# Forward Kinematic Modeling and Analysis of 6-DOF Underwater Manipulator

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Abstract— sometime a system is expressed more clearly and elegantly with an analytical model. For robotic manipulator the analysis consists of its kinematic modeling and its validation. This paper describes the forward kinematic model of a 6-DOF underwater manipulator and analyzes its end-effector motion through joint space. The manipulator is specifically designed for underwater operation of a water pool. The manipulator is accurately described by obtaining the link transformation matrices from each joint using the Denavit-Hartenberg (D-H) notations. Forward kinematic model is developed in MATLAB. The mathematical model of forward kinematics has been validated using Robotics Toolbox. The manipulator forward kinematics model is also simulated in the Robotics toolbox. The method used in this work may also be applicable to solve the forward kinematics problem of other similar kinds of robotic manipulators.

Keywords—Underwater; manipulator; Forward kinematics; dynamics; degree of freedom; Robotics toolbox;

## I. INTRODUCTION

To predict the behaviors of physical systems in some situations is either very complicated or even not possible. Modeling makes it interesting and find out the performance of a system elegantly. Manipulator modeling consists of study of its kinematic behavior. Kinematic modeling is related to manipulator motion without considering forces producing motions. Kinematic study of manipulator consists of how various links move with respect to one another and in time. It gives an analytical description between position and orientation of manipulator end-effector and its joint variables. Problem of kinematic modeling is divided into two subproblems i.e. forward kinematics and inverse kinematics. Forward kinematics is determining the Cartesian position and orientation of a mechanism, given the joint coordinates. Inverse kinematics is computing the joint variables given the manipulator end-effector position and orientation. For serial robotic manipulator, inverse kinematics problem is more expensive than forward kinematic problem [1].

Most of the robotic manipulators usually consist of series of rigid links generally 6 DOF, mounted on a base. They are used in processes like Spot welding, Spraying, Assembling, Manufacturing and pick and place tasks. Kinematics is a widely used research topic for last two decade in the field of robotics. Man C., Fan X., Li C. and Zhao Z. used

mathematical methods to analyze kinematics of humanoid robots [2]. Wang Z., Xilun D., Alberto R. and Alessandro G presented body kinematics of a radial symmetrical six legged robot [3]. An inverse kinematics model was proposed by Cubero that solve all the joint variables of a serial manipulator of any type, based on forward kinematics solution [4].

Kuma [5] presented a virtual model robot containing both forward and inverse kinematic solutions. Clothier K.E. and Shang Y proposed a geometric model to determine unknown joint variables needed for automatic positioning of a robotic system [6]. Mahidzal Dahari, Jian-Ding Tan modeled the forward and inverse kinematics of a KUKA KR-16KS robotic arm for a simple welding process [7].

The kinematic problem is very complex and difficult due to the multi degree of freedom and multilink space mechanisms. Numerous research technologies are developed with some improvements in robots [8]. Jamshed Iqbal, Raza ul Islam, and Hamza Khan modeled a 6 DOF robotic arm manipulator, ED7220C and analyzed its workspace. This robot is used for pick and place applications [9]. Raza ul Islam, J. Iqbal, S. Manzoor, A. Khalid and S. Khan worked on imageguided robot system to make an autonomous robotic system [10].

This paper describes the forward kinematic model of manipulator in section II. Using MATLAB robotics toolbox the forward kinematics model is validated in section III. Section IV presents the results of simulation for manipulator. Finally Section V discusses on conclusion.

## II. FORWARD KINEMATIC MODEL

The robotic manipulator used is 6 DOF robotic arm manipulator based on PUMA 560 robot developed by US Unimation Inc. It is a serial manipulator consisting of 6 revolute joints. This robotic is mostly used in spot/arc welding, material handling and parts assembly. The manipulator looks like human hand, consisting of shoulder elbow and wrist. 3D model of manipulator is shown in fig.1. Wrist can move in three planes roll, yaw and pitch, making the end-effector more flexible for object Handling and manipulation. Each joint consists mainly of DC motor, encoder, Harmonic reducer, casing, waterproof sealing and bearing. We use the O-ring for static sealing. For dynamic sealing we use the SPGO rings. Salient features of manipulator are given in TABLE I.

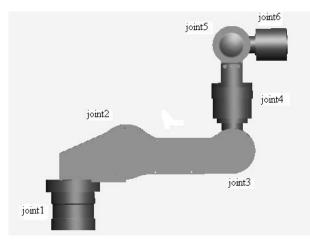


Fig. 1. 3-D CAD model of the manipulator.

TABLE I. SALIENT FEATURES OF UNDERWATER MANIPULATOR

Features	Description			
payload	20 Kg			
precision	±0.5mm			
Weight	60 Kg			
Working depth	20 m			

#### A. Forward Kinematic Modeling

For studying kinematic problem of underwater manipulator we use commonly used method based on Denavit-Hartenberg (DH) parameter. This method is very systematic and more suitable for modeling serial manipulators.

The simplified kinematic model of the manipulator is illustrated in fig.2. Link lengths mentioned in fig.2 are shown in TABLE II.

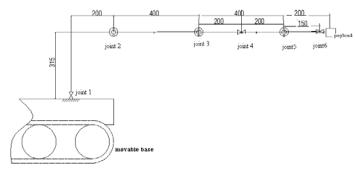


Fig. 2. Underwater manipulator kinematic model

TABLE II. UNDERWATER MANIPULATOR LINK LEJNGTHS

Link No	1	2	3	4	5	6
Link length/mm	200	400	200	200	150	50

Using D-H convention, an orthonormal coordinate system is attached to each link of the manipulator as shown in fig.3. Manipulator D-H parameters are given in TABLE III. Using the homogeneous transformation approach employed in [11] for all links of manipulator. Finally writing the transformation

matrices from base to end-effector by multiplying the individual transformation for each link. This gives the orientation and position of the end-effector (1)

$$T_6^0 = \begin{bmatrix} r_{11} & r_{12} & r_{13} & dx \\ r_{21} & r_{22} & r_{23} & dy \\ r_{31} & r_{23} & r_{33} & dz \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
(1)

where

$$\begin{split} r_{11} &= -s_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3)) - c_6(c_5(s_1s_4 + c_4 + (c_1s_2s_3 - c_1c_2c_3)) \\ &+ s_5(c_1c_2s_3 + c_1c_3s_2) \\ r_{12} &= s_6(c_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) + s_5(c_1c_2s_3 + c_1c_3s_2) - \\ c_6(c_4s_1 - s_4(c_1s_2s_3 - c_1c_2c_3)) \\ r_{13} &= c_5(c_2s_3 + c_1c_3s_2) - s_5(s_1s_4 + c_4(c_1s_2s_3 - c_1c_2c_3)) \\ r_{21} &= s_6(c_1c_4 + s_4(s_1s_2s_3 - c_2c_3s_1)) + c_6(c_5(c_1s_4 - c_4(s_1s_2s_3 - c_2c_3s_1)) - \\ s_5(c_2s_1s_3 + c_3s_1s_2) \\ r_{22} &= c_6(c_1c_4 + s_4(s_1s_2s_3 - c_2c_3s_1)) - s_6(c_5(c_1s_4 - c_4(s_1s_2s_3 - c_2c_3s_1)) - \\ s_5(c_2s_1s_3 + c_3s_1s_2) \\ r_{23} &= s_5(c_1s_4 - c_4(s_1s_2s_3 - c_2c_3s_1)) + c_5(c_2s_1s_3 + c_3s_1s_2) \\ r_{31} &= s_4s_6(c_2s_3 + c_3s_2) - c_6(s_5(c_2c_3 - s_2s_3) + c_4c_5(c_2s_3 + c_3s_2)) + \\ r_{32} &= s_6(s_5(c_2c_3 - s_2s_3) + c_4c_5(c_2s_3 + c_3s_2)) + c_6s_4(c_2s_3 + c_3s_2) \\ r_{33} &= c_5(c_2c_3 - s_2s_3) - c_4s_5(c_2s_3 + c_3s_2) + c_6s_4(c_2s_3 + c_3s_2) \\ dx &= 0.2c_1 + 0.4c_1c_2 - 0.2s_5(s_1s_4 + c_4(s_1s_2s_3 - c_1c_2c_3)) + \\ + 0.2c_5(c_1c_2s_3 + c_1c_3s_2) + 0.4c_2s_1s_3 + 0.4c_3s_1s_2 \\ dy &= 0.2s_1 + 0.4c_2s_1 + 0.2s_5(c_1s_4 - c_4(s_1s_2s_3 - c_2c_3s_1)) + \\ + 0.2c_5(c_2s_1s_3 + c_3s_1s_2) + 0.4c_2s_1s_3 + 0.4c_3s_1s_2 \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_5(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_5(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_5(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_3(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_3(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_3(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_2s_3 - s_2s_3) - 0.2c_3s_3(c_3s_3 + c_3s_2) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_3s_3 - s_3s_3) - 0.2c_3s_3(c_3s_3 + c_3s_3) \\ dz &= 0.4c_3 - 0.4c_3 - 0.4c_3s_3 + 0.2c_3(c_3s_3 - s_3s_3) - 0.2c_3c_3(c_3s_3$$

In equation (1) dx,dy and dz represent the position of the end-effector in x, y and z direction respectively with respect to base. While 3x3 matrix consisting of first three rows and first three columns represent the orientation.

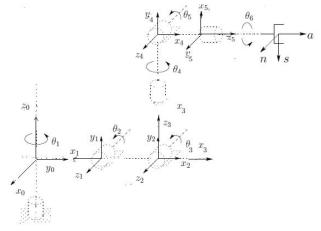


Fig. 3. Coordinate assignment for links of underwater manipulator

TABLE III. D-H PAERAMETRESFOR UNDERWATER MANIPULATOR

Link	$a_{\rm i \ (mm)}$	$lpha_{ m i}$	d <sub>i (mm)</sub>	$ heta_{ m i}$
1	200	-900	0	$ heta_{ ext{l}}$
2	400	00	0	$\theta_2$
3	0	900	0	$\theta_3$
4	0	-900	400	$\theta_{\scriptscriptstyle 4}$
5	0	900	0	$\theta_{\scriptscriptstyle 5}$
6	0	00	200	$\theta_6$

#### III. VALIDATION OF FORWARD KINEMATICS MODEL

MATLAB Robotics Toolbox is used for the validation of forward kinematics model of the underwater manipulator. To check the accuracy of the mathematical model some steps are needed for this purpose. We compare the general position vector in equation (1) with the zero position vector of the manipulator. To get zero position in terms of link parameters, we assume  $\theta 1=\theta 2=\theta 3=\theta 4=\theta 5=\theta 6=0$  degree

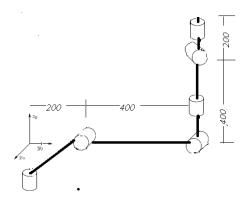


Fig. 4. Zero position of the manipulator

We get the zero position vector of the manipulators as

$$\begin{bmatrix} dx \\ dy \\ dz \end{bmatrix} = \begin{bmatrix} 0.6 \\ 0 \\ 0.6 \end{bmatrix}$$
 (2)

Numerical results and visual plot for position and orientation in MATLAB environment gives clear understanding of the kinematic behavior of manipulator. Given various angle set as input to the developed forward kinematics model and MATLAB toolbox, corresponding results have been compared and plotted. For joint angle configuration [01 02 03 04 05 06] as [0 0 0 0 0 0]; endeffector's position and orientation is expressed in the base coordinates, computed as

$$T_6^0 = \begin{vmatrix} 1 & 0 & 0 & 0.6 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0.6 \\ 0 & 0 & 0 & 1 \end{vmatrix}$$
 (3)

Using MATLAB Robotics Toolbox command "fkine", the same result is obtained. The corresponding MATLAB plot for this joint configuration is shown in Figure 5.

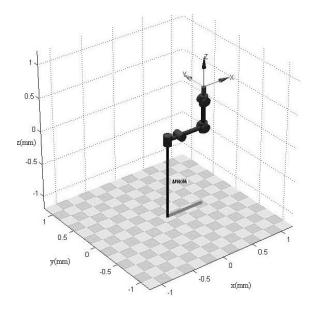


Fig. 5. Plot for joint angle configuration [0 0 0 0 0 0].

Similarly we check the mathematical model of forward kinematics for another set of values,  $\theta 1 = \theta 2 = \theta 3 = \theta 4 = \theta 6 = 0$  and  $\theta 5 = \pi/2$ . Putting these values in equation (1) we get

Plot for this configuration created in Robotics toolbox is shown in figure 6.

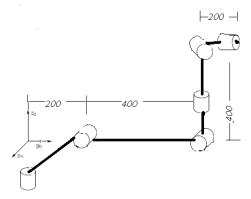


Fig. 6. Manipulator configuration for angle [0 0 0 0 90 0].

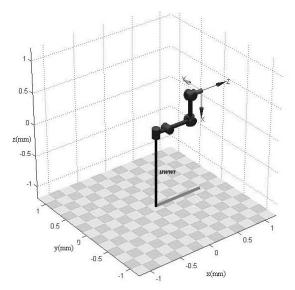


Fig. 7. Plot for joint angle configuration [0 0 0 0 90 0].

#### IV. SIMULATION

Simulation was also done in the MATLAB platform inside the robotic toolbox. The path trajectory of the manipulators was initialized using the "jtraj" function from the robotic toolbox. We know that one of the most important requirements in robotic manipulator is to move the endeffector smoothly from one pose to another pose. We can use this approach for generating such trajectories: straight lines in joint space and straight lines in Cartesian space. We take 2 poses the end-effector moving between these two Cartesian poses, which lie in the xy-plane with the end-effector oriented downward. The initial and final joint coordinate vectors associated with these poses were found....and let the motion to occur over a time period of 2 seconds in 50 ms time steps. A joint-space trajectory is formed by smoothly interpolating between two configurations. The trajectory is best viewed as an animation by plotting the robotic manipulator in toolbox environment. The end-effector position vector is plotted versus time in Fig.8

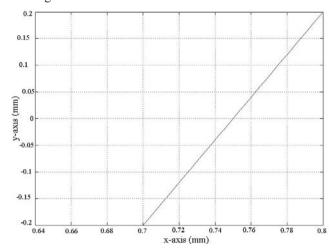


Fig. 8. End-effector position plot in xy-plane

#### V. CONCLUSION

For robotic arm underwater manipulator based on Puma560 robot with some modifications in its structure, its forward kinematic model and motion is analyzed. Forward kinematic model has been validated using MATLAB robotics toolbox. The results from the derived forward kinematic model match exactly with that of MATLAB toolbox. The simulation of the manipulator end-effector motion along a straight line shows that it is modeled correctly. The forward kinematic model we developed may also be extended to other 6-DOF manipulator with some modifications.

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