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Fuzzy Rules
Introduction

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What is an inference rule?

We can consider an inference rule as a model, a way to define a mapping from input to output.

Rules are used to represent inferential relationships among pieces of knowledge.

We consider *forward chaining rules*, having the shape

IF <antecedent> THEN <consequent>

where:

- <antecedent> is a set of <clause>s related by logical operators AND, OR, NOT
- <consequent> is a set of <clause>s related by the logical operators AND, OR, NOT
- <clause>, in general, is either a <proposition>, i.e., a sequence of symbols, or a <pattern>, i.e., a sequence of symbols and variables

What are rules for?

Inference rules are used to implement Knowledge-Based Systems, among which Expert Systems are mostly known as successful AI applications, e.g. for diagnosis, forecast, design, control, classification, planning, etc.

An Expert System is designed upon the experience of somebody to replicate, or improve her/his performance in solving a problem.

Knowledge Acquisition is a complex process bringing to the definition of rule-based systems, implemented and running on computers.

How can rules be used in a computer?

Pattern matching: identify the rules whose antecedents match the known facts (fact base). These can be considered for activation, given the corresponding assignment to variables.

Selection of the rule(s) to be activated: among the rules identified with pattern matching (candidate rules), select the rules that should be activated, i.e., whose consequents have to be asserted as new knowledge in the fact base.

Activation of the selected rules: assert the consequents of the selected rules in the fact base.

With this, rules and information (i.e., knowledge) a system can **generate** new information.

Inference rules example (from Wikipedia)

Suppose that the *rule base* contains the following four rules:

1. **If** *X* croaks and X eats flies - **Then** *X* is a frog
2. **If** *X* chirps and X sings - **Then** *X* is a canary
3. **If** *X* is a frog - **Then** *X* is green
4. **If** *X* is a canary - **Then** *X* is yellow

Suppose to observe the following facts *(fact base*)*:*

a. Fritz croaks
b. Fritz eats flies

From rule *1* and facts *a* and *b* we can add to the fact base the fact c. Fritz is a frog.
Given the new fact base, we can use rule *3* to deduce the fact d. Fritz is green.

What is a fuzzy rule?

A fuzzy rule is a rule whose clauses have the shape

(V is L)

where V is a linguistic variable and L is a label, a value for V associated to a fuzzy set. This is a linguistic clause.

Often, clauses in the antecedent are only related by the AND operator which is not explicitly written.

The antecedent is usually matched against facts that are represented as values of the base variables corresponding to the linguistic variables.

The consequent may be one of two types …

Linguistic rules

Linguistic rules (Mamdani): the consequent is a conjunction of linguistic clauses

IF (A is LA_i) AND (B is LB_k) AND... THEN (U is LU_m) AND ...

E.g.:

IF (Distance is Far) AND (BallDirection is Front)
THEN (Speed is High) AND (Direction is Ahead)

This can be considered as a mapping between

the <u>interpretation</u> of an <u>input</u> configuration
and
a symbolic <u>description</u> of the desired <u>output</u>

Model rules

Model rules (Sugeno, or Takagi-Sugeno-Kosko TSK): bind a model (linear, non linear, NN, …) to the linguistic interpretation of its applicability conditions

IF (A is LAn) AND (B is LBk) AND... THEN U is f(A, B)

E.g.:

IF (Temperature is High) AND (Pressure is High)
THEN Heating = 2000 - 3T -7P

This can be considered as a mapping between

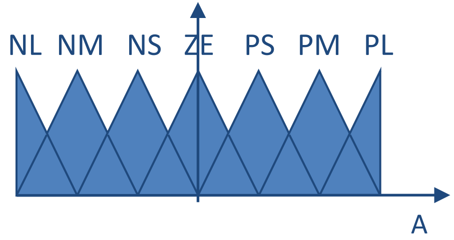
the <u>interpretation</u> of an <u>input</u> configuration (the applicability condition of a model)

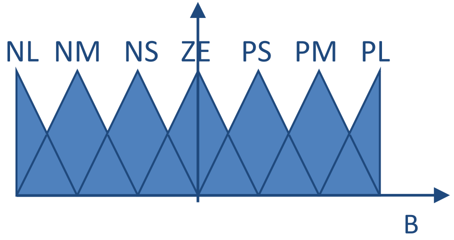
and

a <u>model</u> to be applied to the input real values to obtain the <u>output</u>

How to use fuzzy rules (Mamdani): input and output

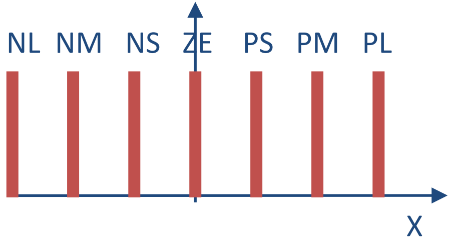
Two input variables: A and B, frame of cognition with equally distributed fuzzy sets (fuzzy partition), from Negative large (NL) to Positive Large (PL).





One output variable: U, equally distributed fuzzy sets from Negative large (NL) to Positive Large (PL).

The fuzzy sets are all singletons, because this makes the computation faster, does not affect the possibility to obtain values in the full range of the output variable, and other reasons defined later.



How to use fuzzy rules (Mamdani): the rules for the example

Usually, we have rules for all the combinations of the possible input values. Here, only 3 rules of the rule base are reported.

The designer can also associate weights to rules, to define the relative relevance of their contribution to the final result.

R1: if A is PL and B is PS Then X is PM

weight 1

R2: if A is PM and B is PS Then X is PS

weight 0.5

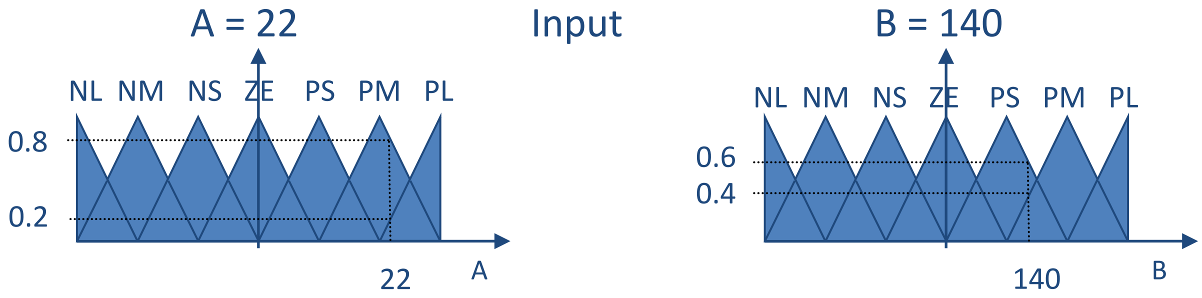
R3: if A is PL and B is PM Then X is PM

weight 1

How to use fuzzy rules (Mamdani): the steps

- Input matching
- Combination of matching degrees
- Combination with rule weight, if present
- Aggregation of output from different rules
- Eventual defuzzyfication of output

Input matching



R1: IF (A is PL) (B is PS) THEN (X is PM)

0.2 0.6

R2: IF (A is PM) (B is PS) THEN (X is PS)

0.8 0.6

R3: IF (A is PM) (B is PM) THEN (X is PM)

0.8 0.4

Combination of matching degrees in the antecedent

R1: IF (A is PL) (B is PS) THEN (X is PM)

$$\underbrace{0.2 \qquad 0.6}_{0.2}$$

R2: IF (A is PM) (B is PS) THEN (X is PS)

$$\underbrace{0.8 \qquad 0.6}_{0.6}$$

R3: IF (A is PM) (B is PM) THEN (X is PM)

$$\underbrace{0.8 \qquad 0.4}_{0.4}$$

We use the **min** operator

The matching degree of the antecedent is a measure of how much the rule fits the current situation

Combination with the rule weight

R1: IF (A is PL) (B is PS) THEN (X is PM)

$\underbrace{\underbrace{0.2 \quad 0.6}_{0.2} \quad 1}_{0.2}$

R2: IF (A is PM) (B is PS) THEN (X is PS)

$\underbrace{\underbrace{0.8 \quad 0.6}_{0.6} \quad 0.5}_{0.5}$

R3: IF (A is PM) (B is PM) THEN (X is PM)

$\underbrace{\underbrace{0.8 \quad 0.4}_{0.4} \quad 1}_{0.4}$

We use the **min** operator

The resulting value is a measure of how much the rule is “good” given the situation and “per se”

Output aggregation

We use the operator **max** to aggregate weights given to the same output value

R1: if (A is PL) (B is PS) Then (X is PM)

0.2

R2: if (A is PM) (B is PS) Then (X is PS)

0.5

R3: if (A is PL) (B is PM) Then (X is PM)

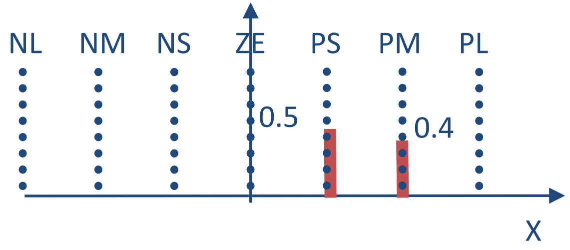
0.4

max

X is PM with weight 0.4

X is PS with weight 0.5

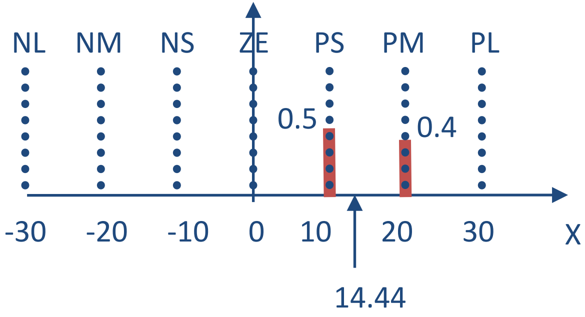
The final weights give the level of cut of the output fuzzy sets



(Eventual, possible) defuzzyfication

The output is given by the fuzzy set composed by the two fuzzy sets identified by the rules, cut at the level provided by the computed weights.

If we would like to have a number, as needed by many applications, we have to defuzzyfy the result.



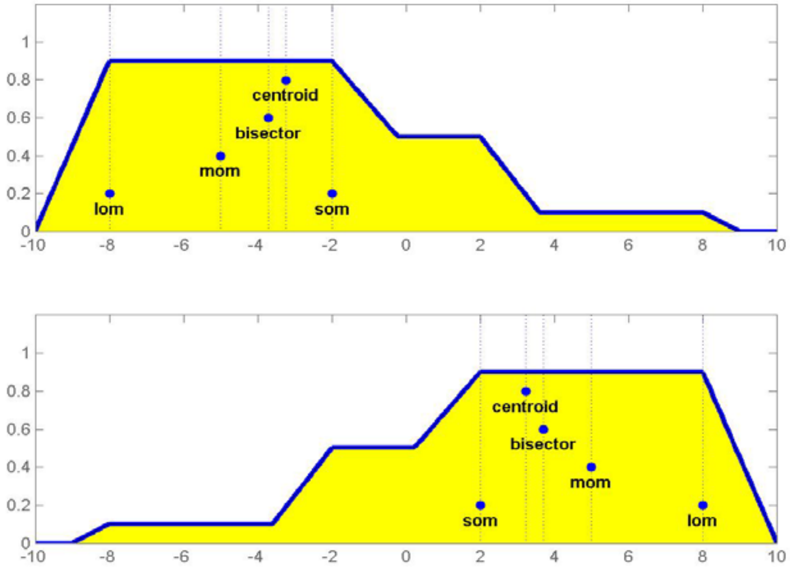
$(10*0.5 + 20*0.4)/(0.5 + 0.4) = \mathbf{14.44}$

We use the operator "weighted media" on the weights of the output values to obtain a numerical value

Defuzzyfication

Also for the defuzzyfication we can have many possible operators to face issues that may arise.
The system will be optimized according to the selected operator.

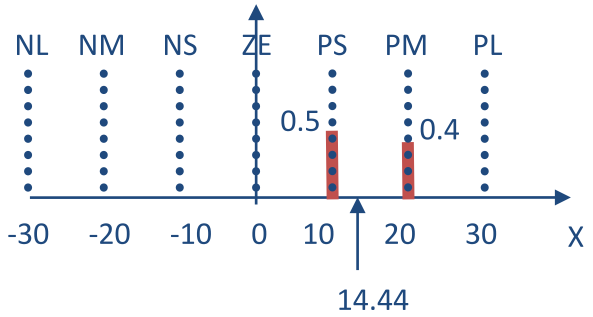
Centroid
Bisector
Average of maxima
Lowest maximum
Highest maximum
Center of the highest area
...



Another possible output: linguistic approximation

X is PM with weight 0.4

X is PS with weight 0.5



(10*0.5 + 20*0.4)/(0.5 + 0.4) = **14.44**

In this case, we would like to have the name of a fuzzy set as an output, e.g. because it corresponds to a decision to take.

The fuzzy set closest to the defuzzyfied output is selected.

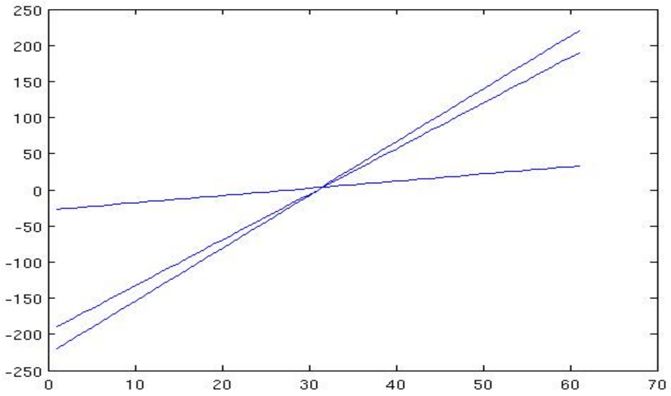
How to use fuzzy rules (Sugeno)

Let’s consider the following fuzzy rules. Inputs are like in the Mamdani example

R1: if A is PL and B is PS Then X is A+2B

R2: if A is PM and B is PS Then X is A +3

R3: if A is PL and B is PM Then X is A+B



The first steps are like in the other example

Output aggregation

R1: if (A is PL) (B is PS) Then (X is A+2B)

0.2

R2: if (A is PM) (B is PS) Then (X is A+3)

0.5

R3: if (A is PL) (B is PM) Then (X is A+B)

0.4

The output is a weighted combination of the models defined in the output of the rules matching the inputs, by using the computed weights

X is (0.2*(A+2B)+0.5*(A+3)+0.4*(A+B))/(0.2+0.5+0.4)

Since A=22 and B=140 then X=125.18

Some exercises

Define the rules to control the light in your room according to the external light coming from a window. What if you can dim a lamp? What if you could operate the shutters?

Define the rules to decide what to do when you see a large, bad-looking dog on your way.

Define the rules to deceive an opponent while bringing the ball in a robot soccer action.

What to remember from these slides?

- Definition of fuzzy rule
- Computation of output from inputs through fuzzy rules
- Defuzzyfication
- Role of operators