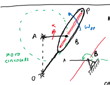
ESERCITAZIONE 4

- CINEMATICA GLIFO
- CINEMATICA QUADRILATERO ARTICOLATO



NOTI:

$OP = 3\,m$

$OA = 1.732\,m$

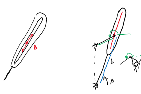
$AB = 1\,m$

$\dot\alpha = 2\ rad/s$

$\ddot\alpha = 0\ rad/s^2$

CALCOLARE

1) ω_{OP}, v_P 2) $\dot\omega_{OP}$, a_P ← per $\alpha = 0$



CALCOLO LA MOBILITA' DEL SISTEMA:

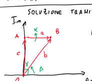
$N°\ CORPI = 2$

$N°\ GDL = N°\ CORPI \cdot 3 = 6$

$N°\ GDL_v = 2\ CERNIERE \times 2\ GDL + 1\ CARRELLO \times 1\ GDL = 5$

$N°\ GDL_r = N°\ GDL - N°\ GDL_v = 6 - 5 = 1$

SOLUZIONE TRAMITE NUMERI COMPLESSI



$(B-O) = (B-A) + (A-O)$

$a e^{i\alpha} + c e^{i\gamma} = b e^{i\beta}$

CHIUSURA CINEMATICA

COSTANTE	a, c, γ
VARIABILE	α, b, β

$\rightarrow a e^{i\alpha} + \underline{ic} = b e^{i\beta}$

$\begin{cases} a\cos\alpha = b\cos\beta \\ a\sin\alpha + c = b\sin\beta \end{cases} \Rightarrow \begin{cases} b = 2\,m \\ \beta = 60° \end{cases}$

VELOCITA'

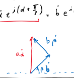
$i a\dot\alpha e^{i\alpha} = \dot b e^{i\beta} + b i\dot\beta e^{i\beta}$

$\begin{cases} -a\dot\alpha\sin\alpha = \dot b\cos\beta - b\dot\beta\sin\beta \\ a\dot\alpha\cos\alpha = \dot b\sin\beta + b\dot\beta\cos\beta \end{cases}$

$\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix} \begin{Bmatrix} \dot b \\ \dot\beta \end{Bmatrix} = \begin{Bmatrix} -a\dot\alpha\sin\alpha \\ a\dot\alpha\cos\alpha \end{Bmatrix}$

$\Rightarrow \begin{Bmatrix} \dot b \\ \dot\beta \end{Bmatrix} = \begin{Bmatrix} 1.73\ m/s \\ 0.5\ rad/s \end{Bmatrix}$

$\underbrace{a\dot\alpha e^{i(\alpha+\frac{\pi}{2})}} = \dot b e^{i\beta} + b\dot\beta e^{i(\beta+\frac{\pi}{2})}$



ACCELERAZIONE

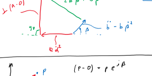
$i a\ddot\alpha e^{i\alpha} - a\dot\alpha^2 e^{i\alpha} = \ddot b e^{i\beta} + i2\dot b\dot\beta e^{i\beta} + bi\ddot\beta e^{i\beta} - b\dot\beta^2 e^{i\beta}$

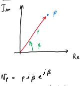
$\begin{cases} -\cancel{a\ddot\alpha\sin\alpha} - a\dot\alpha^2\cos\alpha = \ddot b\cos\beta - 2\dot b\dot\beta\sin\beta - b\ddot\beta\sin\beta - b\dot\beta^2\cos\beta \\ \cancel{a\ddot\alpha\cos\alpha} - a\dot\alpha^2\sin\alpha = \ddot b\sin\beta + 2\dot b\dot\beta\cos\beta + b\ddot\beta\cos\beta - b\dot\beta^2\sin\beta \end{cases}$

$\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix} \begin{Bmatrix} \ddot b \\ \ddot\beta \end{Bmatrix} = \begin{Bmatrix} -a\ddot\alpha\sin\alpha - a\dot\alpha^2\cos\alpha + 2\dot b\dot\beta\sin\beta + b\dot\beta^2\cos\beta \\ a\ddot\alpha\cos\alpha - a\dot\alpha^2\sin\alpha - 2\dot b\dot\beta\cos\beta + b\dot\beta^2\sin\beta \end{Bmatrix}$

$\Rightarrow \begin{cases} \ddot b = -1.5\ m/s^2 \\ \ddot\beta = 0.8667\ rad/s^2 \end{cases}$

$\underbrace{a\ddot\alpha e^{i(\alpha+\frac{\pi}{2})}} + \underbrace{a\dot\alpha^2 e^{i(\alpha+\pi)}} = \underbrace{\ddot b e^{i\beta}} + \underbrace{2\dot b\dot\beta e^{i(\beta+\frac{\pi}{2})}} + \overbrace{b\ddot\beta e^{i(\beta+\frac{\pi}{2})}} + \underbrace{b\dot\beta^2} e^{i(\beta+\pi)}$





$(P-O) = \rho e^{i\beta}$

$\begin{cases} x_P = \rho\cos\beta \\ y_P = \rho\sin\beta \end{cases}$

$\rho \longrightarrow$ COSTANTE

$\dot\beta, \ddot\beta$ NOTI

$v_P = \rho i\dot\beta e^{i\beta}$

$\begin{cases} v_{Px} = -\rho\dot\beta\sin\beta = -1.3\ m/s \\ v_{Py} = \rho\dot\beta\cos\beta = 0.75\ m/s \end{cases}$

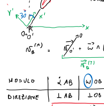
$|v_P| = \sqrt{v_{Px}^2 + v_{Py}^2} = 1.5\ m/s$

$a_P = -\rho\dot\beta^2 e^{i\beta} + i\rho\ddot\beta e^{i\beta}$

$\begin{cases} a_{Px} = -\rho\dot\beta^2\cos\beta - \rho\ddot\beta\sin\beta = -2.63\ m/s^2 \\ a_{Py} = -\rho\dot\beta^2\sin\beta + \rho\ddot\beta\cos\beta = 0.651\ m/s^2 \end{cases}$

$|a_P| = \sqrt{a_{Px}^2 + a_{Py}^2} = 2.71\ m/s^2$

SOLUZIONE TRAMITE MOTI RELATIVI

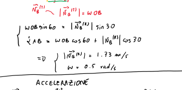


TERNA X'O'Y' ROTANTE

$v_B^{(A)} = \underbrace{\cancel{\vec v_{O'}} + \vec\omega \wedge (B-O)}_{\vec v_B^{(T)}} + \vec v_B^{(R)}$

MODULO	$\dot\alpha AB$	ωOB	?
DIREZIONE	⊥AB	⊥OB	// OB

$\boxed{v_B^{(A)} = \vec v_B^{(T)} + \vec v_B^{(R)}}$



$\begin{cases} \omega OB\sin 60 = |\vec v_B^{(R)}|\sin 30 \\ \dot\alpha AB = \omega OB\cos 60 + |v_B^{(R)}|\cos 30 \end{cases}$

$\Rightarrow \begin{cases} |\vec v_B^{(R)}| = 1.73\ m/s \\ \omega = 0.5\ rad/s \end{cases}$

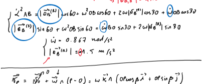
ACCELERAZIONE

$\vec a_B^{(A)} = \cancel{\vec a_{O'}} + \underbrace{\dot\omega \wedge (B-O) - \omega^2(B-O)}_{\vec a_{B,t}^{(T)} + \vec a_{B,n}^{(T)}} + \vec a_B^{(R)} + \underbrace{2\vec\omega \wedge \vec v_B^{(R)}}_{\vec a_B^{(c)}}$

$\vec a_B^{(A)} \rightarrow \vec a_{B,t}^{(A)} + \vec a_{B,n}^{(A)}$

MODULO	$\cancel{\ddot\alpha AB}$ 0	$\dot\alpha^2 AB$	$\dot\omega OB$	$\omega^2 OB$	?	$2\omega v_B$
DIREZIONE	⊥AB	// AB	⊥OB	// OB	// OB	⊥OB

$\cancel{\vec a_{B,t}^{(A)}} + \underbrace{\vec a_{B,n}^{(A)}} = \underbrace{\vec a_{B,t}^{(T)}} + \underbrace{\vec a_{B,n}^{(T)}} + \underbrace{\vec a_B^{(R)}} + \underbrace{\vec a_B^{(c)}}$



$\begin{cases} \dot\alpha^2 AB = |\vec a_B^{(R)}|\cos 60 + \omega^2 OB\cos 60 + 2\omega|\vec v_B^{(R)}|\cos 30 + \dot\omega OB\cos 30 \\ |\vec a_B^{(R)}|\sin 60 + \omega^2 OB\sin 60 = \dot\omega OB\sin 30 + 2\omega|\vec v_B^{(R)}|\sin 30 \end{cases}$

$\begin{cases} \dot\omega = 0.867\ rad/s^2 \\ |\vec a_B^{(R)}| = -1.5\ m/s^2 \end{cases}$

$\vec v_P = \cancel{\vec v_O} + \vec\omega \wedge (P-O) = \omega\vec k \wedge (OP\cos\beta\,\vec i + OP\sin\beta\,\vec j)$

$= \omega OP\cos\beta\,\vec j - \omega OP\sin\beta\,\vec i$

$= -1.3\,\vec i + 0.75\,\vec j\ [m/s]$

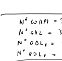
$\vec a_P = \cancel{\vec a_O} + \dot{\vec\omega} \wedge (P-O) - \omega^2(P-O) =$

$= \dot\omega\vec k \wedge (OP\cos\beta\,\vec i + OP\sin\beta\,\vec j) - \omega^2(OP\cos\beta\,\vec i + OP\sin\beta\,\vec j)$

$= (\dot\omega OP\cos\beta - \omega^2 OP\sin\beta)\,\vec j + (-\dot\omega OP\sin\beta - \omega^2 OP\cos\beta)\,\vec i =$

$= -2.63\,\vec i + 0.651\,\vec j\ [m/s^2]$

CINEMATICA DEL QUADRILATERO ARTICOLATO



NOTA LA GEOMETRIA

$\begin{cases} OA = 1\,m \\ AB = 2.35\,m \\ O'B = 2\,m \\ OO' = 1.41\,m \end{cases}$ $\begin{cases} \beta = 6.08\ rad \\ \alpha = 0\ rad \\ \gamma = 0.536\ rad \\ \delta = 0.785\ rad \end{cases}$

$\dot\beta = 0.5\ rad/s$

$\ddot\beta = 0.1\ rad/s^2$

DETERMINARE

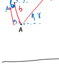
1) ω_{AB} $\omega_{O'B}$ 2) $\dot\omega_{AB}$ $\dot\omega_{O'B}$

$N°\ CORPI = 3$

$N°\ GDL = 3 \times 3 = 9$

$N°\ GDL_v = 4 \times 2 = 8$

$N°\ GDL_r = 9 - 8 = 1$



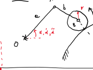
$d e^{i\delta} + a e^{i\alpha} = b e^{i\beta} + c e^{i\gamma}$

COSTANTE	a, b, c, d, δ
VARIABILE	α, β, γ

$\begin{cases} \dot\alpha = 0.33\ rad/s \\ \dot\gamma = 0.0833\ rad/s \end{cases}$

$\begin{cases} \ddot\alpha = 0.052\ rad/s^2 \\ \ddot\gamma = 0.0174 \end{cases}$

ESE 1



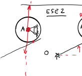
DATI

$\alpha, \dot\alpha, \ddot\alpha$

CALCOLARE

$\vec\omega, \dot{\vec\omega}$

ESE 2



DATI $\alpha, \dot\alpha, \ddot\alpha$

CALCOLARE $\vec v_A$, $\vec a_A$