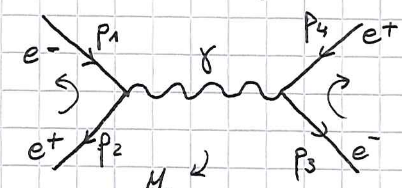
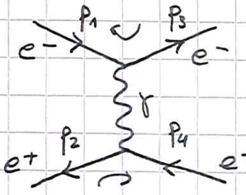
1. Lowest order diagrams of process $e^+e^- \longrightarrow e^+e^-$ & matrix elements



M_a



$\rightarrow M_s$

$$M_a = -\frac{e^2}{q^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right] g_{\mu\nu}\left[\bar{u}(p_3)\gamma^\nu v(p_4)\right]$$

$$= -\frac{e^2}{q^2}\left[\bar{v}(p_2)\gamma^\mu u(p_1)\right]\left[\bar{u}(p_3)\gamma_\mu v(p_4)\right]$$

$$M_s = -\frac{e^2}{q^2}\left[\bar{u}(p_3)\gamma^\mu u(p_1)\right]\left[\bar{v}(p_4)\gamma_\mu v(p_2)\right]$$

2.

I. $$\sum_{spins}|M_a|^2 = Tr\left[(\not{p}_2 - m)\gamma^\mu(\not{p}_1+m)\gamma^\nu\right] Tr\left[(\not{p}_3+m)\gamma_\mu(\not{p}_4-m)\gamma_\nu\right]\frac{e^4}{q^4}$$

↳ Since we are interested in $\sqrt{s} = 14\,GeV$, $m \ll E$, then we can neglect masses of e^- and e^+ (ultrarelativistic limit)

$$\simeq Tr\left[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\right] Tr\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right]$$

$$Tr\left[\not{p}_2\gamma^\mu\not{p}_1\gamma^\nu\right] = p_{2\alpha}p_{1\beta}\, Tr\left[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\right]$$

$$= p_{2\alpha}p_{1\beta}\, 4\left(g^{\alpha\mu}g^{\beta\nu} - g^{\alpha\beta}g^{\mu\nu} + g^{\alpha\nu}g^{\mu\beta}\right)$$

$$= 4\left[p_2^\mu p_1^\nu - g^{\mu\nu}(p_1\cdot p_2) + p_2^\nu p_1^\mu\right]$$

$$Tr\left[\not{p}_3\gamma_\mu\not{p}_4\gamma_\nu\right] = p_3^\alpha p_4^\beta\, Tr\left[\gamma_\alpha\gamma_\mu\gamma_\beta\gamma_\nu\right]$$

$$= 4\left[p_{3\mu}p_{4\nu} - g_{\mu\nu}(p_3\cdot p_4) + p_3^\nu p_4^\mu\right]$$

$$\Rightarrow \sum_{spins}|M_a|^2 = 16\Big[(p_2\cdot p_3)(p_1\cdot p_4) - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + (p_1\cdot p_3)(p_2\cdot p_4) +$$

$$- \cancel{(p_3\cdot p_4)(p_1\cdot p_2)} + 4\cancel{(p_1\cdot p_2)(p_3\cdot p_4)} - \cancel{(p_3\cdot p_4)(p_1\cdot p_2)} +$$

$$+ (p_2\cdot p_4)(p_1\cdot p_3) - \cancel{(p_1\cdot p_2)(p_3\cdot p_4)} + (p_2\cdot p_3)(p_1\cdot p_4)\Big]\frac{e^4}{q^4}$$

$$= 32\left[(p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4)\right]\frac{e^4}{q^4}$$

$$\langle|M_a|^2\rangle = \frac{1}{4}\sum_{spins}|M_a|^2 = 8\left[(p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4)\right]\frac{e^4}{q^4}$$

$$= \frac{8e^4}{s^2}\left[(p_2\cdot p_3)(p_1\cdot p_4) + (p_1\cdot p_3)(p_2\cdot p_4)\right]$$