

$$\mathcal{M}_\gamma = \frac{e^2}{3q^2} [\bar{v}(k) \gamma^\mu u(p)] [\bar{u}(p') \gamma_\mu v(k')]$$

$$4 \langle |\mathcal{M}_\gamma|^2 \rangle = \frac{9s^2}{e^4} =$$

$$\sum_{p,q,r,s} \bar{v}_a^p(k) \gamma_{ab}^\mu u_b^q(p) \bar{u}_c^r(p') \gamma_{cd}^\nu v_d^s(k') \bar{v}_e^s(k') \gamma_{ef}^\mu u_f^r(p') \bar{u}_g^q(p) \gamma_{gh}^\nu v_h^p(k)$$

$$= \text{Tr} \{ (\not{k} - m) \gamma^\mu (\not{p} + m) \gamma^\nu \} \text{Tr} \{ (\not{k}' - m) \gamma_\nu (\not{p}' + m) \gamma_\mu \}$$

$$\text{Tr} \{ (\not{k} - m) \gamma^\mu (\not{p} + m) \gamma^\nu \} =$$

$$= \text{Tr} \{ (\not{k} \gamma^\mu - m \gamma^\mu) (\not{p} \gamma^\nu + m \gamma^\nu) \} =$$

$$= \text{Tr} \{ \not{k} \gamma^\mu \not{p} \gamma^\nu - m^2 \gamma^\mu \gamma^\nu \} = k_\alpha p_\beta \text{Tr} \{ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\nu \} - 4m^2 g^{\mu\nu}$$

$$= 4 \{ k^\mu p^\nu - (k \cdot p) g^{\mu\nu} + k^\nu p^\mu - m^2 g^{\mu\nu} \}$$

$$= 16 \{ k^\mu p^\nu - (k \cdot p) g^{\mu\nu} + k^\nu p^\mu - m^2 g^{\mu\nu} \} \{ k'_\nu p'_\mu - (k' \cdot p') g_{\mu\nu} + k'_\mu p'_\nu - M^2 g_{\mu\nu} \}$$

$$= 16 \{ (k \cdot p') (k' \cdot p) - \cancel{(k' \cdot p') (k \cdot p)} + (k \cdot k') (p \cdot p') - M^2 (k \cdot p)$$

$$- \cancel{(k \cdot p) (k' \cdot p')} + 4 \cancel{(k \cdot p) (k' \cdot p')} - \cancel{(k \cdot p) (k' \cdot p')} + 4M^2 (k \cdot p)$$

$$+ (k \cdot k') (p \cdot p') - \cancel{(k' \cdot p') (k \cdot p)} + (k \cdot p') (k' \cdot p) - M^2 (k \cdot p)$$

$$- m^2 (k' \cdot p') + 4m^2 (k' \cdot p') - m^2 (k' \cdot p') + 4M^2 m^2 \}$$

$$= 32 \{ (k \cdot p') (k' \cdot p) + (k \cdot k') (p \cdot p') + M^2 (k \cdot p) + m^2 (k' \cdot p') + 2M^2 m^2 \}$$

* In CMS frame

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} \frac{p_+}{p_-} |\mathcal{M}|^2$$

$$p = (E, 0, 0, p)$$

$$p' = (E, 0, p' \sin \theta, p' \cos \theta)$$

$$k = (E, 0, 0, -p)$$

$$k' = (E, 0, -p' \sin \theta, -p' \cos \theta)$$

$$(p \cdot k) = E^2 + p^2$$

$$(p' \cdot k') = E^2 + p'^2$$

$$(p \cdot p') = E^2 - pp' \cos \theta \quad (k \cdot k') = E^2 - pp' \cos \theta =$$

$$(p \cdot k') = E^2 + pp' \cos \theta = (p' \cdot k)$$

$$p = \sqrt{E^2 - m^2}$$

$$M_2 = \frac{1}{q^2 - m_2^2} \{ \bar{v}(k) \gamma^\mu (c_v - c_A \gamma^5) u(p) \} \{ \bar{u}(p') \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) v(k') \}$$

→ Spin sums

$$4 \langle |M_2|^2 \rangle (S - M_2^2)^2 =$$

$$= \sum_{p q r s} \left[\bar{v}^\dagger(k) \gamma^\mu (c_v - c_A \gamma^5) u^p(p) \bar{u}^r(p') \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) v^s(k') \right] \times \\ \left[\bar{v}^\dagger(k) \gamma^\mu (c_v - c_A \gamma^5) u^p(p) \bar{u}^r(p') \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) v^s(k') \right]^\dagger \quad (\neq)$$

$$= \sum_{p q r s} \left\{ \bar{v}_a^\dagger(k) \gamma_{ab}^\mu (c_v - c_A \gamma^5) u_{bc}^p(p) \bar{u}_d^r(p') \gamma_{de}^\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) v_{ef}^s(k') \right\} \times \\ \left\{ \bar{v}_\alpha^s(k') \gamma_{\beta\gamma}^\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) u_\gamma^r(p') \bar{u}_\delta^p(p) \gamma_{\delta\epsilon}^\mu (c_v - c_A \gamma^5) v_{\epsilon\eta}^p(k) \right\}$$

$$= \sum_p v_\eta^p(k) \bar{v}_a^\dagger(k) \sum_q u_{bc}^p(p) \bar{u}_\delta^q(p) \sum_r u_\gamma^r(p') \bar{u}_d^r(p') \sum_s v_\epsilon^s(k') \bar{v}_f^s(k') \times \\ \gamma_{ab}^\mu (c_v - c_A \gamma^5)_{bc} \gamma_{de}^\mu (c_v - c_A \gamma^5)_{ef} \gamma_{\beta\gamma}^\mu (c_v + c_A \gamma^5)_{\beta\gamma} \gamma_{\delta\epsilon}^\mu (c_v + c_A \gamma^5)_{\delta\epsilon}$$

$$= \text{Tr} \left\{ (\not{k} - m) \gamma^\mu (c_v - c_A \gamma^5) (\not{p} + m) \gamma^\mu (c_v - c_A \gamma^5) \right\} \times \\ \times \text{Tr} \left\{ (\not{k}' - M) \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) (\not{p}' + M) \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) \right\}$$

$$\bullet \text{Tr} \left\{ (\not{k} - m) \gamma^\mu (c_v - c_A \gamma^5) (\not{p} + m) \gamma^\mu (c_v - c_A \gamma^5) \right\} =$$

$$= \text{Tr} \left\{ (\not{k} \gamma^\mu - m \gamma^\mu) (c_v - c_A \gamma^5) (\not{p} \gamma^\mu + m \gamma^\mu) (c_v - c_A \gamma^5) \right\} =$$

$$= \text{Tr} \left\{ [c_v \not{k} \gamma^\mu - c_A \not{k} \gamma^\mu \gamma^5 - m c_v \gamma^\mu + m c_A \gamma^\mu \gamma^5] \times \right.$$

$$\left. \times [c_v \not{p} \gamma^\mu - c_A \not{p} \gamma^\mu \gamma^5 + m c_v \gamma^\mu - m c_A \gamma^\mu \gamma^5] \right\} =$$

$$= \text{Tr} \left\{ c_v^2 \not{k} \gamma^\mu \not{p} \gamma^\mu - c_A c_v \not{k} \gamma^\mu \not{p} \gamma^\mu \gamma^5 + \cancel{m c_v^2 \not{k} \gamma^\mu \gamma^5} + \cancel{m c_v c_A \not{k} \gamma^\mu \gamma^5 \gamma^5} \right. \\ \left. - c_A c_v \not{k} \gamma^\mu \gamma^5 \not{p} \gamma^\mu + c_A^2 \not{k} \gamma^\mu \gamma^5 \not{p} \gamma^\mu \gamma^5 - \cancel{m c_A c_v \not{k} \gamma^\mu \gamma^5} - \cancel{m c_A^2 \not{k} \gamma^\mu \gamma^5 \gamma^5} \right. \\ \left. - \cancel{m c_v^2 \not{k} \gamma^\mu \gamma^5} + \cancel{m c_A c_v \not{k} \gamma^\mu \gamma^5 \gamma^5} - m^2 c_v^2 \gamma^\mu \gamma^\mu - m^2 c_A c_v \gamma^\mu \gamma^\mu \gamma^5 \right\}$$

$$+ m C_A C_V \cancel{\gamma^\mu \gamma^5 \not{p} \gamma^\beta} + m C_A^2 \cancel{\gamma^\mu \gamma^5 \not{p} \gamma^\beta \gamma^5} + m^2 C_A C_V \gamma^\mu \gamma^5 \gamma^\beta - m^2 C_A^2 \gamma^\mu \gamma^5 \gamma^\beta \gamma^5 \}$$

$$- \text{Tr} \{ c_v^2 \cancel{K \gamma^\mu \not{p} \gamma^\beta} - c_A c_v K \gamma^\mu \not{p} \gamma^\beta \gamma^5 - c_A c_v K \gamma^\mu \gamma^5 \not{p} \gamma^\beta + \underbrace{c_A^2 K \gamma^\mu \gamma^5 \not{p} \gamma^\beta \gamma^5}_{\substack{c_A^2 K \gamma^\mu \not{p} \gamma^\beta \gamma^5 \\ - c_A^2 K \gamma^\mu \not{p} \gamma^\beta}}$$

$$\{\gamma^\mu, \gamma^\nu\} = 0, (\gamma^5)^2 = 1, \text{Tr}(\gamma^\alpha \gamma^\beta \gamma^5) = 0$$

$$- m^2 c_v^2 \gamma^\mu \gamma^\beta + \cancel{m^2 C_A C_V \gamma^\mu \gamma^\beta \gamma^5} + \cancel{m^2 C_A C_V \gamma^\mu \gamma^\beta \gamma^5} - m^2 C_A^2 \gamma^\mu \gamma^\beta \gamma^5 \gamma^5 \underbrace{\gamma^\beta \gamma^5}_{+ m^2 C_A^2 \gamma^\mu \gamma^\beta} \}$$

$$= \text{Tr} \{ c_v^2 K \gamma^\mu \not{p} \gamma^\beta - c_A c_v K \gamma^\mu \not{p} \gamma^\beta \gamma^5 - \underbrace{c_A c_v K \gamma^\mu \gamma^5 \not{p} \gamma^\beta}_{+ c_A c_v K \gamma^\mu \not{p} \gamma^\beta \gamma^5} + c_A^2 K \gamma^\mu \not{p} \gamma^\beta \\ - m^2 c_v^2 \gamma^\mu \gamma^\beta - m^2 C_A^2 \gamma^\mu \gamma^\beta \} - c_A c_v K \gamma^\mu \not{p} \gamma^\beta \gamma^5$$

$$= \text{Tr} \{ (K \gamma^\mu \not{p} \gamma^\beta) (c_v^2 + c_A^2) - m^2 \gamma^\mu \gamma^\beta (c_v^2 - c_A^2) - 2 c_A c_v K \gamma^\mu \not{p} \gamma^\beta \gamma^5 \}$$

$$\text{Tr}(\gamma^\alpha \not{p} \gamma^\beta) = 4 (g^{\alpha\beta} p^\beta - g^{\alpha\beta} p^\beta + g^{\alpha\beta} p^\beta)$$

$$= (c_v^2 - c_A^2) k_\alpha p_\beta \underbrace{\text{Tr} \{ \gamma^\alpha \gamma^\mu \gamma^\beta \gamma^\beta \}}_{4(g^{\alpha\mu} g^{\beta\beta} - g^{\alpha\beta} g^{\mu\beta} + g^{\alpha\beta} g^{\mu\beta})} - 4 m^2 (c_v^2 - c_A^2) g^{\mu\beta} - 2 c_A c_v \text{Tr} \{ K \gamma^\mu \not{p} \gamma^\beta \gamma^5 \}$$

$$= 4 \{ (c_v^2 + c_A^2) [k^\mu p^\beta - (k \cdot p) g^{\mu\beta} + k^\beta p^\mu] - m^2 (c_v^2 - c_A^2) g^{\mu\beta} \\ - 2 i c_A c_v k_\alpha p_\beta \underbrace{\epsilon^{\alpha\mu\beta\rho}}_{\text{antisymmetric in } \mu\beta} \}$$

$$(A) \text{ Cross term: } 4 c_A c_v \tilde{C}_A \tilde{C}_V k_\alpha p_\beta p'^\alpha k'^\beta \epsilon^{\alpha\mu\beta\rho} \epsilon_{\rho\mu\tau\sigma}$$

$$= 8 c_A c_v \tilde{C}_A \tilde{C}_V \{ (k \cdot p')(k' \cdot p) - (p \cdot p')(k \cdot k') \} \epsilon^{\mu\beta\alpha\rho} \epsilon_{\rho\mu\tau\sigma} = -2 (\delta_\tau^\rho \delta_\sigma^\alpha - \delta_\sigma^\rho \delta_\tau^\alpha)$$

$$\Rightarrow 4 \langle |M_2|^2 \rangle (S - M_i^2)^2 =$$

$$\frac{4}{16} \{ \underbrace{(c_v^2 + c_A^2)}_{\equiv \tilde{\Sigma}} \underbrace{[k^\mu p^\beta - (k \cdot p) g^{\mu\beta} + k^\beta p^\mu]}_{\text{fully symmetric in } \mu\beta} - m^2 \underbrace{(c_v^2 - c_A^2)}_{\equiv \tilde{\Delta}} g^{\mu\beta} \} \times \\ \times \{ \underbrace{(c_v^2 + c_A^2)}_{\equiv \tilde{\Sigma}} [k'_\mu p'_\beta - (k' \cdot p') g_{\mu\beta} + k'_\beta p'_\mu] - M^2 \underbrace{(c_v^2 - c_A^2)}_{\equiv \tilde{\Delta}} g_{\mu\beta} \} =$$

$$= 4 \left\{ \sum \tilde{\Sigma} (k^\mu p^\nu - (k \cdot p) g^{\mu\nu} + k^\nu p^\mu) (k'_\mu p'_\nu - (k' \cdot p') g_{\mu\nu} + k'_\nu p'_\mu) + \right. \\ \left. + \sum \tilde{\Delta} M^2 (\underbrace{k \cdot p - 4 k \cdot p + k \cdot p}_{-2 k \cdot p}) + 2 \Delta \sum m^2 (k' \cdot p') \right. \\ \left. + \Delta \tilde{\Delta} 4 m^2 M^2 \right\}$$

$$\leadsto (k^\mu p^\nu - (k \cdot p) g^{\mu\nu} + k^\nu p^\mu) (k'_\mu p'_\nu - (k' \cdot p') g_{\mu\nu} + k'_\nu p'_\mu) \\ \left| \begin{aligned} &= \left\{ (k \cdot k') (p \cdot p') - \cancel{(k' \cdot p') (k \cdot p)} + (k' \cdot p) (k \cdot p') \right. \\ &\quad - \cancel{(k \cdot p) (k' \cdot p')} + 4 \cancel{(k \cdot p) (k' \cdot p')} - \cancel{(k \cdot p) (k' \cdot p')} \\ &\quad \left. + (k \cdot p') (k' \cdot p) - \cancel{(k' \cdot p') (k \cdot p)} + (k \cdot k') (p \cdot p') \right\} \\ &= 2 (k \cdot k') (p \cdot p') + 2 (k \cdot p') (k' \cdot p) \end{aligned} \right.$$

$$\Rightarrow \langle |M_Z|^2 \rangle (s - m_Z^2) =$$

$$= 8 \left\{ \sum \tilde{\Sigma} \left[(k \cdot k') (p \cdot p') + (k \cdot p') (k' \cdot p) \right] - M_Z^2 \sum \tilde{\Delta} (k \cdot p) + m^2 \Delta \sum (k' \cdot p') \right. \\ \left. + 2 \Delta \tilde{\Delta} m^2 M^2 - 4 c_A c_V \tilde{c}_A \tilde{c}_V \left[(k \cdot p') (k' \cdot p) - (p \cdot p') (k \cdot k') \right] \right\}$$

$$(*) \left[\bar{v}(b) \gamma^\mu (c_V - c_A \gamma^5) u(p) \right]^\dagger =$$

$$= u(p)^\dagger (c_V - c_A \gamma^5)^\dagger (\gamma^\mu)^\dagger (\gamma^0)^\dagger v(b)$$

$$= u(p)^\dagger (c_V - c_A \gamma^5) (v^\mu)^\dagger v^0 v(b)$$

$$= \bar{u}(p) \gamma^\mu (c_v + c_A \gamma^5) \gamma^\nu v(k)$$

$$= \bar{u}(p) \gamma^\mu (c_v + c_A \gamma^5) \gamma^\nu v(k)$$

$$= \bar{u}(p) \gamma^\mu (c_v - c_A \gamma^5) v(k)$$

$$\mathcal{M}_h = -\frac{g_w^2}{4m_w^2} \frac{m_\mu m_b}{s - m_h^2} \bar{v}(k) u(p) \bar{u}(p') v(k')$$

$$4 \langle |\mathcal{M}_h|^2 \rangle =$$

$$\left(\frac{g_w^2}{4m_w^2} \frac{m_\mu m_b}{s - m_h^2} \right)^2 \sum_{p,q,r,s} \bar{v}_a^p(k) u_b^q(p) \bar{u}_c^r(p') v_d^s(k') \bar{v}_e^s(k') u_f^r(p') \bar{u}_g^r(p) v_h^p(k)$$

$$= k_H^2 \sum_p v_h^p(k) \bar{v}_a^p(k) \sum_q u_b^q(p) \bar{u}_g^q(p) \sum_r u_f^r(p') \bar{u}_c^r(p') \sum_s v_d^s(k') \bar{v}_e^s(k')$$

$$= k_H^2 (\not{k} - m)_{ha} \not{p}_{bg} (\not{p}' + m)_{fc} (\not{k}' - m)_{de}$$

$$= k_H^2 \text{Tr} \{ (\not{k} - m)(\not{p} + m) \} \text{Tr} \{ (\not{p}' + m)(\not{k}' - m) \}$$

$$\text{Tr} \{ (\not{k} - m)(\not{p} + m) \} = \text{Tr} \{ \not{k}\not{p} - m\not{k} - m\not{p} - m^2 \} = 4 \{ (k \cdot p) - m^2 \}$$

$$= 16 k_H [(k \cdot p) - m^2] [(k' \cdot p') - M^2]$$

$$= 16 k_H [(k \cdot p)(k' \cdot p') - M^2(k \cdot p) - m^2(k' \cdot p') + M^2 m^2]$$

→ w/ Mandelstam variables

$$(k \cdot p) = \frac{s}{2} - m^2 \quad (k' \cdot p') = \frac{s}{2} - M^2$$

$$\Rightarrow = 16 k_H \left\{ \left(\frac{s}{2} - m^2 \right) \left(\frac{s}{2} - M^2 \right) - M^2 \left(\frac{s}{2} - m^2 \right) - m^2 \left(\frac{s}{2} - M^2 \right) + M^2 m^2 \right\}$$

$$= 16 k_H \left\{ \frac{s^2}{4} - \frac{M^2 s}{2} - \frac{m^2 s}{2} + M^2 m^2 - M^2 \frac{s}{2} + M^2 m^2 - \frac{m^2 s}{2} + M^2 m^2 + M^2 m^2 \right\}$$

$$= 16 k_H \left\{ \frac{s^2}{4} - s(M^2 + m^2) + 4M^2 m^2 \right\}$$

$$\mathcal{M}_Z = \frac{1}{s - m_Z^2} \left\{ \bar{u}(k) \gamma^\mu (c_V - c_A \gamma^5) u(p) \right\} \left\{ \bar{u}(p') \gamma_\mu (c_V - c_A \gamma^5) v(k') \right\}$$

$$\mathcal{M}_\gamma = \frac{e^2}{s} \left\{ \bar{u}(k) \gamma^\mu u(p) \right\} \left\{ \bar{u}(p') \gamma_\mu v(k') \right\}$$

$$\rightarrow 4 \langle \mathcal{M}_Z \mathcal{M}_\gamma^* \rangle 3s(s - m_Z^2) =$$

$$\sum_{p,q,r,s} \left\{ \bar{u}^p(k) \gamma^\mu (c_V - c_A \gamma^5) u^q(p) \right\} \left\{ \bar{u}^r(p') \gamma_\mu (c_V - c_A \gamma^5) v^s(k') \right\} \\ \left\{ \left[\bar{u}^p(k) \gamma^\mu u^q(p) \right] \left[\bar{u}^r(p') \gamma_\mu v^s(k') \right] \right\}^+ =$$

$$= \sum_{p,q,r,s} \left\{ \bar{u}_a^p(k) (\gamma^\mu (c_V - c_A \gamma^5))_{ab} u_b^q(p) \right\} \left\{ \bar{u}_c^r(p') (\gamma_\mu (\tilde{c}_V - \tilde{c}_A \gamma^5))_{cd} v_d^s(k') \right\} \\ \times \left\{ \bar{v}_e^s(k') \gamma_\mu u_f^r(p') \bar{u}_g^q(p) \gamma_\mu v_h^p(k) \right\}$$

$$= (k - m)_{ba} (\not{p} + m)_{bg} (k' - m)_{de} (\not{p}' + m)_{fc}$$

$$(\gamma^\mu (c_V - c_A \gamma^5))_{ab} (\gamma_\mu (\tilde{c}_V - \tilde{c}_A \gamma^5))_{cd} \gamma_{ef}^g \gamma_{gh}^g$$

$$= \text{Tr} \left\{ (k - m) \gamma^\mu (c_V - c_A \gamma^5) (\not{p} + m) \gamma^\mu \right\} \times \\ \times \text{Tr} \left\{ (k' - m) \gamma_\mu (\tilde{c}_V - \tilde{c}_A \gamma^5) \right\}$$

$$\bullet \text{Tr} \left\{ (k - m) \gamma^\mu (c_V - c_A \gamma^5) (\not{p} + m) \gamma^\mu \right\} =$$

$$= \text{Tr} \left\{ (c_V \not{k} \gamma^\mu - m c_V \gamma^\mu - c_A \not{k} \gamma^\mu \gamma^5 + m c_A \gamma^\mu \gamma^5) (\not{p} \gamma^\mu + m \gamma^\mu) \right\}$$

$$= \text{Tr} \left\{ c_V \not{k} \gamma^\mu \not{p} \gamma^\mu - m c_V \cancel{\not{p} \gamma^\mu \gamma^\mu}^{\text{odd}} - c_A \not{k} \gamma^\mu \gamma^5 \not{p} \gamma^\mu + m c_A \cancel{\gamma^\mu \gamma^5 \not{p} \gamma^\mu}^{\text{odd}} \right. \\ \left. + m c_V \cancel{\not{k} \gamma^\mu \gamma^\mu}^{\text{odd}} - m^2 c_V \gamma^\mu \gamma^\mu - m c_A \cancel{\not{k} \gamma^\mu \gamma^5 \gamma^\mu}^{\text{odd}} + m^2 c_A \cancel{\gamma^\mu \gamma^5 \gamma^\mu}^{\text{odd}} \right\}$$

$$= \text{Tr} \left\{ \underbrace{c_V \not{k} \gamma^\mu \not{p} \gamma^\mu}_{c_V k_\alpha p_\beta \gamma^{\alpha\mu\beta\gamma}} - \underbrace{c_A \not{k} \gamma^\mu \gamma^5 \not{p} \gamma^\mu}_{k_\alpha p_\beta \gamma^\alpha \gamma^\mu \gamma^5 \gamma^\beta \gamma^\gamma} - m^2 c_V \underbrace{\gamma^\mu \gamma^\mu}_{4g^{\mu\mu}} \right\} \\ = i 4 k_\alpha p_\beta \epsilon^{\alpha\mu\beta\gamma}$$

$$= C_V 4 k_\epsilon p_\tau (g^{\epsilon\mu} g^{\tau\beta} - g^{\epsilon\tau} g^{\mu\beta} + g^{\epsilon\beta} g^{\mu\tau}) - i 4 C_A k_\alpha p_\beta \epsilon^{\alpha\tau\beta\delta} - 4 m^2 C_V g^{\mu\beta}$$

$$= 4 \left\{ C_V \left[k^\mu p^\beta - (k \cdot p) g^{\mu\beta} + k^\beta p^\mu \right] - C_A k_\alpha p_\beta \epsilon^{\alpha\mu\beta\delta} - C_V m^2 g^{\mu\beta} \right\}$$

- $$\text{Tr} \{ (K' - M) \gamma_\beta (\not{p}' + M) \gamma_\mu (\tilde{C}_V - \tilde{C}_A \gamma^5) \}$$

$$\left| \begin{array}{l} \leftarrow \text{cyclic} \\ - \text{Tr} \{ (\not{p}' + M) \gamma_\mu (\tilde{C}_V - \tilde{C}_A \gamma^5) (K' - M) \gamma_\beta \} \end{array} \right.$$

$$= 4 \left\{ \tilde{C}_V \left[p'_\mu k'_\beta - (k' \cdot p') g_{\mu\beta} + p'_\beta k'_\mu \right] - i \tilde{C}_A p'^\alpha k'^\tau \epsilon_{\alpha\mu\tau\beta} - \tilde{C}_V M^2 g_{\mu\beta} \right\}$$

$$\Rightarrow \langle M_Z M_Y^* \rangle 3s(s - m_\tau^2) =$$

$$4 \left\{ \tilde{C}_V \left[p'_\mu k'_\beta - (k' \cdot p') g_{\mu\beta} + p'_\beta k'_\mu \right] - i \tilde{C}_A p'^\alpha k'^\tau \epsilon_{\alpha\mu\tau\beta} - \tilde{C}_V M^2 g_{\mu\beta} \right\} \times$$

$$\left\{ C_V \left[k^\mu p^\beta - (k \cdot p) g^{\mu\beta} + k^\beta p^\mu \right] - C_A k_\alpha p_\beta \epsilon^{\alpha\mu\beta\delta} - C_V m^2 g^{\mu\beta} \right\}$$

$$= 4 \left\{ 2 C_V \tilde{C}_V \left[(p' \cdot k)(p \cdot k') + (p' \cdot p)(k \cdot k') \right] - 2 C_V \tilde{C}_V m^2 (k' \cdot p') \right.$$

$$\left. - C_A \tilde{C}_A k_\alpha p_\beta p'^\alpha k'^\tau \underbrace{\epsilon^{\alpha\mu\beta\delta} \epsilon_{\delta\mu\tau\beta}}_{= \epsilon^{\alpha\mu\beta\delta} \epsilon_{\delta\mu\tau\beta}} - 2 C_V \tilde{C}_V M^2 (k \cdot p) + 4 C_V \tilde{C}_V m^2 M^2 \right\}$$

$$\left| \begin{array}{l} \epsilon^{\alpha\mu\beta\delta} \epsilon_{\delta\mu\tau\beta} = -2 (\delta_\tau^\beta \delta_\delta^\alpha - \delta_\delta^\beta \delta_\tau^\alpha) \end{array} \right.$$

$$= 4 \left\{ 2 C_V \tilde{C}_V \left[(p' \cdot k)(p \cdot k') + (p' \cdot p)(k \cdot k') \right] - 2 C_V \tilde{C}_V \left[m^2 (k' \cdot p') + M^2 (k \cdot p) \right] \right.$$

$$\left. + 2 C_A \tilde{C}_A \left[(p \cdot k')(p' \cdot k) - (p \cdot p')(k \cdot k') \right] + 4 C_V \tilde{C}_V m^2 M^2 \right\}$$

$$= 4 \left\{ 2 C_V \tilde{C}_V \left[\frac{(m^2 + M^2 - u)^2}{4} + \frac{(m^2 + M^2 - t)^2}{4} - m^2 \left(\frac{s}{2} - M^2 \right) - M^2 \left(\frac{s}{2} - m^2 \right) \right] \right.$$

$$\left. + 2 C_A \tilde{C}_A \left[\frac{(m^2 + M^2 - u)^2}{4} + \frac{(m^2 + M^2 - t)^2}{4} \right] \right\}$$

$$\mathcal{M}_H = -\frac{g^2}{4m_W^2} \frac{mM}{s-m_H^2} \bar{v}(k) u(p) \bar{u}(p') v(k')$$

$$\mathcal{M}_\gamma = \frac{e^2}{3q^2} \left[\bar{v}(k) \gamma^\mu u(p) \right] \left[\bar{u}(p') \gamma_\mu v(k') \right]$$

$$4 \langle \mathcal{M}_\gamma \mathcal{M}_H^* \rangle \frac{12 m_W^2 s(s-m_H^2)}{e^2 g^2 m M} (-1) =$$

$$\sum_{p,q,r,s} \bar{v}_a^p(k) \gamma_{ab}^\mu u_b^q(p) \bar{u}_c^r(p') \gamma_{cd}^\nu v_d^s(k') \bar{v}_e^s(k') \not{1}_{ef} u_f^r(p') \bar{u}_g^r(p) \not{1}_{gh} v_h^p(k)$$

$$= \text{Tr} \left\{ (\not{k} - m) \gamma^\mu (\not{p} + m) \right\} \text{Tr} \left\{ (\not{p}' + m) \gamma_\mu (\not{k}' - m) \right\}$$

$$\left| \begin{aligned} \text{Tr} \left\{ (\not{k} \gamma^\mu - m \gamma^\mu) (\not{p} + m) \right\} &= \text{Tr} \left\{ \cancel{k} \gamma^\mu + m \cancel{k} \gamma^\mu - m \not{p} \gamma^\mu + m^2 \cancel{p} \gamma^\mu \right\} \\ &= 4m(k^\mu - p^\mu) \qquad 4M(k'_\mu - p'_\mu) \end{aligned} \right|$$

$$= 16 M m (k^\mu - p^\mu)(k'_\mu - p'_\mu) = 16 M m \left[(k \cdot k') - (k \cdot p') - (p \cdot k') + (p \cdot p') \right]$$

$$\left| \begin{aligned} &= 16 M m [u - t] \end{aligned} \right|$$

$$\mathcal{M}_t = \frac{1}{s - m_Z^2} \left[\bar{v}(k) \gamma^\mu (c_v - c_A \gamma^5) u(p) \right] \left[\bar{u}(p') (c_v - c_A \gamma^5) v(k') \right]$$

$$\mathcal{M}_t = - \frac{g^2}{4m_W^2} \frac{mM}{s - m_h^2} \bar{v}(k) u(p) \bar{u}(p') v(k')$$

$$\langle \mathcal{M}_Z \mathcal{M}_t^* \rangle = \frac{(s - m_h^2)(s - m_Z^2) 4m_W^2}{g^2 M M} (-1) =$$

$$= \sum_{pqrs} \bar{v}_a^p(k) \gamma^\mu (c_v - c_A \gamma^5) u_{ab}^q(p) \bar{u}_c^r(p') \gamma_\mu (c_v - c_A \gamma^5) v_{cd}^s(k') \\ \bar{v}_a^s(k') \uparrow u_p^r(p') \bar{u}_r^q(p) \uparrow v_{rs}^p(k) =$$

$$= \text{Tr} \{ (\not{k} - m) \gamma^\mu (c_v - c_A \gamma^5) (\not{p} + m) \} \text{Tr} \{ (\not{p}' + M) \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) (\not{k}' - M) \}$$

$$\bullet \text{Tr} \{ (\not{k} - m) \gamma^\mu (c_v - c_A \gamma^5) (\not{p} + m) \} =$$

$$= \text{Tr} \{ (\not{k} \gamma^\mu - m \gamma^\mu) (c_v \not{p} + m c_v - c_A \gamma^5 \not{p} - m c_A \gamma^5) \} =$$

$$= \text{Tr} \{ \cancel{c_v k \gamma^\mu \not{p}} + m c_v \not{k} \gamma^\mu - \cancel{c_A k \gamma^\mu \gamma^5 \not{p}} - m c_A \cancel{k \gamma^\mu \gamma^5} + \text{tr}(\gamma^\mu \gamma^5 \gamma^\nu \gamma^\nu) = 0 \\ + m c_v \gamma^\mu \not{p} - m^2 \cancel{c_v \gamma^\mu} + m c_A \cancel{\gamma^5 \not{p}} + m^2 \cancel{c_A \gamma^\mu \gamma^5} \}$$

$$= 4 m c_v k^\mu - 4 m c_v p^\mu = 4 m c_v (k^\mu - p^\mu)$$

$$\bullet \text{Tr} \{ (\not{p}' + M) \gamma_\mu (\tilde{c}_v - \tilde{c}_A \gamma^5) (\not{k}' - M) \} = -4 M \tilde{c}_v (p'_\mu - k'_\mu) = 4 M \tilde{c}_v (k'_\mu - p'_\mu)$$

$$= 16 M m c_v \tilde{c}_v (k^\mu - p^\mu) (k'_\mu - p'_\mu)$$

$$= 16 M m c_v \tilde{c}_v [(k \cdot k') - (k \cdot p') - (k' \cdot p) + (p \cdot p')]$$

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$$= 16 M m c_v \tilde{c}_v \left[\cancel{m^2 + M^2} - t - \cancel{m^2 + M^2} + u \right]$$

$$= 16 M m c_v \tilde{c}_v [u - t]$$