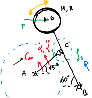
ESERCITAZIONE 8

ESERCIZIO 1



NOTI:

$M = 10\ Kg$ $\quad$ $F = 100\ N$

$AC = 0.3\ m$ $\quad$ $\dot\alpha = 10\ rad/s$

$DB = 0.76\ m$ $\quad$ $\ddot\alpha = 0\ rad/s^2$

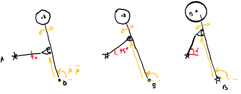
$BC = 0.4\ m$ $\quad$ $f_D = 0.2$

$R = 0.1\ m$

1) CALCOLARE $\vec{v}_D$, $\vec{a}_D$

→ 2) CALCOLARE $\boxed{C_m}$ PER GARANTIRE IL MOTO

3) REAZIONI VINCOLARI IN A



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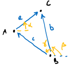
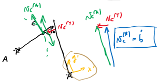
1) $(\dot\beta, \ddot\beta, \dot b, \ddot b) \rightarrow \vec{v}_D, \vec{a}_D$





↘ APPROCCIO TRAMITE NUMERI COMPLESSI

COSTANTE: a, c, γ

VARIABILI: α, β, b

NOTO

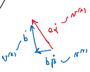
$$\vec{v}_C^{(A)} = \vec{v}_C^{(T)} + \vec{v}_C^{(R)}$$

$$\underset{\dot\alpha AC}{\perp AC} = \underset{\dot\beta BC}{\perp BC} + \underset{\dot b}{\parallel BC}$$

$$\boxed{b e^{i\beta} = c\,\cancel{e^{i\gamma}} + a e^{i\alpha}} \leftarrow$$

VELOCITA':
$$\dot b e^{i\beta} + i b \dot\beta e^{i\beta} = i a \dot\alpha e^{i\alpha}$$

$$\underbrace{\dot b e^{i\beta}} + \underbrace{b\dot\beta e^{i(\beta+\frac{\pi}{2})}} = \underbrace{a\dot\alpha e^{i(\alpha+\frac{\pi}{2})}}$$



$$\begin{cases} \dot b\cos\beta - b\dot\beta\sin\beta = -a\dot\alpha\sin\alpha \\ \dot b\sin\beta + b\dot\beta\cos\beta = a\dot\alpha\cos\alpha \end{cases}$$

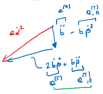
$$\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix} \begin{Bmatrix} \dot b \\ \dot\beta \end{Bmatrix} = \begin{Bmatrix} -a\dot\alpha\sin\alpha \\ a\dot\alpha\cos\alpha \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} \dot b \\ \dot\beta \end{Bmatrix} = \begin{Bmatrix} 2.9\ m/s \\ 1.94\ rad/s \end{Bmatrix}$$

ACCELERAZIONE:

$$\ddot b e^{i\beta} + i2\dot b\dot\beta e^{i\beta} + i b\ddot\beta e^{i\beta} - b\dot\beta^2 e^{i\beta} = -a\dot\alpha^2 e^{i\alpha} + i a\cancelto{0}{\ddot\alpha} e^{i\alpha}$$

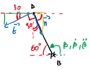
$$\underbrace{\ddot b e^{i\beta}} + \underbrace{2\dot b\dot\beta e^{i(\beta+\frac{\pi}{2})}} + \underbrace{b\ddot\beta e^{i(\beta+\frac{\pi}{2})}} \underbrace{- b\dot\beta^2 e^{i\beta}} = \underbrace{-a\dot\alpha^2 e^{i\alpha}}$$



$$\begin{cases} \ddot b\cos\beta - 2\dot b\dot\beta\sin\beta - b\ddot\beta\sin\beta - b\dot\beta^2\cos\beta = -a\dot\alpha^2\cos\alpha - a\cancelto{0}{\ddot\alpha}\sin\alpha \\ \ddot b\sin\beta + 2\dot b\dot\beta\cos\beta + b\ddot\beta\cos\beta - b\dot\beta^2\sin\beta = -a\dot\alpha^2\sin\alpha + a\cancelto{0}{\ddot\alpha}\cos\alpha \end{cases}$$

$$\begin{bmatrix} \cos\beta & -b\sin\beta \\ \sin\beta & b\cos\beta \end{bmatrix} \begin{Bmatrix} \ddot b \\ \ddot\beta \end{Bmatrix} = \begin{Bmatrix} 2\dot b\dot\beta\sin\beta + b\dot\beta^2\cos\beta - a\dot\alpha^2\cos\alpha \\ -2\dot b\dot\beta\cos\beta + b\dot\beta^2\sin\beta - a\dot\alpha^2\sin\alpha \end{Bmatrix}$$

$$\Rightarrow \begin{cases} \ddot b = -6.26\ m/s^2 \\ \ddot\beta = 44.3\ rad/s^2 \end{cases}$$



TEOREMA DI RIVALS:

$$\vec{v}_D = \cancelto{0}{\vec{v}_B} + \vec{\omega}_{BD} \wedge (D-B) = \dot\beta\, DB\, \vec t$$

dove $\vec t = (-\cos 30, -\sin 30) = \left(-\frac{\sqrt3}{2}, -\frac12\right)$

$\vec n = (\sin 30, -\cos 30) = \left(\frac12, -\frac{\sqrt3}{2}\right)$

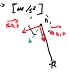
$$\Rightarrow \vec{v}_D = -\dot\beta DB\frac{\sqrt3}{2}\vec i - \frac{\dot\beta DB}{2}\vec j = -1.27\vec i \underbrace{- 0.735}_{\dot y_D}\vec j\ [m/s]$$

$$|v_D| = \sqrt{v_{D,x}^2 + v_{D,y}^2} = \dot\beta DB = 1.47\ m/s$$

$$\Rightarrow \vec{a}_D = \cancelto{0}{\vec{a}_B} + \dot{\vec\omega}_{BD} \wedge (D-B) - \omega_{BD}^2 (D-B) = \underbrace{\ddot\beta DB}\vec t + \underbrace{\dot\beta^2 DB}\vec n$$

$a_{D,t} = 33.7\ m/s^2$ $\quad$ $a_{D,n} = 6.4\ \frac{m}{s^2}$

$$\vec{a}_D = \ddot\beta DB\left(-\frac{\sqrt3}{2}, -\frac12\right) + \dot\beta^2 DB\left(\frac12, -\frac{\sqrt3}{2}\right) =$$
$$= -26\vec i - 22.4\vec j\ [m/s^2]$$



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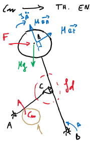
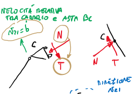
2) $C_m \longrightarrow$ TH. ENERGIA CINETICA

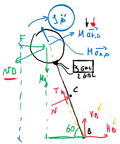




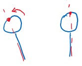
$$T = -f_d |N| \frac{v_{21}}{|v_{21}|}$$

DIREZIONE v_{21}

- OPPOSTA ALLA VELOCITÀ RELATIVA
- FUNZIONE DI N
- COMPONENTE DISSIPATIVA



$$\boxed{W_{DISS,T} = -|T|\cdot|v_{21}|}$$



$$\overset{+}{\circlearrowleft} \sum M_B = 0 \quad -N\,BC - Ma_{tD}BD + Mg\,BD\cdot\cos 60 - F\,BD\sin 60 - J\ddot\beta = 0$$

$$N = \frac{-Ma_{tD}BD + Mg\,BD\cos 60 - F\,BD\sin 60 - J\ddot\beta}{BC} = -717\ N$$

$$T = f_d\cdot|N| = 143.3\ N$$

$$\sum W_K = \frac{d}{dt}\{E_C\}$$

$v_D = \dot y_D\vec j + \dot x_D\vec i$ $\quad$ $M\vec g = -Mg\vec j + 0\vec i$ $\quad$ $W_{DISS,T}$

$$\sum W_K = \vec{C}_m \times \dot{\vec\alpha} + M\vec g \times \vec{v}_D + \vec F \times \vec{v}_D - |T||v_{21}| =$$
$$= C_m\dot\alpha - Mg\dot y_D + F\dot x_D - |T\cdot\dot b|$$

$$E_C = \frac12 M v_D^2 + \frac12 J\omega_D^2 = \frac12 M\vec{v}_D\times\vec{v}_D + \frac12 J\vec\omega_D\times\vec\omega_D$$

$$\frac{d}{dt}\{E_C\} = M\vec{v}_D\times\vec{a}_D + J\vec\omega_D\times\dot{\vec\omega}_D =$$
$$= (M\dot\beta DB\,\vec t)\times(\ddot\beta DB\,\vec t + \cancelto{0}{\dot\beta^2 DB\,\vec n}) + \overset{\frac12 MR^2}{J}\ddot\beta\dot\beta =$$
$$= \left(M\,DB^2 + \frac12 MR^2\right)\dot\beta\ddot\beta$$

$$\Rightarrow C_m\dot\alpha - Mg\dot y_D + F\dot x_D - |T\cdot\dot b| = \left(M\,BD^2 + \frac12 MR^2\right)\dot\beta\ddot\beta$$

$$C_m = \frac{\left(M\,BD^2 + \frac12 MR^2\right)\dot\beta\ddot\beta + |T\cdot\dot b| + Mg\dot y_D - F\dot x_D}{\dot\alpha} = 97.11\ Nm$$

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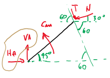
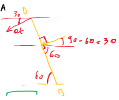
3) REAZIONI VINCOLARI IN A





$\sum Fx = 0$ $\qquad$ $H_A - N\cos 30 + T\cos 60 = 0$

$H_A = N\cos 30 - T\cos 60 = -652.6\ N$

$\sum Fy = 0$ $\qquad$ $V_A - T\sin 60 - N\sin 30 = 0$

$V_A = T\sin 60 + N\sin 30 = -234.4\ N$