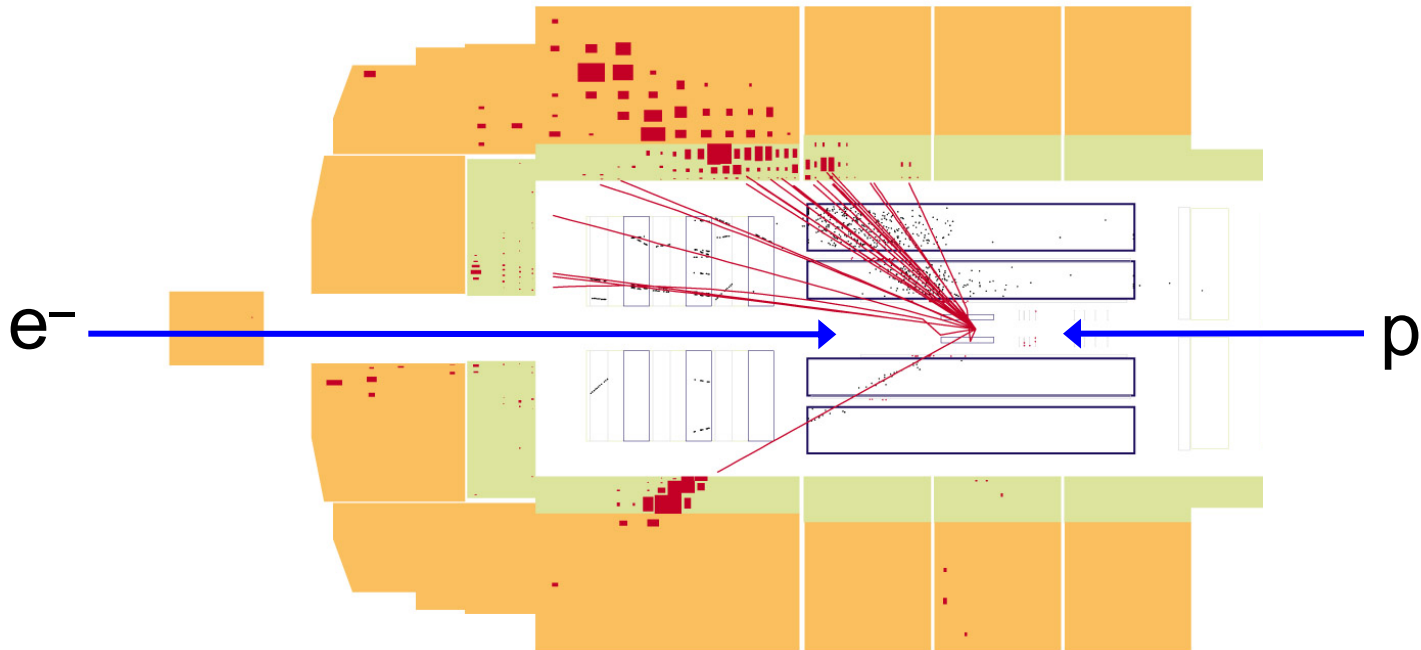


Particle Physics

FYS4555 – UiO – 2018

Based on Prof Mark Thomson's slides



Handout 6 : Deep Inelastic Scattering

$e^- p$ Elastic Scattering at Very High q^2

- ★ At high q^2 the Rosenbluth expression for elastic scattering becomes

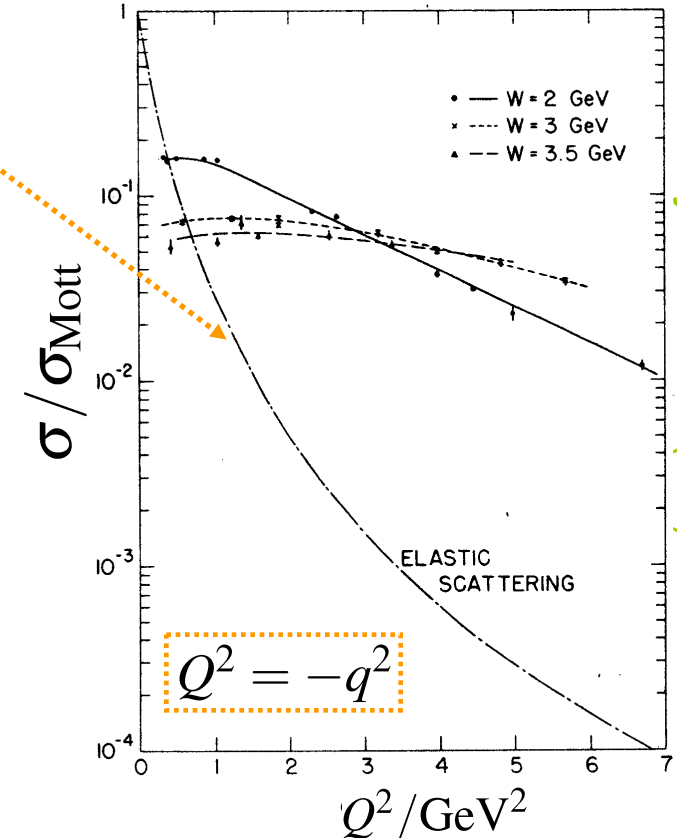
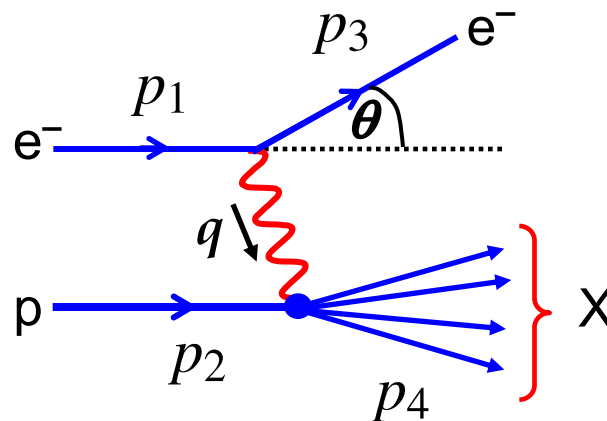
$$\left(\frac{d\sigma}{d\Omega}\right)_{elastic} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{q^2}{2M^2} G_M^2 \sin^2 \frac{\theta}{2}\right) \quad \tau = -\frac{q^2}{4M^2} \gg 1$$

- From $e^- p$ elastic scattering, the proton magnetic form factor is

$$G_M(q^2) \approx \frac{1}{(1 + q^2/0.71 \text{ GeV}^2)^2} \quad \rightarrow \quad G_M(q^2) \propto q^{-4} \quad \text{at high } q^2$$

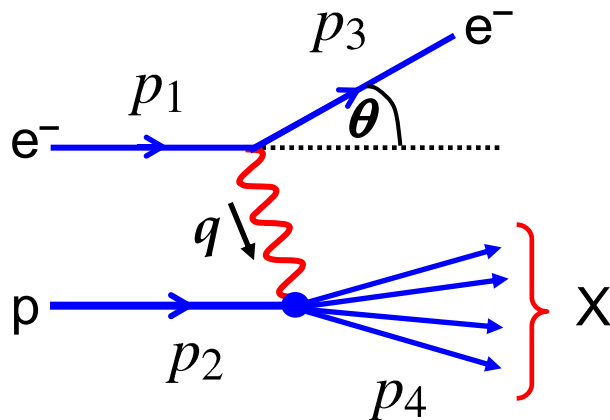
$$\Rightarrow \left(\frac{d\sigma}{d\Omega}\right)_{elastic} \propto q^{-6}$$

- Due to the finite proton size, **elastic scattering** at high q^2 is unlikely and **inelastic reactions** where the proton breaks up dominate.



M. Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935

Kinematics of Inelastic Scattering



- For inelastic scattering the mass of the final state hadronic system is no longer the proton mass, M
- The final state hadronic system must contain at least one **baryon** which implies the final state invariant mass $M_X > M$

$$M_X^2 = p_4^2 = (E_4^2 - |\vec{p}_4|^2)$$

★ For inelastic scattering introduce four new kinematic variables: x, y, ν, Q^2

★ Define:

$$x \equiv \frac{Q^2}{2p_2 \cdot q}$$

Bjorken x

(Lorentz Invariant)

where

$$Q^2 \equiv -q^2$$

$$Q^2 > 0$$

• Here $M_X^2 = p_4^2 = (q + p_2)^2 = -Q^2 + 2p_2 \cdot q + M^2$

$$\Rightarrow Q^2 = 2p_2 \cdot q + M^2 - M_X^2 \Rightarrow Q^2 \leq 2p_2 \cdot q$$

Note: in many text books W is often used in place of M_X

hence

$$0 < x < 1 \text{ inelastic}$$

$$x = 1 \text{ elastic}$$

Proton intact
 $M_X = M$

★ Define:

$$y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad (\text{Lorentz Invariant})$$

• In the Lab. Frame:

$$p_1 = (E_1, 0, 0, E_1) \quad p_2 = (M, 0, 0, 0)$$

$$q = (E_1 - E_3, \vec{p}_1 - \vec{p}_3)$$

$$\rightarrow y = \frac{M(E_1 - E_3)}{ME_1} = 1 - \frac{E_3}{E_1}$$

So y is the fractional energy loss of the incoming particle

$$0 < y < 1$$

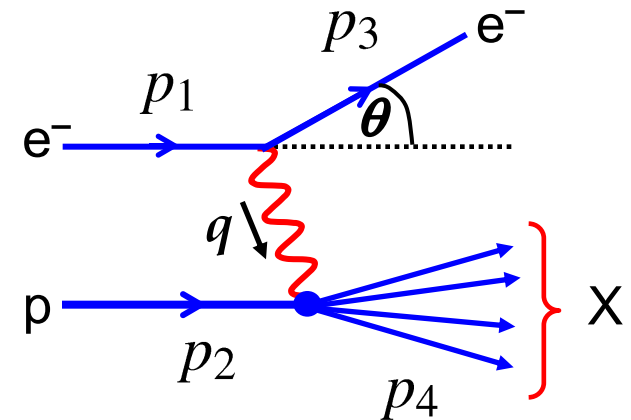
• In the C.o.M. Frame (neglecting the electron and proton masses):

$$p_1 = (E, 0, 0, E); \quad p_2 = (E, 0, 0, -E); \quad p_3 = (E, E \sin \theta^*, 0, E \cos \theta^*)$$

$$\rightarrow y = \frac{1}{2}(1 - \cos \theta^*) \quad \text{for } E \gg M$$

★ Finally Define:

$$v \equiv \frac{p_2 \cdot q}{M} \quad (\text{Lorentz Invariant})$$

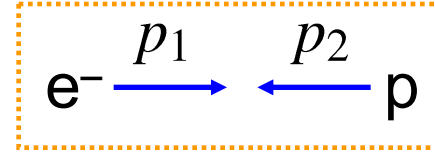


• In the Lab. Frame: $v = E_1 - E_3$

v is the energy lost by the incoming particle

Relationships between Kinematic Variables

- Can rewrite the new kinematic variables in terms of the squared centre-of-mass energy, s , for the electron-proton collision



$$s = (p_1 + p_2)^2 = p_1^2 + p_2^2 + 2p_1 \cdot p_2 = 2p_1 \cdot p_2 + M^2 + \cancel{m_e^2}$$

Neglect mass of electron

$$2p_1 \cdot p_2 = s - M^2$$

- For a fixed centre-of-mass energy, it can then be shown that the four kinematic variables

$$Q^2 \equiv -q^2 \quad x \equiv \frac{Q^2}{2p_2 \cdot q} \quad y \equiv \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad v \equiv \frac{p_2 \cdot q}{M}$$

are not independent.

- i.e. the scaling variables x and y can be expressed as

$$x = \frac{Q^2}{2Mv}$$

$$y = \frac{2M}{s - M^2} v$$

Note the simple relationship between y and v

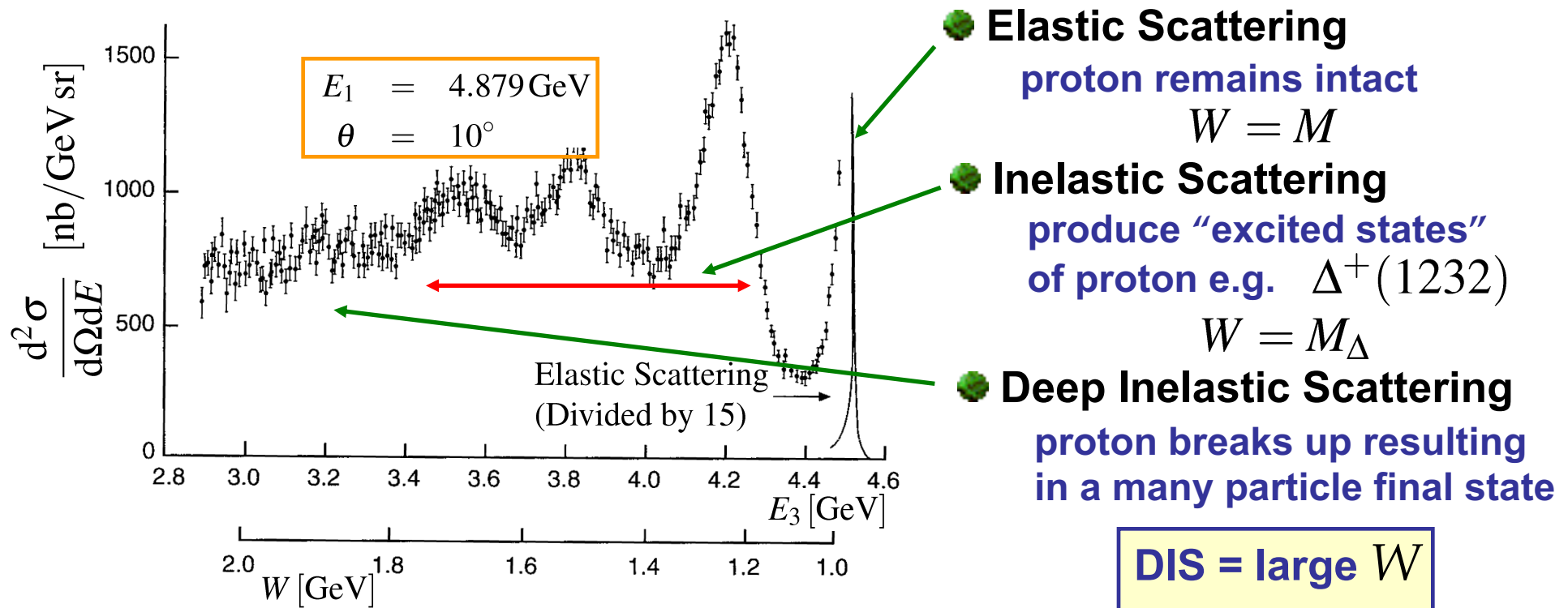
and $xy = \frac{Q^2}{s - M^2} \Rightarrow Q^2 = (s - M^2)xy$

- For a fixed centre of mass energy, the interaction kinematics are completely defined by **any two** of the above kinematic variables (except y and v)
- For elastic scattering ($x = 1$) there is only one independent variable. As we saw previously if you measure electron scattering angle know everything else.

Inelastic Scattering

Example: Scattering of 4.879 GeV electrons from protons at rest

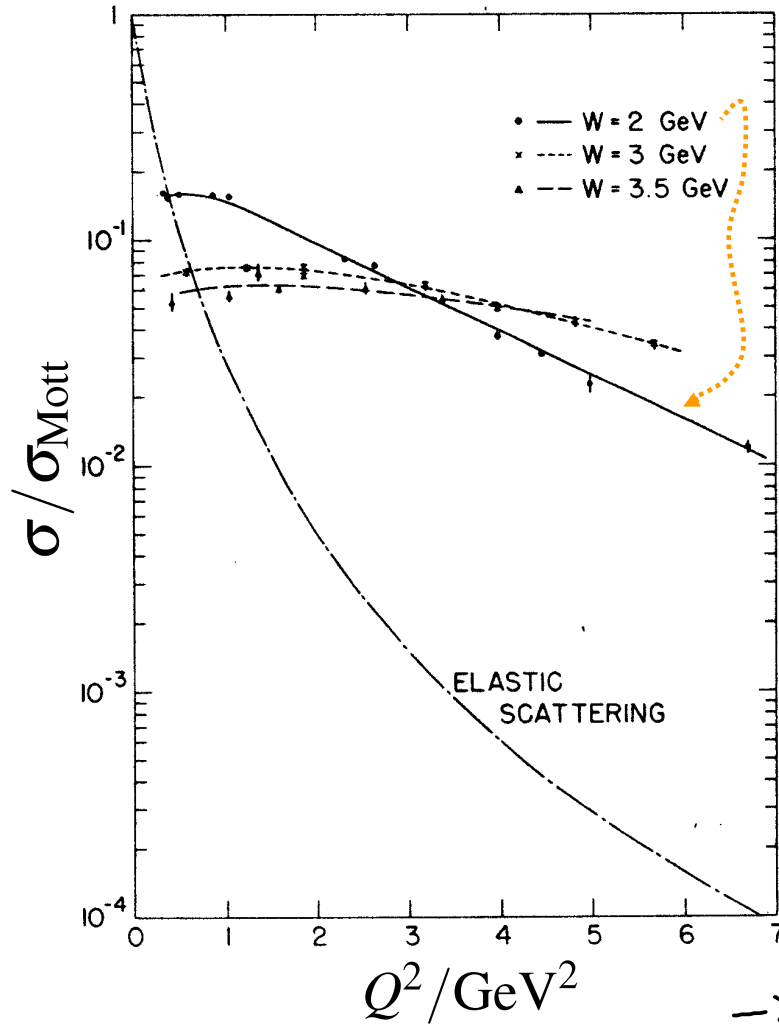
- Place detector at 10° to beam and measure the energies of scattered e^-
- Kinematics fully determined from the electron energy **and** angle !
- e.g. for **this energy and angle** : the invariant mass of the final state hadronic system $W^2 = M_X^2 = 10.06 - 2.03E_3$ (try and show this)



Inelastic Cross Sections

- Repeat experiments at different angles/beam energies and determine q^2 dependence of elastic and inelastic cross-sections

M. Breidenbach et al.,
Phys. Rev. Lett. 23 (1969) 935



- Elastic scattering falls off rapidly with q^2 due to the proton not being point-like (i.e. form factors)
- Inelastic scattering cross sections only weakly dependent on q^2
- Deep Inelastic scattering cross sections almost independent of q^2 !

i.e. "Form factor" $\rightarrow 1$

→ Scattering from point-like objects within the proton !

→ probably hitting sth. inside

Elastic → Inelastic Scattering

★ Recall: Elastic scattering (Handout 5)

- Only **one independent variable**. In Lab. frame express differential cross section in terms of the electron scattering angle (Rosenbluth formula)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta/2} \frac{E_3}{E_1} \left(\frac{G_E^2 + \tau G_M^2}{(1 + \tau)} \cos^2 \frac{\theta}{2} + 2\tau G_M^2 \sin^2 \frac{\theta}{2} \right) \quad \tau = \frac{Q^2}{4M^2}$$

Note: here the energy of the scattered electron is determined by the angle.

- In terms of the Lorentz invariant kinematic variables can express this differential cross section in terms of Q^2 (problem 8.2)

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\frac{G_E^2 + \tau G_M^2}{(1 + \tau)} \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 G_M^2 \right]$$

which can be written as:

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[f_2(Q^2) \left(1 - y - \frac{M^2 y^2}{Q^2} \right) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

★ Inelastic scattering

- For Deep Inelastic Scattering have **two independent variables**. Therefore need a double differential cross section

Deep Inelastic Scattering

- ★ It can be shown that the most general Lorentz Invariant expression for $e^-p \rightarrow e^-X$ inelastic scattering (via a single exchanged photon is):

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (1)$$

INELASTIC
SCATTERING

c.f.
$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[\left(1 - y - \frac{M^2 y^2}{Q^2} \right) f_2(Q^2) + \frac{1}{2} y^2 f_1(Q^2) \right]$$

ELASTIC
SCATTERING

We will soon see how this connects to the quark model of the proton

- **NOTE:** The form factors have been replaced by the **STRUCTURE FUNCTIONS**

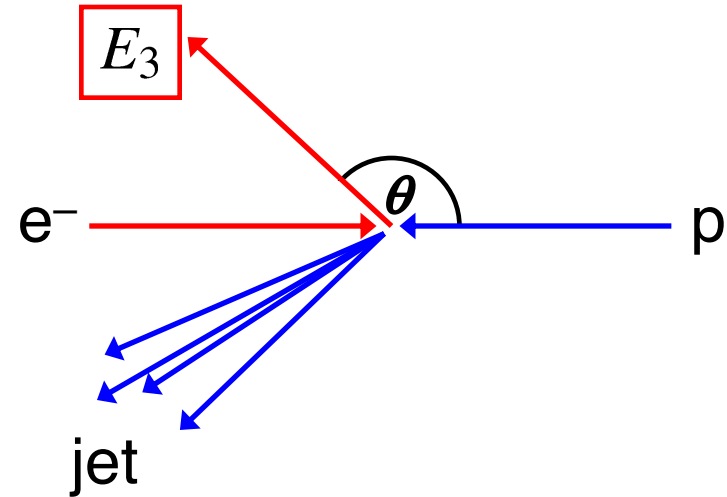
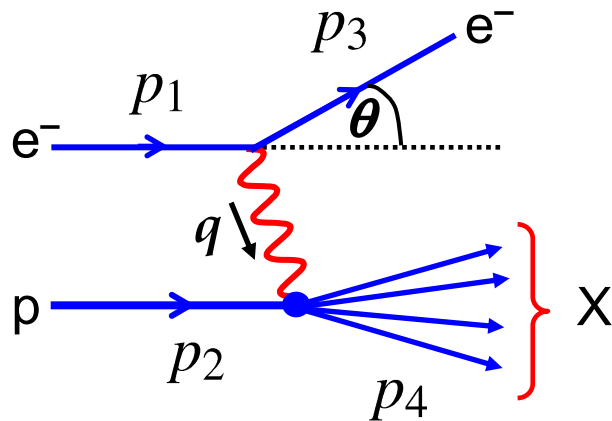
$$F_1(x, Q^2) \quad \text{and} \quad F_2(x, Q^2)$$

which are a function of x and Q^2 : can not be interpreted as the Fourier transforms of the charge and magnetic moment distributions. We shall soon see that they describe the **momentum distribution** of the quarks within the proton

- ★ In the limit of high energy (or more correctly $Q^2 \gg M^2 y^2$) eqn. (1) becomes:

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1 - y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right] \quad (2)$$

- In the Lab. frame it is convenient to express the cross section in terms of the angle, θ , and energy, E_3 , of the scattered electron – experimentally well measured.



$$Q^2 = 4E_1 E_3 \sin^2 \theta / 2; \quad x = \frac{Q^2}{2M(E_1 - E_3)}; \quad y = 1 - \frac{E_3}{E_1}; \quad v = E_1 - E_3$$

- In the Lab. frame, Equation (2) becomes:

(problems 8.2, 8.3)

$$\frac{d^2 \sigma}{dE_3 d\Omega} = \frac{\alpha^2}{4E_1^2 \sin^4 \theta / 2} \left[\frac{1}{v} F_2(x, Q^2) \cos^2 \frac{\theta}{2} + \frac{2}{M} F_1(x, Q^2) \sin^2 \frac{\theta}{2} \right] \quad (3)$$

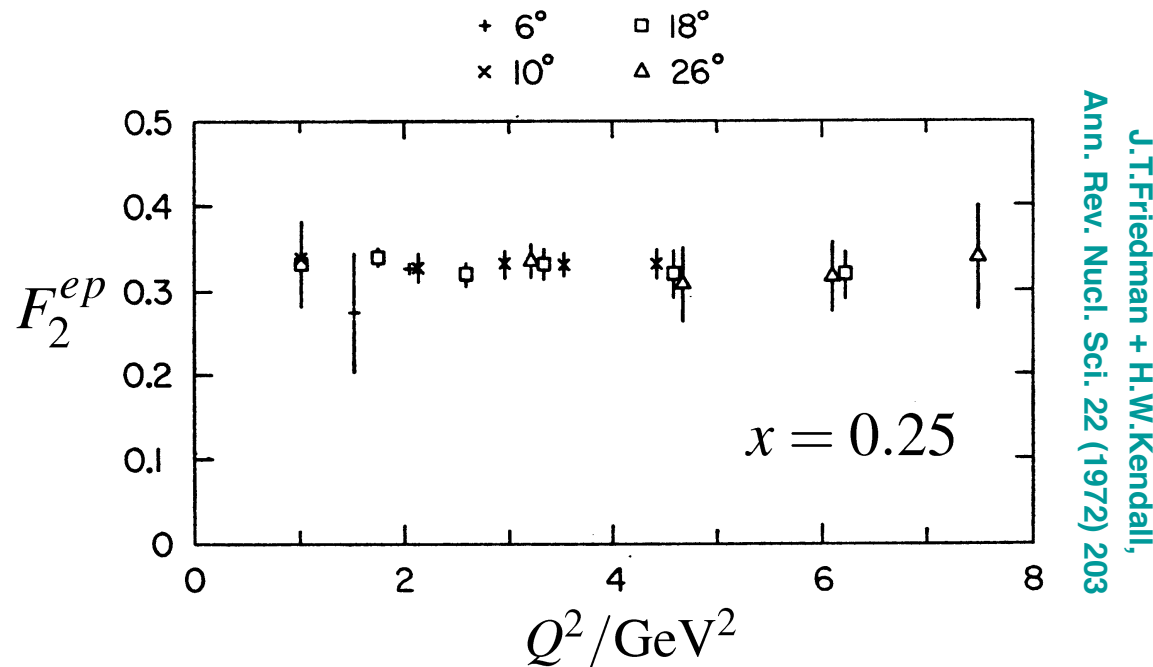
Electromagnetic Structure Function

Pure Magnetic Structure Function

Measuring the Structure Functions

- ★ To determine $F_1(x, Q^2)$ and $F_2(x, Q^2)$ for a given x and Q^2 need measurements of the differential cross section at several different scattering angles and incoming electron beam energies

Example: electron-proton scattering F_2 vs. Q^2 at fixed x



- ♦ Experimentally it is observed that both F_1 and F_2 are (almost) independent of Q^2

Bjorken Scaling and the Callan-Gross Relation

- ★ The near (see later) independence of the structure functions on Q^2 is known as **Bjorken Scaling**, i.e.

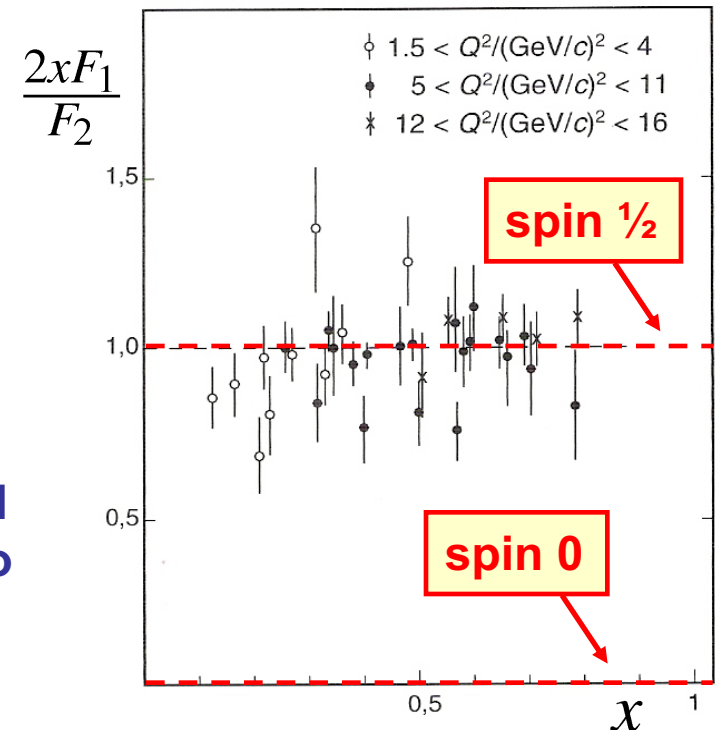
$$F_1(x, Q^2) \rightarrow F_1(x) \qquad F_2(x, Q^2) \rightarrow F_2(x)$$

- It is strongly suggestive of scattering from **point-like** constituents within the proton

- ★ It is also observed that $F_1(x)$ and $F_2(x)$ are not independent but satisfy the **Callan-Gross relation**

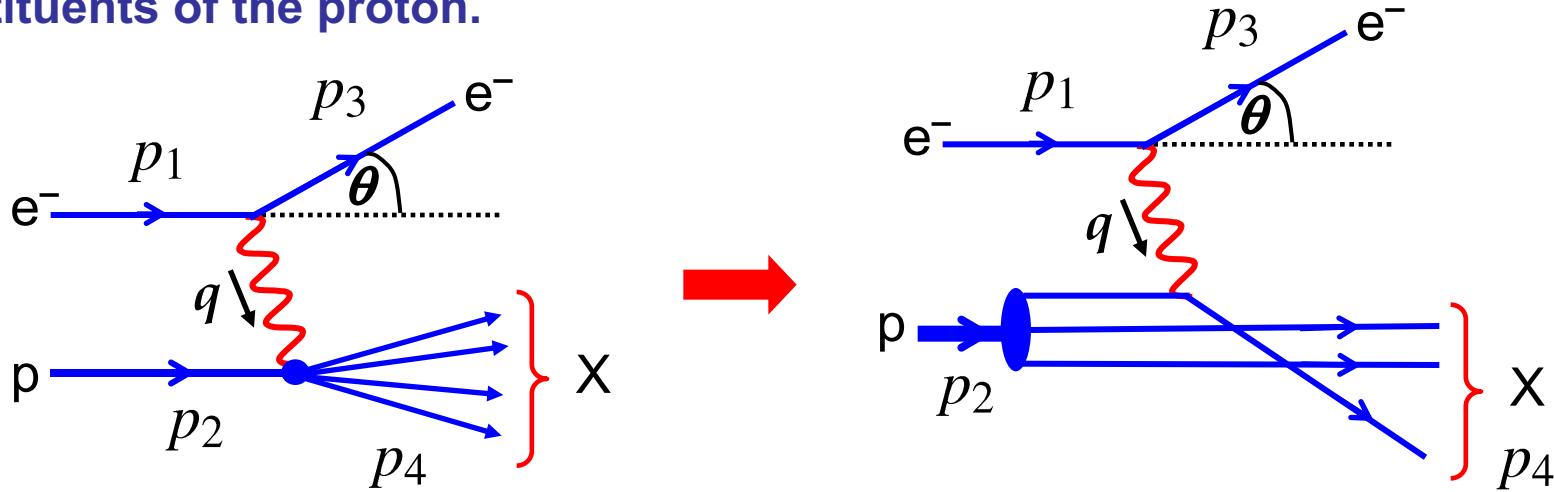
$$F_2(x) = 2xF_1(x)$$

- As we shall soon see this is exactly what is expected for scattering from **spin-half** quarks.
- **Note** if quarks were spin zero particles we would expect the purely magnetic structure function to be zero, i.e. $F_1(x) = 0$



The Quark-Parton Model

- Before quarks and gluons were generally accepted Feynman proposed that the proton was made up of point-like constituents “**partons**”
- Both Bjorken Scaling and the Callan-Gross relationship can be explained by assuming that Deep Inelastic Scattering is dominated by the scattering of a single virtual photon from point-like spin-half constituents of the proton.

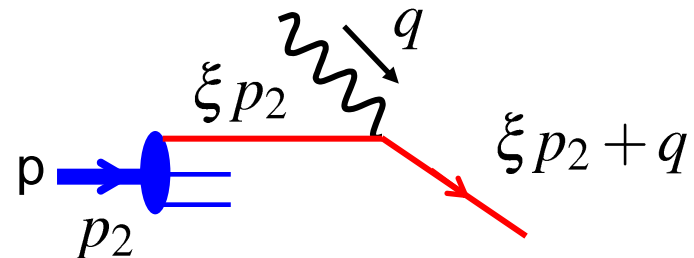
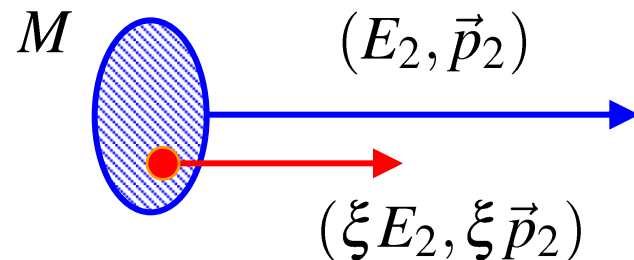


Scattering from a proton
with structure functions

Scattering from a point-like
quark within the proton

★ How do these two pictures of the interaction relate to each other?

- In the parton model the basic interaction is **ELASTIC** scattering from a “**quasi-free**” spin- $\frac{1}{2}$ quark in the proton, i.e. treat the quark as a free particle!
- The parton model is most easily formulated in a frame where the proton has very high energy, often referred to as the “**infinite momentum frame**”, where we can neglect the proton mass and $p_2 = (E_2, 0, 0, E_2)$
- In this frame can also neglect the mass of the quark and any momentum transverse to the direction of the proton.
- Let the quark carry a fraction ξ of the proton's four-momentum.



- After the interaction the struck quark's four-momentum is $\xi p_2 + q$

$$(\xi p_2 + q)^2 = m_q^2 \approx 0 \quad \rightarrow \quad \cancel{\xi^2 p_2^2} + q^2 + 2\xi p_2 \cdot q = 0 \quad (\xi^2 p_2^2 = m_q^2 \approx 0)$$

$$\rightarrow \xi = \frac{Q^2}{2p_2 \cdot q} = x$$

Bjorken x can be identified as the fraction of the proton momentum carried by the struck quark (in a frame where the proton has very high energy)

- In terms of the proton momentum

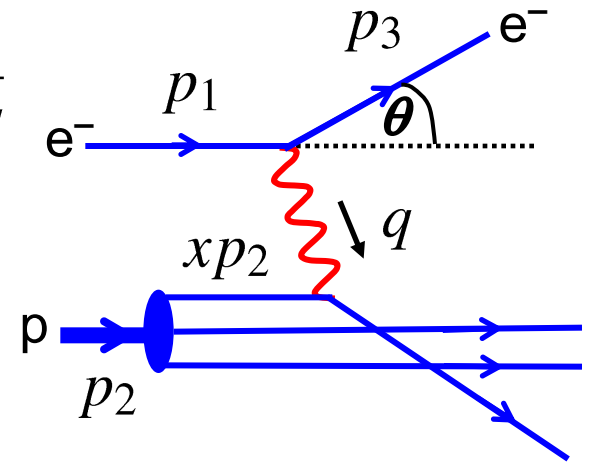
$$s = (p_1 + p_2)^2 \simeq 2p_1 \cdot p_2 \quad y = \frac{p_2 \cdot q}{p_2 \cdot p_1} \quad x = \frac{Q^2}{2p_2 \cdot q}$$

- But for the underlying quark interaction

$$s^q = (p_1 + xp_2)^2 = 2xp_1 \cdot p_2 = xs$$

$$y_q = \frac{p_q \cdot q}{p_q \cdot p_1} = \frac{xp_2 \cdot q}{xp_2 \cdot p_1} = y$$

$$x_q = 1 \quad \text{(elastic, i.e. assume quark does not break up)}$$



- Previously derived the Lorentz Invariant cross section for $e^- \mu^- \rightarrow e^- \mu^-$ elastic scattering in the ultra-relativistic limit (handout 4 + problem 6.7).

Now apply this to $e^- q \rightarrow e^- q$

$$\frac{d\sigma}{dq^2} = \frac{2\pi\alpha^2 e_q^2}{q^4} \left[1 + \left(1 + \frac{q^2}{s_q} \right)^2 \right]$$

e_q is quark charge, i.e.
 $e_u = +2/3; \quad e_d = -1/3$

- Using (slide 5) $-q^2 = Q^2 = (s_q - m^2)x_q y_q \rightarrow \frac{q^2}{s_q} = -y_q = -y$

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4} \left[1 + (1 - y)^2 \right]$$

$$\frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2 e_q^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right]$$

★ This is the expression for the differential cross-section for **elastic** e^-q scattering from a quark carrying a fraction x of the proton momentum.

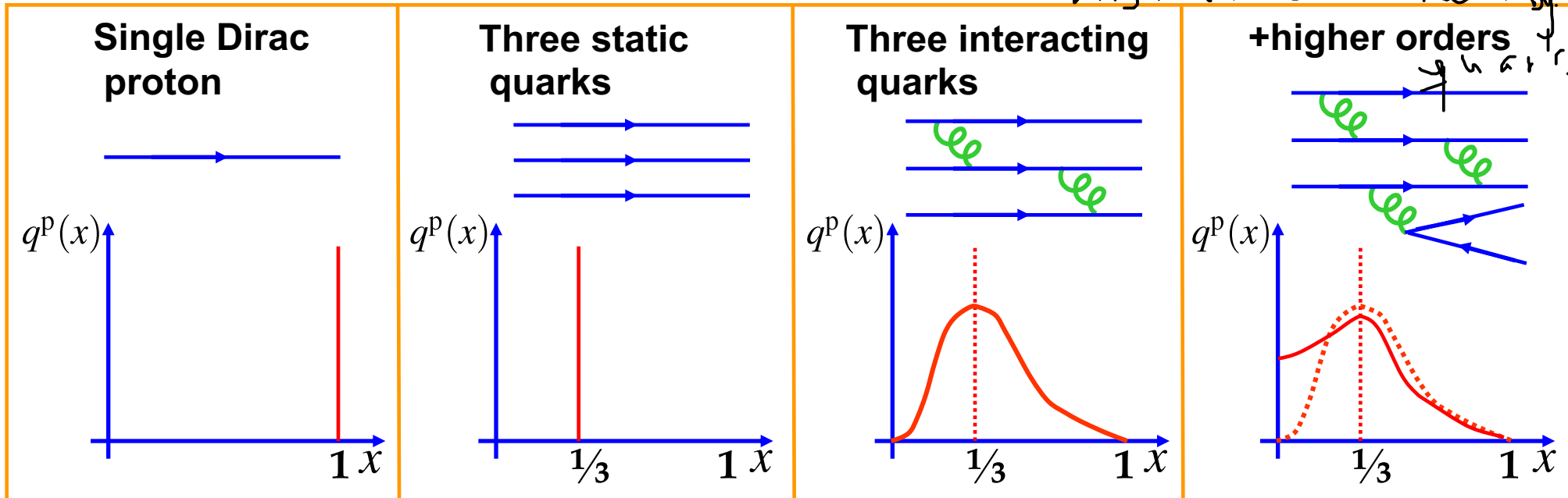
• Now need to account for distribution of quark momenta within proton

★ Introduce parton distribution functions such that $q^p(x)dx$ is the number of quarks of type q within a proton with momenta between $x \rightarrow x + dx$

● Expected form of the parton distribution function ?

(3)
 will be a sum of
 1/2

→ Don't know exactly, but I can try



- ★ The cross section for scattering from a particular quark type within the proton which in the range $x \rightarrow x + dx$ is

$$\frac{d^2\sigma}{dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right] \times e_q^2 q^p(x) dx$$

quarks with momentum $\in [x, x+dx]$

- ★ Summing over all types of quark within the proton gives the expression for the **electron-proton** scattering cross section

$$\frac{d^2\sigma^{ep}}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) + \frac{y^2}{2} \right] \sum_q e_q^2 q^p(x)$$

Quark q
Distribution related (5)

- ★ Compare with the **electron-proton** scattering cross section in terms of structure functions (equation (2)):

$$\frac{d^2\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4} \left[(1-y) \frac{F_2(x, Q^2)}{x} + y^2 F_1(x, Q^2) \right]$$

Can identify F_1, F_2 (6)

- ★ By comparing (5) and (6) obtain the parton model prediction for the structure functions in the general L.I. form for the differential cross section

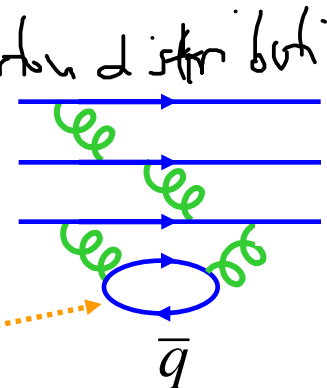
$$F_2^p(x, Q^2) = 2xF_1^p(x, Q^2) = x \sum_q e_q^2 q^p(x)$$

⇒ Can relate measured structure functions to the underlying quark distributions

The parton model predicts:

- **Bjorken Scaling** $F_1(x, Q^2) \rightarrow F_1(x)$ $F_2(x, Q^2) \rightarrow F_2(x)$ *high energies*
 - ★ Due to scattering from **point-like particles** within the proton
- **Callan-Gross Relation** $F_2(x) = 2xF_1(x)$
 - ★ Due to scattering from **spin half Dirac particles** where the magnetic moment is directly related to the charge; hence the “electro-magnetic” and “pure magnetic” terms are fixed with respect to each other. *Not perturbative*
- ★ At present parton distributions **cannot be calculated from QCD**
 - Can't use perturbation theory due to large coupling constant
- ★ Measurements of the structure functions enable us to determine the parton distribution functions !
- ★ For **electron-proton scattering** we have: *Measure F_2 → get parton distribution*

$$F_2^p(x) = x \sum_q e_q^2 q^p(x)$$
 - Due to higher orders, the proton contains not only up and down quarks but also anti-up and anti-down quarks (will neglect the small contributions from heavier quarks)



- For electron-proton scattering have: just write down

uud: $F_2^{\text{ep}}(x) = x \sum_q e_q^2 q^{\text{p}}(x) = x \left(\frac{4}{9} u^{\text{p}}(x) + \frac{1}{9} d^{\text{p}}(x) + \frac{4}{9} \bar{u}^{\text{p}}(x) + \frac{1}{9} \bar{d}^{\text{p}}(x) \right)$

- For electron-neutron scattering have:

udd: $F_2^{\text{en}}(x) = x \sum_q e_q^2 q^{\text{n}}(x) = x \left(\frac{4}{9} u^{\text{n}}(x) + \frac{1}{9} d^{\text{n}}(x) + \frac{4}{9} \bar{u}^{\text{n}}(x) + \frac{1}{9} \bar{d}^{\text{n}}(x) \right)$

- ★ Now assume “isospin symmetry”, i.e. that the neutron (ddu) is the same

as a proton (uud) with up and down quarks interchanged, i.e. \rightarrow not different for strong interact.

up \downarrow 95% correct

$$d^{\text{n}}(x) = u^{\text{p}}(x); \quad u^{\text{n}}(x) = d^{\text{p}}(x)$$

and define the neutron distributions functions in terms of those of the proton

$$u(x) \equiv u^{\text{p}}(x) = d^{\text{n}}(x); \quad d(x) \equiv d^{\text{p}}(x) = u^{\text{n}}(x)$$

$$\bar{u}(x) \equiv \bar{u}^{\text{p}}(x) = \bar{d}^{\text{n}}(x); \quad \bar{d}(x) \equiv \bar{d}^{\text{p}}(x) = \bar{u}^{\text{n}}(x)$$

giving:

$$F_2^{\text{ep}}(x) = 2xF_1^{\text{ep}}(x) = x \left(\frac{4}{9} u(x) + \frac{1}{9} d(x) + \frac{4}{9} \bar{u}(x) + \frac{1}{9} \bar{d}(x) \right) \quad (7)$$

$$F_2^{\text{en}}(x) = 2xF_1^{\text{en}}(x) = x \left(\frac{4}{9} d(x) + \frac{1}{9} u(x) + \frac{4}{9} \bar{d}(x) + \frac{1}{9} \bar{u}(x) \right) \quad (8)$$

• **Integrating (7) and (8) :**

$$\int_0^1 F_2^{\text{ep}}(x) dx = \int_0^1 x \left(\frac{4}{9} [u(x) + \bar{u}(x)] + \frac{1}{9} [d(x) + \bar{d}(x)] \right) dx = \frac{4}{9} f_u + \frac{1}{9} f_d$$

$$\int_0^1 F_2^{\text{en}}(x) dx = \int_0^1 x \left(\frac{4}{9} [d(x) + \bar{d}(x)] + \frac{1}{9} [u(x) + \bar{u}(x)] \right) dx = \frac{4}{9} f_d + \frac{1}{9} f_u$$

★ $f_u = \int_0^1 [xu(x) + x\bar{u}(x)] dx$ **is the fraction of the proton momentum carried by the up and anti-up quarks**

Experimentally

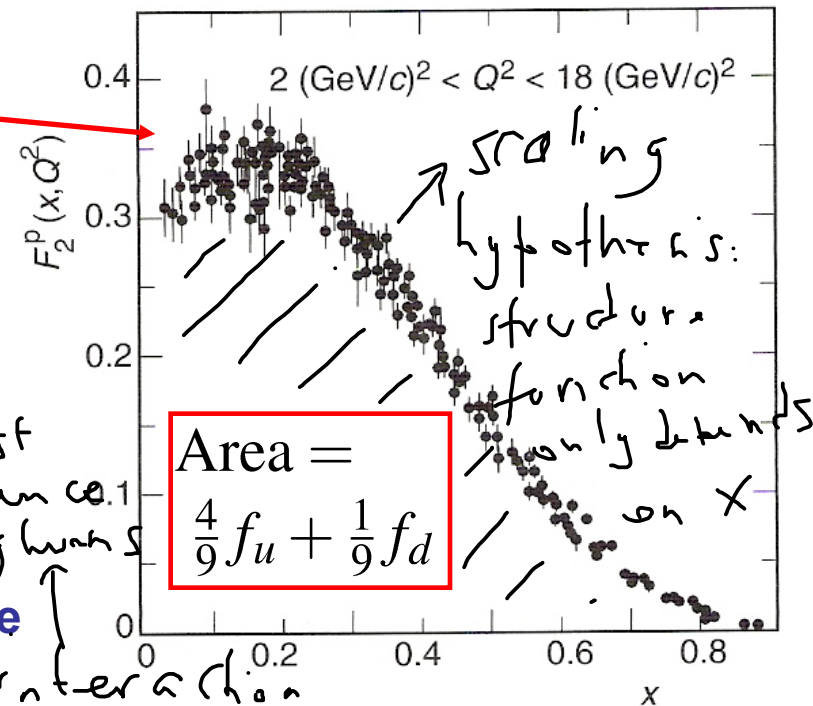
$$\int F_2^{\text{ep}}(x) dx \approx 0.18$$

$$\int F_2^{\text{en}}(x) dx \approx 0.12$$

➔ $f_u \approx 0.36 \quad f_d \approx 0.18$

★ **In the proton, as expected, the up quarks carry twice the momentum of the down quarks**

★ **The quarks carry just over 50 % of the total proton momentum. The rest is carried by gluons (which being neutral doesn't contribute to electron-nucleon scattering) → Must strong interaction**



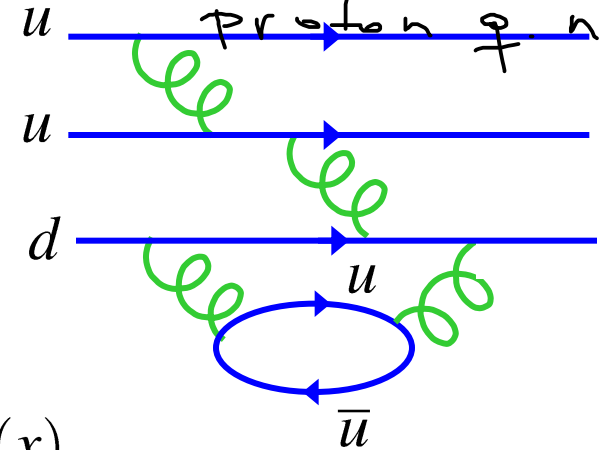
Carry QW

← Valence and Sea Quarks

g, don't it contribute for proton?

- As we are beginning to see the proton is complex...

- The parton distribution function $u^p(x) = u(x)$ includes contributions from the **"valence"** quarks and the virtual quarks produced by gluons: the **"sea"**



- Resolving into valence and sea contributions:

$$u(x) = u_V(x) + u_S(x) \quad d(x) = d_V(x) + d_S(x)$$

$$\bar{u}(x) = \bar{u}_S(x) \quad \bar{d}(x) = \bar{d}_S(x)$$

No valence anti-quarks in proton

- The proton contains two valence up quarks and one valence down quark and would expect:

$$\int_0^1 u_V(x) dx = 2 \quad \int_0^1 d_V(x) dx = 1$$

- But no **a priori** expectation for the total number of sea quarks !
- But **sea quarks** arise from **gluon quark/anti-quark pair production** and with $m_u = m_d$ it is reasonable to expect *same*

$$u_S(x) = d_S(x) = \bar{u}_S(x) = \bar{d}_S(x) = S(x)$$

- With these relations (7) and (8) become

$$F_2^{\text{ep}}(x) = x \left(\frac{4}{9} u_V(x) + \frac{1}{9} d_V(x) + \frac{10}{9} S(x) \right) \quad F_2^{\text{en}}(x) = x \left(\frac{4}{9} d_V(x) + \frac{1}{9} u_V(x) + \frac{10}{9} S(x) \right)$$

Giving the ratio

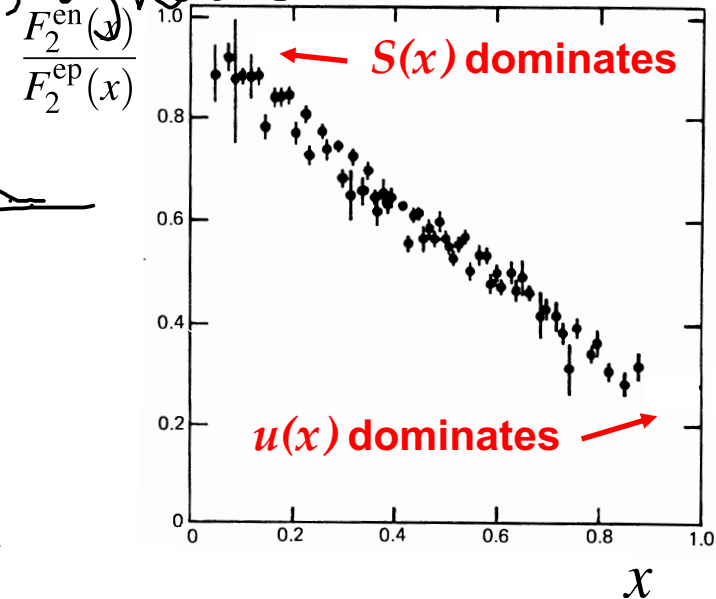
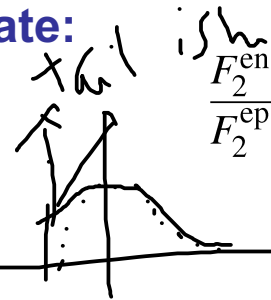
$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} = \frac{4d_V(x) + u_V(x) + 10S(x)}{4u_V(x) + d_V(x) + 10S(x)} \rightarrow \frac{1}{q^2}$$

- The sea component arises from processes such as $g \rightarrow \bar{u}u$. Due to the $1/q^2$ dependence of the gluon propagator, much more likely to produce low energy gluons. Expect the sea to comprise of **low energy** q/\bar{q}
- Therefore at low x expect the sea to dominate:

$d \sim u \ll S$

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \rightarrow 1 \quad \text{as } x \rightarrow 0$$

Observed experimentally



- At high x expect the sea contribution to be small

$$\frac{F_2^{\text{en}}(x)}{F_2^{\text{ep}}(x)} \rightarrow \frac{4d_V(x) + u_V(x)}{4u_V(x) + d_V(x)} \quad \text{as } x \rightarrow 1$$

Note: $u_V = 2d_V$ would give ratio **2/3** as $x \rightarrow 1$

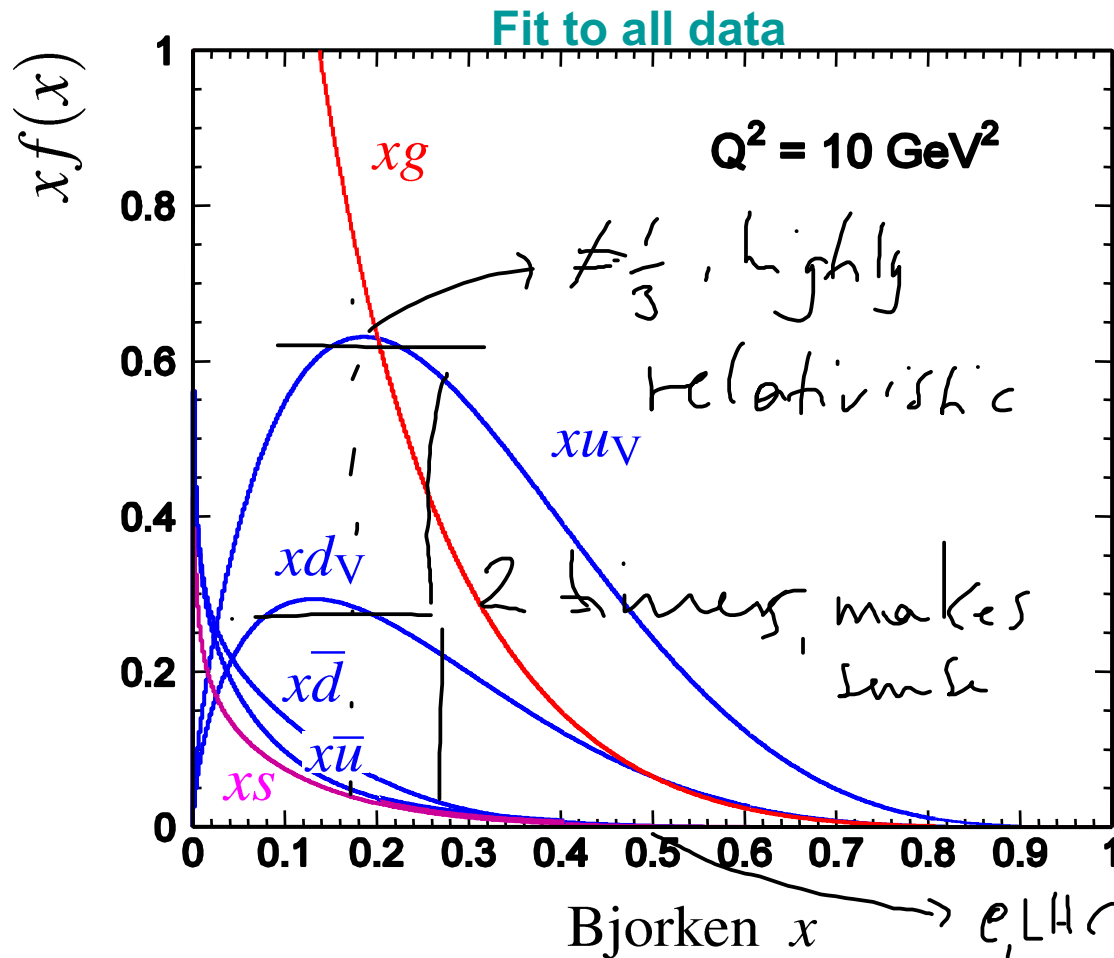
Experimentally $F_2^{\text{en}}(x)/F_2^{\text{ep}}(x) \rightarrow 1/4$ as $x \rightarrow 1$

$$\rightarrow d(x)/u(x) \rightarrow 0 \quad \text{as } x \rightarrow 1$$

This behaviour is not understood. (many assumptions...)

Parton Distribution Functions

- Ultimately the parton distribution functions are obtained from a fit to all experimental data including neutrino scattering (see handout 10)
- Hadron-hadron collisions give information on gluon pdf $g(x)$



Note:

- Apart from at large x
 $u_V(x) \approx 2d_V(x)$
- For $x < 0.2$
gluons dominate
- In fits to data assume
 $u_s(x) = \bar{u}(x)$
- $\bar{d}(x) > \bar{u}(x)$
not understood –
exclusion principle?
- Small strange quark
component $s(x)$

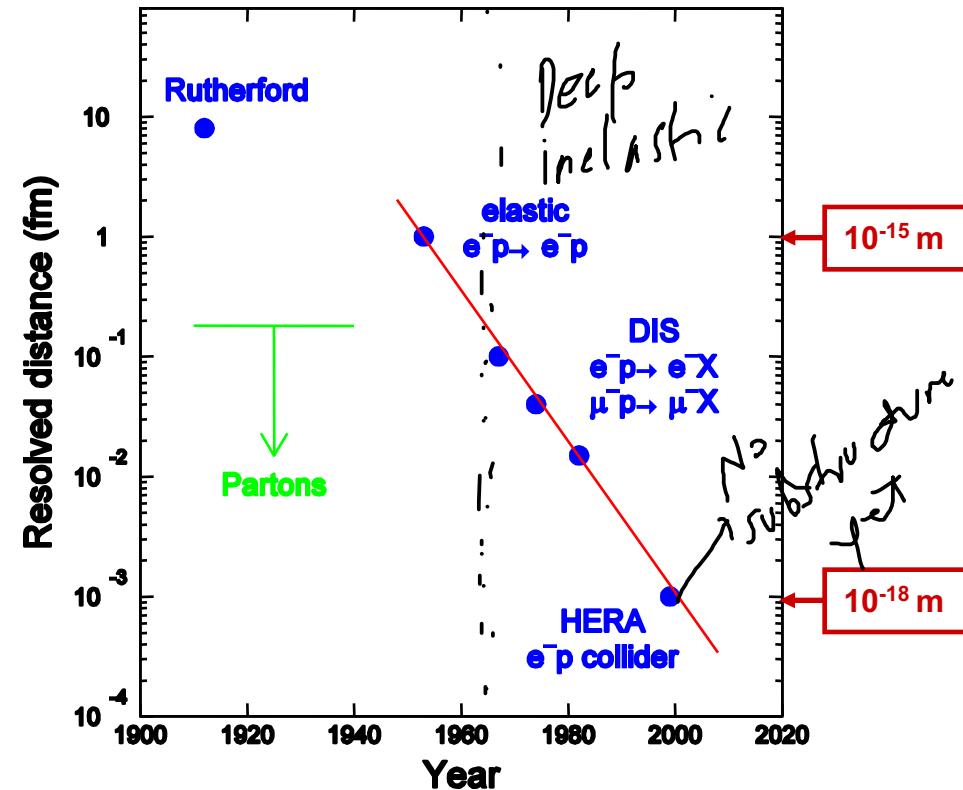
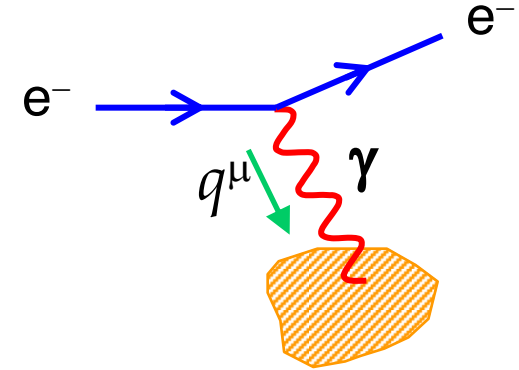
Scaling Violations

- In last 40 years, experiments have probed the proton with virtual photons of ever increasing energy
- Non-point like nature of the scattering becomes apparent when $\lambda_\gamma \sim$ size of scattering centre

$$\lambda_\gamma = \frac{h}{|\vec{q}|} \sim \frac{1 \text{ GeV fm}}{|\vec{q}|(\text{GeV})}$$

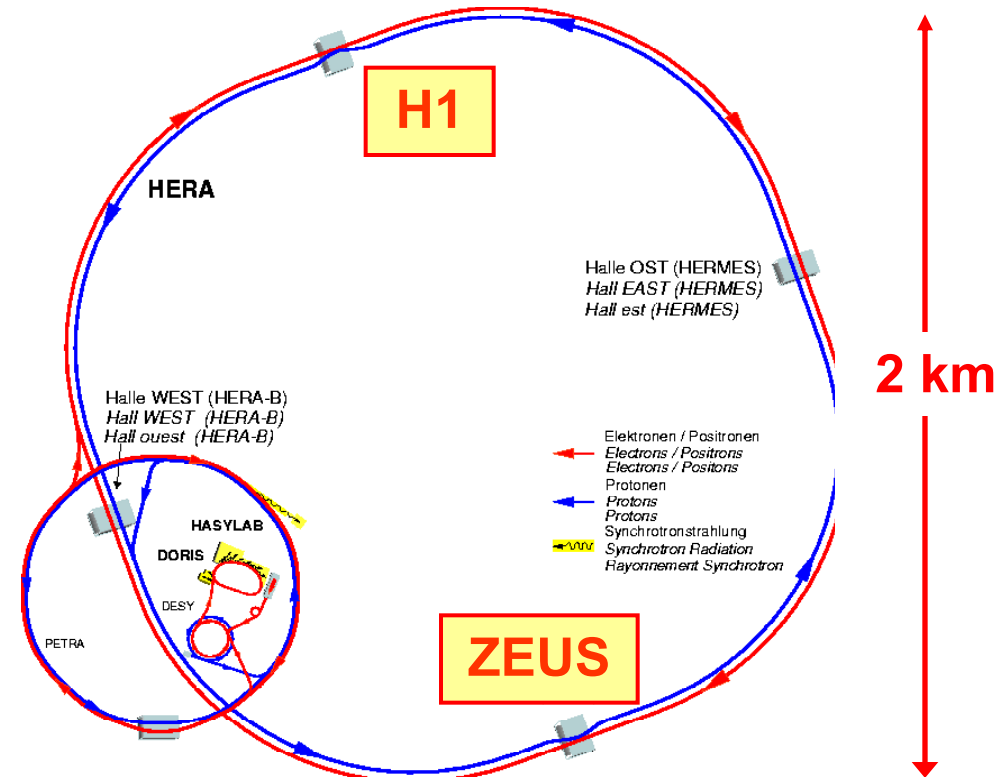
- Scattering from point-like quarks gives rise to **Bjorken scaling**: no q^2 cross section dependence
- IF quarks were not point-like, at high q^2 (when the wavelength of the virtual photon \sim size of quark) would observe rapid decrease in cross section with increasing q^2 .
- To search for quark sub-structure want to go to highest q^2

HERA



HERA $e^{\pm}p$ Collider : 1991-2007

★ DESY (Deutsches Elektronen-Synchrotron) Laboratory, Hamburg, Germany

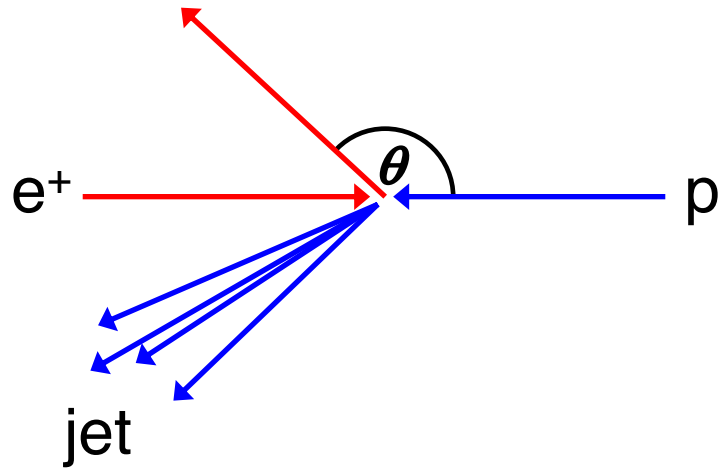


★ Two large experiments : H1 and ZEUS

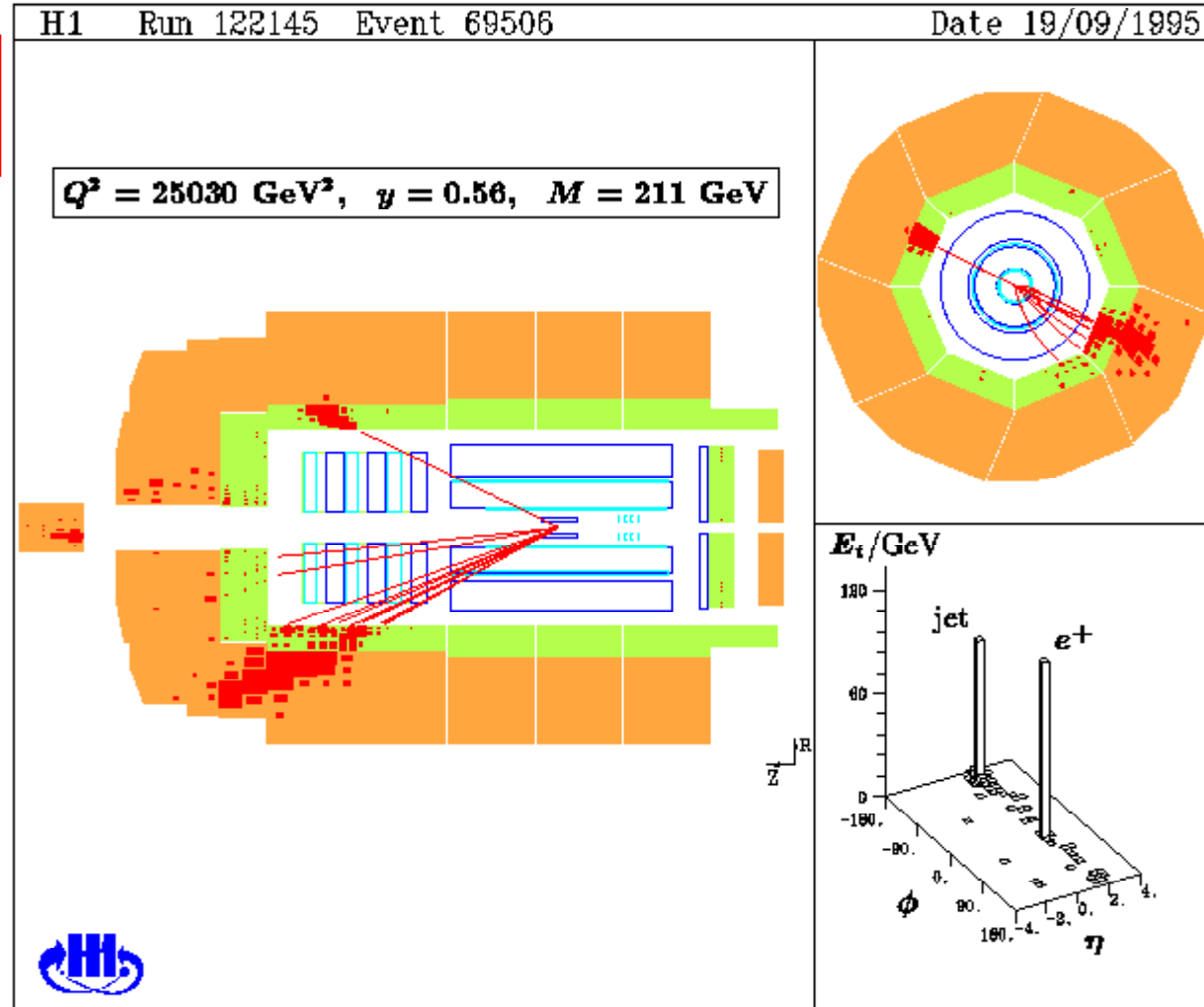
★ Probe proton at very high Q^2 and very low x

Example of a High Q^2 Event in H1

★ Event kinematics determined from electron angle and energy



★ Also measure hadronic system (although not as precisely) – gives some redundancy



4-momentum transfer $F_2(x, Q^2)$ Results

- ★ No evidence of rapid decrease of cross section at highest Q^2

→ $R_{\text{quark}} < 10^{-18} \text{ m}$ *No substructure*

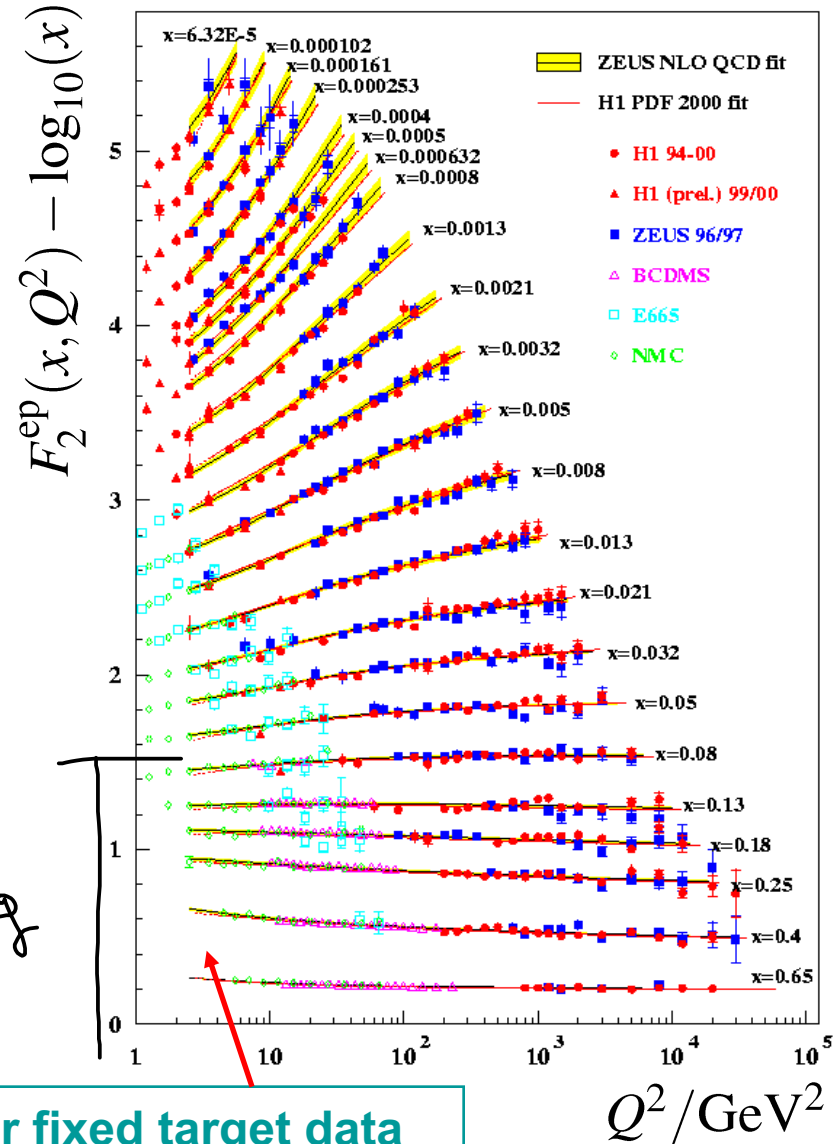
- ★ For $x > 0.05$, only weak dependence of F_2 on Q^2 : consistent with the expectation from the quark-parton model

- ★ But observe clear scaling violations, particularly at low x

$$F_2(x, Q^2) \neq F_2(x)$$

→ limitations of parton model afterwards in QCD, gluons govern all

Bjorken scaling
OK

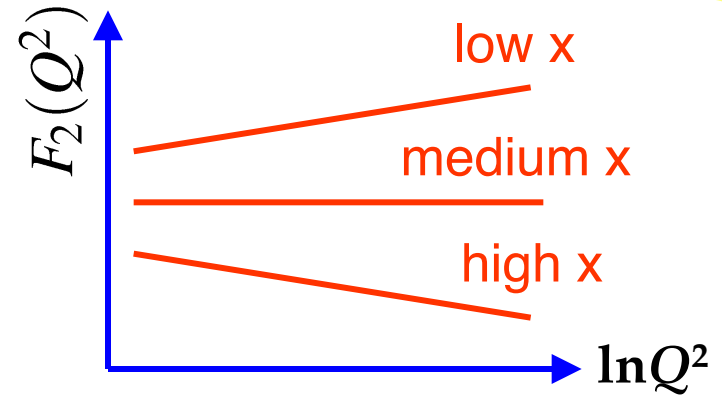
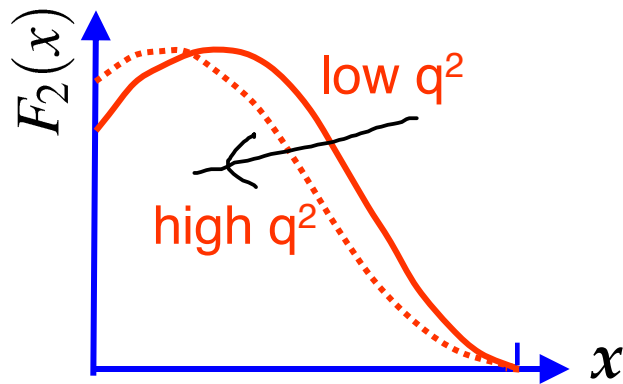


Earlier fixed target data

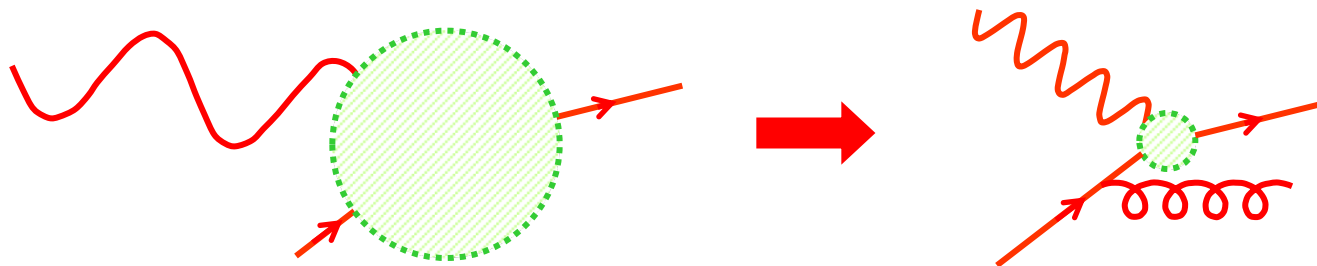
Q^2 / GeV^2

Origin of Scaling Violations

- ★ Observe “small” deviations from **exact Bjorken scaling** $F_2(x) \rightarrow F_2(x, Q^2)$



- ★ At high Q^2 observe more low x quarks
- ★ “Explanation”: at high Q^2 (shorter wave-length) resolve finer structure: i.e. reveal quark is sharing momentum with gluons. At higher Q^2 expect to “see” more low x quarks



- ★ QCD cannot predict the x dependence of $F_2(x, Q^2)$

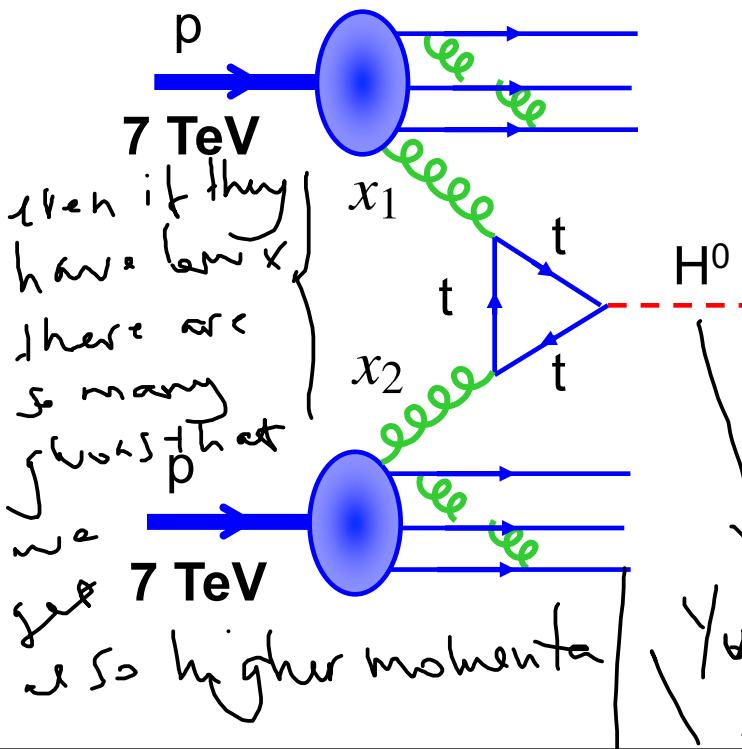
★ But QCD can predict the Q^2 dependence of $F_2(x, Q^2)$

Proton-Proton Collisions at the LHC

- ★ Measurements of structure functions not only provide a powerful test of QCD, the **parton distribution functions** are essential for the calculation of cross sections at pp and p \bar{p} colliders.

- **Example:** Higgs production at the Large Hadron Collider **LHC** (2009-)

- The LHC will collide 7 TeV protons on 7 TeV protons ~ 6.5
- However underlying collisions are between partons
- Higgs production the LHC dominated by “**gluon-gluon fusion**” *distributions measured*



- Cross section depends on gluon PDFs

$$\sigma(pp \rightarrow HX) \sim \int_0^1 \int_0^1 g(x_1)g(x_2)\sigma(gg \rightarrow H)dx_1dx_2$$

- Uncertainty in gluon PDFs lead to a $\pm 5\%$ uncertainty in Higgs production cross section
- Prior to HERA data uncertainty was $\pm 25\%$

*Yukawa coupling
→ mass comes in a mix
→ Need higher order since gluon is massless*

Summary

- ♦ At **very high** electron energies $\lambda \ll r_p$:
the proton appears to be a sea of quarks and gluons.
- ♦ Deep Inelastic Scattering = Elastic scattering from the quasi-free constituent quarks

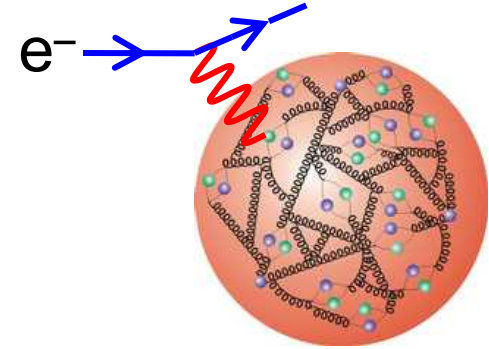
⇒ Bjorken Scaling $F_1(x, Q^2) \rightarrow F_1(x)$

⇒ Callan-Gross $F_2(x) = 2xF_1(x)$

point-like scattering

Scattering from spin-1/2

- ♦ Describe scattering in terms of parton distribution functions $u(x), d(x), \dots$ which describe momentum distribution inside a nucleon
- ♦ The proton is much more complex than just uud - sea of anti-quarks/gluons
- ♦ Quarks carry only 50 % of the protons momentum – the rest is due to low energy gluons
- ♦ We will come back to this topic when we discuss neutrino scattering...



Solve

- **Problems** 8.2, 8.3, 8.6, 8.7