

06/04/2020

E4) Dato il SD LTI a TD SISO

$$\begin{cases} x(k) = \begin{bmatrix} -1 & 0,5 \\ 2 & -1 \end{bmatrix} x(k-1) + \begin{bmatrix} 0,5 \\ 1 \end{bmatrix} u(k-1) \\ y(k) = \begin{bmatrix} 2 & 1 \end{bmatrix} x(k) + u(k) \end{cases}$$

1) e' SP?

2) e' AS/S/I?

3) primi 3 valori di y per $x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ e $u(k) = \cos(k) + 3 \sin(k)$
 \uparrow
 $k=0,1,2$

1) No perché compare nell'eq. d'uscita

2) Autovalori di A:

$$\det(sI - A) = 0$$

$$\det \begin{bmatrix} s+1 & -0.5 \\ -2 & s+1 \end{bmatrix} = 0$$

$$s^2 + 2s = 0 \quad s = \begin{cases} 0 \\ -2 \end{cases}$$

Autovalore con MODULO > 1 \Rightarrow sistema !

SIAMO A TD!!!

$$3) \quad U(k) = \text{scd}(k) + 3 \text{ber}(k)$$

k	0	1	2
scd(k)	1	1	1
ber(k)	0	1	2
3 ber(k)	0	3	6

U(k)	1	4	7
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$k=0$)

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y(0) = \begin{bmatrix} 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \overset{\downarrow u(0)}{1} = 3$$

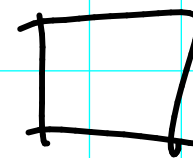
$k=1$)

$$x(1) = \begin{bmatrix} -1 & 0.5 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 1 \end{bmatrix} 1 = \begin{bmatrix} -0.5 \\ 3 \end{bmatrix}$$

$$y(1) = \overset{A}{\begin{bmatrix} 2 & 1 \end{bmatrix}} \overset{x(0)}{\begin{bmatrix} -0.5 \\ 3 \end{bmatrix}} + \overset{b}{4} \overset{u(0)}{1} = 6$$

$k=2$)

...



E5] Dato

$$\begin{cases} \dot{x} = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u \\ y = \begin{bmatrix} 0 & 2 \end{bmatrix} x \end{cases}$$

- 1) SP?
- 2) AS/S/I?
- 3) FOT?
- 4) Regg? Oss?
- 5) $y(t)$ per
 $x(0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$ e
 $u(t) = \sin(t)$

1) Si perché $d=0$

2) AS: A triangolare inf, autovalori -1 e -2 , sempre con $\text{Re} < 0$

$$3) \quad G(s) = c (sI - A)^{-1} b + d$$

$$= \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} s+1 & 0 \\ -1 & s+2 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)} \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} s+2 & 0 \\ 1 & s+1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \frac{1}{1} \begin{bmatrix} 2 & s+1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{2}{(s+1)(s+2)}$$

4) È RZO perché nel calcolo di $G(s)$ non vi sono state cancellazioni.

5) ML con e^{At} e TF con $G(s)$

- Autovalori di A : $s_1 = -1$, $s_2 = -2$

- Autovettori

$s_1)$ $Az = -z$

$$\begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = - \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$\begin{cases} -z_1 = -z_1 \\ z_1 - 2z_2 = -z_2 \end{cases}$$

$$\begin{cases} z_1 = z_2 \\ \forall z_1 \end{cases}$$

$$\Rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$S_2) \quad Az = -2z$$

$$\begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = -2 \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$$

$$-z_1 = -2z_1 \Rightarrow \begin{cases} z_1 = 0 \\ \forall z_2 \end{cases} \Rightarrow \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

• Π . diagonalisierbar

$$T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \Rightarrow T^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$\text{NB } T^{-1}AT = \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$$

$$\bullet \quad e^{At} = e^{T^{-1} \begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} T^{-1} t} = T e^{\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix} t} T^{-1}$$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} e^{-t} & 0 \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ e^{-t} & e^{-2t} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ e^{-t} - e^{-2t} & e^{-2t} \end{bmatrix}$$

• ML di x e di y

$$x(t) = e^{At} x(0) = \begin{bmatrix} e^{-t} & 0 \\ e^{-t} - e^{-2t} & e^{-2t} \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 2e^{-2t} \end{bmatrix}$$

$$y_L(t) = C x_L(t) = \begin{bmatrix} 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2e^{-2t} \end{bmatrix} = 4e^{-2t}, t \geq 0$$

equiv. \rightarrow
 opp. $4e^{-2t} \sin(t)$

• TF di y

$$Y_F(s) = G(s) U(s) = \frac{2}{(s+1)(s+2)} \cdot \frac{1}{s}$$

$u(t) = \text{scz}(t)$

Herzviside:

$$Y_F(s) = \frac{\alpha}{s} + \frac{\beta}{s+1} + \frac{\gamma}{s+2}$$

$$\alpha(s+1)(s+2) + \beta s(s+2) + \gamma s(s+1) = 2$$

$$s=0 \Rightarrow 2\alpha = 2 \Rightarrow \alpha = 1$$

$$s=-1 \Rightarrow -\beta = 2 \Rightarrow \beta = -2$$

$$s=-2 \Rightarrow 2\gamma = 2 \Rightarrow \gamma = 1$$

$$Y_F(s) = \frac{1}{s} - \frac{2}{s+1} + \frac{1}{s+2}$$

$\downarrow \mathcal{L}^{-1}$

$$\begin{aligned} y_F(t) &= \cos(t) - 2e^{-t} \cos(t) + e^{-2t} \cos(t) \\ &= (1 - 2e^{-t} + e^{-2t}) \cos(t) \end{aligned}$$

Terminato complessivo:

$$y(t) = y_L(t) + y_F(t) = 4e^{-2t} \cos(t) + \cos(t)$$

$$= (1 - 2e^{-t} + 5e^{-2t}) \cos(t) \quad \square$$

Alternative: Scrittura con TDL e trasformata

$$\begin{cases} \dot{x} = Ax + bU \\ y = cx + dU \end{cases}$$

$$sX - x(0) = AX + bU$$

$$(sI - A)X = x(0) + bU$$

$$X(s) = (sI - A)^{-1} (x(0) + bU)$$

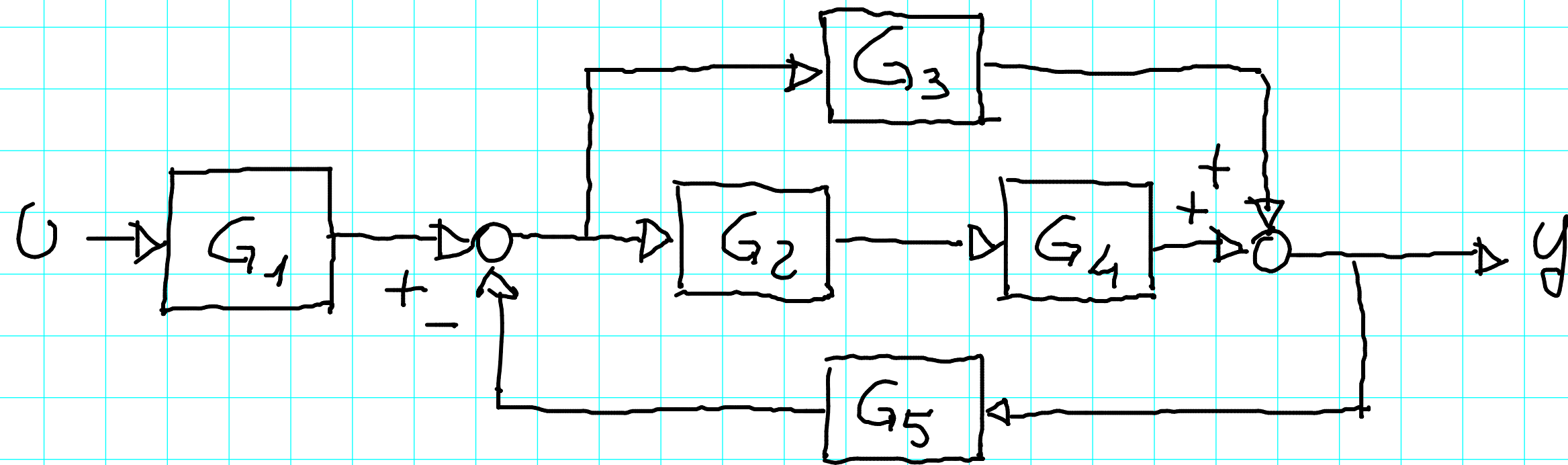
$$Y(s) = c(sI - A)^{-1} (x(0) + bU) + dU$$

$$= c(sI - A)^{-1} x(0) + G(s) U(s)$$

\Rightarrow Teorema

□

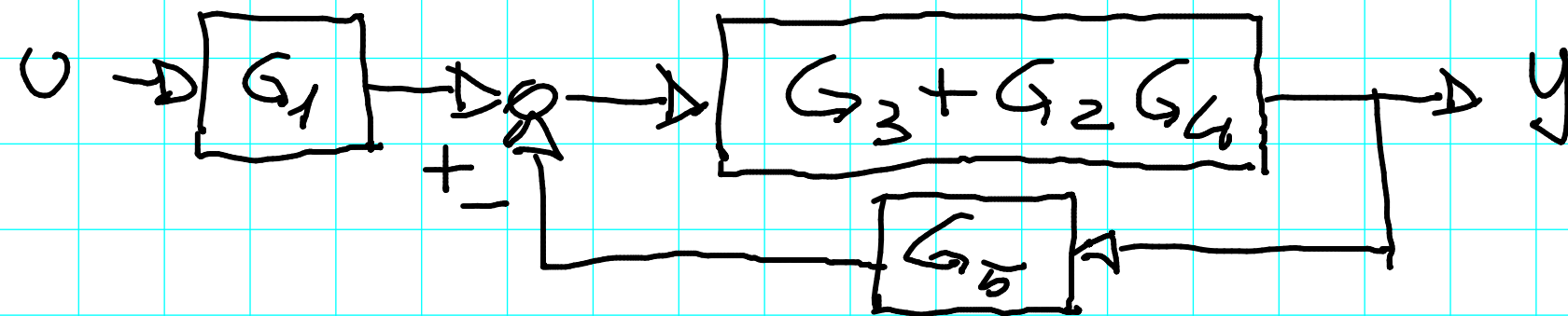
E6] Dato lo schema a blocchi



1) Esprimere la FdT $G(s) = \frac{Y(s)}{U(s)}$ in funzione di $G_1(s) \dots G_5(s)$

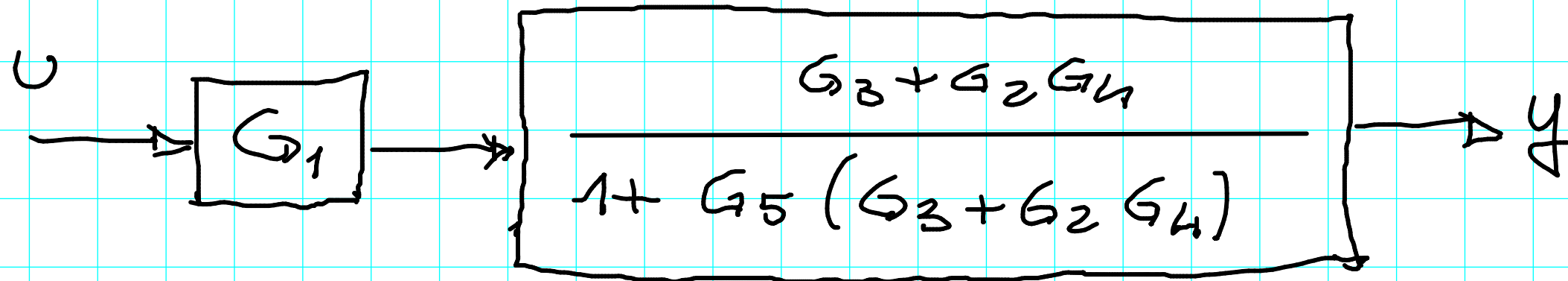
2) Dire se la stabilità (asintotica) di polo dei blocchi $G_1 \dots G_5$ è necessaria e/o sufficiente per quella del sistema complessivo

- 1) • Serie di G_2 e G_4 in parallelo a G_3



- Anello di retroazione negativa

$$G = \frac{\text{Aperto}}{1 + \text{Anello}}$$



- Serie con G_1

$$G(s) = \frac{Y(s)}{U(s)} = \frac{G_1(G_3 + G_2G_4)}{1 + G_5(G_3 + G_2G_4)}$$

2) E' necessario b stab. (as.) di G_1 in presenza il blocco non e' parte di alcun anello

Verifica:

$$G_i = \frac{N_i}{D_i} \quad N_i, D_i \text{ polinomi}$$

$$G(s) = \frac{\frac{N_1}{D_1} \left(\frac{N_3}{D_3} + \frac{N_2}{D_2} \frac{N_4}{D_4} \right)}{1 + \frac{N_5}{D_5} \left(\frac{N_3}{D_3} + \frac{N_2}{D_2} \frac{N_4}{D_4} \right)} = \frac{N_1}{D_1} \frac{\frac{N_3 D_2 D_4 + N_2 N_4 D_3}{D_2 D_3 D_4}}{1 + \frac{N_5 (N_3 D_2 D_4 + N_2 N_4 D_3)}{D_5 D_2 D_3 D_4}}$$

$$= \frac{N_1 D_5 (N_3 D_2 D_4 + N_2 N_4 D_3)}{D_1 (D_2 D_3 D_4 D_5 + N_5 (N_3 D_2 D_4 + N_2 N_4 D_3))} \quad \square$$

E7) Dato il SLD NL a TC SISO

$$\begin{cases} \dot{x}_1 = x_1 + x_1^2 u \\ \dot{x}_2 = x_1 - x_2 u \\ y = 2x_1 x_2 \end{cases}$$

1) π ?

2) $\Sigma \Phi$?

3) \bar{x} e \bar{y} per $u(t) = \bar{u} = 2$?

4) stab. equilibri?

-
- 1) Σ , il tempo non compare esplicitamente in f e g
2) Σ , g non dipende da u

3) Calcolo \overline{x}

$$\begin{cases} 0 = \overline{x}_1 + \overline{x}_1^2 \overline{v} \\ 0 = \overline{x}_1 - \overline{x}_2 \overline{v} \end{cases} \Rightarrow \begin{cases} \overline{x}_1 (1 + \overline{x}_1 \overline{v}) = 0 \Rightarrow \overline{x}_1 \begin{cases} 0 \\ -1/\overline{v} \end{cases} \\ \overline{x}_2 = \overline{x}_1 / \overline{v} \end{cases}$$

Con $\overline{v} = 2$ vi sono i due stati di equilibrio

$$\overline{x}_a = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\overline{x}_b = \begin{bmatrix} -1/2 \\ -1/4 \end{bmatrix}$$

e in corrispondenza essendo $\overline{y} = 2 \overline{x}_1 \overline{x}_2$

$$\overline{y}_a = 0$$

$$\overline{y}_b = 1/4$$

4) Osservo la m. dinamica f_n del sist. linearizzato

$$f_n = \begin{bmatrix} 1 + 2x_1 v & 0 \\ 1 & -v \end{bmatrix}$$

$$\text{eq. a: } x_1 = x_2 = 0 \quad v = 2 \Rightarrow f_n|_{\text{eq. a}} = \begin{bmatrix} 1 & 0 \\ 1 & -2 \end{bmatrix}$$

Autovale in 1 cioè con $\text{Re} > 0 \Rightarrow$ equilibrio !

$$\text{eq. b: } x_1 = -1/2, x_2 = -1/4, v = 2$$

$$f_n|_{\text{eq. b}} = \begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix}$$

Autovalei -1 e -2
autovale con $\text{Re} < 0$
 \Rightarrow equilibrio AS \square

E8

Dato

$$\begin{cases} \dot{x} = \begin{bmatrix} -1 & 1 \\ -1 & 2 \end{bmatrix} x + \begin{bmatrix} -3 \\ -3 \end{bmatrix} u \\ y = \begin{bmatrix} -2 & 1 \end{bmatrix} x \end{cases}$$

1) AS/S/?

2) $G(s)$?

3) $R \neq 0$?

4) Schema = blocchi equivalenti

1) Autovalori di A

$$\det(sI - A) = 0$$

$$\det \begin{bmatrix} s+11 & -9 \\ 12 & s-10 \end{bmatrix} = 0$$

$$s^2 + s - 2 = 0 \quad \Rightarrow \quad 2^\circ \text{ grado e 1 variabile separata } \Rightarrow \quad |$$

$$\text{Autovalori: } s = \frac{-1 \pm \sqrt{1+8}}{2} = \begin{matrix} -2 \\ 1 \end{matrix}$$

$$2) \quad G(s) = c (sI - A)^{-1} b + d$$

$$= [-2 \ 1] \begin{bmatrix} s+11 & -9 \\ 12 & s-10 \end{bmatrix}^{-1} \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \frac{1}{(s+2)(s-1)} [-2 \ 1] \begin{bmatrix} s-10 & 9 \\ -12 & s+11 \end{bmatrix} \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \frac{1}{1} [-2s+8 \quad s-7] \begin{bmatrix} -3 \\ -3 \end{bmatrix}$$

$$= \frac{6s - 24 - 3s + 21}{1} = \frac{3\cancel{(s-1)}}{(s+2)\cancel{(s-1)}} = \frac{3}{s+2}$$

cancel the same \Rightarrow na RDP

3) Reggiungibilite'

$$M_R = [b \quad Ab] = \begin{bmatrix} -3 & 6 \\ -3 & 6 \end{bmatrix} \quad \text{Singolare} \Rightarrow \text{NR}$$

Osservabilita'

$$M_o = [c' \quad A'c'] = \begin{bmatrix} -2 & 10 \\ 1 & -8 \end{bmatrix} \quad \text{non sing.} \Rightarrow \text{O}$$

4) Schenke a block hi

