Assignment I - Bhabha scattering

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The solution to the first two problems is presented in the attached sheets at the end of the paper. The γ matrices properites used are from [1].

The code used for this project can be found here:
`https://github.com/FedericoNardi/ParticlePhysics.git`

1 Differential Cross Section

Starting from the expression for the matrix element

$$< |M_{fi}|^2 >= 4e^4\left[2\frac{(p_1\cdot p_4)(p_2\cdot p_3)+(p_1\cdot p_3)(p_2\cdot p_4)}{s^2}+2\frac{(p_3\cdot p_4)(p_1\cdot p_2)+(p_1\cdot p_4)(p_3\cdot p_2)}{t^2}+ \right.$$
$$\left. -\frac{(p_1\cdot p_4)(p_3\cdot p_2)}{st}\right]$$
$$\text{where} \quad s=(p_1+p_2)^2=(p_3+p_4)^2 \quad \text{and} \quad t=(p_1-p_3)^2=(p_2-p_4)^2$$

and inserting the expressions for momenta in the **center-mass frame**

$$\begin{aligned}
p_1 &= (E,0,0,E)\\
p_2 &= (E,0,0,-E)\\
p_3 &= (E,0,E\sin\theta,E\cos\theta)\\
p_4 &= (E,0,-E\sin\theta,-E\cos\theta)
\end{aligned}$$

we get the identities

$$\begin{aligned}
&p_2\cdot p_3 = E^2(1-\cos\theta) = p_1\cdot p_4\\
&p_1\cdot p_3 = E^2(1-\cos\theta) = p_2\cdot p_4\\
&p_3\cdot p_4 = 2E^2 = p_1\cdot p_2\\
&s = 4E^2\\
&t = 2E^2(1-\cos\theta)
\end{aligned}$$

and therefore the expression for the matrix element, with $e^2=4\pi\alpha$ becomes

$$< |M_{fi}|^2 >= (4\pi\alpha)^2\left[\underbrace{\frac{(1+\cos\theta)^2+(1-\cos\theta)^2}{2}}_{\text{Annihilation contribution}}+\underbrace{\frac{8+2(1+\cos\theta)^2}{(1-\cos\theta)^2}}_{\text{scattering contribution}}-\underbrace{\frac{(1+\cos\theta)^2}{1-\cos\theta}}_{\text{cross term contr.}}\right].$$

The expression for the Lorentz-invariant differential cross section is

$$\frac{d\sigma}{dt} = \frac{1}{64\pi\, s|\vec{p_1}|^2} < |M_{fi}|^2 >$$

we can get an expression for our reference system by substituting the differential $dt = 2|\vec{p_1}|\,|\vec{p_3}|d(\cos\theta)$:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{32\pi\, s} < |M_{fi}|^2 > \tag{1}$$

since in our frame $|\vec{p_1}| = |\vec{p_3}| = E$.

Using the values $E = \frac{\sqrt{s}}{2} = 7\text{GeV}$, $e^2 = 4\pi\alpha = \frac{4\pi}{137}$, equation 1 has been implemented on a Python code evaluating it for various values of $\cos\theta$. Figure 1 shows the differential cross section from equation 1 evaluated at different angles, while in figure 2 each component of the total matrix element contribution is isolated.

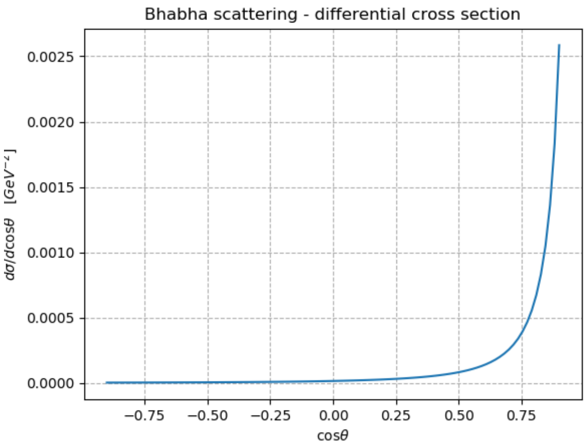


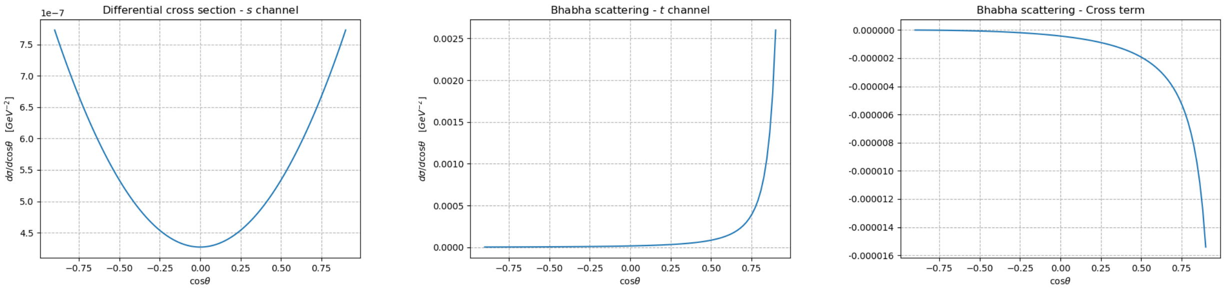
Figure 1: Angular dependence of differential cross section for electron-positron scattering

It is clear that the dominant contribution to the differential cross section for the process comes from the t-channel (scattering), while the interference cross term has a soft damping effect only for small angles $\cos\theta \sim 1$.

Using the result derived in the lectures for $e^+e^- \to \mu^+\mu^-$

$$\frac{d\sigma}{d(\cos\theta)} = 2\pi\frac{d\sigma}{d\Omega} = \frac{(4\pi\alpha^2)}{64\pi\, s}\left[(1 + cos^2\theta) + (1 - \cos^2\theta)\right]$$

we can note that it is the same expression obtained by combining equation 1 with the first term of the matrix element $< |M_{fi}|^2 >$. As expected, in the ultrarelativistic limit $m_{e^\pm} \sim m_{\mu^\pm} << E$ the masses of the leptons can be neglected and the s-channel processes themselves can produce muons-antimuons and electrons-antielectrons with the same yield.



(a) s-channel (b) t-channel (c) cross term

Figure 2: Different contribution for the differential cross section.

2 Total Bhabha Cross Section

The total cross section for the process is obtained as

$$\sigma = \int_{-1}^{1} d(\cos\theta) \frac{d\sigma}{d(\cos\theta)}.$$

Since the integral diverges for $cos\theta \to 1$, a cut-off ϵ has been introduced in the integration domain that becomes $[-1, 1-\epsilon]$. The integral has been evaluated numerically for $\epsilon = 0.001$ and for different values of center mass energies, and the results are shown in figure 3.

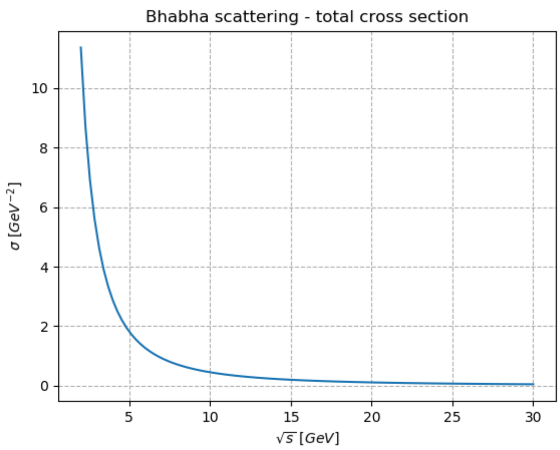


Figure 3: Energy dependence of total cross section for electron-positron scattering

The behaviour of the total cross section is as expected $\propto E^{-2}$, since the only energy dependence is in the prefactor of equation 1, that goes as $1/s$.
Due to the fact that the differential cross section diverges for small angles, the result is strongly dependent on the chosen cut off, i.e. the effective (small) scat-

tering angles that an hypothetical detector is sensitive to.

For $\sqrt{s} = 14\text{GeV}$ the total cross section is

$$\sigma = 0.232\,\text{GeV}^{-2} = 90.3\,\mu\text{b}$$

being $1\text{GeV}^{-2} = 0.3894\,\text{pb}$. For a beam with integrated luminosity $\mathcal{L} = 10\,\text{pb}^{-1}$ and a 50% acceptance efficiency, the expected number of events detected is

$$N = \sigma\,\mathcal{L}\,\times \text{efficiency} = 4.5 \times 10^8\,\text{events}$$

assuming that our setup detects events up to very small scattering angles.

The total cross section for the muon pair production process $e^+e^- \to \mu^+\mu^-$ is shown in figure 4. The behaviour is the same as for Bhabha scattering for the same argument as before, however for this process the total cross section is much smaller: the differential cross section does not diverge for any angle (see figure 2a) and there was no need to introduce a cut-off in the integral. This implies a much smaller contribution to the total cross section for small angles.

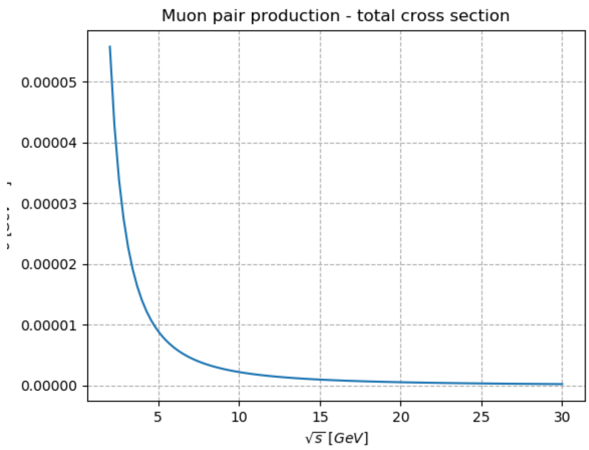


Figure 4: Energy dependence of total cross section for electron-positron scattering

In fact, for this process the total cross section at $\sqrt{s} = 14\text{GeV}$ is

$$\sigma = 1.14 \times 10^{-6}\,\text{GeV}^{-2} = 443\,\text{pb}$$

3 Comparison with experimental results

A comparison with the number of events for Bhabha scattering and muon pair production has been made from [2].
In the article is considered a center mass energy of 29 GeV with integrated luminosity $\mathcal{L} = 19.6\text{pb}^{-1}$ with angular acceptance $|\cos\theta| < 0.55$ and $0.75 < |\cos\theta| < 0.85$. With a 99% events sensitivity and a 12% dead time, a 88%

acceptance sensitivity has been considered. The expected number of events is therefore:

$$\begin{aligned} 10^5 \,\text{events} \qquad &\text{for } e^+e^- \to e^+e^- \\ 200 \,\text{events} \qquad &\text{for } e^+e^- \to \mu^+\mu^- \end{aligned}$$

The sample in the article consists in 8915 e^+e^- events and 811 $\mu^+\mu^-$ events. While the first one can be compatible with our estimation, the latter is 4 times higher. This may be due to a too shallow analysis of the detector system from my part.

References

[1] `http://bolvan.ph.utexas.edu/~vadim/classes/2008f.homeworks/traceology.pdf`

[2] D. Bender et al., *Tests of QED at 29 GeV center-of-mass energy*, Phys. Rev. D 30, 515 – Published 1 August 1984