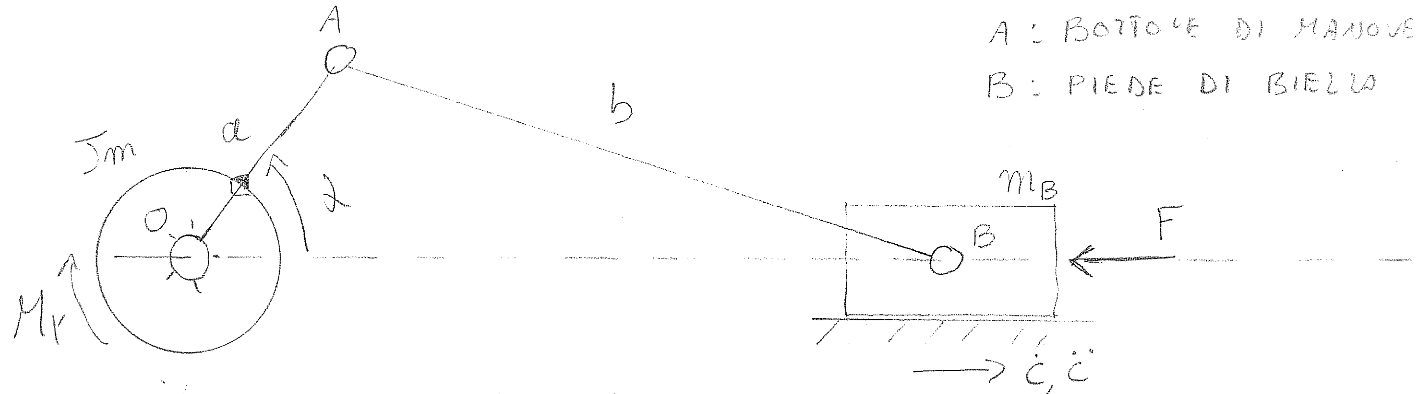
DINAMICA DI UN MOTORE A COMBUSTIONE INTERNA

ANALISI CINETOSTATICA

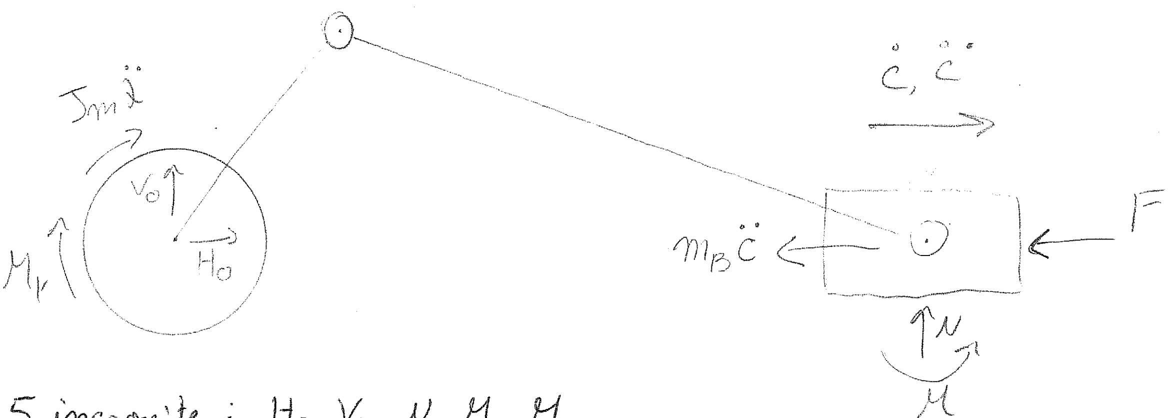
O: CENTRO DI MANOVELLA
A: BOTTONE DI MANOVELLA
B: PIEDE DI BIELLA



NOTI $\alpha, \dot{\alpha}, \ddot{\alpha}$ e F calcolare M_r

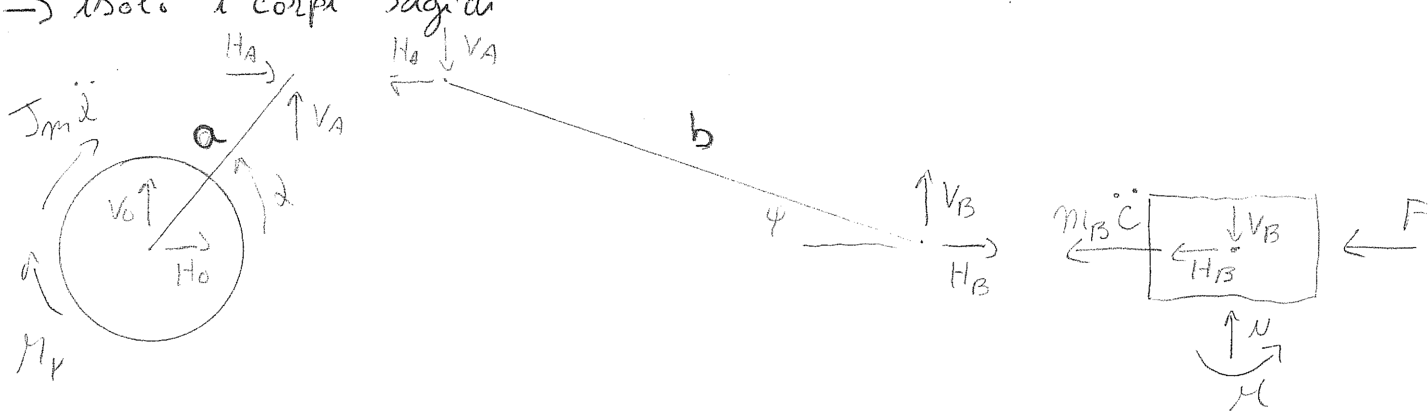
Inerzie solo su manovella e corsoio / Trascuro forze peso (manovellismo veloce)

- risolvo con equilibri dinamici
  → evidenzio forze esterne (attive e reattive) e forze d'inerzia



5 incognite : H_o, V_o, N, M, M_r

→ isolo i corpi rigidi



MANOVELLA

$$\begin{cases} F_x^* = H_o + H_A = 0 \\ F_y^* = V_o + V_A = 0 \\ M_o^* = -M_r - H_A\, a \operatorname{sen}\alpha + V_A\, a \cos\alpha - J_m \ddot{\alpha} = 0 \end{cases}$$

BIELLA

$$\begin{cases} F_x^* = -H_A + H_B = 0 \\ F_y^* = -V_A + V_B = 0 \\ M_A^* = V_B\, b \cos\varphi + H_B\, b \operatorname{sen}\varphi = 0 \end{cases}$$

CORSO10

$$\begin{cases} F_x^* = -F - H_B - m_B \ddot{c} = 0 \\ F_y^* = N - V_B = 0 \\ M_B^* = M = 0 \end{cases}$$

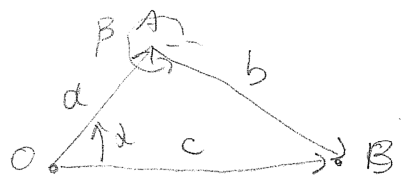
HO SCRITTO 9 EQUAZIONI IN 9 INC. : $H_0, H_A, V_0, V_A, M_r, H_B, V_B, N, M$

$$H_A = H_B = -F - m_B \ddot{c} \qquad \underline{\underline{A}}\,\underline{x} = \underline{b}$$

$$V_A = V_B = -H_B \tan\varphi = (F + m_B \ddot{c}) \tan\varphi$$

$$M_r = (F + m_B \ddot{c})\, a \,\mathrm{sen}\,\alpha + (F + m_B \ddot{c}) \tan\varphi \, a \cos\alpha - J_m \ddot{\alpha} =$$
$$= (F + m_B \ddot{c})\, a \,(\mathrm{sen}\,\alpha + \cos\alpha \tan\varphi) - J_m \ddot{\alpha}$$

il legame tra $\ddot{c}$ e la c.l. α è da ricavare dall'analisi cinematica



In alternativa utilizzo approccio energetico: bilancio di potenze

$$\sum W_k = \frac{d E_c}{dt}$$

$$W_k = \vec{M_r} \times \vec{\dot{\alpha}} + \vec{F} \times \vec{v_B} = -M_r \dot{\alpha} - F \dot{c}$$

$$E_c = \frac{1}{2} J_m \dot{\alpha}^2 + \frac{1}{2} m_B \dot{c}^2$$

$$-M_r \dot{\alpha} - F\dot{c} = J_m \dot{\alpha}\ddot{\alpha} + m_B \dot{c}\ddot{c} \qquad 1 \text{ incognita: } M_r$$

[DA CINEMATICA :

$$\begin{cases} a \cos\alpha + b \cos\beta = c \\ a \,\mathrm{sen}\,\alpha + b \,\mathrm{sen}\,\beta = 0 \end{cases} \qquad \begin{cases} -a \dot{\alpha} \,\mathrm{sen}\,\alpha - b \dot{\beta} \,\mathrm{sen}\,\beta = \dot{c} \\ a \dot{\alpha} \cancel{\mathrm{sen}}\cos\alpha + b \dot{\beta} \cos\beta = 0 \end{cases}$$

$$\dot{\beta} = -\frac{a \dot{\alpha} \cos\alpha}{b \cos\beta} \quad \Rightarrow \quad \dot{c} = -a\dot{\alpha} \,\mathrm{sen}\,\alpha - b \,\mathrm{sen}\,\beta \cdot -\frac{a \dot{\alpha} \cos\alpha}{b \cos\beta} = -a\dot{\alpha}(\mathrm{sen}\,\alpha + \cos\alpha \tan\beta)$$

$\tan\varphi = -\tan\beta$]

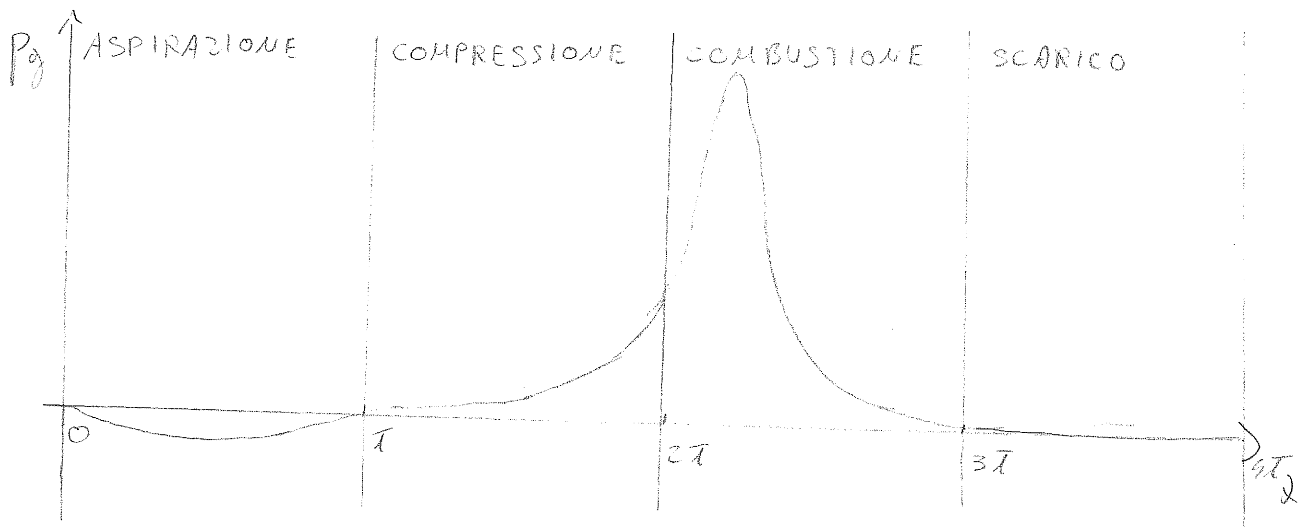
$$+M_r \dot{\alpha} = -(F + m_B \ddot{c})\dot{c} - J_m \dot{\alpha}\ddot{\alpha} \quad \Rightarrow \quad M_r \dot{\alpha} = (F + m_B \ddot{c})\, a \dot{\alpha} (\mathrm{sen}\,\alpha + \cos\alpha \tan\varphi) - J_m \dot{\alpha}\ddot{\alpha}$$

$$+ \frac{d E_c}{dt} = -W_i$$

Nel motore a combustione interna la forza sul corsoio / pistone dipende dalla pressione del gas nella camera di scoppio.

$$F = \pi \frac{D^2}{4} P_{gas} \qquad D: \text{alesaggio}$$

L'andamento della pressione in funzione dell'angolo di manovella è:



SOLUZIONE CON PLV

$$\delta L = \vec{M}_r \times \delta \vec{\theta}_{OA} + \vec{F} \times \vec{\delta_B} + (-J \vec{\dot{\omega}}_{OA}) \times \delta \vec{\theta}_{OA} + (-m_B \vec{a}_B) \times \vec{\delta_B}$$

$$\vec{\delta_B} = \delta_B \vec{i}$$

$$\delta_B = \frac{\partial x_B}{\partial \alpha} \delta \alpha \qquad \frac{\partial x_B}{\partial \alpha} = \frac{\partial \dot{x}_B}{\partial \dot{\alpha}} = \frac{\partial \dot{c}}{\partial \dot{\alpha}} = -a (\sin\alpha + \cos\alpha \tan\varphi)$$

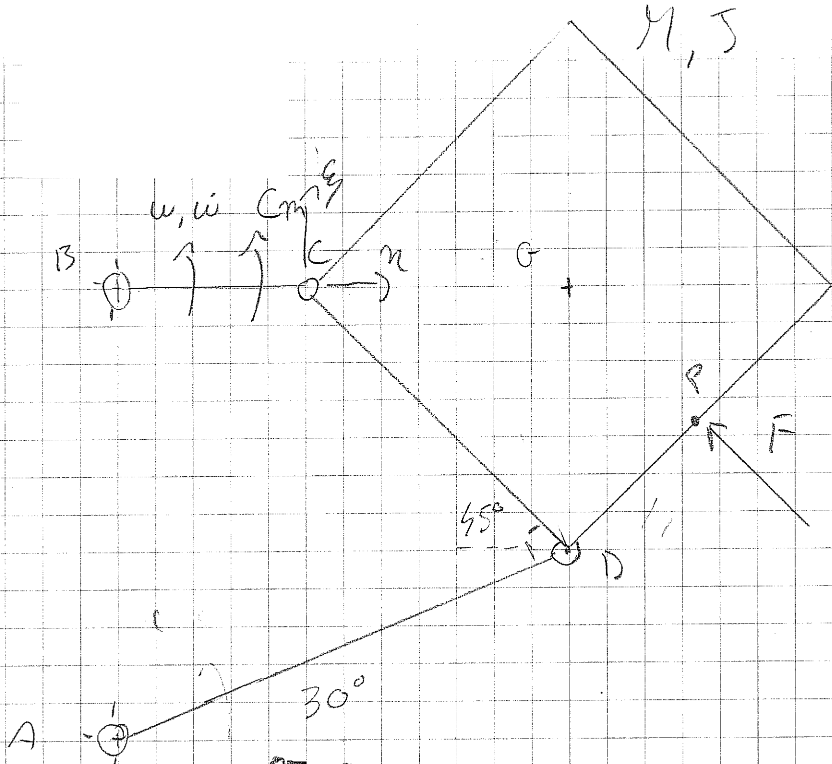
$$\delta_B = -a (\sin\alpha + \cos\alpha \tan\varphi)\, \delta\alpha$$

$$\delta L = -M_r \delta\alpha + F a (\sin\alpha + \cos\alpha \tan\varphi) \delta\alpha - J\ddot{\alpha}\, \delta\alpha + m_B \ddot{c}\, a (\sin\alpha + \cos\alpha \tan\varphi) \delta\alpha$$

$$= \Big[\underbrace{-M_r + Fa(\sin\alpha + \cos\alpha \tan\varphi)}_{Q} \underbrace{- J\ddot{\alpha} + m_B \ddot{c}\, a(\sin\alpha + \cos\alpha \tan\varphi)}_{Q_{In}}\Big] \delta\alpha = 0$$

$$Q + Q_{IN} = 0$$

$$\Rightarrow M_r = -J\ddot{\alpha} + (F + m_B \ddot{c})\, a (\sin\alpha + \cos\alpha \tan\varphi)$$



DATI
a $AB = 0.5$ m
b $BC = 0.183$
c $CD = 0.353$
d $AD = 0.5$
$M = 10$ kg
$J = 0.1$ kg m^2
$\omega = 37.32$ rad/s
$\dot{\omega} = 110$ rad/s
$F = 50$ N

CALCOLARE
- v_G, a_G
- Cm
- REAZ. A, B

- GDL

$3 \times 3 = 9$ gdl $\quad 4 \times 2 = 8$ gdv $\quad$ **1 gdl** residuo $(\omega, \dot{\omega})$ ASSEGNATI

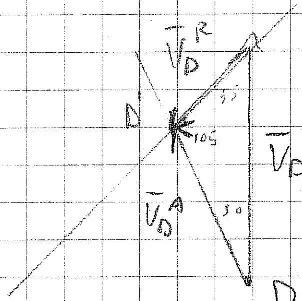
- CINEMATICA

TERNA MOBILE TRASLANTE IN C (QUADRILATERO ARTICOLATO)

STUDIO IL MOTO DI D

ASSOLUTO: ROT A $\quad$ TRASC.: TRASL. $\bar{v}_D^{(T)} = \vec{v}_C$ $\quad$ REL.: ROT C

$\bar{v}_D^{(A)} =$	$\bar{v}_D^{(T)} +$	$\bar{v}_D^{R}$
$\omega_{AD}\, AD$?	$\omega\, BC$	$\omega_{CD}\, CD$?
$\perp AD$	$\perp BC$	$\perp CD$
(5)	6,83	(3.53)



$v_D^A = 5$ m/s

$v_D^R = 3.53$ m/s

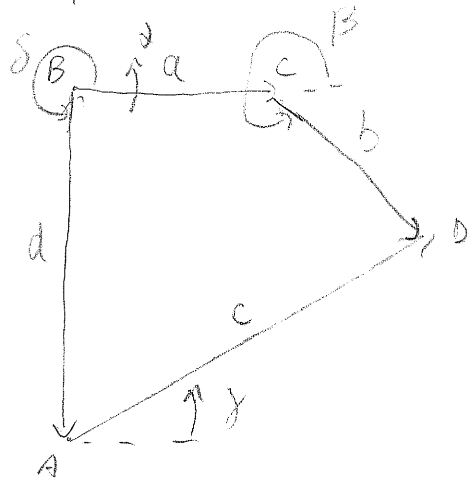
$$\omega_{AD} = \frac{v_D^A}{AD} = 10 \text{ rad/s}$$

$$\omega_{CD} = \frac{v_D^R}{CD} = 10 \text{ rad/s}$$

SOLUZIONE CON NUMERI COMPLESSI

$$(D-C)+(C-B)=(D-A)+(A-B)$$
$$b e^{i\beta}+a e^{i\alpha}=c e^{i\gamma}+i d$$

COST	b a c d
VAR	β α γ



$$\begin{cases} b\cos\beta + a\cos\alpha = c\cos\gamma \\ b\,\text{sen}\,\beta + a\,\text{sen}\,\alpha = c\,\text{sen}\,\gamma - d \end{cases}$$

$\Rightarrow$ VERIFICO I DATI DEL PROBLEMA

VELOCITÀ

$$\begin{cases} -b\dot\beta\,\text{sen}\,\beta - a\dot\alpha\,\text{sen}\,\alpha = -c\dot\gamma\,\text{sen}\,\gamma \\ b\dot\beta\cos\beta + a\dot\alpha\cos\alpha = c\dot\gamma\cos\gamma \end{cases}$$

$$\begin{bmatrix} -b\,\text{sen}\,\beta & c\,\text{sen}\,\gamma \\ b\cos\beta & -c\cos\gamma \end{bmatrix}\begin{Bmatrix}\dot\beta \\ \dot\gamma\end{Bmatrix}=\begin{Bmatrix} a\dot\alpha\,\text{sen}\,\alpha \\ -a\dot\alpha\cos\alpha\end{Bmatrix}$$

$\dot\beta = -10\ \text{rad/s}$ $\quad\dot\gamma = 10\ \text{rad/s}$

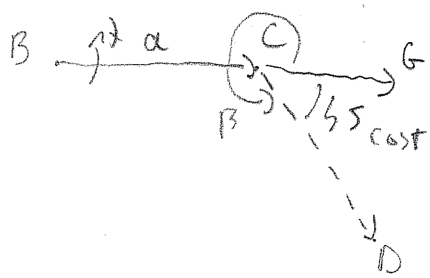
ACCELERAZIONE

$$\begin{cases} -b\ddot\beta\,\text{sen}\,\beta - b\dot\beta^2\cos\beta - a\ddot\alpha\,\text{sen}\,\alpha - a\dot\alpha^2\cos\alpha = -c\ddot\gamma\,\text{sen}\,\gamma - c\dot\gamma^2\cos\gamma \\ b\ddot\beta\cos\beta - b\dot\beta^2\,\text{sen}\,\beta + a\ddot\alpha\cos\alpha - a\dot\alpha^2\,\text{sen}\,\alpha = c\ddot\gamma\cos\gamma - c\dot\gamma^2\,\text{sen}\,\gamma \end{cases}$$

$$\begin{bmatrix} -b\,\text{sen}\,\beta & c\,\text{sen}\,\gamma \\ b\cos\beta & -c\cos\gamma \end{bmatrix}\begin{Bmatrix}\ddot\beta \\ \ddot\gamma\end{Bmatrix}=\begin{Bmatrix} b\dot\beta^2\cos\beta + a\ddot\alpha\,\text{sen}\,\alpha + a\dot\alpha^2\cos\alpha - c\dot\gamma^2\cos\gamma \\ b\dot\beta^2\,\text{sen}\,\beta - a\ddot\alpha\cos\alpha + a\dot\alpha^2\,\text{sen}\,\alpha - c\dot\gamma^2\,\text{sen}\,\gamma\end{Bmatrix}$$

$\ddot\beta = 498\ \text{rad/s}^2$ $\quad\ddot\gamma = 449\ \text{rad/s}^2$

$$(G-B)=(G-C)+(C-B)$$
$$(G-B)=f e^{i(\beta+\frac{\pi}{4})}+a e^{i\alpha}$$



$CG = f$

$$\begin{cases} x_G = f\cos\left(\beta+\frac{\pi}{4}\right) + a\cos\alpha \\ y_G = f\,\text{sen}\left(\beta+\frac{\pi}{4}\right) + a\,\text{sen}\,\alpha \end{cases}$$

$$\begin{cases} \dot{x}_G = -f\dot{\beta}\,\mathrm{sen}\left(\beta+\frac{\pi}{4}\right) - a\dot{\alpha}\,\mathrm{sen}\,\alpha \\ \dot{y}_G = f\dot{\beta}\cos\left(\beta+\frac{\pi}{4}\right) + a\dot{\alpha}\cos\alpha \end{cases} \qquad \begin{aligned} \dot{x}_G &= 0 \\ \dot{y}_G &= 4{,}33\ m/s \end{aligned}$$

$$\begin{cases} \ddot{x}_G = -f\ddot{\beta}\,\mathrm{sen}\left(\beta+\frac{\pi}{4}\right) - f\dot{\beta}^2\cos\left(\beta+\frac{\pi}{4}\right) - a\ddot{\alpha}\,\mathrm{sen}\,\alpha - a\dot{\alpha}^2\cos\alpha \\ \ddot{y}_G = f\ddot{\beta}\cos\left(\beta+\frac{\pi}{4}\right) - f\dot{\beta}^2\,\mathrm{sen}\left(\beta+\frac{\pi}{4}\right) + a\ddot{\alpha}\cos\alpha - a\dot{\alpha}^2\,\mathrm{sen}\,\alpha \end{cases} \qquad \begin{aligned} \ddot{x}_G &= -280\ m/s^2 \\ \ddot{y}_G &= 155\ m/s^2 \end{aligned}$$

TEOREMA E_c

$$\sum W_K = \frac{dE_c}{dt}$$

$$W_K = \vec{C}_m \times \vec{\omega} + \vec{F} \times \vec{v}_G + M\vec{g} \times \vec{v}_G = C_m\omega_m - F_x\dot{x}_G + F_y\dot{y}_G - mg\dot{y}_G$$

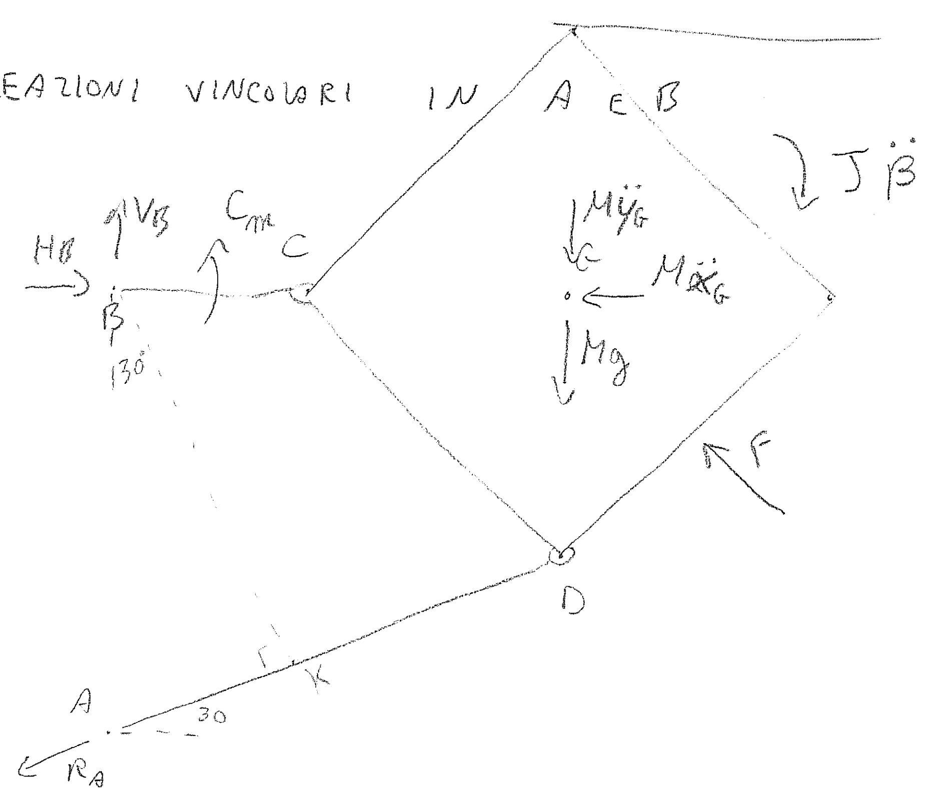
$$E_c = \frac{1}{2}M v_G^2 + \frac{1}{2}J\omega_G^2 = \frac{1}{2}M\dot{x}_G^2 + \frac{1}{2}M\dot{y}_G^2 + \frac{1}{2}J\omega_G^2$$

$$C_m\dot{\alpha} - F\frac{\sqrt{2}}{2}\dot{x}_G + F\frac{\sqrt{2}}{2}\dot{y}_G - Mg\dot{y}_G = M\dot{x}_G\ddot{x}_G + M\dot{y}_G\ddot{y}_G + J\dot{\beta}\ddot{\beta}$$

$$\Rightarrow C_m = \frac{1}{\dot{\alpha}}\left[F\frac{\sqrt{2}}{2}\underbrace{\cancel{\dot{x}_G}}_{0} - F\frac{\sqrt{2}}{2}\dot{y}_G + Mg\dot{y}_G + M\underbrace{\cancel{\dot{x}_G}}_{0}\ddot{x}_G + M\dot{y}_G\ddot{y}_G + J\dot{\beta}\ddot{\beta}\right]$$

$$C_m = 161\ Nm$$

REAZIONI VINCOLARI IN A E B



$$\overset{+\curvearrowleft}{} M_B^* = -Mg\,BG + F\frac{\sqrt{2}}{2}BG + C_m - R_A\,BA\cos 30 - M\ddot{y}_G\,BG - J\ddot{\beta} = 0$$

$$\Rightarrow R_A = -1244\ N$$

$$F_x^* = -F\frac{\sqrt{2}}{2} + H_B - R_A\cos 30 - M\ddot{x}_G = 0 \quad \Rightarrow H_B = -3842\ N$$

$$F_y^* = -Mg + F\frac{\sqrt{2}}{2} + V_B - R_A\,\mathrm{sen}\,30 - M\ddot{y}_G = 0 \quad \Rightarrow V_B = 880\ N$$