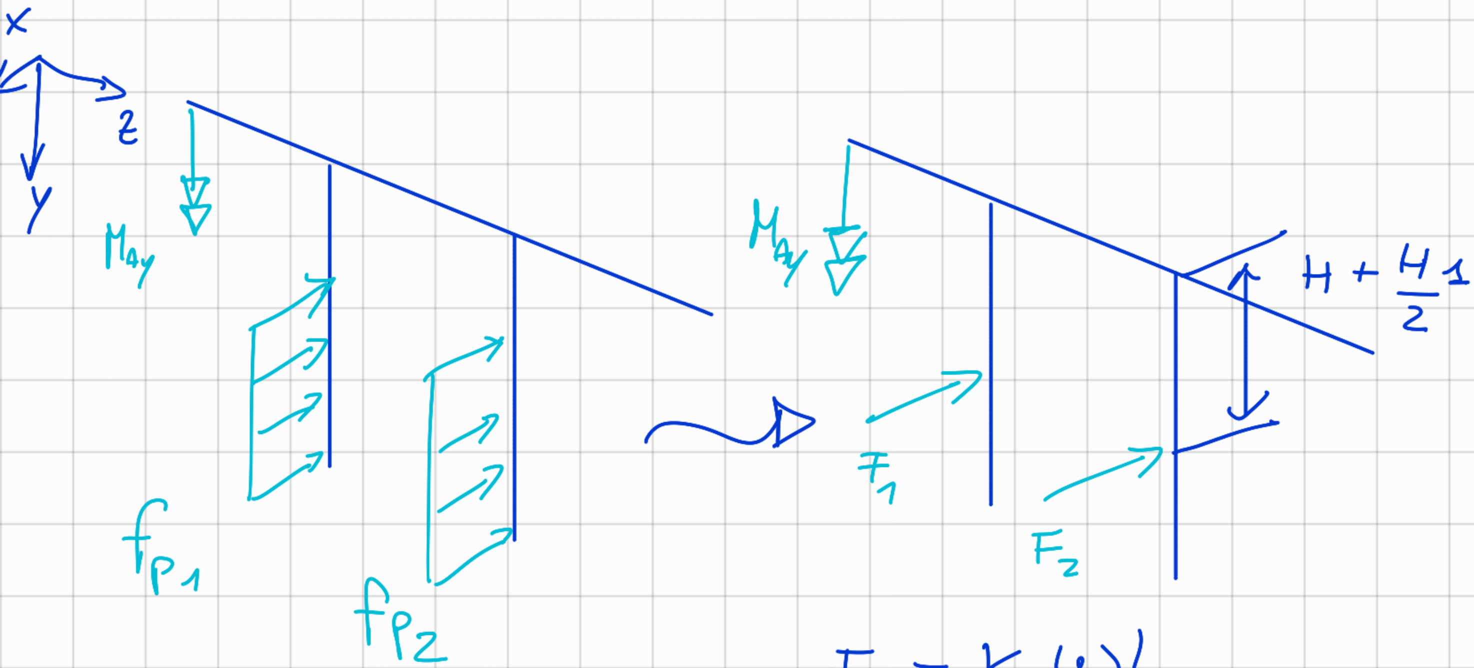
SOTTOPOSTO ad AZIONE TORSIONALE COSTANTE



① AZ. PALE

$$\omega = \frac{2\pi n}{60} = 0{,}42 \ Rad/s$$

$$P = M_{Ay} \cdot \omega \implies M_{Ay} = \frac{P}{\omega} = 17{,}9 \cdot 10^6 \ Nmm$$



$$F_1 = K \omega L_1$$

$$F_2 = K \omega (L_1 + L_2)$$

$$M_{Ay} = F_1 L_1 + F_2 (L_1 + L_2)$$

$$M_{AY} = F_1 L_1 + F_2 (L_1 + L_2)$$

$$= k\omega\left(L_1^2 + (L_1+L_2)^2\right)$$

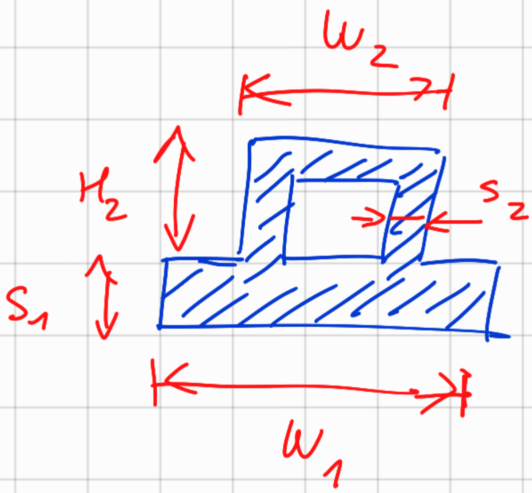
$$K = \frac{M_{AY}}{\omega\left(L_1^2 + (L_1+L_2)^2\right)}$$

$$f_{P_1} = \frac{F_1}{H_1} = 2390\,\frac{N}{m} \longleftarrow \left\{ F_1 = \frac{L_1}{L_1^2 + (L_1+L_2)^2} M_{AY} = 2870\,N \right.$$

$$f_{P_2} = \frac{F_2}{H_2} = 4450\,\frac{N}{m} \longleftarrow \left\{ F_2 = \frac{L_1 + L_2}{L_1^2 + (L_1+L_2)^2} M_{AY} = 5340\,N \right.$$

2 PESI

caso a) Pala



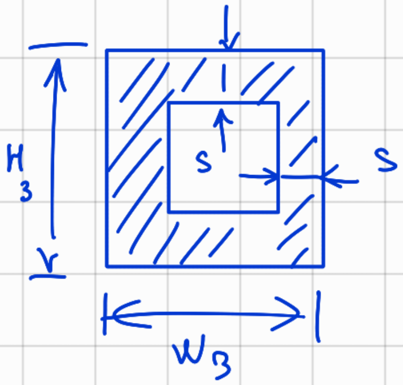
$$A_P = W_1 S_1 + H_2 W_2 - (H_2 - S_2)(W_2 - 2S_2)$$

$$= 3888\,mm^2$$

$q_p = A_p \cdot \rho \cdot g = 299{,}4 \ N/m$ (Peso distribuito)

$P_p = q_p \,(H + H_1) = 449{,}1 \ N$

b) TRAVE

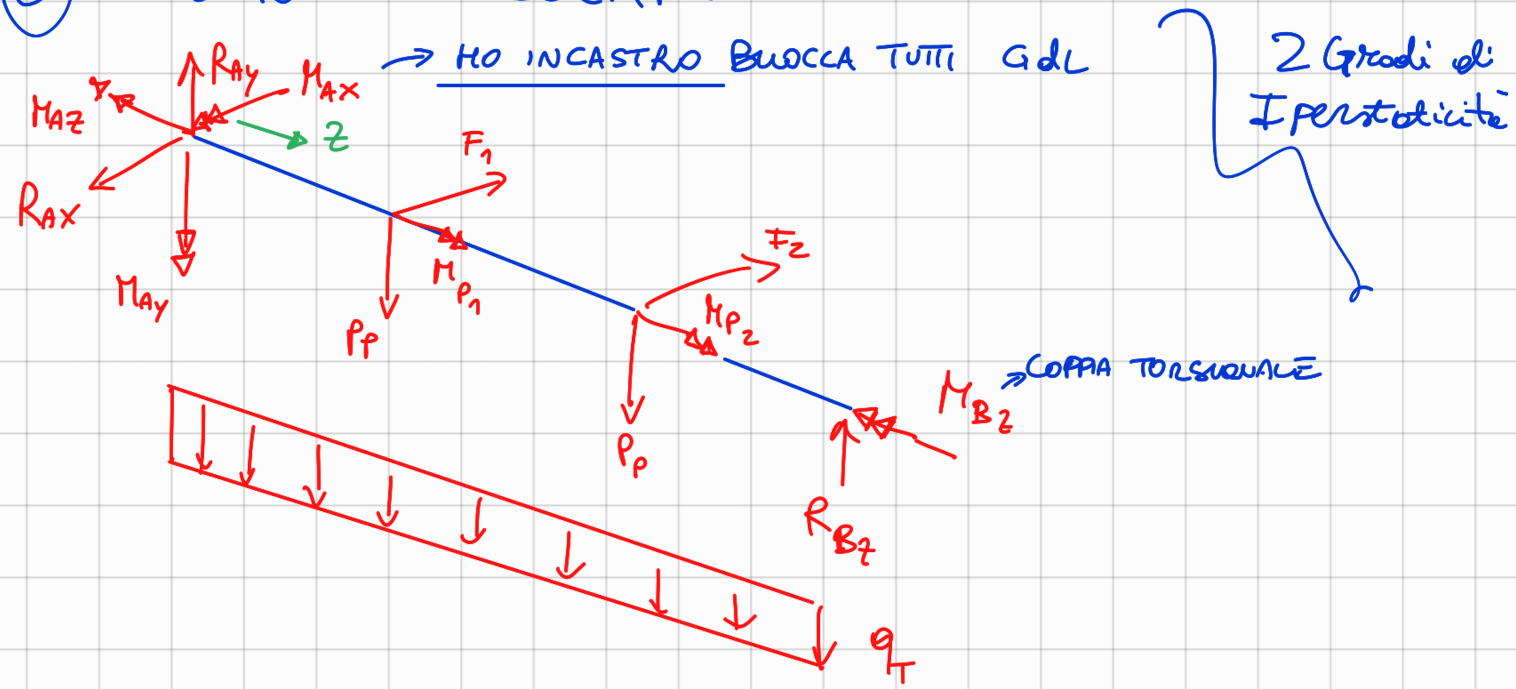


$A_T = H_3 W_3 - (H_3 - 2S)(W_3 - 2S) = 6200 \ mm^2$

$q_T = A_T \cdot \rho \cdot g = 477{,}4 \ N/m$

$P_T = q_T \cdot L = 1910 \ N$

3) REAZIONI VINCOLARI



a) Torsionale

$\theta(0) = 0 \quad , \quad \theta(L) = 0$ Conoscenze sistema x definire incognita iperstatica

$$M_z = \begin{cases} M_{Az} & 0 \le z \le L_1 \\ M_{Az} - M_{P_1} & L_1 \le z \le L_1 + L_2 \\ M_{Az} - M_{P_1} - M_{P_2} & L_1 + L_2 \le z \le L \end{cases}$$

$$M_{Az} + M_{Bz} = M_{P_1} + M_{P_2}$$

o castigliano o $\underbrace{\text{linea elastica}}_{+\text{ facile}}$

$$\theta(L) = \int_0^L \frac{M_z}{G J_T} \, dz = 0$$

semplifico per "$G J_T$"

$$\rightarrow M_{Az} \cdot L_1 + (M_{Az} - M_{P_1}) L_2 + (M_{Az} - M_{P_1} - M_{P_2}) L_3 = 0$$

condizione x cui $\theta(L) = 0$

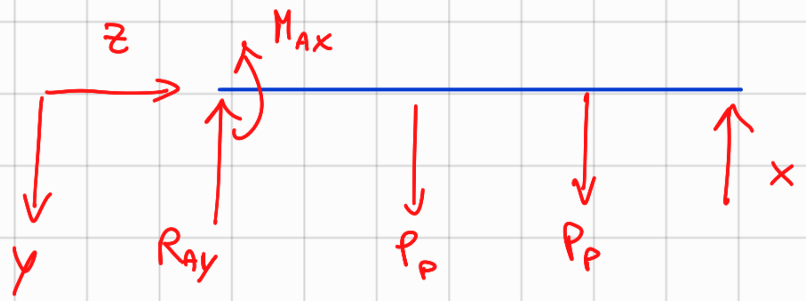
$$M_{Az} \cdot L = M_{P_1}(L_2 + L_3) + M_{P_2}(L_3)$$

$$M_{Az} = 3,37 \cdot 10^6 \text{ N mm}$$

$$M_{Bz} = 4,03 \cdot 10^6 \text{ N} \cdot \text{mm}$$

b) 2° INCOGNITA IPERSTATICA $\quad R_{By}$ in + rispetto a incastro

$R_{By} = X$ (incognita iperstatica)



trovo R_{Ay} M_{Ax} in funz di x
Azioni interne in funz. di x

equilibri

$$y: \quad R_{Ay} = 2P_P - X + P_T$$

$$rot: \quad M_{Ax} = P_P\,(2L_1 + L_2) - xL$$

a) $0 \leq z \leq L_1$

$$M_x = -M_{Ax} + R_{Ay}\cdot z - q_T\,\frac{z^2}{2}$$

b) $L_1 \leq z \leq L_1 + L_2$

$$M_x = -M_{Ax} + R_{Ay}\cdot z - P_P\,(z - L_1) - q_T\,\frac{z^2}{2}$$

c) $L_1 + L_2 \leq z \leq L$

$$M_x = -M_{Ax} + R_{Ay}\cdot z - P_P\,(z - L_1) - P_P(z - (L_1 + L_2)) - q_T\frac{z^2}{2}$$

USO CASTIGLIANO

$$S_{By} = \frac{\partial U}{\partial X} = 0$$

$$= \int_0^L \frac{M_x}{EI_x} \frac{\partial M_x}{\partial X} dz$$

$$\frac{\partial(-M_{Ax})}{\partial X}$$

$$= \int_0^{L_1} \frac{\boxed{R_{Ay} \cdot z - M_{Ax} - q_T \frac{z^2}{2}}}{EI_x} (-z + L) dz +$$

derivata

$$\frac{\partial R_{Ay,z}}{\partial X}$$

$$+ \int_{L_1}^{L_1+L_2} \frac{\boxed{R_{Ay} \cdot z - M_{Ax} - q_T \cdot \frac{z^2}{2}} - P_P (z - L_1)}{EI_x} (-z + L) dz +$$

$$+ \int_{L_1+L_2}^{L} \frac{\boxed{R_{Ay} - M_{Ax} - q_T \frac{z^2}{2}} - P_P (z - L_1) - P_P(z - (L_1 + L_2))}{EI_x} (-z + L) dz = 0$$

$$\int_0^L \left(R_{Ay} - M_{Ax} - q_T \frac{z^2}{2}\right)(L - z) dz - \int_{L_1}^{L} P_P (z - L_1)(L - z) dz - \int_{L_1+L_2}^{L} P_P (z - (L_1 + L_2))(L - z) dz$$

$$= \left[q_T \frac{z^4}{8} + \left(-\frac{q_T}{2} - R_{Ay}\right) \frac{z^3}{3} + \left(R_{Ay} L + M_{Ax}\right) \frac{z^2}{2} - M_{Ax} L \right]_0^L -$$

$$- P_P \left[-\frac{z^3}{3} + (L_1 + L) \frac{z^2}{2} - L_1 L_2 \right]_{L_1}^{L} -$$

$$- P_P \left[-\frac{z^3}{3} + (L_1 + L_2 + L) \frac{z^2}{2} - (L_1 + L_2) L_2 \right]_{L_1+L_2}^{L}$$

$$= q_T \frac{L}{8} - \left(\frac{q_T}{2} + R_{Ay}\right)\frac{L}{3} + \left(R_{Ay} L + M_{Ax}\right)\frac{L}{2} - M_{Ax} L +$$

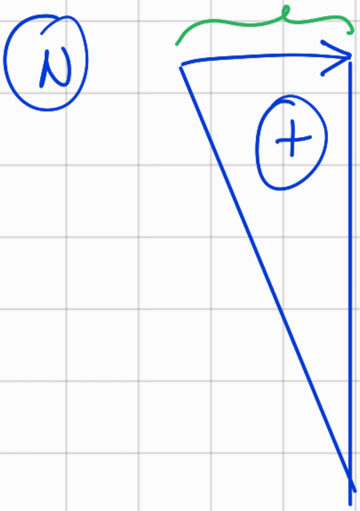
$$P_p \left(\frac{L^3}{3} - (L_1 + L)\frac{L^2}{2} + L_1 L^2 - \frac{L_1^3}{3} + (L_1 + L)\frac{L_1^2}{2} - L_1^2 L + \right.$$

$$\left. + P_p \left(\frac{L^3}{3} - (L_1 + L_2 + L)\frac{L^2}{2} + (L_1+L_2)L^2 - \frac{(L_1+L_2)^3}{3} + \frac{(L_1+L_2+L)(L_1+L_2)^2}{2} - (L_1+L_2)^2 L \right) = 0 \right.$$

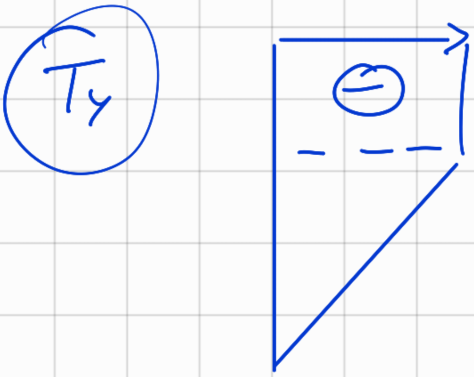
$X = 1012\ N$

$R_{By} = 1012\ N$, $R_{Ay} = 1796\ N$, $M_{Ax} = 1{,}57 \cdot 10^6\ Nmm$

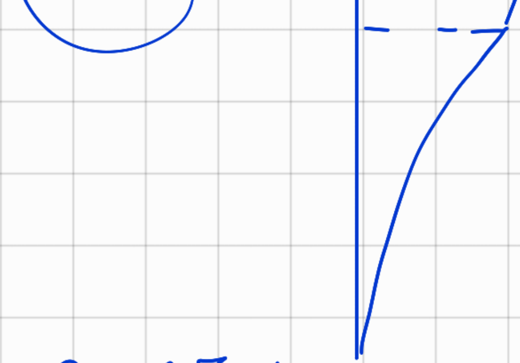
5 AZIONI INTERNE



$-F_1 = -2870\ N$

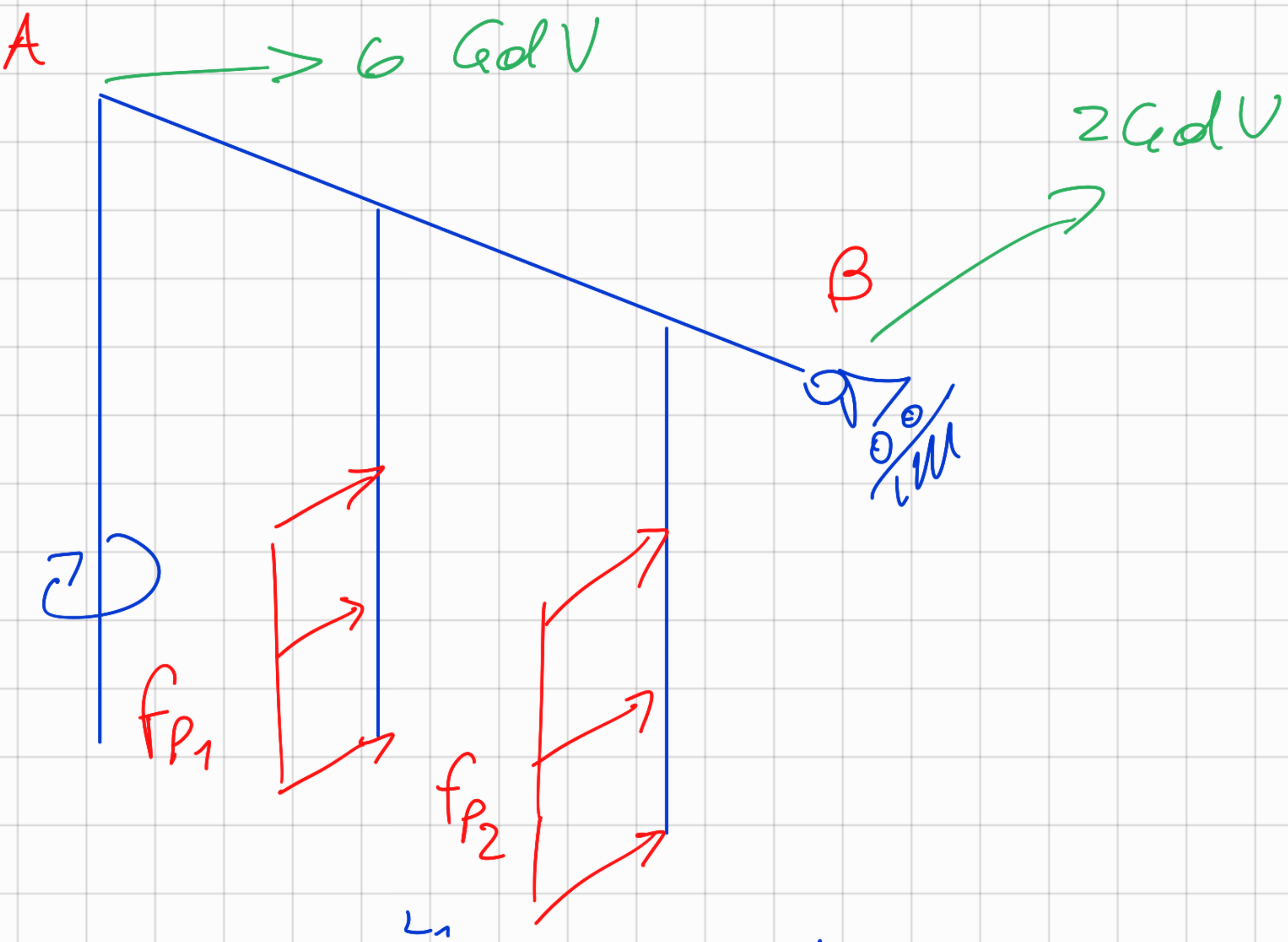


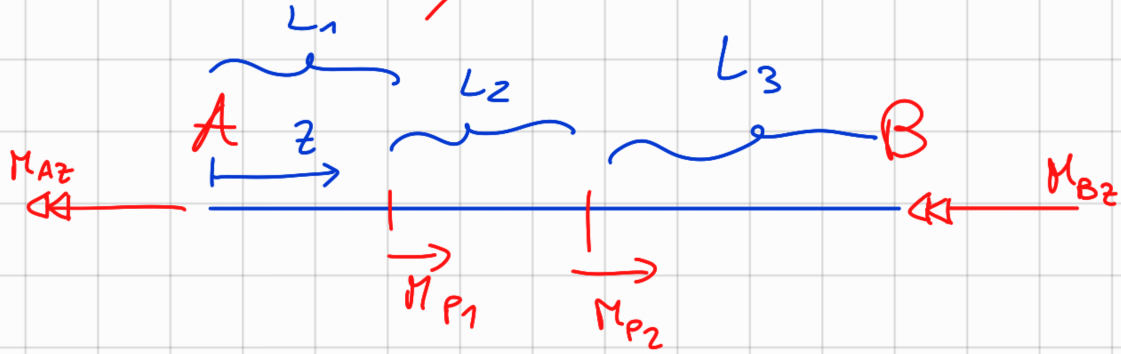
$M_{p2} = 4{,}81 \cdot 10^6\ N\,mm$



Punto d'immersione

RIPRENDO ES. PRECEDENTE





Linea elastica

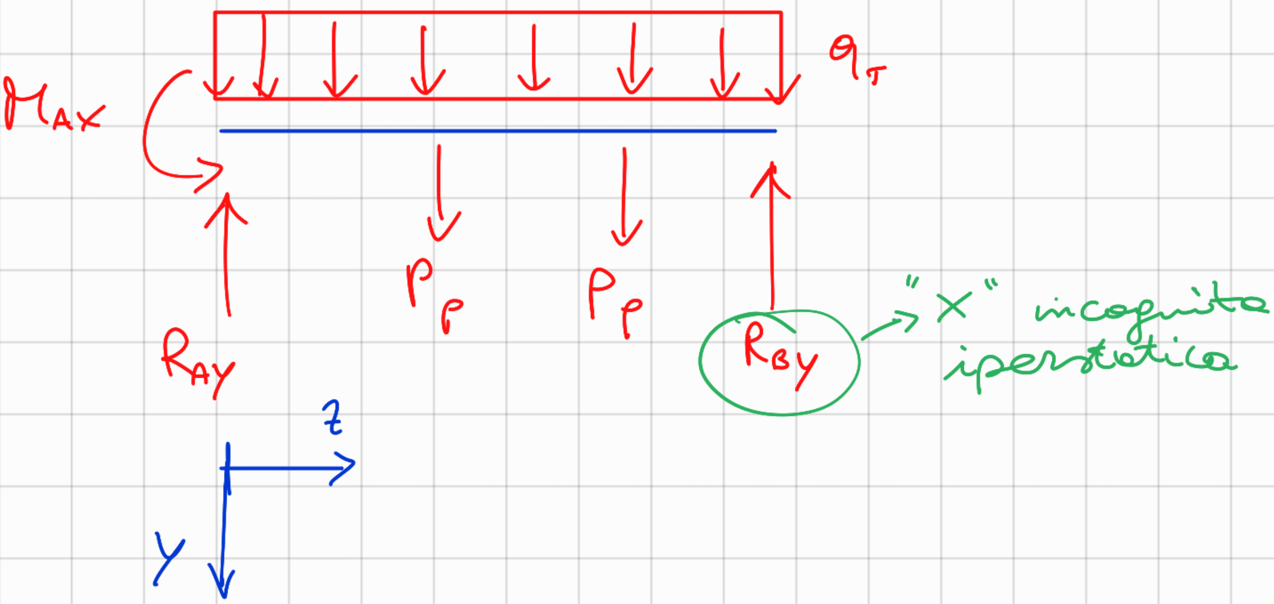
$$\theta_z(0) = 0$$

$$\theta_z(L) = 0$$

$$M_z = \begin{cases} - & [0, L_1] \\ - & [L_1, L_1+L_2] \\ - & [L_1+L_2, L] \end{cases}$$

$$\frac{\partial \theta}{\partial z} = \frac{M_z}{G J_t}$$

② FLESSIONE



equi in y: $$2 P_p = R_{Ay} + \overbrace{R_{By}}^{X} - q_T \cdot L$$

$$R_{Ay} = 2P_p - X + q_T \cdot L$$

rot A $$M_{AX} = \underbrace{P_p}_{\text{Peso della pala}} L_1 + P_p (L_1 + L_2) - X L + q_T \frac{L^2}{2}$$

$$z \in [0, L_1]$$

$$T_y = + R_{Ay} - q_T \cdot z$$

$$\rightarrow M_x = - M_{AX} + R_{Ay} \cdot z - q_T \cdot \frac{z^2}{2}$$

$z \in [L_1, L_1 + L_2]$

$$T_y = +R_{Ay} - q_T \cdot z + P_p$$

$$\to M_x = -M_{Ax} + R_{Ay} z - q_T \frac{z^2}{2} + P_p (z - L_1)$$

$z \in [L_1 + L_2, L]$

$$T_y = +R_{Ay} - q_T \cdot z + 2P_p$$

$$\to M_x = -M_{Ax} + R_{Ay} \cdot z - q_T \frac{z^2}{2} + P_p(z - L_1) + P_p (z - (L_1 + L_2))$$

$V(L) = 0 \to \frac{\partial^2 V}{\partial z^2} = -\frac{M_x}{EI_x} \to V(0) = 0$

$\theta(0) = \frac{\partial V}{\partial z}(0) = 0$ } COND.IZ. x eq. linea elastica

$\downarrow$

ottima x se ho un contributo solo

$\Downarrow$

$$\boxed{\frac{\partial U}{\partial X} = 0}$$ } CONDIZIONE CASTIGLIANO (no cedimento strutturale)

$$\frac{\partial U}{\partial X} = \int_{STR} \frac{M_x}{EI_x} \frac{\partial M_x}{\partial X} \, dz = 0$$

$\uparrow$

le metto qua dentro

1° SPEZZ.

$$\frac{\partial U}{\partial X} = \int^{L_1} \left(R_{Ay} z - q_T \frac{z^2}{2} - M_{Ax} \right) (-z + L) \, dz +$$

$\frac{\partial}{\partial x} \quad 0$ levo EI_x (tanto va a zero)

$$\int_{L_1}^{L_1+L_2} \Big(\underline{R_{Ay} z} \; . \; \underline{. \; .}\Big)(L-z)\,dz + \int_{L_1+L_2}^{L} \Big(\underline{R_{Ay} z} \ldots\Big)(L-z)\,dz = 0$$

qui compare x

$$M_x = R_{Ay} z - q_T \frac{z^2}{2} - M_{Ax} - P_p(z-L_1) - P_p\big(z-(L_1+L_2)\big)$$

$$R_{Ay} = 2P_p + q_T \cdot L - x$$

$$M_{Ax} = P_p\big(2(L_1+L_2)\big) - xL + q_T \frac{L^2}{2}$$

parti che entrano nella derivata

$$\frac{\partial U}{\partial x} = \int_0^L \Big(R_{Ay} z - q_T \frac{z^2}{2} - M_{Ax}\Big)(L-z)\,dz + \int_{L_1}^{L} P_p \cdot (z-L_1)(L-z)\,dz + \int_{L_1+L_2}^{L} P_p\big(z-(L_1+L_2)\big)(L-z)\,dz$$

$$= \Big[q_T \frac{z^4}{8} + \Big(-R_{Ay} - \frac{q_T}{2}L\Big)\frac{z^3}{3} + \Big(R_{Ay} L + M_{Ax}\Big)\frac{z^2}{2} - M_{Ax} L z\Big]_0^L +$$

$$+ \Big[P_p\Big(-\frac{z^3}{3} + (L_1+L)\frac{z^2}{3} - L_1 L z\Big)\Big]_{L_1}^{L} +$$

$$+ \Big[P_p\Big(-\frac{z^3}{3} + (L_1+L_2+L)\frac{z^2}{2} - (L_1+L_2)L z\Big)\Big]_{L_1+L_2}^{L} = 0$$

$$\frac{\partial U}{\partial x} = \underline{q_T \frac{L^4}{8}} - \Big(R_{Ay} + \underline{\frac{q_T}{2}L}\Big)\frac{L^3}{3} + \big(R_{Ay} L + M_{Ax}\big)\frac{L^2}{2} - M_{Ax} L^2 +$$

$$+P_P\left[-\frac{L^3}{3}+(L_1+L)\frac{L^2}{2}-L_1L^2+\frac{L_1^3}{3}-(L_1+L)\frac{L_1^2}{2}+L_1^2L\right.$$

$$-\frac{L^3}{3}+(L_1+L_2+L)\frac{L^2}{2}-(L_1+L_2)L^2+\frac{(L_1+L_2)^3}{3}-$$

$$\left.-(L_1+L_2+L)\frac{(L_1+L_2)^2}{2}+(L_1+L_2^2)L\right]$$

$$R_{Ay}=2P_P+q_TL-x$$

$$M_{Ax}=P_P(2L_1+L_2)-xL+q_T\frac{L^2}{2}$$

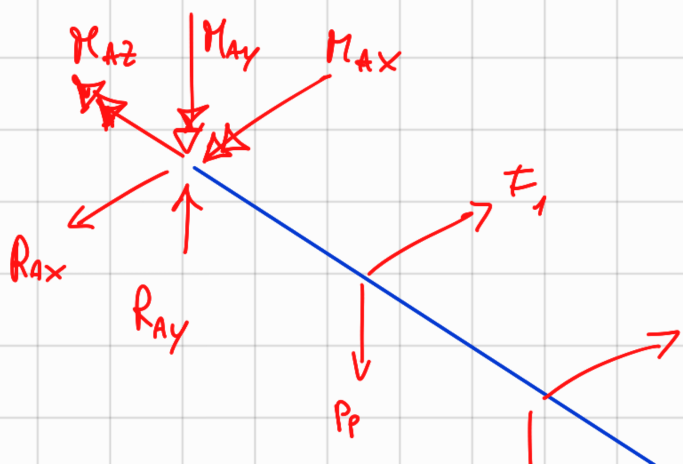
$$-R_{Ay}\frac{L^3}{3}+R_{Ay}\frac{L^3}{2}+M_{Ax}\frac{L^2}{2}-M_{Ax}L^2=-q_T\frac{L^4}{8}-q_T\frac{L^4}{6}-P_P[\dots]$$

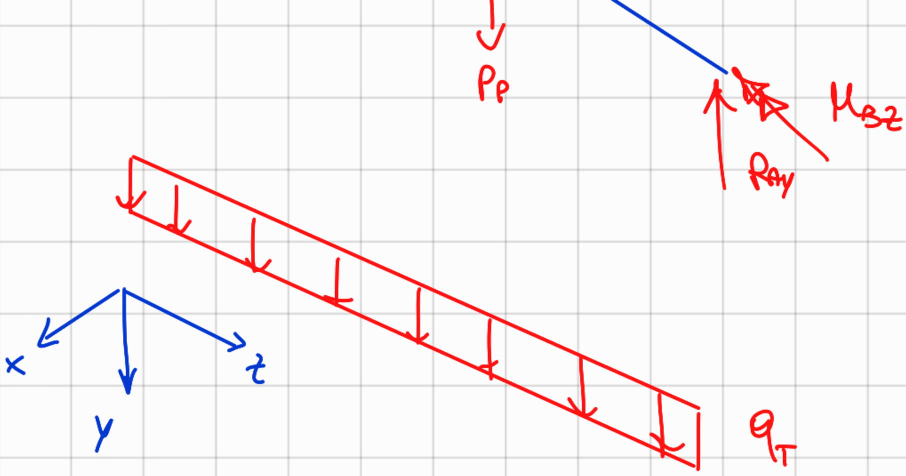
$$R_{Ay}\frac{L^3}{6}-M_{Ax}\frac{L^2}{2}=[\dots]$$

$$\underbrace{-x\frac{L^3}{6}+xL\frac{L^2}{2}}=[\dots]-(2P_P+q_TL)\frac{L^3}{6}+\left(P_P(2L_1+L_2)+q_T\frac{L^2}{2}\right)\frac{L^2}{2}$$

$$x\frac{L^3}{3}=[\dots]$$

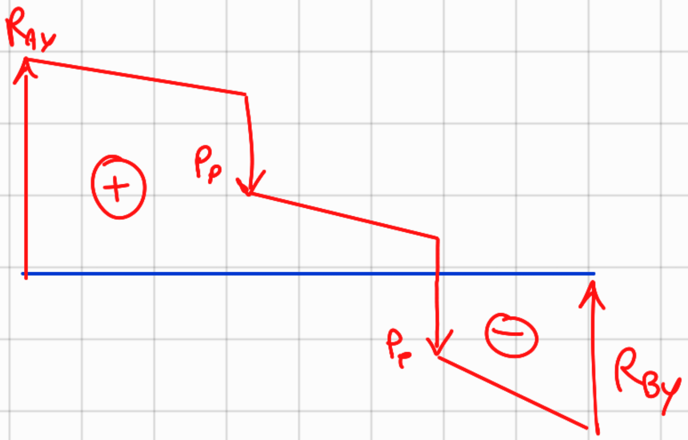
$$\boxed{x=1012\,N}$$



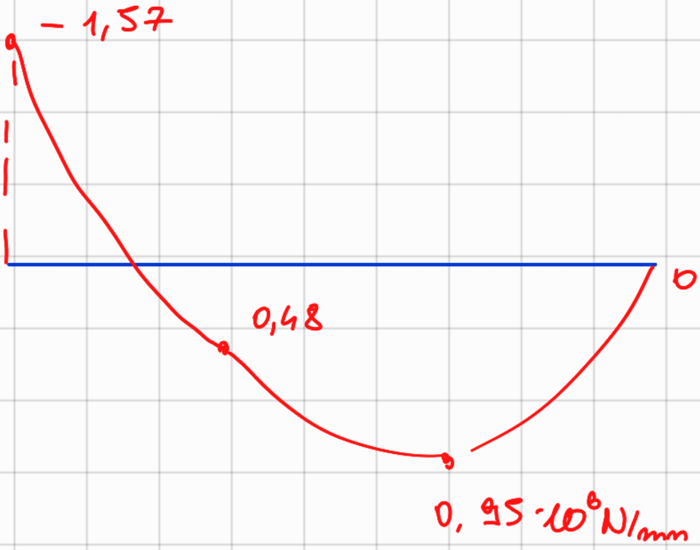


$N = 0$

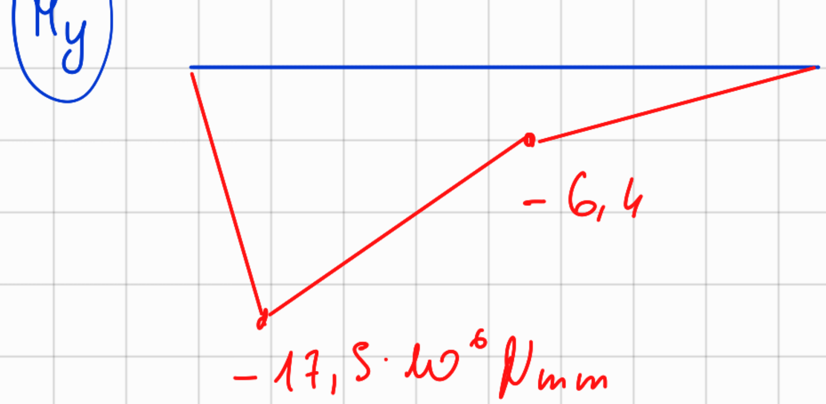
T_y

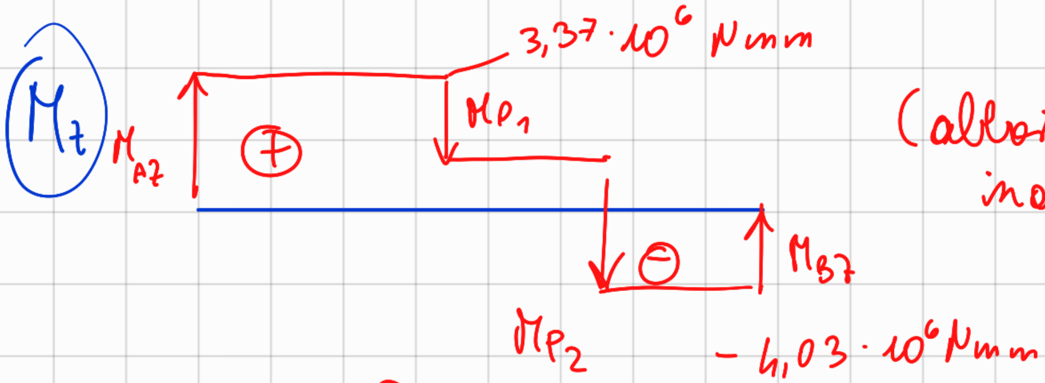


M_x

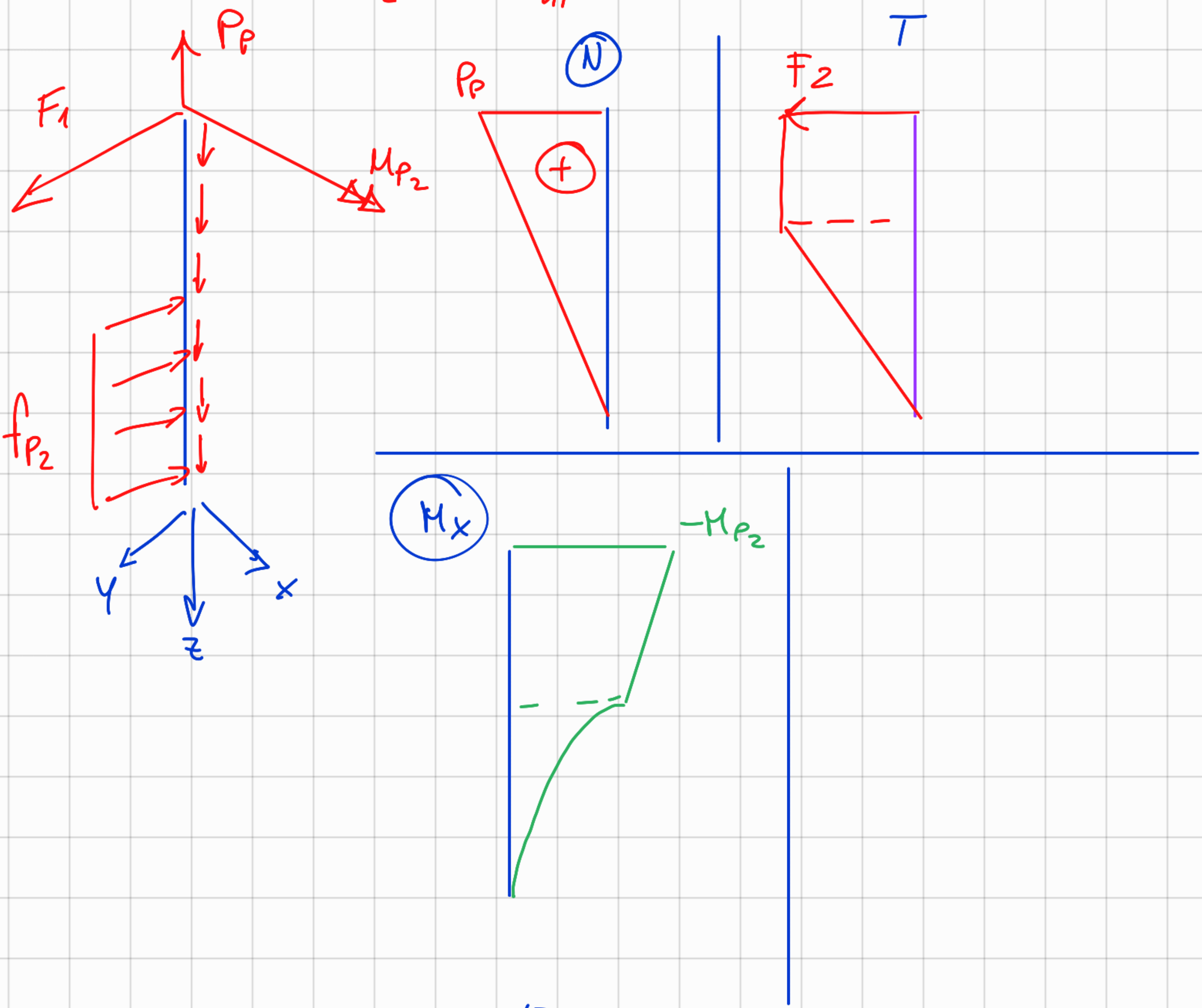


T_x

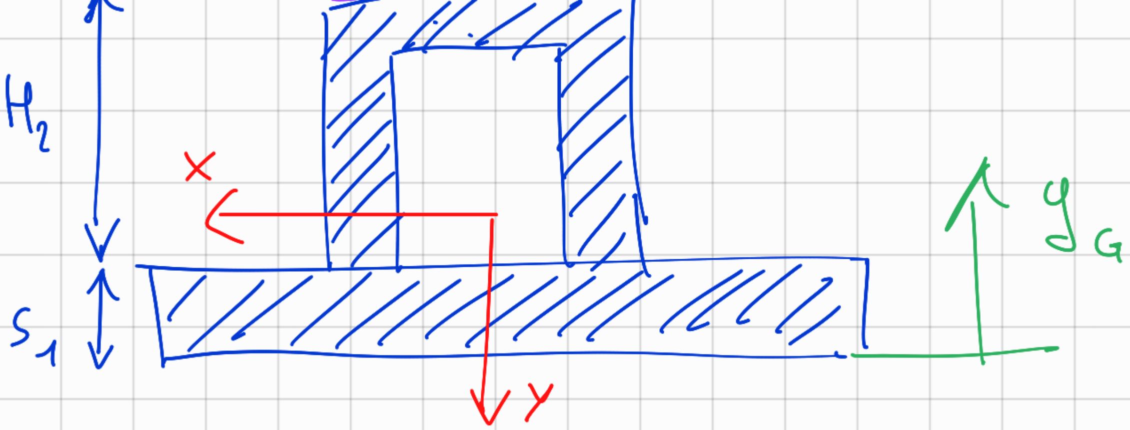




(albero tenuto indifferente segno su M_z)



Punto critico in $\boxed{z=0}$



$$\begin{cases} N = 449,1\ N \\ M_x = 4,81 \cdot 10^6\ Nmm \end{cases}$$

$\rightarrow A = 3888\ mm^2$

$\rightarrow y_G$ (ho solo M_x) $= \dfrac{S_x}{A} = 39,6mm$

$\rightarrow I_x = 8,01 \cdot 10^6\ mm^4$

$- \sigma_{zz} = \dfrac{N}{A} + \dfrac{M_x}{I_x}\, y$

$y_{MAX} = y_G$

$y_{MIN} = -92,4\ mm$

$= -(s_1 + H_2 - y_G)$

$\sigma_{zz,MAX} = 23,63$ MPa
$\sigma_{zz,MIN} = -55,3$ MPa

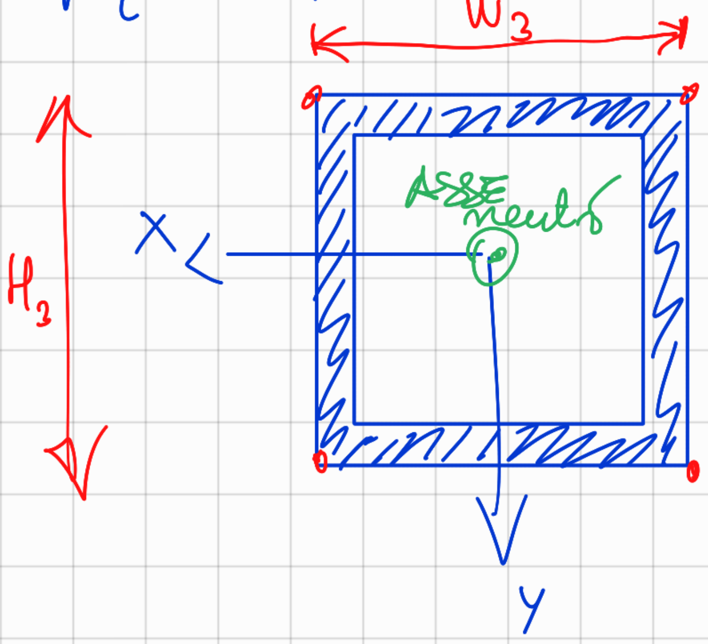
} appartenenti

$$\varphi = \frac{\sigma_{ys}}{\sigma_{zz,MIN}} = 7,2$$

$$\boxed{t = 0}$$

$M_x = -1,57 \cdot 10^6$ Nmm
$M_y = -17,9 \cdot 10^6$ Nmm } flessione → sforzi normali

$M_z = 3,37 \cdot 10^6$ Nmm } Torsione → sforzi taglio



$I_x = 28,5 \cdot 10^6$ N mm
$I_y = 21,3 \cdot 10^6$ N·mm

$$\sigma_{zz,MAX} = \frac{|M_x|}{I_x}\frac{H_3}{2} + \frac{|M_y|}{I_y}\frac{W_3}{2} =$$

$= 30,1$ MPa

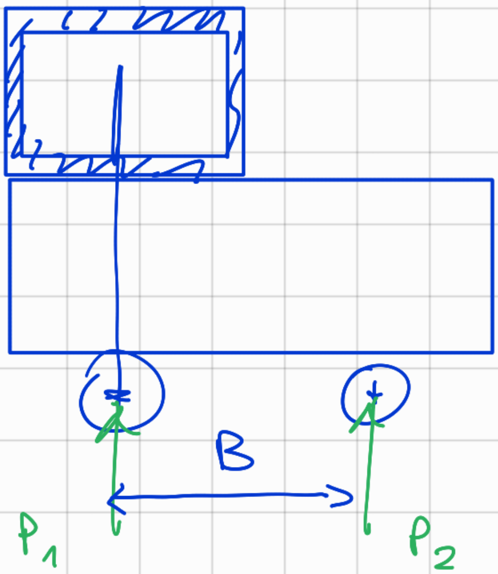
$$\tau_{xz} = \frac{M_z}{2\Omega_s} = 7,1\ MPa$$

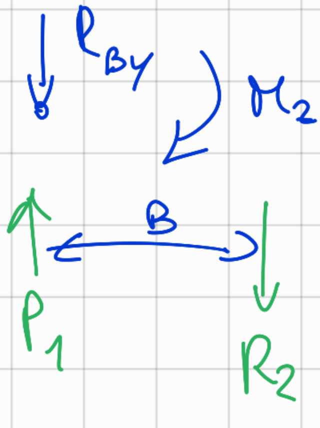
$$\sigma_{VM} = \sqrt{\sigma^2 + 3\tau^2} = 32,5\ MPa$$

$$\varphi = 12,3$$

Trovo reazione carrello

$\boxed{z = L}$





equi lungo y: $R_1 + R_2 = R_{By}$

Rotaz : $R_2 \cdot B = M_z$

$$R_2 = 8,96\ kN$$

$$R_1 = 7,24\ kN$$