1 One step methods for ODEs

Exercise 7.1. Consider the following Cauchy problem

$$\begin{cases} y' = t - y + 1 & 0 \le t \le 1 \\ y(0) = 1 \end{cases}$$

whose solution is $y(t) = t + e^{-t}$.

a. Write on paper, and then implement, the Forward Euler method to approximate the solution in $T = 1$. Analyze convergence with varying h.

b. Write on paper, and then implement, the Backward Euler method to approximate the solution in $T = 1$. Analyze convergence with varying h.

Exercise 7.2. Consider the following Cauchy problem

$$\begin{cases} y' = -ty^2 & 0 \le t \le 2 \\ y(0) = 1 \end{cases}$$

whose solution is $y(t) = \dfrac{2}{t^2 + 2}$.

a. Write on paper, and then implement, the Backward Euler method in a suitable function. Note that the system to be solved is now nonlinear.

b. Determine the step size h such that the unitary local truncation error after the first step is smaller than 10^{-3}. Solve the problem using such step.

Exercise 7.3. Consider the Cauchy problem

$$\begin{cases} y' = -2ty & 0 \le t \le 1 \\ y(0) = 1 \end{cases}$$

with exact solution $y(t) = e^{-t^2}$.

a. Write on paper, and then implement, the θ-method.

b. Consider the cases $\theta = 0, 1/4, 1/2, 3/4$ and 1. Apply the method with $h = 2^{-n}$ for $n = 4, \dots, 8$ and approximate the convergence order.