31/03/2020

Gb)

$$G_b(s) = \frac{1}{s^g} \qquad (\omega > 0)$$

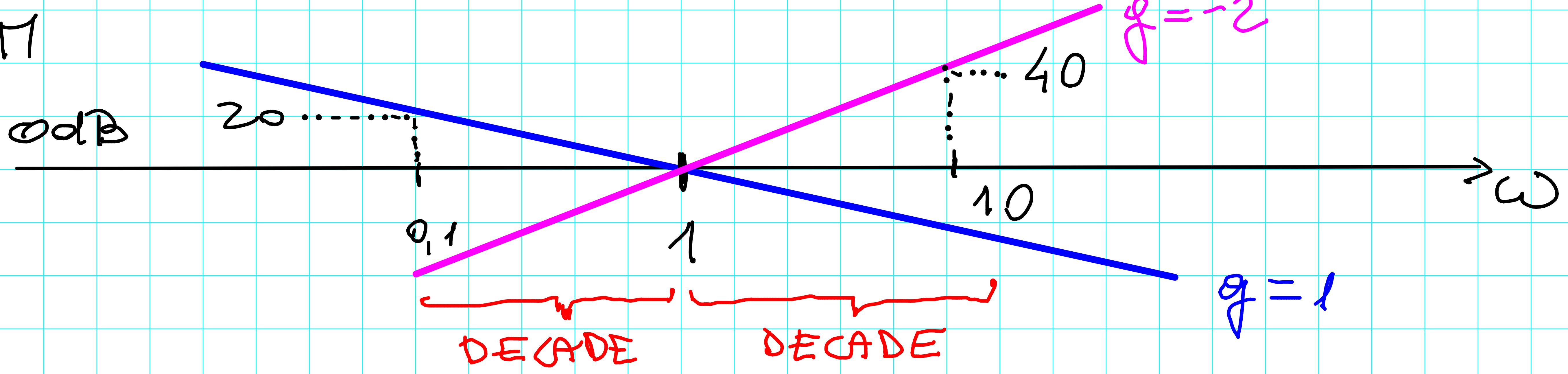
ecco perchè usiamo asse ω logaritmico

$$G_b(j\omega) = \frac{1}{(j\omega)^g}$$

$$|G_b(j\omega)| = \frac{1}{\omega^g} \Rightarrow |G_b(j\omega)|_{dB} = -20\, g \, \underbrace{\log \omega}$$

$$\angle^\circ G_b(j\omega) = -g \cdot 90^\circ$$

DBM



Pendenza $= -20\,g \;\text{dB}/\text{decade}$

(ABBREV. "pendenza $-g$")

distanza corrispondente al rapporto 10

DBF Orizzontale al valore $-g \cdot 90°$

NB Finora nessuna approssimazione

G_c)

$$G_c(s) = 1 + sT$$

$$G_c(j\omega) = 1 + j\omega T$$

Esatto:

$$|G_c(j\omega)| = \sqrt{1 + (\omega T)^2}$$

$$\angle^\circ G_c(j\omega) = \mathrm{arctg}^\circ(\omega T)$$

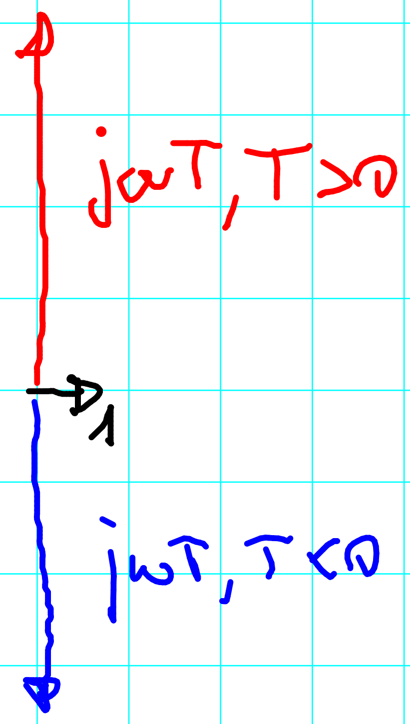
APPROX :

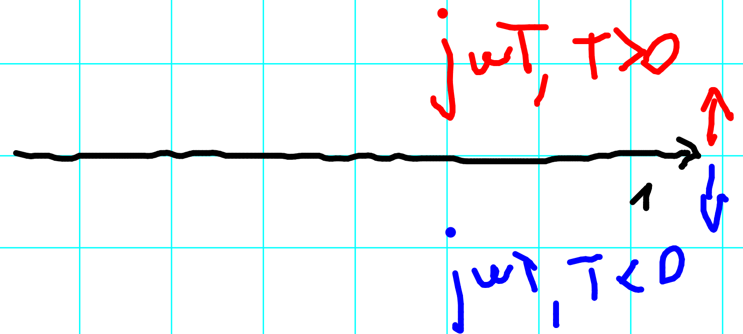
D. di Bode ASINTOTICA

• $|\omega T| >> 1$

$$G_c(j\omega) \simeq j\omega T \Rightarrow |G_c(j\omega)| \simeq |\omega T|$$

$$\angle^\circ G_c(j\omega) \simeq \begin{cases} 90^\circ & T>0 \\ -90^\circ & T<0 \end{cases}$$



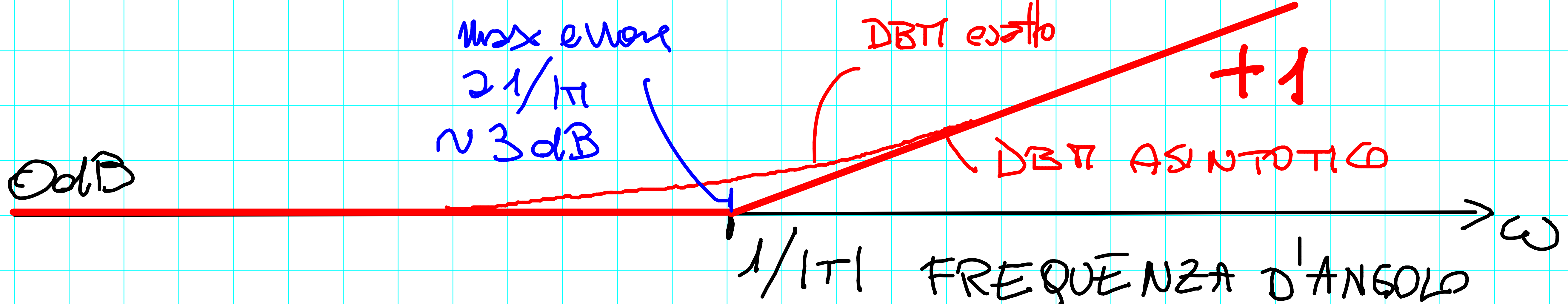


• $|\omega T| << 1$

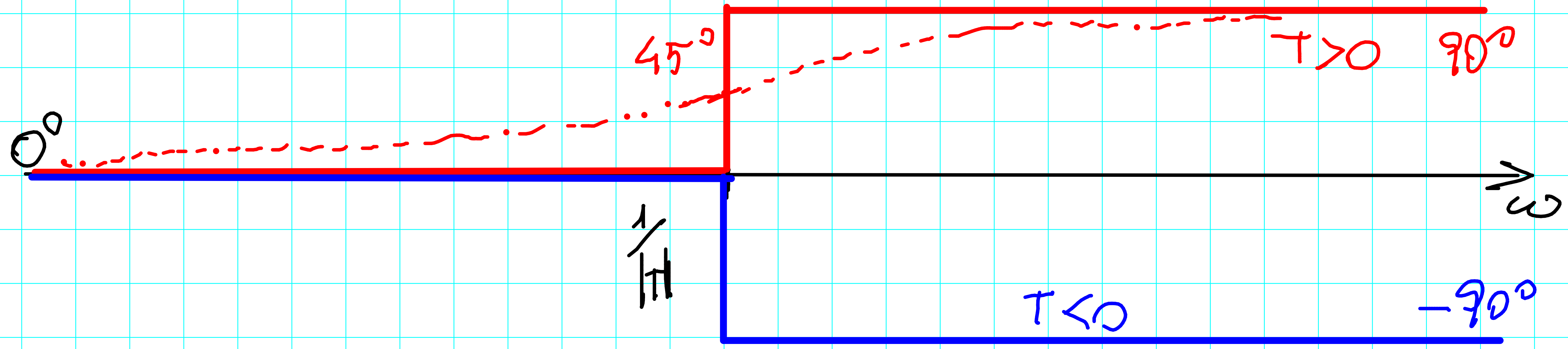
$G_c(j\omega) \simeq 1 \Rightarrow |G_c(j\omega)| \simeq 1$

$\measuredangle^{\circ} G_c(j\omega) \simeq 0^{\circ}$

DBM



DBF



Gd)

$$G_d(s) = 1 + 2\frac{\xi}{\omega_n}s + \frac{1}{\omega_n^2}s^2$$

$$G_d(j\omega) = 1 + 2\frac{\xi}{\omega_n}j\omega + \frac{1}{\omega_n^2}(j\omega)^2$$

$$= \underbrace{1 - \frac{\omega^2}{\omega_n^2}}_{\mathbb{R}} + j\underbrace{2\xi\frac{\omega}{\omega_n}}_{\mathbb{I}}$$

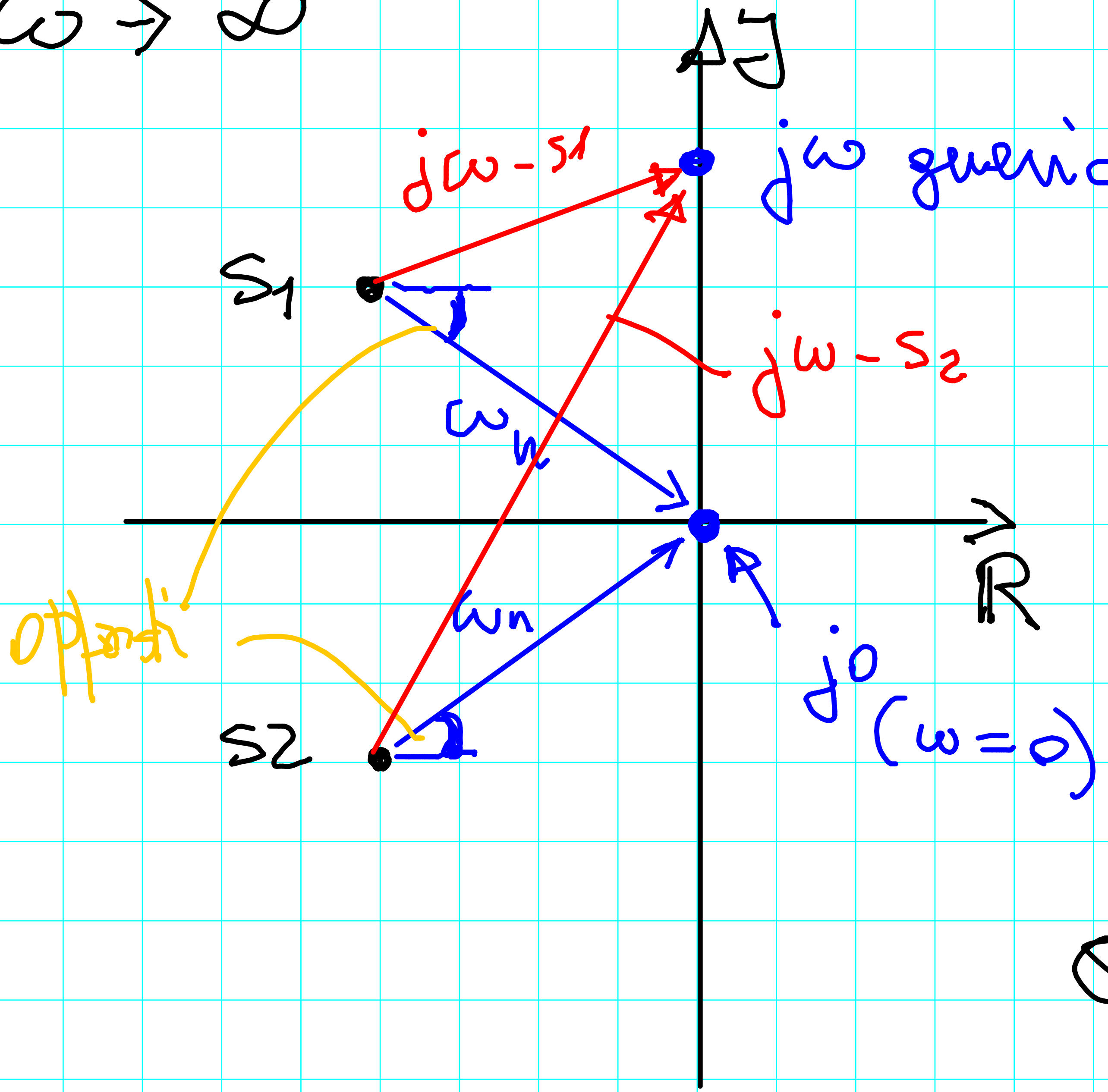
$\omega \to 0$

p. reale $\to 1$, p. imm. $\to 0$

$\Rightarrow |G_d(j\omega)| \to 1$, $|G_d(j\omega)|_{dB} \to 0$

$\angle^\circ G_d(j\omega) \to 0^\circ$

$\omega \to \infty$

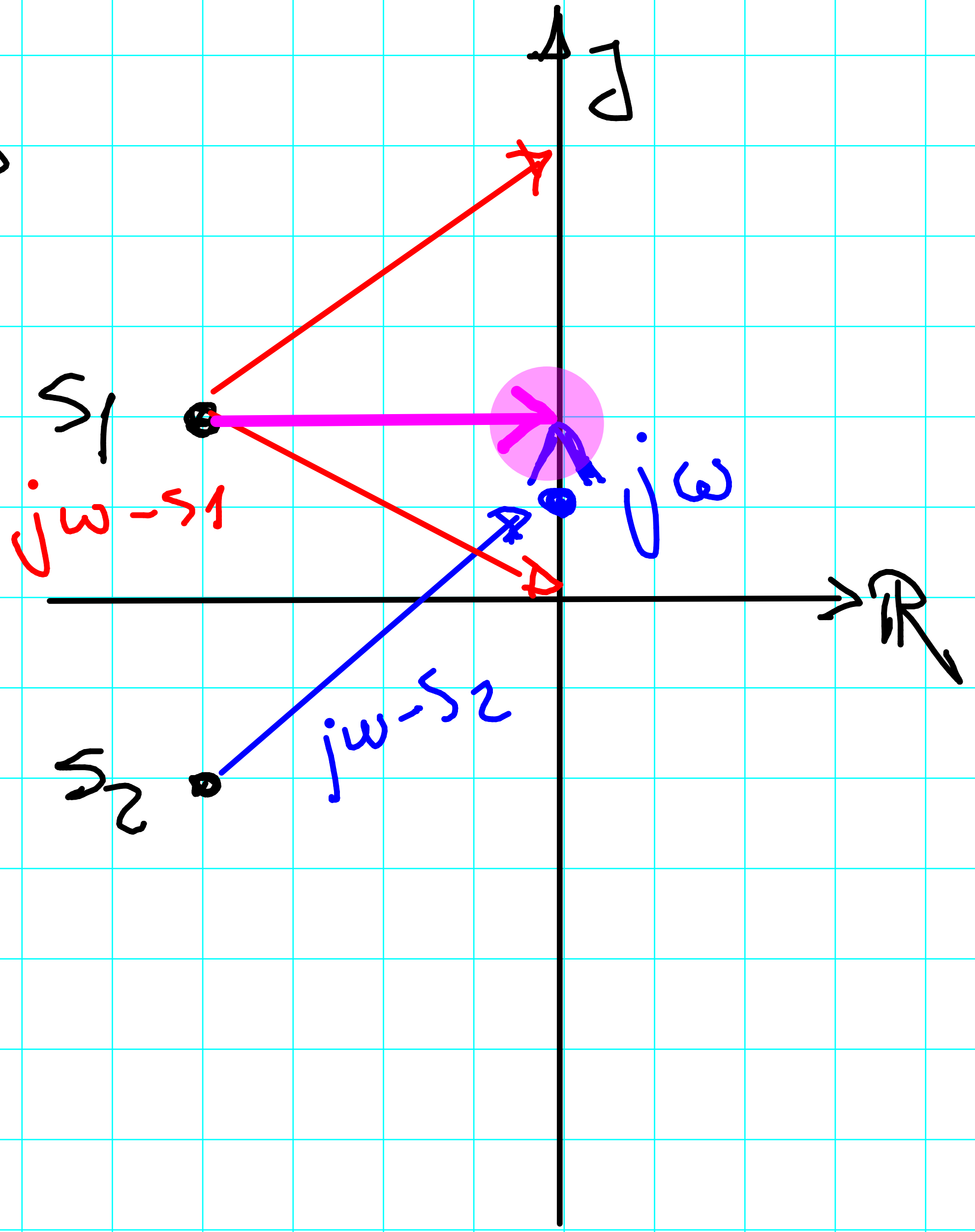


$$G_d(s) = \frac{1}{\omega_n^2}(s - s_1)(s - s_2)$$

i 2 vettori $j\omega - s_i$
per $\omega \to \infty$ hanno ambedue
modulo $\to \infty$
fase $\to 90°$

Quindi $|G_d| \to \infty$ come ω^2
e $\measuredangle° G_d \to 180°$

OSS

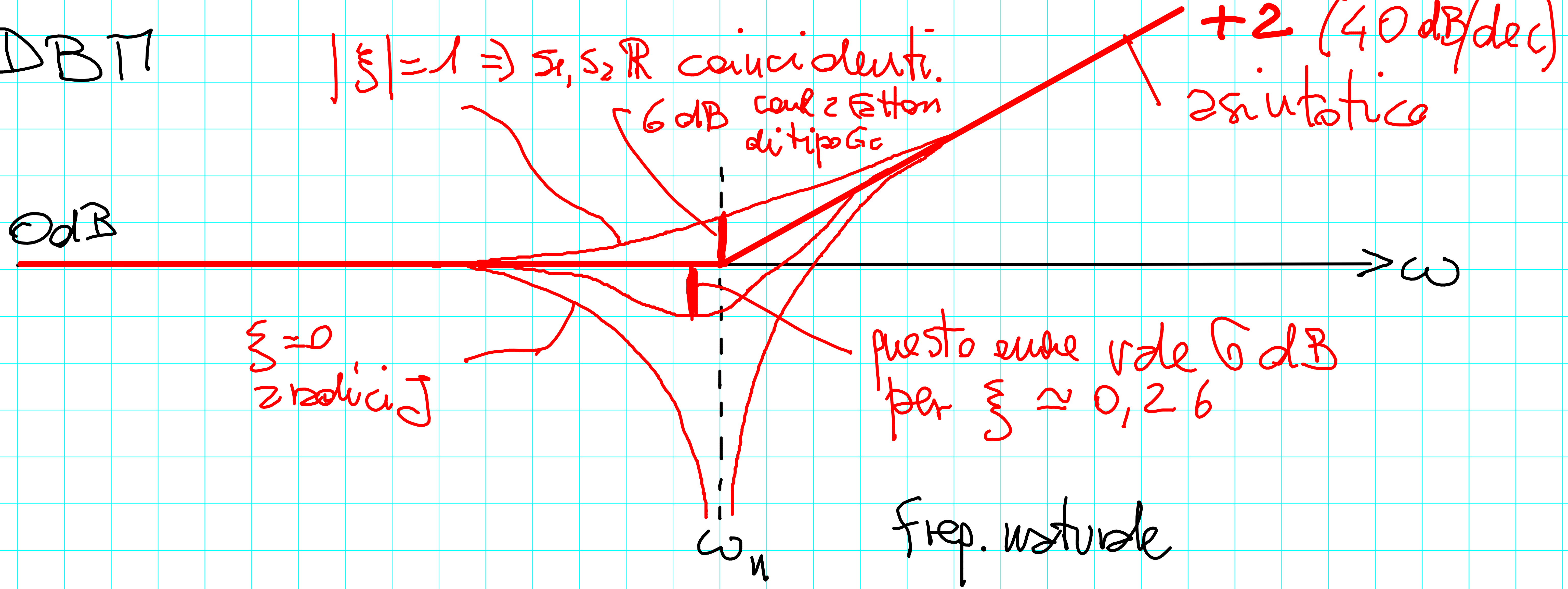


$|j\omega - s_2|$ monotono crescente

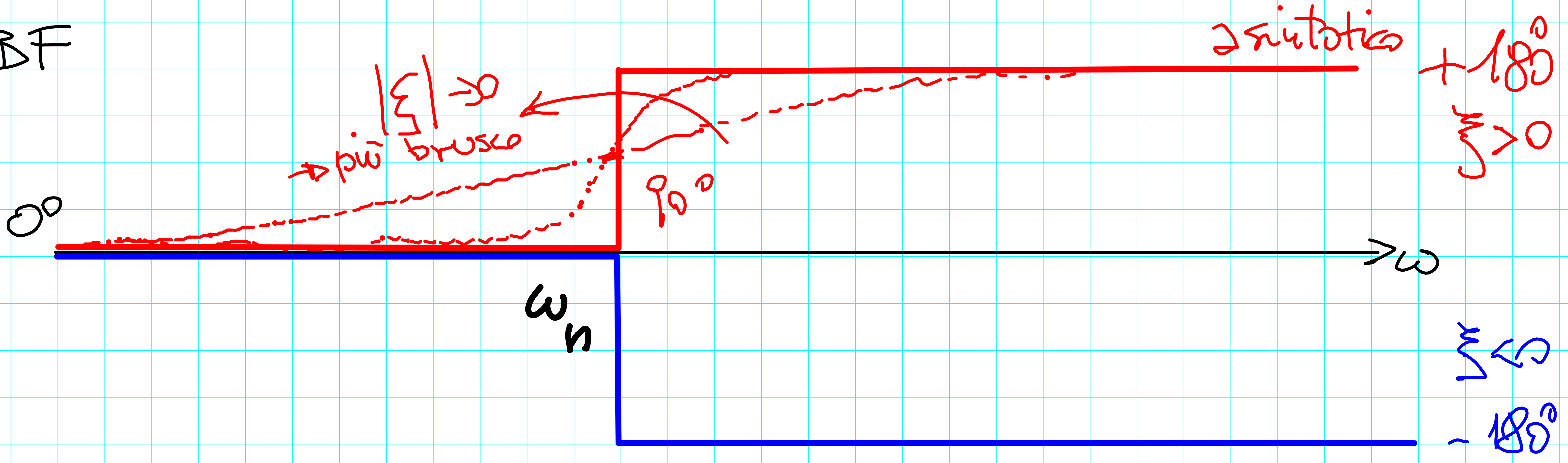
$|j\omega - s_1|$ ha un MINIMO per $\omega = \Im m(s_1)$

NB più s_1, s_2 sono vicini all'asse J, più tale minimo è pronunciato e la variazione di fase avviene bruscamente

DBM


$|\xi| = 1 \Rightarrow s_1, s_2 \in \mathbb{R}$ coincidenti.
6 dB con i 2 fattori di tipo Gc
+2 (40 dB/dec)
asintotico
0dB
ω
$\xi = 0$
2 radici J
questo errore vale 6 dB per $\xi \simeq 0,26$
ω_n
freq. naturale

DBF



(MIT mathlets)

□

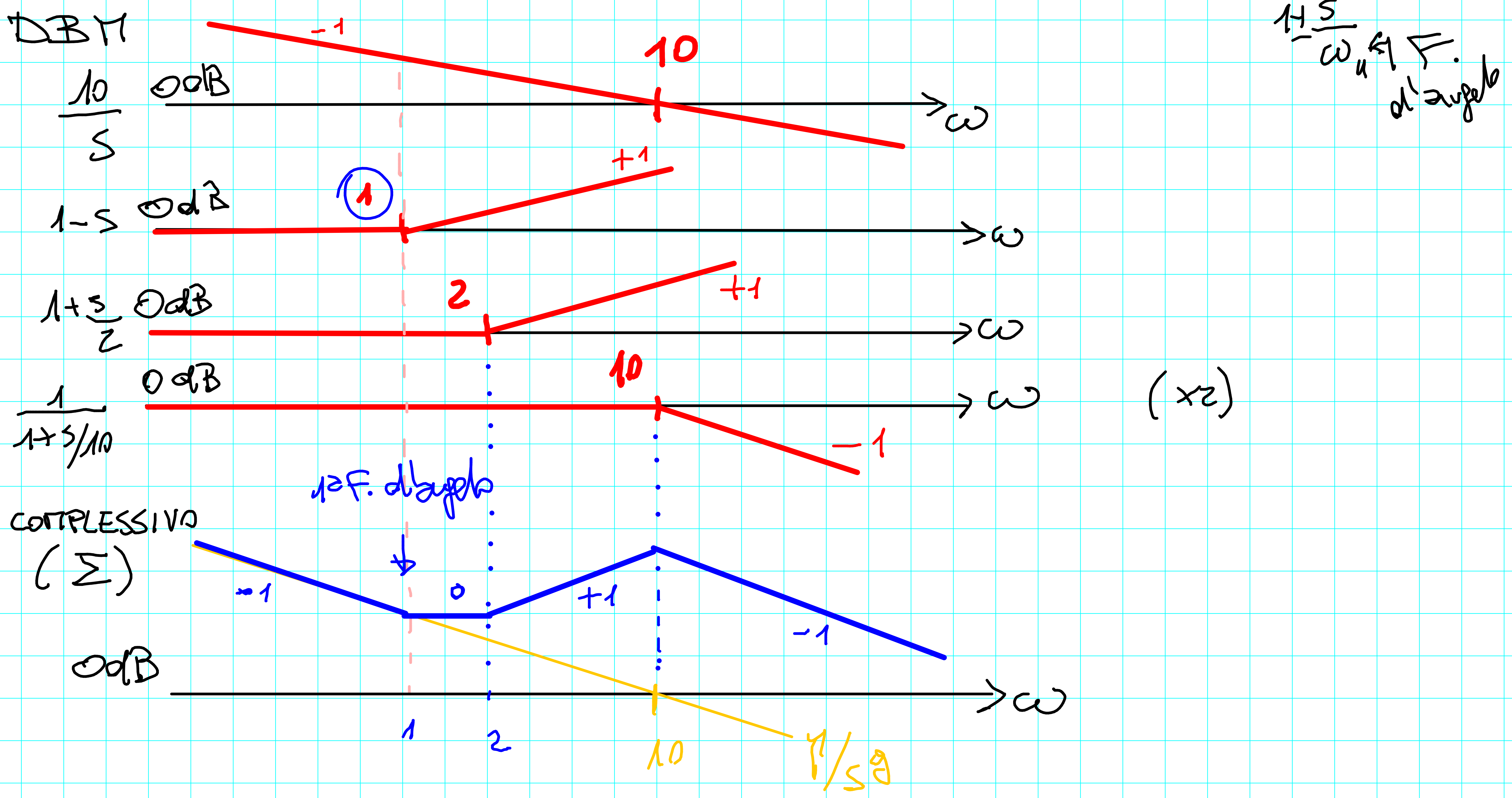
Tracciamento complessivo

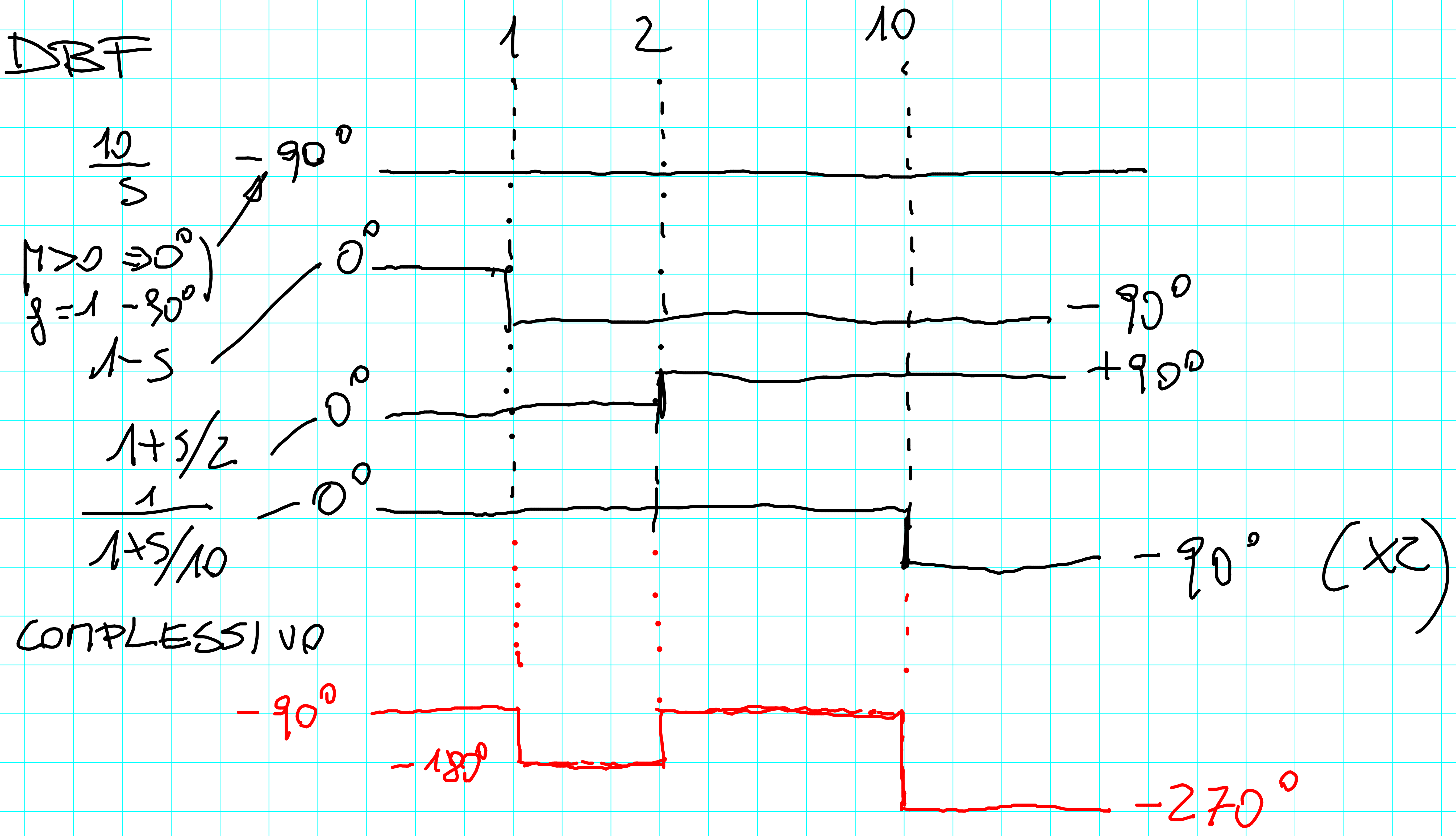
ES)

$$G(s) = \frac{10(1-s)(1+s/2)}{s(1+s/10)^2} \qquad \begin{array}{l} \mu = 10 \\ g = 1 \end{array}$$

$$= 10 \cdot \frac{1}{s} \quad (1-s) \quad (1+s/2) \quad \frac{1}{1+s/10} \quad \frac{1}{1+s/10}$$

Facciamo DBM e DBF di tutti questi termini e sommiamo


DBM
1+s/ωn → 1 F. d'angolo
10/s 0dB
-1
10
ω
1−s 0dB
1
+1
ω
1+s/2 0dB
2
+1
ω
1/(1+s/10) 0dB
10
ω
−1
(x2)
1ª F. d'angolo
COMPLESSIVO (Σ)
-1
0
+1
-1
0dB
ω
1
2
10
1/s


DBF
1
2
10
10/s
-90°
μ>0 ⇒ 0°
g=1 -90°
1-s
0°
-90°
+90°
1+s/2
0°
1/(1+s/10)
0°
-90°
(XC)
COMPLESSIVO
-90°
-180°
-270°

METODO DI TRACCIAMENTO

DBM

- traccia DBM di μ / s^g
- segna su asse ω le freq. d'angolo di poli e zeri non in $s = 0$

zero $\Rightarrow$ pendenza aumenta di 1

polo $\Rightarrow$ " diminuisce di 1

DBF

- parte al valore X° $\mu/(j\omega)^g$

  μ ↗ 0° se >0 ; ↘ -180° se <0

  $\frac{1}{s^g} \to -g \cdot 90^\circ$

"SX" = nel semipiano sinistro, cioè con Re <0

- Zero SX fase aumenta di 90°

  Zero DX " diminuisce " "

  polo SX " diminuisce " "

  polo DX " aumenta " "

$\square$

ES

$$G(s) = \frac{100\,(1-s)\,(1+s/5)}{s^2\,(1-s/10)\,(1+s/100)^2}$$

D. Bode asintotici?

$\mu = 100$

$g = 2$

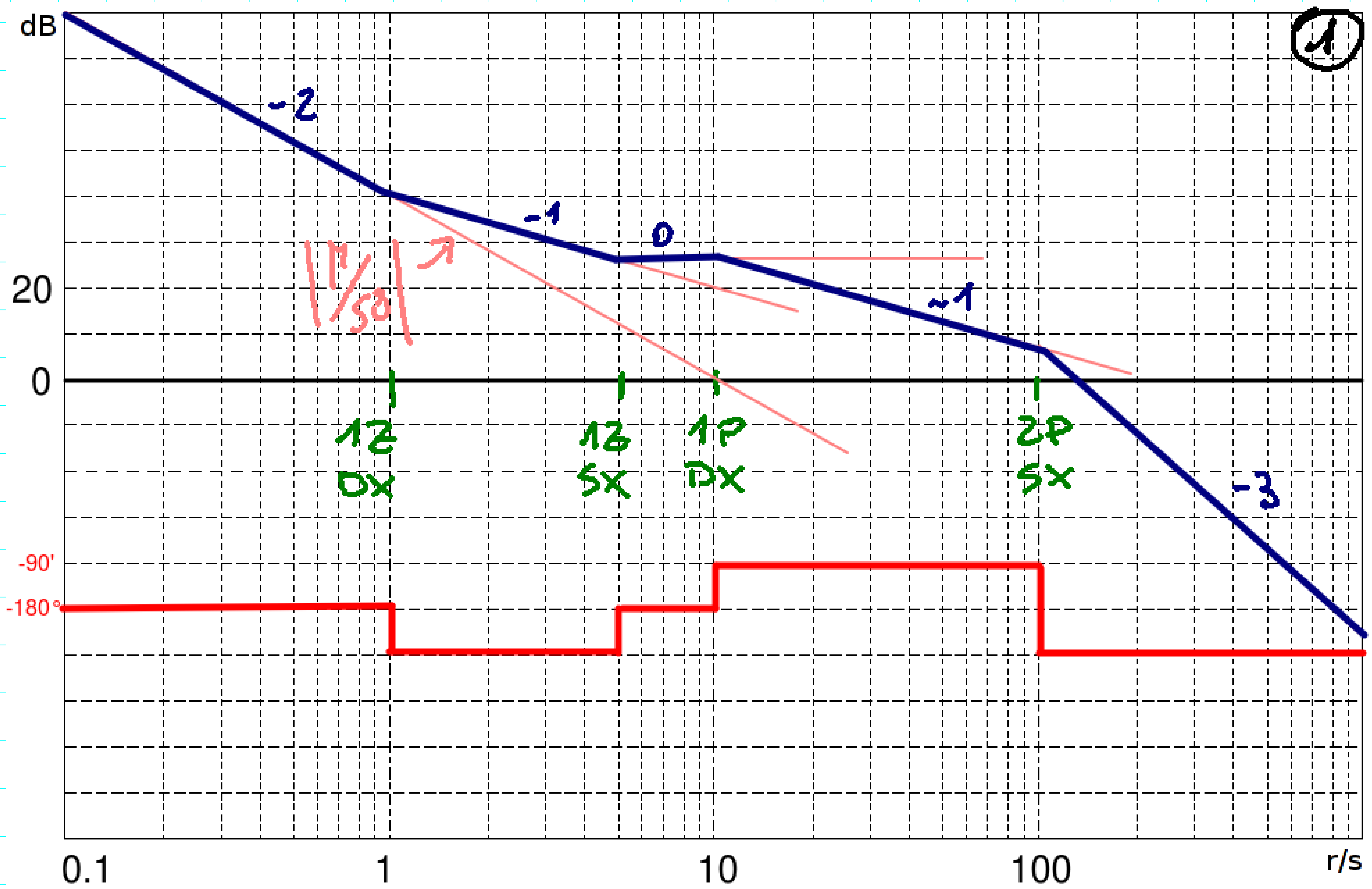
$\Rightarrow \dfrac{\mu}{s^g}$ ha pendenza -2 e taglia l'asse 0 dB per $\dfrac{100}{\omega^2} = 1$ cioè per $\omega = 10$

Freq. d'angolo di poli e zeri non nell'origine:

$\omega = 1$	1	Z	DX
$\omega = 5$	1	Z	SX
$\omega = 10$	1	P	DX
$\omega = 100$	2	P	SX

$\Rightarrow$ Foglio semilog 1

□



dB
20
0
-90'
-180°
0.1
1
10
100
r/s
1
-2
-1
0
-1
-3
M/50
1Z
DX
1Z
SX
1P
DX
2P
SX