Modeling and Control of Cyberphysical systems

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Project report

Part 2

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1 - Communication Network Structure Selection

1.1 - Preliminary Considerations

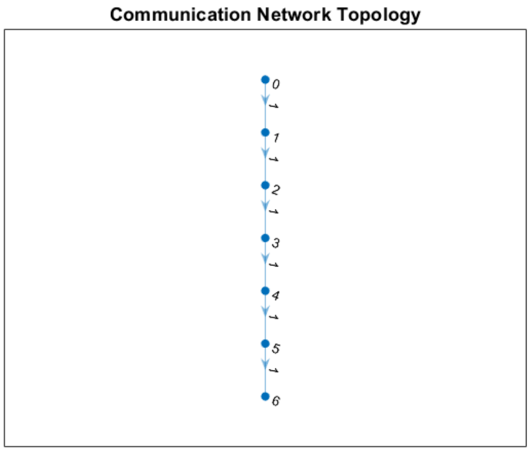
We considered several parameters to select the optimal configuration taking into account both the command activity and the output response of the agents. The results are based on a unitary step response analysis, imposed by the leader initial conditions, given A matrices with eigenvalues: [0, -0.5]. The zero eigenvalue is needed to have a marginally stable system (as opposed to an asymptotically stable one) outputting a constant response, and the magnitude of the negative eigenvalue is purposely low to have a more responsive system.

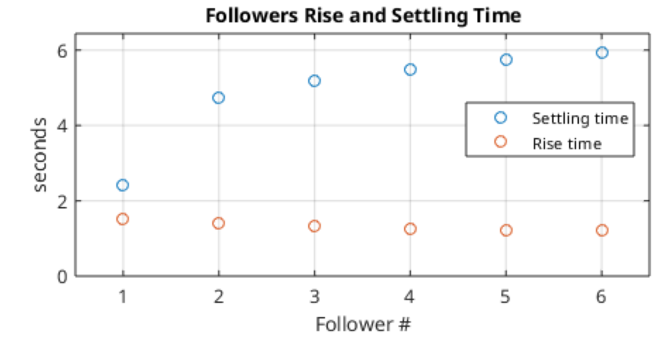
Regarding the command activity we only considered the signal l2-norm, whereas for the output response we took into account the root mean square error w.r.t. the leader output, the 10% - 90% rise time and the 2% settling time.

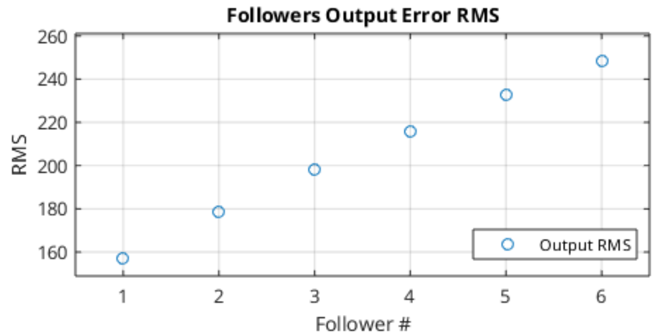
In this preliminary part, we considered fixed values for both the control and observers coupling gain and weight matrices R and Q, since we noticed that those values do not drastically affect the choice of the topology.

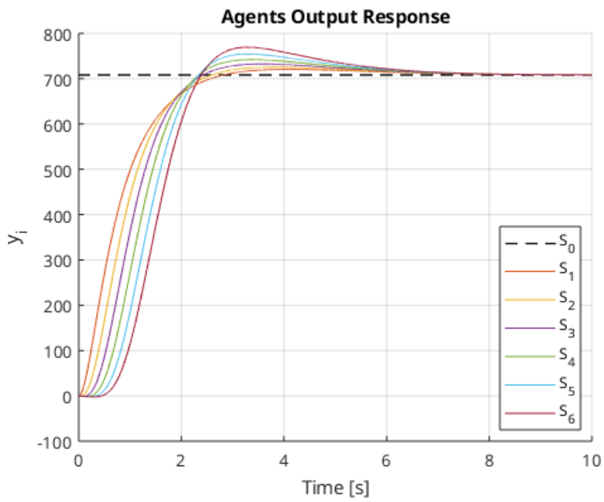
In the following, we report the schemes of the analyzed topologies and the obtained results.

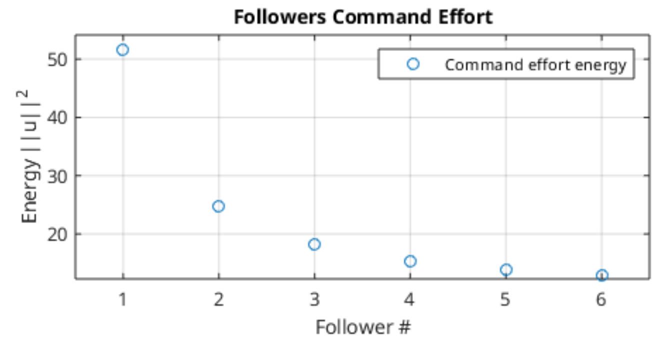
1.2 - Linear Topology











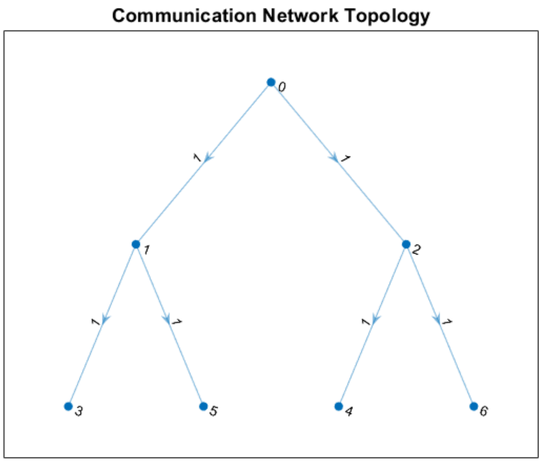
Analyzing this topology it is clear that each node can only rely on the previous one, so the information has an increasing delay in reaching the last node, which results in settling time which linearly increases when traversing the topology and in a marked difference between the outputs.

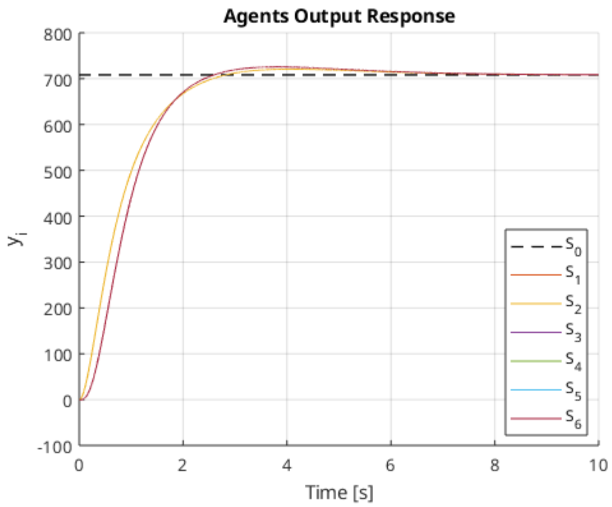
The RMS also follows a similar trend, as also highlighted by the fact that the command effort decreases going along the topology. This is again due to the delay needed for the information about the leader’s behavior to reach each node in the network.

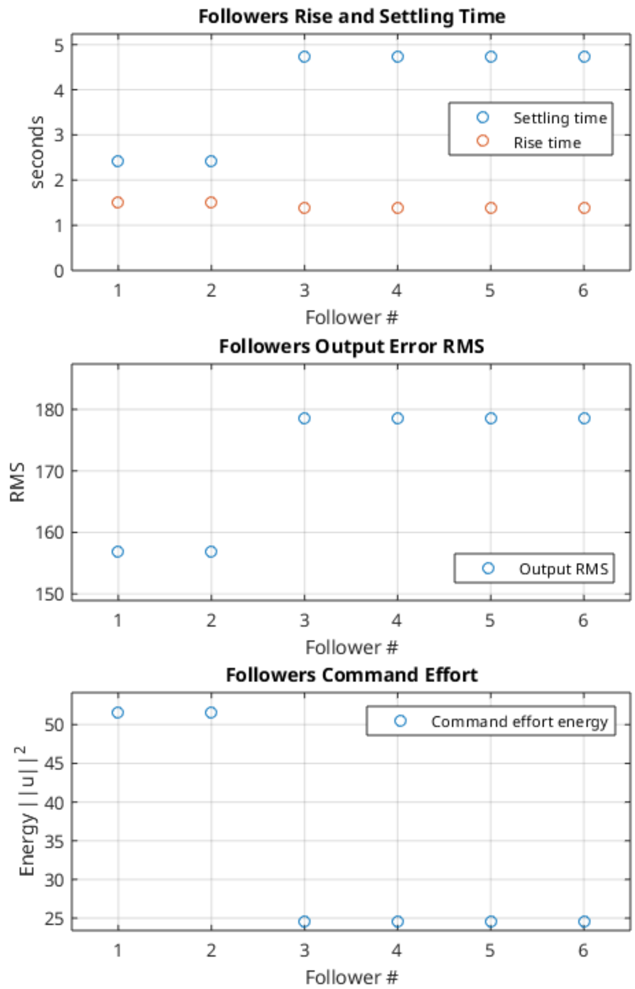
1.3 - Binary Tree Topology

Unlike the linear topology, this tree configuration ensures a maximum distance from the leader node equal to 2, while still keeping a quite small number of edges and network links.

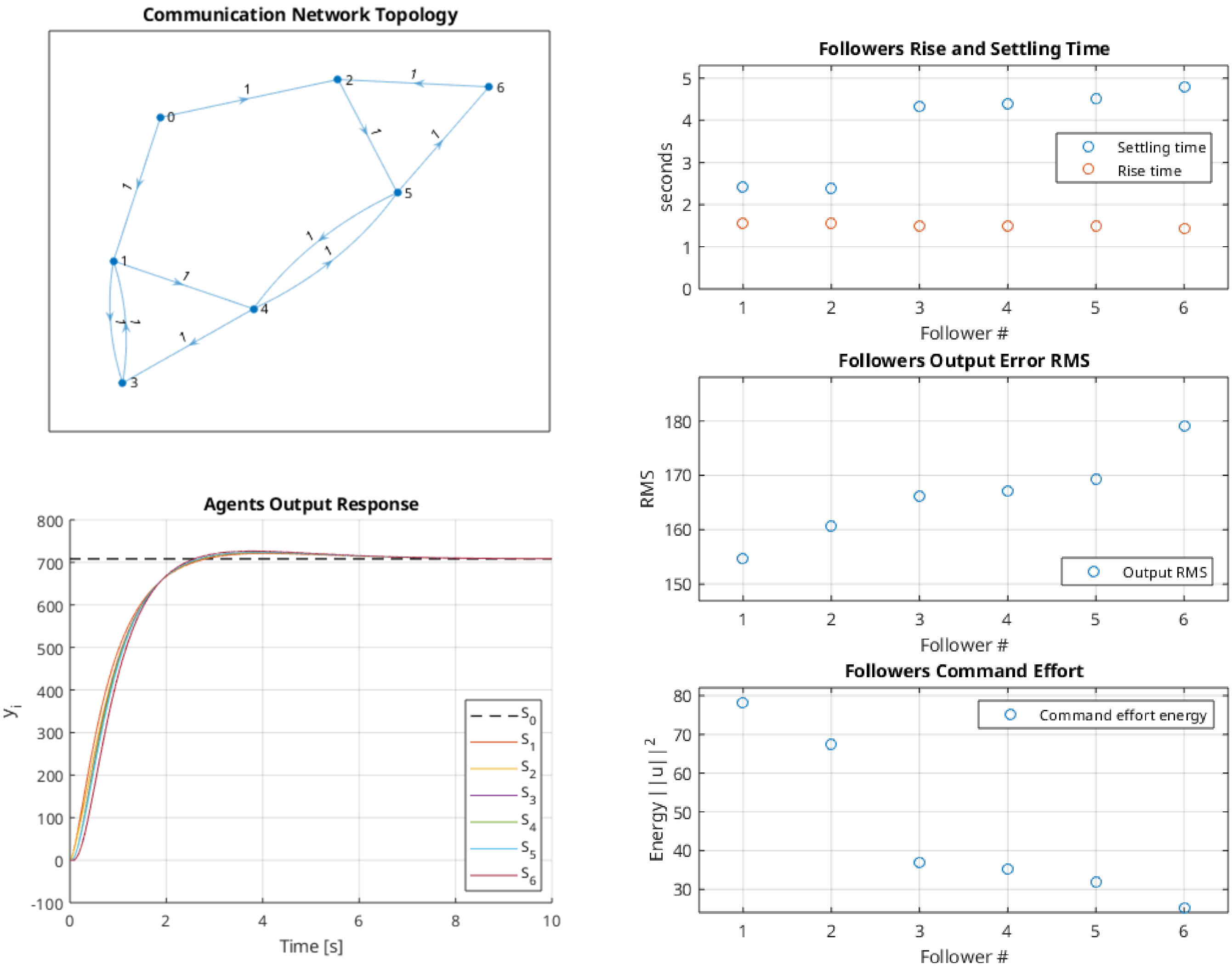
We can observe a distinctly different behavior for nodes #1 and #2, which are directly connected to the leader, compared to the leaf nodes: as expected, the first ones have a smaller value of output error RMS, along with a higher command activity, which globally results in a greater responsiveness. On the contrary, the agents corresponding to children nodes show the opposite trend. It is quite interesting to notice that the regulation performances, measured in terms of rise and settling time, are almost the same for all nodes at the same depth.







1.4 - Selected Topology



Starting from the previous configuration, we tried to modify it in order to increase diversity for some nodes. In particular, we added one link between each couple of leaf nodes, to enforce their communication, and also two "feedback" links from agents #3 and #6 towards their parent nodes #1 and #2.

In the topology selection, we also considered the differences that would be obtained by employing a distributed observer approach as opposed to a local one, and the possible effect of noise on agents.

In order to develop subsequent analyses, we finally settled on this topology because it's an excellent case of study, as it demonstrates a higher resilience to noise affecting the measurements and overall better performances when considering local observers, while maintaining a structure that could be reasonably implemented in a CPS framework.

In fact, in such a context we can investigate the impact of an “isolated” agent (#6) receiving information only from one node (#5) which is quite far from the leader and sending information to one node (#2) quite close to it. This choice enables us to further analyze the algorithm behavior in suboptimal conditions, by studying how noise corruption may affect the performances of node #6 and eventually of node #2.

Furthermore, we only considered unitary weights in the graph, since this choice leads to lower command inputs and to less uniform responses by different agents, enabling us to have a setup which is more sensitive to parameters tunings.

2 - Constant Reference

2.1 - Metrics

We analyzed the response of the system when we impose a unitary constant position to the leader, corresponding to an output of 708.27. The analysis was initially performed in an ideal, noiseless environment, we will discuss the effect of noise in the dedicated section.

Regarding the parameters choice, we considered the coupling gain c and the matrices Q, Q_o and R, R_o , used in the optimization problem in order to compute the observer and the controller gain. Each one was modified individually while keeping the others constant. The quantities mentioned in the following sections are relative to a simulation time of 10 seconds, and are:

- $RMS(\varepsilon)$: the RMS of the tracking error $\varepsilon = (y_0 - y_i)$, averaged between followers
- t_{ga} : the time after which the RMS of the global disagreement error falls below 10^{-2} $\forall t > t_{ga}$
- $||u||_2^2$: the energy of the control input, as per Parseval’s theorem, averaged across the followers
- $t_{s,2\%}$: the 2%-settling time, averaged between followers
- δy_{max}: $max_i \sum_{j \neq i} ||y_i - y_j||_2$, the highest sum of the distances between outputs of one one agent w.r.t. the others, averaged over time
- $\bar{x}$: the estimation error, defined as $||\hat{x} - x||_2$, where x is the true value of the state and $\hat{x}$ is the observer’s estimate, averaged over time

The first 5 metrics have been considered when varying control parameters, whereas the last one replaces the command effort for the distributed observer parameters, to have an insight about observers behavior.

2.2 - Distributed Observers

2.2.1 - Control Parameters Tuning

c	$RMS\{\varepsilon\}$	$t_{ga}(s)$	$\|\|u\|\|_2^2$	$t_{s,2\%}(s)$	δy_{max}
$c = c_{min}$	166.16	5.89	45.84	3.81	38.54
$c = 2\,c_{min}$	151.14	5.62	100.35	2.47	18.42
$c = 5\,c_{min}$	140.42	5.43	263.62	2.53	7.12
$c = 10\,c_{min}$	136.45	5.36	517.31	2.55	3.53

constant parameters:

$c_o = c_{min} \quad R = R_o = 1 \quad Q = Q_o = I$

c_{min}: the minimum value for the coupling gain, in accordance with the algorithm hypotheses

As expected, the main parameter affected by the coupling gain is δy_{max}, as it represents the highest difference between outputs. A higher value of c makes it so that the outputs are packed closer together and results in a lower RMS, at the cost of a stark increase in the command effort; in a real-world application the latter would be a limiting factor.

Since the value $c = 2\,c_{min}$ seems to be the best trade-off between output performances and command effort, the subsequent analysis will be performed considering this parameter setting.

Q / R	$RMS\{\varepsilon\}$	$t_{ga}(s)$	$\|\|u\|\|_2^2$	$t_{s,2\%}(s)$	δy_{max}
$Q = 0.01\,R * I$	237.29	8.28	6.29	7.21	159.51
$Q = 0.1\,R * I$	182.00	6.23	26.40	4.65	57.39
$Q = R * I$	151.14	5.62	100.35	2.47	18.42
$Q = 10\,R * I$	137.39	5.42	336.07	2.47	5.80
$Q = 100\,R * I$	132.32	5.36	961.74	2.46	1.87

constant parameters:

$c = 2\,c_{min} \quad c_o = c_{min} \quad R_o = 1 \quad Q_o = I$

Q and R regulate the tradeoff between command effort and reference tracking: as the table shows, a lower value of Q (relative to R) will result in poorer tracking, but also in a lower effort. Besides obtaining a more precise tracking, the spread of the outputs and the time to reach global agreement are affected positively by higher values of Q.relative to a smaller subset of the system.

2.2.2 - Observer Parameters Tuning

co	$RMS\{\varepsilon\}$	$t_{ga}(s)$	$\bar{x}$	$t_{s,2\%}(s)$	δy_{max}
$c_o = c_{o,min}$	151.14	5.62	5.23e-2	2.47	18.42
$c_o = 10^3 c_{o,min}$	150.94	5.62	5.22e-2	2.47	18.42
$c_o = 10^6 c_{o,min}$	150.95	5.62	5.22e-2	2.47	18.43

constant parameters:
$c = 2c_{min}$ $R = R_o = 1$ $Q = Q_o = I$

It's quite clear that the value of c_o does not significantly affect the system's behavior. We tried to investigate values several order of magnitude bigger than c_{min} without obtaining significant differences in considered metrics nor in CPS behavior.

Qo / Ro	$RMS\{\varepsilon\}$	$t_{ga}(s)$	$\bar{x}$	$t_{s,2\%}(s)$	δy_{max}
$Q_o = 10^{-6} R_o * I$	191.56	6.62	2.87e-2	5.82	91.52
$Q_o = 10^{-3} R_o * I$	156.85	5.46	5.36e-2	2.40	25.20
$Q_o = R_o * I$	151.05	5.62	5.23e-2	2.47	18.42
$Q_o = 10^3 R_o * I$	150.95	5.62	5.22e-2	2.47	18.42
$Q_o = 10^6 R * I$	150.94	5.62	5.22e-2	2.47	18.42

constant parameters:
$c = 2c_{min}$ $c_o = c_{min}$ $R = 1$ $Q = I$

It can be noticed that the tracking performances reaches a platooning for parameter values around $R_o = 1$ and $Q_o = I$.

As expected, very low values of Q_o decrease precision and increase global settling and agreement time. On the other hand, low values of Q_o lead to a better average tracking of the states of the agents, as highlighted by the $\bar{x}$ metric; a further analysis would be required to understand this particular feature.

2.3 - Local Observers

2.3.1 - Control Parameters Tuning

c	$RMS\{\varepsilon\}$	$t_{ga}\,(s)$	$\|\|u\|\|_2^2$	$t_{s,2\%}(s)$	δy_{max}
$c = c_{min}$	210.10	6.32	9.77	5.52	41.62
$c = 2\,c_{min}$	197.86	6.12	12.32	5.28	20.07
$c = 5\,c_{min}$	189.48	6.00	14.48	5.14	7.82
$c = 10\,c_{min}$	186.50	5.96	15.34	5.09	3.88

constant parameters:
$R = 1 \qquad Q = I$

As in the case of the cooperative observer, higher values of the coupling gain lead to a lower spread in the outputs, in this case, however, the command effort doesn't scale nearly as aggressively as for the cooperative observer. This can be explained by the fact that the error used to compute the corrective command is lower, as it's not computed considering several nodes of the system, but is only self-related.

Q / R	$RMS\{\varepsilon\}$	$t_{ga}\,(s)$	$\|\|u\|\|_2^2$	$t_{s,2\%}(s)$	δy_{max}
$Q = 0.01\,R * I$	270.39	8.64	2.94	7.71	168.08
$Q = 0.1\,R * I$	223.36	6.66	7.31	5.85	62.33
$Q = R * I$	197.86	6.12	12.32	5.28	20.07
$Q = 10\,R * I$	187.11	5.97	15.37	5.11	6.32
$Q = 100\,R * I$	183.33	5.92	16.62	5.05	1.99

constant parameter:
$c = 2\,c_{min}$

The same considerations made for the cooperative observer can be made for the local, with the clear exception of the command effort: once again, the values are much lower, making higher values of Q feasible.

3 - Other References

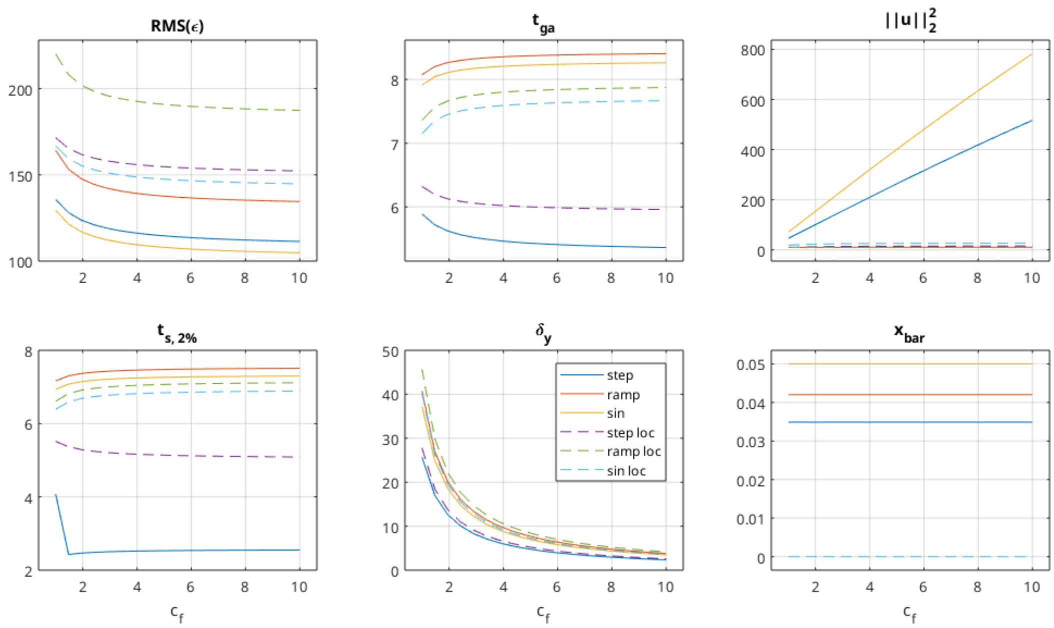
We can proceed with our discussion focusing on the main differences of the system behavior when tracking other kinds of reference signals, compared to the previous one.

We will analyze the previously considered constant reference, a ramp reference, with unitary slope, and a sinusoidal reference, with unitary amplitude and frequency of 0.5 Hz.

$r_1(t) = 1 \qquad\qquad r_2(t) = t \qquad\qquad r_3(t) = sin(\pi t)$

In the following plots the previously defined metrics are reported, considering the same ranges of parameters values, compared for the three reference signals. The dashed graphs correspond to the implementation with local observers.

3.1 - Control Coupling Gain

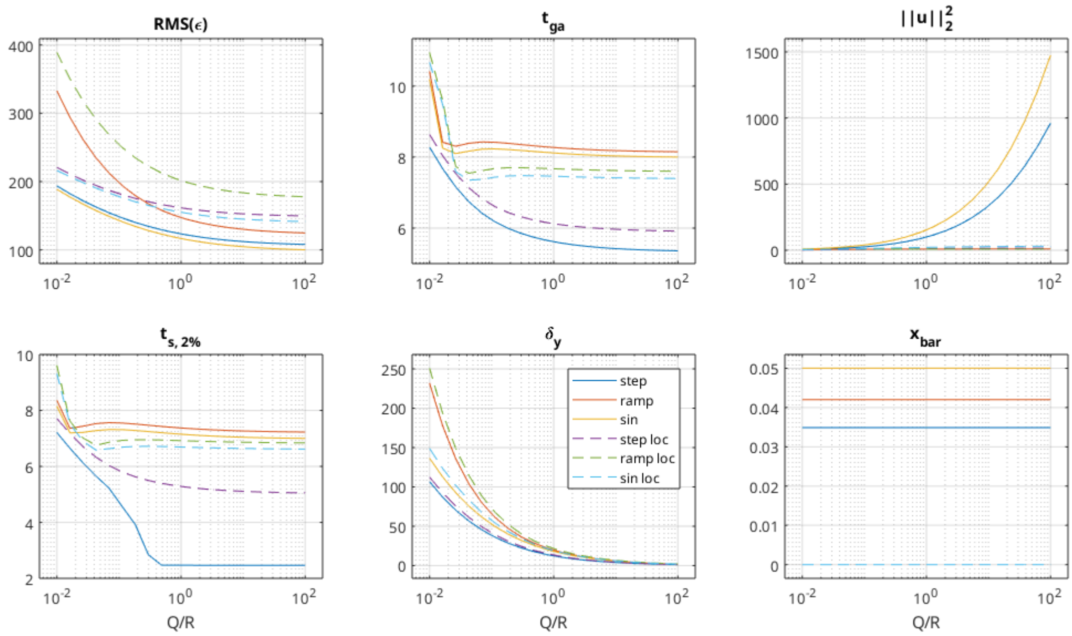


The trends of the analyzed metrics do not significantly vary changing the reference type, showing a descending trend for output error RMS and maximum distance δy_{max}, while settling and global agreement time slightly increase with the coupling gain. This is due to the fact that each node takes more and more into consideration the estimation error of all the other followers.

The only noticeable differences between references are the time at which the global disagreement falls below a certain threshold (close to 0), since the step is the only one which has a decreasing behavior, and the command effort, which significantly increases only for non-ramp references.

Another interesting conclusion is that the local observers provide a higher RMS w.r.t. the distributed ones, for all the references under investigation.

3.2 - Control Weight Matrices



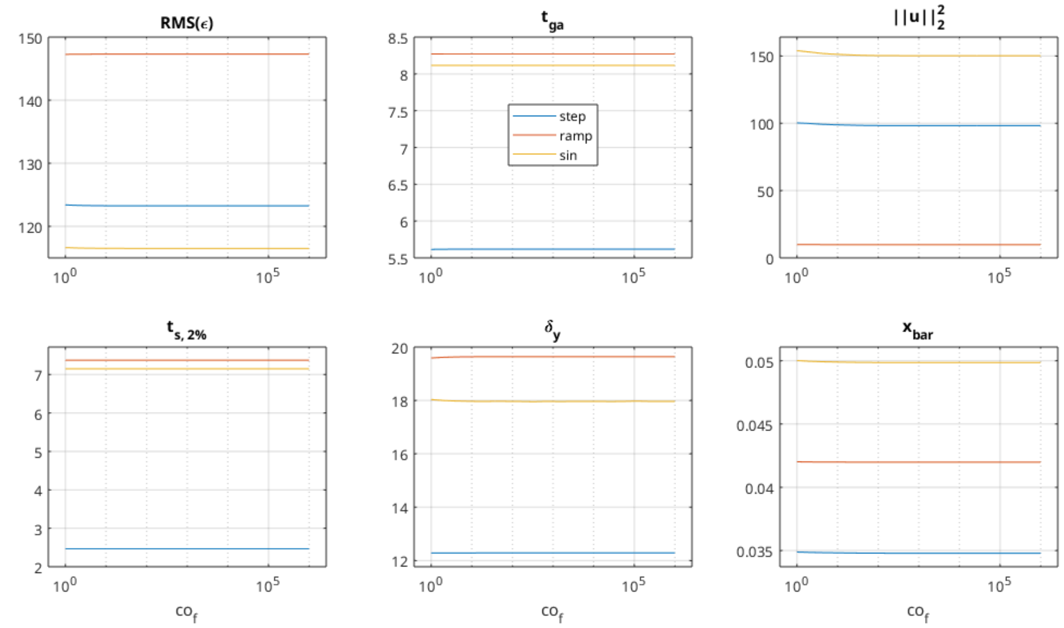
Now we focus on the Q / R ratio: as expected, when Q has a negligible magnitude w.r.t. R (e.g 10^{-2}), the output error performances (in terms of error RMS, global agreement and settling time) are quite bad while favoring a low command activity. On the other hand, a platooning of tracking behavior is reached when Q increases, along with a stronger command effort, even with an exponential behavior for constant and sinusoidal reference.

It's worth noticing that the step reference keeps the best overall performance compared with the other types of reference.

Finally, the local observers' configuration has a better behavior than the cooperative one for both ramp and sinusoidal reference tracking, when taking into account time related metrics.

3.3 - Observer Coupling Gain

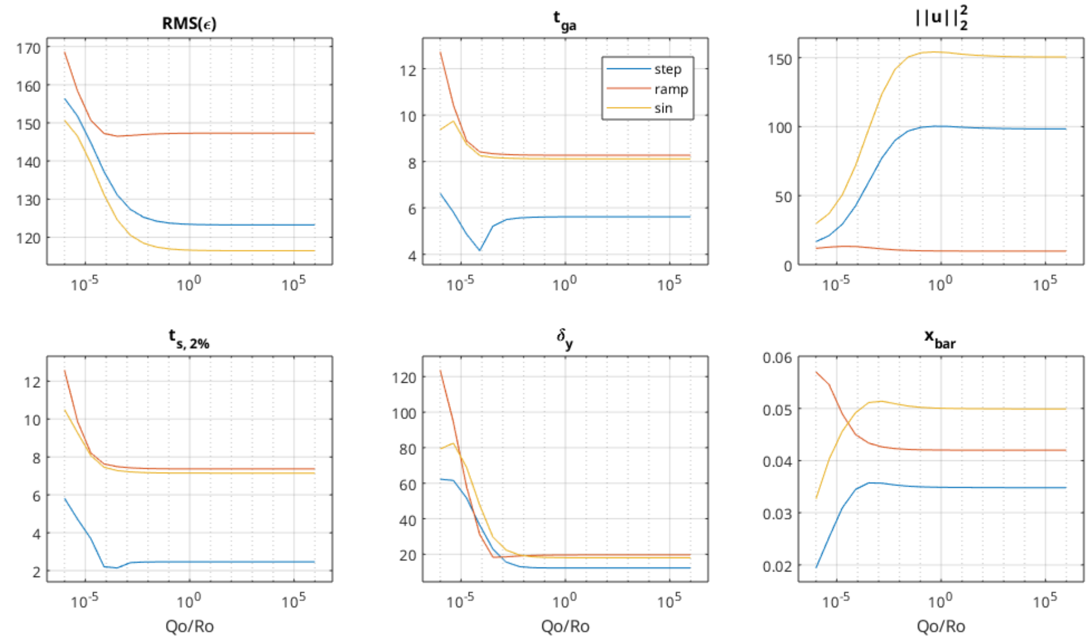
As already highlighted by the analysis regarding the step reference performed in the previous section, the observer coupling gain seems to play little to no role in the algorithm performances even when varying its values by several orders of magnitude.



3.4 - Observer Weight Matrices

Regarding the observer weights matrices, there is an evident flattening behavior for all metrics and for all references when crossing the 1 ratio. Before it, the performance follows an increasing trend when the ratio reaches lower values at the cost of command effort, especially for non-ramp references.

It's worth noticing that the estimation error follows an opposite behavior considering a ramp w.r.t. others reference signals, since is the only one decreasing at ratio increase.



4 - Output Noise Effects

4.1 - Topology-based Considerations

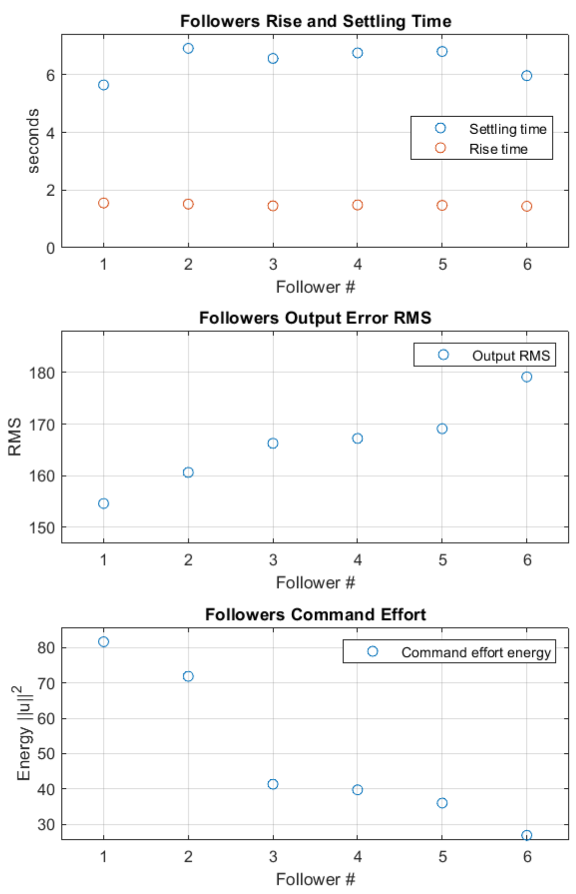
As said before, the selected topology ensures a quite good resilience to disturbances, because of its robust structure:

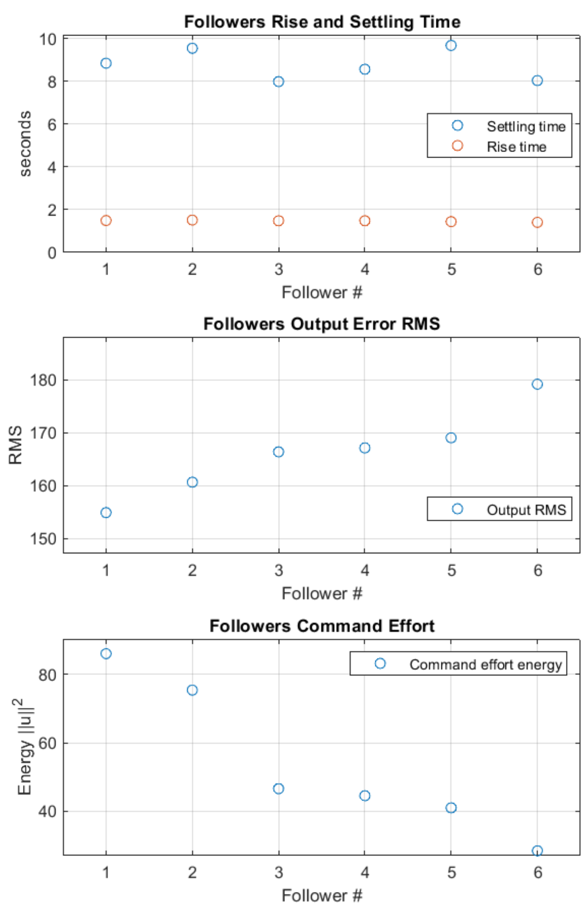
- the high number of connections between nodes enforces their estimation error's convergence;
- the general performance seems not to be affected by the presence of "feedback" links from children to parent nodes, which should represent a suboptimal configuration.

The following sections provide simulation results which confirms our topology based predictions. We chose a zero mean gaussian noise, with a value of 30 for variance.

4.2 - Distributed Noise Injection

In the first simulations we injected the selected noise into all the followers' and/or the leader to simulate general context involving output measurement disturbances.

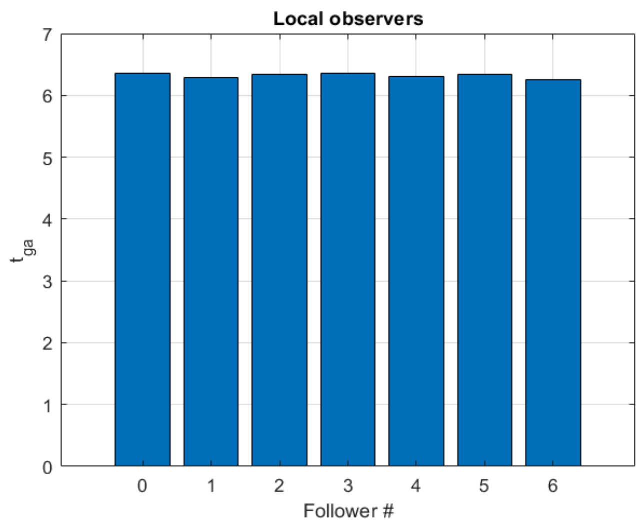




On the left figure is considered a configuration with all follower agents affected by noise: it’s possible to notice a general behavior similar to the noiseless configuration displayed in section 1, except for the settling time, which reasonably increases.
Moreover, we have found a really close performance trend in the opposite situation, when the noise is affecting only the leader but not the followers.

In the right figure we considered instead the noise affecting all the agents, both leader and followers: this simulation shows an even higher settling time for all nodes, close to simulation time for most of them.

4.3 - Single Follower Node Noise Injection



The graph above shows the effect of gaussian, white, noise with a variance of 50 on the time to reach global agreement (as it was defined in section 2.1) when applied to each node individually; only the data relative to the local observers was included as in the case of cooperative observers the norm of the global disagreement is always above the threshold (1e-2), due to the spreading of the noise across nodes.

We can see that the results are also extremely uniform, having a standard deviation of only 0.035, and can therefore conclude that using this approach there is not a single critical node.

5 - Final considerations

These simulations highlight how both the local and cooperative approach to observers have their own advantages and disadvantages: a local observer will result in a lower command effort and in a better tolerance to noise, however a cooperative observer can provide a more precise tracking and faster convergence times, at the cost of a higher command effort.

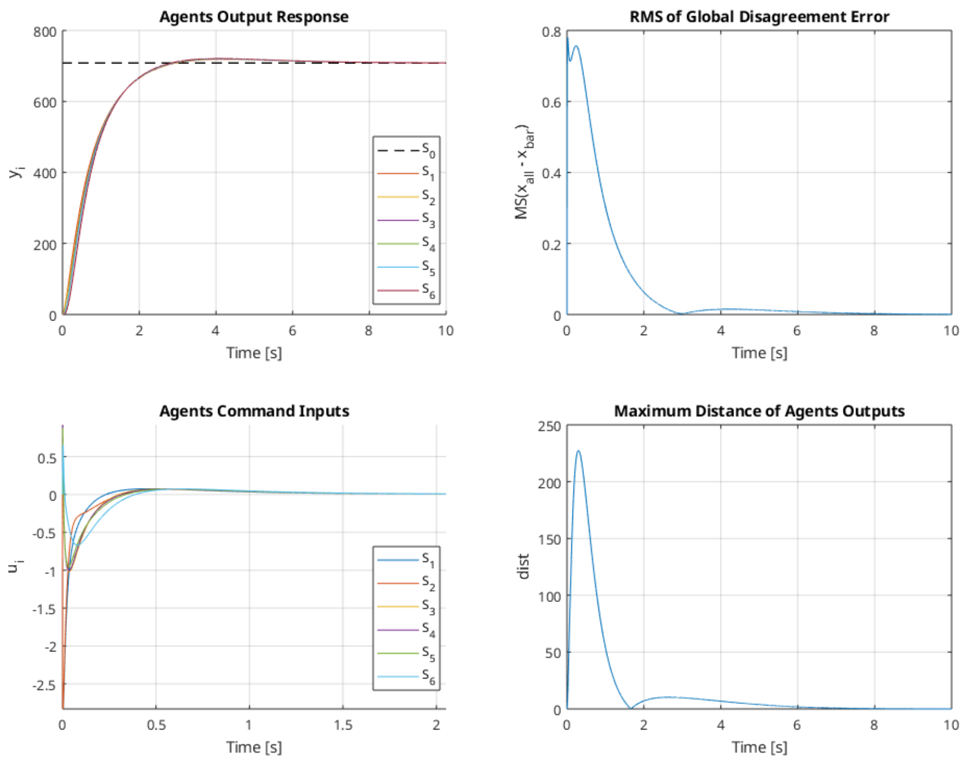
The different references provide different insights about the system behavior: the transient behavior which is present both in the constant and in the sinusoidal reference, due to the different initial position of the followers w.r.t. to the leader, leads to higher command effort in all configurations. On the other hand, the pure tracking phase, which constitutes most of the ramp reference behavior, due to the same initial position but different initial velocity, requires much less energy, but does not always translate into better performances.

Moreover, when considering the effect of noise in the system, we detected a quite resilient behavior provided by both the structure of the selected network and the effectiveness of the cooperative observer and tracking system. In fact, assuming reasonable and realistic parameters for noise typology, it is possible to overcome its disturbance action without losing so much effectiveness in tracking performance, especially when employing local observer configuration.

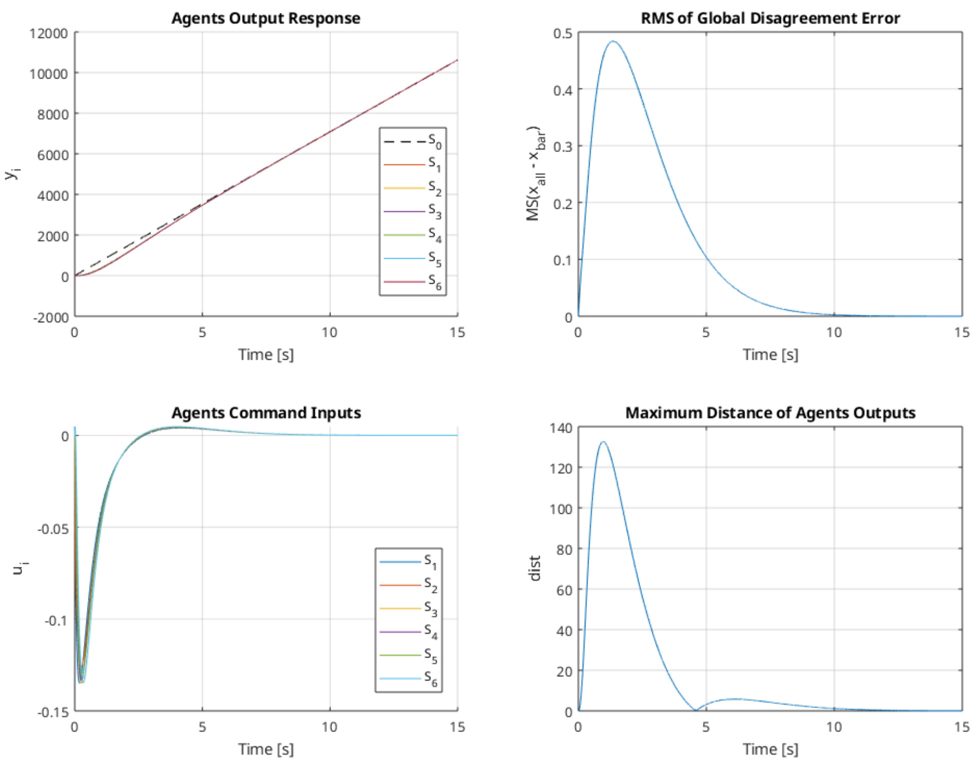
6 - Appendix

In this appendix, we display some of the plots produced and analyzed to derive the conclusions presented in the report.

6.1 - Constant Reference



6.2 - Ramp Reference



6.3 - Sinusoidal Reference