ESERCITAZIONE 3

$A \longrightarrow$ lancio x

$B \longrightarrow$ lancio y

NOTI:

$AB = 300\ m$

$v_A = 216.7\ m/s$

$a_A = 5\ m/s$

$a_B = 0.3\ m/s^2$

1) $\vec{v}_A, \vec{a}_A$ 2) $\vec{v}_B, \vec{a}_B$

SOLUZIONE TRAMITE MANICI CARTESIANI

NB SCEGLI I ANGOLI!

$(B-O) + (B-A) = (A-O)$

$x e^{i\theta} + b e^{i\theta} = a e^{i\alpha}$

$\xi = \frac{b}{x} \qquad b = \xi x \qquad x > 0 \qquad a = x_A$

CONTATTO x, b, θ

VINCOLI ① ②

$x \in b e^{i\theta} = a$

① $\begin{cases} a - b\cos\theta = a \\ -b\sin\theta = 0 \end{cases} \Rightarrow \begin{cases} b = \cos\theta\left(-\frac{\pi}{2}\right) = 180° \\ \theta = 0 \end{cases}$

② $b = x_A \qquad \Rightarrow x = b\cos\theta_A = 350\ m$

VELOCITA'

$\dot{x} \in = \dot{\rho} b e^{i\theta} + i\dot{\theta} b$

$\dot{x} = \dot{\rho} b e^{i\theta} + i\dot{\theta} b$

① $\begin{cases} \dot{a} - \dot{b}\cos\theta = 0 \\ -\dot{b}\sin\theta = 0 \end{cases}$

② $\dot{b} = \dot{x}_A = \dot{a}$

$\Rightarrow \begin{cases} \dot{b} \\ \dot{\theta} \end{cases} = \begin{bmatrix} \cos\theta & -b\sin\theta \\ \sin\theta & b\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} \dot{a} \\ 0 \end{bmatrix}$

$[A]^{-1} = \frac{[A^*]^T}{\det(A)} \qquad \det(A) = b\cos\theta$

$[A^*] = \begin{bmatrix} 1 & \frac{b\cos\theta}{b} \\ 0 & \frac{b\cos\theta}{\cos\theta} \end{bmatrix} \qquad [A^*]^T = \begin{bmatrix} 1 & 0 \\ b\cos\theta & b\cos\theta \end{bmatrix}$

$[A]^{-1} = \begin{bmatrix} 1 & 0 \\ \frac{-b\sin\theta}{b\cos\theta} & 1 \end{bmatrix} = \frac{[A^*]^T}{\det(A)}$

$\begin{bmatrix} \dot{b} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{-\sin\theta}{\cos\theta} & 1 \end{bmatrix} \begin{bmatrix} \dot{a} \\ 0 \end{bmatrix} \Rightarrow \begin{cases} \dot{b} = 14.78\ m/s \\ \dot{\theta} = -5.66\ m/s \end{cases}$

$\dot{a} = -b\dot{\theta}\sin\theta = \dot{x} \Rightarrow \dot{\theta} = \frac{\dot{a}}{b\sin\theta}$

$\dot{x} = \dot{\rho} b e^{i\theta} + i\dot{\theta} b \qquad \dot{x} = \frac{\dot{a}}{\sin\theta}$

$\dot{x} = 0$

$\dot{x} > 0$

ACCELERAZIONE $(\ddot{x} \in = \ddot{\rho} b e^{i\theta} + i\dot{\theta} b)$

$\ddot{x} \in = \ddot{\rho} b e^{i\theta} - b\dot{\theta}^2 e^{i\theta} + i\ddot{\theta} b$

$\ddot{x} = \ddot{\rho} b e^{i\theta} - b\dot{\theta}^2 \cos\theta + i\ddot{\theta} b$

$\ddot{a} = \ddot{\rho} b\cos\theta - b\dot{\theta}^2\cos\theta$

$\ddot{a} = \ddot{\rho} b\sin\theta + \ddot{\theta} b$

$\begin{bmatrix} b\cos\theta & 0 \\ -b\sin\theta & b \end{bmatrix} \begin{bmatrix} \ddot{b} \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} -b\dot{\theta}^2\cos\theta + \ddot{a} \\ -b\dot{\theta}^2\sin\theta \end{bmatrix}$

$\Rightarrow \begin{cases} \ddot{b} = 314.2\ m/s^2 \\ \ddot{\theta} = -238.7\ m/s^2 \end{cases}$

$(B-O) = (B-A) + (A-O)$

$= y e^{i\theta} + a$

$x e^{i\theta} = y e^{i\theta} + a$

$x = y\cos\theta + a$

$y = y\sin\theta$

$\xi = \frac{AB}{y}$

VELOCITA' $\vec{v}_B = \dot{y} e^{i\theta} + i\dot{\theta} y e^{i\theta}$

① $\dot{x} = -\dot{y}\sin\theta + \dot{\theta} y\cos\theta = 0.5\ m/s$

② $\dot{y} = \dot{y}\cos\theta - \dot{\theta} y\sin\theta = -1.93\ m/s$

$\vec{v}_B = \dot{x}_B \vec{i} + \dot{y}_B \vec{j} = -2.5\ \vec{i} - 4.93\ \vec{j}\ [m/s]$

$|\vec{v}_B| = \sqrt{\dot{x}_B^2 + \dot{y}_B^2} = 5\ m/s$

$a_B = \ddot{y} e^{i\theta} - \dot{\theta}^2 y e^{i\theta} + i\ddot{\theta} y e^{i\theta}$

$\ddot{x} = -\ddot{y}\sin\theta - \dot{\theta} y\cos\theta - \dot{\theta} y\cos\theta = 0.01\ m/s$

$\ddot{y} = \ddot{y}\cos\theta - \dot{\theta} y\sin\theta - \dot{\theta} y\sin\theta = -1.93\ m/s$

$\vec{a}_B = \ddot{x}_B \vec{i} + \ddot{y}_B \vec{j} = 0.01\ \vec{i} - 1.93\ \vec{j}\ [m/s]$

$|\vec{a}_B| = \sqrt{\ddot{x}_B^2 + \ddot{y}_B^2} = 1.93\ m/s$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad \vec{a}_A = 0$

$\vec{a}_{B/A} = (\ddot{r} - r\dot{\theta}^2)\vec{i} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{j}$

$\vec{a}_B = \vec{a}_{B/A} = \vec{a}_{B/A}(-\cos\theta\vec{i} - \sin\theta\vec{j})$

$\vec{a}_B = \vec{a}_B(-\vec{i}) = \vec{a}_B\left(-\cos\left(-\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\vec{j}\right)$

$= -4.93\ \vec{j} = 2.5\ \vec{i} - 4.93\ \vec{j}\ [m/s]$

SOLUZIONE CON TERNE MOBILI

x' y' TERNA TRASLANTE

$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} = \vec{v}_A + \vec{\omega}(B-A)$

$\vec{v}_B$ MOBILE — $\vec{v}_B(t) = \vec{v}_A(t) + \vec{\omega}(t)\times(B-A)$ — $\vec{v}_B(t)$ ASSOLUTA

VELOCITA' IN TRASLAZIONE

$\vec{v}_B(t) = \vec{v}_A(t)$

$\vec{v}_B(t) = \vec{v}_A(t) + \vec{\omega}(t)\times(B-A)$

MODULO	①	$\lvert v_A \rvert$	$\lvert \omega \rvert$	$\lvert AB \rvert$
DIREZIONE	$\vec{v}_A$	$\vec{v}_A$	$\vec{\omega}$	$\vec{AB}$

$\vec{v}_A = \vec{v}_A(t) = \vec{v}_A(t)\vec{i}$

$\vec{\omega} = \vec{\omega}(t) = \vec{\omega}(t)\vec{k}$

$\vec{AB} = \vec{AB}(t) = \vec{AB}(t)\vec{j}$

$|\vec{AB}(t)| = \sqrt{\vec{AB}^2(t)} = \sqrt{\frac{300^2}{3/2}} = 245\ m$

$|\vec{v}_B(t)| = \sqrt{|\vec{v}_A(t)|^2 + |\vec{v}_{B/A}(t)|^2} = 12\ \frac{m}{s}$

$\vec{v}_B(t) = -5.66\ \vec{i}\ [m/s] \qquad \vec{v}_B = 14.78\ \vec{i}\ [m/s]$

$\frac{AB}{2} = 0.95\ m$

$\vec{\omega}\vec{AB} = \vec{v}_A = \vec{\omega}\times(B-A)$

NOTO $\vec{v}$ $|v|$ ω

NOTO $\vec{\omega}$ $|\omega|$ v

$\vec{\omega}\vec{AB} = \vec{v}_A \Rightarrow |\omega|\,|AB|\cos\theta = |v_A|\cos\theta$

$= (-2.5)\vec{i} - 4.93\ \vec{j} = 2.5\ \vec{i} - 4.93\ \vec{j}\ [m/s]$

ACCELERAZIONE

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} + \vec{\omega}\times(\vec{\omega}\times(B-A)) + 2\vec{\omega}\times\vec{v}_{B/A}$

NOTO $\vec{v}$ — $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

NOTO $\vec{\omega}$ — $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} + \vec{\omega}\times(\vec{\omega}\times(B-A)) + 2\vec{\omega}\times\vec{v}_{B/A}$

TRASLAZIONE

$\vec{a}_B(t) = \vec{a}_A(t)$

ROTAZIONE

$\vec{a}_B(t) = \vec{a}_A(t) + \vec{\omega}(t)\times(\vec{\omega}(t)\times(B-A)) + 2\vec{\omega}(t)\times\vec{v}_{B/A}(t)$

MODULO	①	$\lvert a_A \rvert$	$\lvert \omega \rvert$	$\lvert \omega A \rvert$
DIREZIONE	$\vec{a}_A$	$\vec{a}_A$	$\vec{\omega}$	$\vec{\omega A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} + \vec{a}_{Cor}$

(1) $\omega^2 A\cos\alpha = a_A \Rightarrow \omega = \sqrt{\frac{a_A}{A\cos\alpha}}$

$\vec{\omega} = \frac{\omega^2 A\sin\alpha + a_A}{A\cos\alpha} = 3.5\ \frac{rad}{s^2} \Rightarrow \vec{\alpha}$

(2) $|v_{B/A}| = \omega^2 A\sin\alpha + a_A\cos\alpha = 2.5\ m/s$

$\vec{a}_B = -2.5\ \vec{i}\ [m/s]$

TERNA DI RIFERIMENTO

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} + \vec{a}_{Cor}$

MODULO	①	$\lvert a_A \rvert$	$\lvert a_{B/A} \rvert$	$\lvert a_{Cor} \rvert$
DIREZIONE	$\vec{a}_A$	$\vec{a}_A$	$\vec{a}_{B/A}$	$\vec{a}_{Cor}$

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} + \vec{a}_{Cor}$

(1) $|a_{Cor}| = a_A = \omega^2 A\sin\alpha \Rightarrow \frac{a_A}{A\sin\alpha} = 0.157\ \frac{rad}{s^2}$

(2) $|v_{B/A}| = \omega^2 A\cos\alpha = \frac{a_A}{A\sin\alpha}\cos\alpha = 0.157\ \frac{m}{s}$

$\vec{a}_B = 0.157\ \vec{i} - 1.93\ \vec{j}\ [m/s^2]$