ESERCITAZIONE 1

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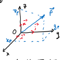
ESERCITAZIONI: GIOVEDÍ h. 16:30

- RICHIAMI DI CALCOLO VETTORIALE
- CINEMATICA DEL PUNTO

GRANDEZZE
- SCALARE → densità ρ, massa m, volume V, Energia E
- VETTORIALE
  - CINEMATICHE: velocità v, accelerazione a
  - DINAMICHE: Forze F, Coppie C

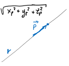
Definizione VETTORE $\in \mathbb{R}^3$



$\vec{P} = (P-O) = x_P \vec{i} + y_P \vec{j} + z_P \vec{k}$

$\vec{i} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \quad \vec{j} = \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \quad \vec{k} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$

UN VETTORE É DEFINITO DA:
- MODULO $|\vec{P}| = P = \sqrt{x_P^2 + y_P^2 + z_P^2}$
- DIREZIONE r
- VERSO (+/-)
- PUNTO DI APPLICAZIONE



OSS $|\vec{i}| = 1 = \sqrt{1^2 + 0 + 0} = 1$

OPERAZIONI TRA VETTORI

PRODOTTO SCALARE $c = \vec{a} \times \vec{b}$ SCALARE

$c = |a||b| \cos\alpha = a\, b \cos\alpha$

ESEMPIO

$W = \vec{F} \times \vec{v} = |F||v| \cos\gamma$

$\gamma = 90 - \alpha$

$W = |F||v| \sin\alpha$

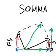
$F_{//} = F \sin\alpha$

$\vec{F} = 3\vec{i} + 2\vec{j}$ [N]

$\vec{v} = -\vec{i} + 4\vec{j}$ [m/s]

$W = (3\vec{i} + 2\vec{j}) \times (-\vec{i} + 4\vec{j}) = -3 + 0 + 0 + 8 = 5\ W$

SOMMA DI VETTORI

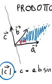


$\vec{a} + \vec{b} = \vec{c}$

$\vec{c} = (a_x \vec{i} + a_y \vec{j}) + (b_x \vec{i} + b_y \vec{j}) = c_x \vec{i} + c_y \vec{j}$

PRODOTTO VETTORIALE

$\vec{c} = \vec{a} \wedge \vec{b}$ VETTORE



$|\vec{c}| = c = a b \sin\alpha$

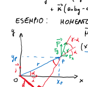
$\vec{c} = \vec{a} \wedge \vec{b} = \det \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} =$

$= \vec{i}(a_y b_z - b_y a_z) - \vec{j}(a_x b_z - a_z b_x) + \vec{k}(a_x b_y - b_x a_y)$

$\vec{c} = \vec{a} \wedge \vec{b} = (a_x \vec{i} + a_y \vec{j} + a_z \vec{k}) \wedge (b_x \vec{i} + b_y \vec{j} + b_z \vec{k}) =$

$= a_x b_y \vec{k} - a_x b_z \vec{j} - a_y b_x \vec{k} + a_y b_z \vec{i} + a_z b_x \vec{j} - a_z b_y \vec{i} = \vec{i}(a_y b_z - a_z b_y) - \vec{j}(a_x b_z - a_z b_x) + \vec{k}(a_x b_y - a_y b_x)$

ESEMPIO: MOMENTO DI UNA FORZA



$\vec{M} = (P-O) \wedge \vec{F} = (x_P \vec{i} + y_P \vec{j}) \wedge (F_x \vec{i} + F_y \vec{j})$

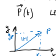
$\begin{cases} x_P = r\cos\alpha \\ y_P = r\sin\alpha \end{cases} \quad \begin{cases} F_x = F\cos\gamma \\ F_y = F\sin\gamma \end{cases}$

$\vec{M} = \vec{k}(rF\cos\alpha \sin\gamma - rF\sin\alpha\cos\gamma) = \vec{k}(-rF\sin(\alpha-\gamma)) = \vec{k}\, rF\sin(\gamma-\alpha) =$

$\vec{M} = F d \vec{k}$

CINEMATICA DEL PUNTO

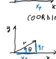
$\vec{P}(t)$ LEGGE ORARIA



$(P-O) = x_P \vec{i} + y_P \vec{j}$

$|P-O| = \sqrt{x_P^2 + y_P^2} = r$

COORDINATE CARTESIANE



$(P-O) = r\cos\theta\, \vec{i} + r\sin\theta\, \vec{j}$

$tg(\theta) = \frac{y_P}{x_P}$

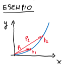
COORDINATE POLARI

TRAIETTORIA $y(x)$ CURVA PERCORSA DA $P(t)$ DURANTE IL MOTO

$P(t) = (P-O) = x(t)\vec{i} + y(t)\vec{j}$

$\begin{cases} x(t) \\ y(t) \end{cases} \longrightarrow y(x)$ TRAIETTORIA

ESEMPIO

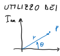


$\begin{cases} x(t) = 3t \\ y(t) = 5t^2 \end{cases}$ FORMA PARAMETRICA

$t = \frac{x}{3} \rightarrow y = 5\left(\frac{x}{3}\right)^2 = \frac{5}{9}x^2$

$\begin{cases} \vec{P}_1 = \vec{P}(t_1) \\ \vec{P}_2 = \vec{P}(t_2) \end{cases}$

UTILIZZO DEI NUMERI COMPLESSI



$\vec{P} = r e^{j\theta}$ $\quad j = \sqrt{-1}$

FORMA ESPONENZIALE

$\vec{P} = r\cos\theta + j r\sin\theta$

FORMA TRIGONOMETRICA

Re, Im ⟷ x, y

RELAZIONI DI EULERO
$e^{j\theta} = \cos\theta + j\sin\theta$

$e^{j0} = 1;\ e^{j\frac{\pi}{2}} = j;\ e^{j\pi} = -1$

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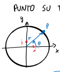
1) $\vec{P} = (P-O)$ POSIZIONE
2) $\vec{v} = \frac{d(P-O)}{dt}$ VELOCITÁ
3) $\vec{a} = \frac{d\vec{v}}{dt}$ ACCELERAZIONE

PUNTO SU TRAIETTORIA CIRCOLARE



$\begin{cases} r \text{ COSTANTE} \\ \theta(t) \text{ VARIABILE E NOTO} \end{cases}$

1) $\vec{v}_P$
2) $\vec{a}_P$

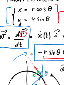
$\vec{P} = (P-O) = x(t)\vec{i} + y(t)\vec{j} = x(\theta(t))\vec{i} + y(\theta(t))\vec{j}$

FORMA PARAMETRICA
$\begin{cases} x = r\cos\theta \\ y = r\sin\theta \end{cases}$

FORMA VETTORIALE
$(P-O) = r\cos\theta\,\vec{i} + r\sin\theta\,\vec{j}$

$\vec{v} = \frac{d\vec{P}}{dt} = \dot{x}(t)\vec{i} + \dot{y}(t)\vec{j} = \frac{\partial x}{\partial\theta}\frac{\partial\theta}{\partial t}\vec{i} + \frac{\partial y}{\partial\theta}\frac{\partial\theta}{\partial t}\vec{j} =$

$= -r\sin\theta\,\dot\theta\,\vec{i} + r\cos\theta\,\dot\theta\,\vec{j} = v_x\vec{i} + v_y\vec{j}$



$\vec{v} = r\dot\theta(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j})$

$|v|$ MODULO; $\vec{t}$ VERSORE TANGENTE $\frac{\vec{v}}{|\vec{v}|}$

$\dot\theta \gtrless 0$

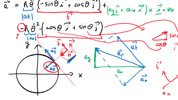
$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{x}(t)\vec{i} + \ddot{y}(t)\vec{j} = (-R\cos\theta\,\dot\theta^2 - R\sin\theta\,\ddot\theta)\vec{i} + (-R\sin\theta\,\dot\theta^2 + R\cos\theta\,\ddot\theta)\vec{j} = a_x\vec{i} + a_y\vec{j}$

$\vec{a} = R\ddot\theta(-\sin\theta\,\vec{i} + \cos\theta\,\vec{j}) + (a_y\vec{j} + a_x\vec{i}) \times \vec{i} = a_x$

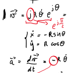
$|a_t|$, $\vec{t}$

$-R\dot\theta^2(\cos\theta\,\vec{i} + \sin\theta\,\vec{j})$

$|a_n|$



APPROCCIO CON NUMERI COMPLESSI



$\vec{P} = (P-O) = R e^{j\theta}$

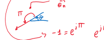
$\begin{cases} x = Re\{\vec{P}\} = R\cos\theta \\ y = Im\{\vec{P}\} = R\sin\theta \end{cases}$

$\vec{v} = jR\dot\theta e^{j\theta} = R\dot\theta e^{j(\theta + \frac{\pi}{2})}$ $\quad (j = e^{j\frac{\pi}{2}},\ |v| = R\dot\theta)$

$\begin{cases} \dot{x} = -R\sin\theta\,\dot\theta \\ \dot{y} = R\cos\theta\,\dot\theta \end{cases}$

$\vec{a} = \frac{d\vec{v}}{dt} = -R\dot\theta^2 e^{j\theta} + jR\ddot\theta e^{j\theta} = R\dot\theta^2 e^{j(\theta+\pi)} + R\ddot\theta e^{j(\theta+\frac{\pi}{2})}$

$\vec{a}_n$, $\vec{a}_t$



$-1 = e^{j\pi}$ $\quad e^{j\theta} e^{j\pi} = e^{j(\theta+\pi)}$