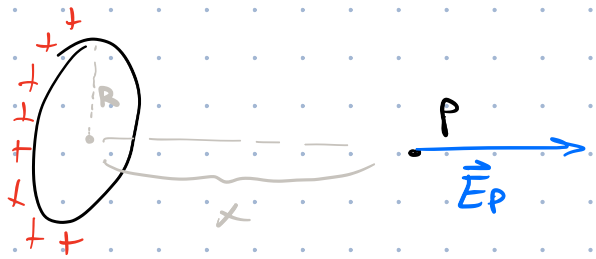
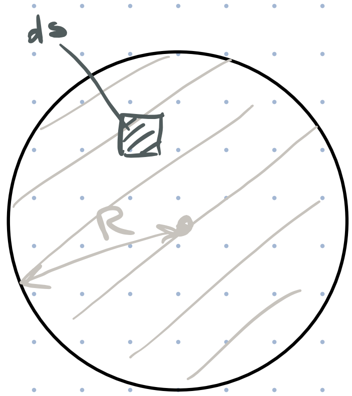
RICORDIAMO



$$E_P = \frac{1}{4\pi\varepsilon_0} \frac{Qx}{(x^2+R^2)^{3/2}}$$



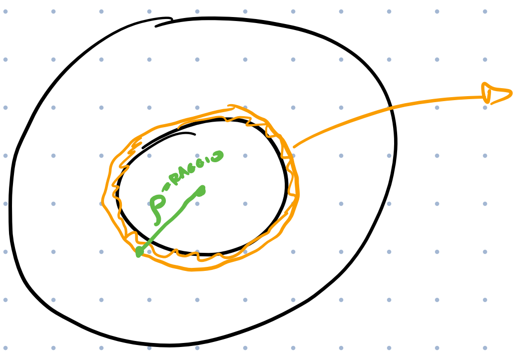
DISCO PIENO

$\sigma > 0$

$dq = \sigma ds$

$$\vec{E}_{TOT} = \int_S d\vec{E} = \int_S \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma ds}{r^2} \hat{r}$$

DOBBIAMO CERCARE DI SFRUTTARE LA SIMMERIA DEL PROBLEMA



ANELLO DI AREA INFINITESIMO

↓

il campo prodotto da un ANELLO spinge lungo l' asse

Prendo $\rho \in [0, R]$ ⤷ tutti i ρ formano anelli

⇓

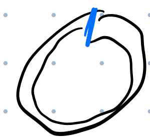
la somma vettoriale sarà tra vettori lungo la stessa direzione e lo stesso verso

$$E_p = \int_S dE = \int_S \frac{1}{4\pi\varepsilon_0} \cdot \frac{dq\, x}{(x^2+\rho^2)^{\frac{3}{2}}}$$

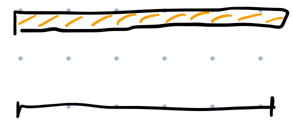
del disco

$dq = \sigma\, ds$

ds:



TAGLIO L' ANELLO



$I\, d\rho$

$2\pi\rho$

$ds = 2\pi\rho \cdot d\rho$

$dq = \sigma \cdot 2\pi\rho \cdot d\rho$

$$= \int \frac{1}{4\pi\varepsilon_0} \cdot \frac{\sigma\, 2\pi\rho\, d\rho\, x}{(x^2+\rho^2)^{3/2}} \quad \text{con } \rho \in [0,R]$$

$$E_{TOT} = \int_0^R \frac{1}{4\pi\varepsilon_0} \cdot \frac{2\pi\sigma\rho\, d\rho\, x}{(x^2+\rho^2)^{3/2}}$$

$$= \frac{\sigma}{2\varepsilon_0} x \int_0^R \frac{\rho\, d\rho}{(x^2+\rho^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0} \left(- \frac{1}{\sqrt{x^2+\rho^2}} \Bigg|_0^R \right)$$

$$\frac{\sigma}{2\varepsilon_0} \cdot \left\{ 1 - \frac{x}{\sqrt{x^2+R^2}} \right\} \hat{u}$$

versore lungo la direzione dell' asse

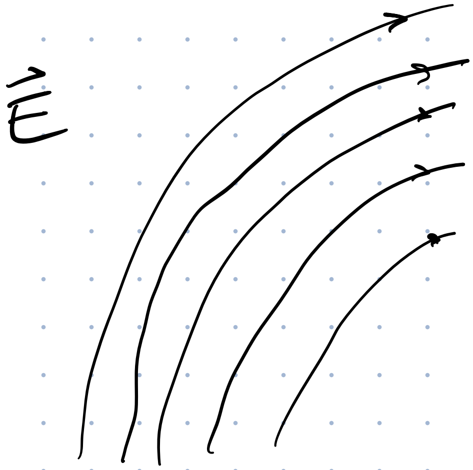
SIAMO IN GRADO DI CALCOLARE IL CAMPO GENERATO DA UN DISCO AD UNA DISTANZA x

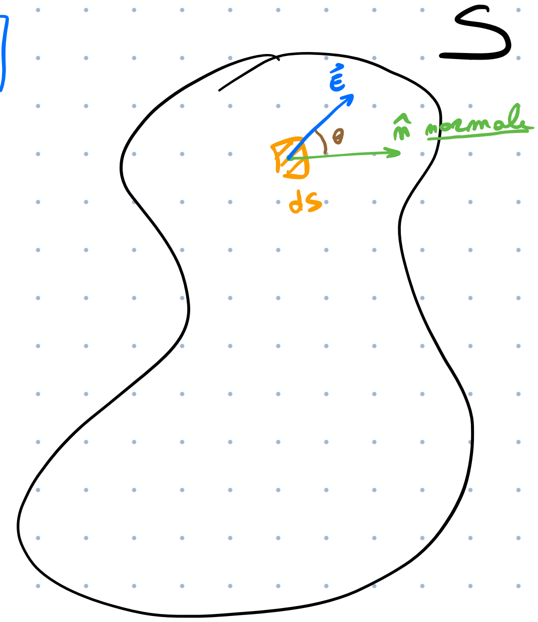
Posso valutare il campo generato con $R \to \infty$

$$\vec{E}_p = \lim_{R \to \infty} (\quad) \hat{u} = \frac{\sigma}{2\varepsilon_0} \hat{n}$$

CERCHIAMO UN ALTRA SOLUZIONE

FLUSSO $\rightarrow \Phi(\vec{E}) \left[\frac{N}{C} m^2\right]$





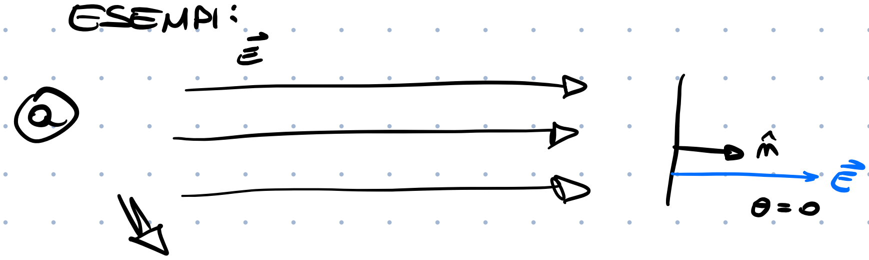
SUPEFICIE
- CHIUSA: racchiude un Volume (es. valigetta)
- APERTA (es. foglio)

$$\Phi \stackrel{DEF}{:=} \underbrace{\vec{E}\cdot\hat{n}}\cdot ds$$

$$\Phi = E\cdot\cos\theta \, ds$$

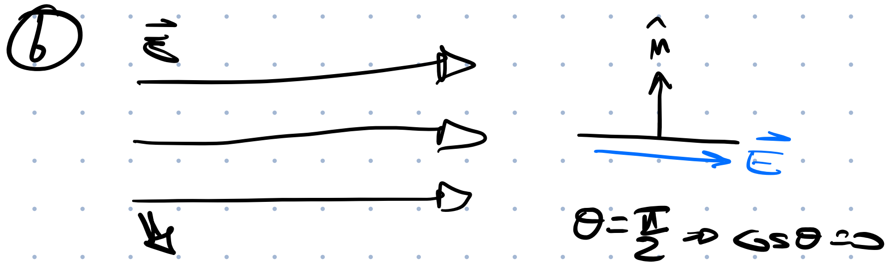
ESEMPI:

(a)



Φ_{MAX}

(b)

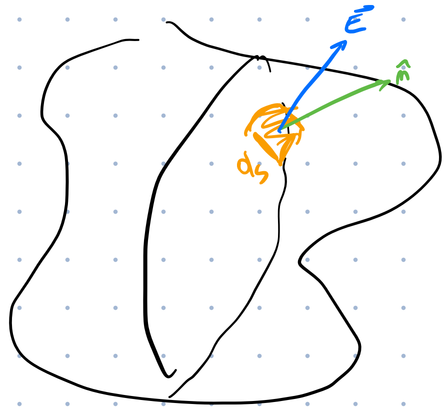


$\theta = \frac{\pi}{2} \rightarrow \cos\theta = 0$

$\Phi = 0$

Il flusso capisce se il campo investe il piano e con quale entità

Sup. Chiusa



Il **verso** di $\hat{n}$ va da dentro verso l'esterno

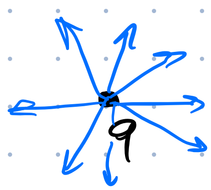
$$d\Phi = \vec{E}\cdot\hat{n}\,ds$$

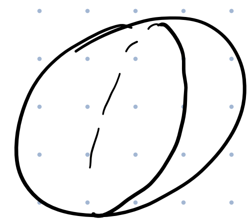
la sup è chiusa

$$\boxed{\Phi(\vec{E}) = \oint_S d\Phi = \oint_S \vec{E}\cdot\hat{n}\,ds}$$

Esercizio

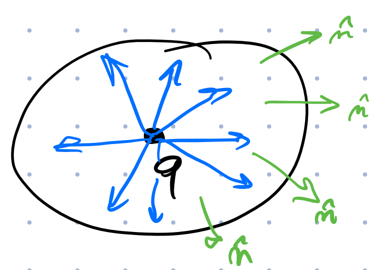
Prendiamo una carica puntiforme





superficie sferica

$q > 0$



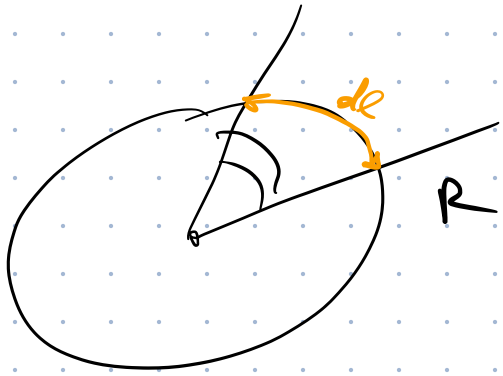
La sfera contiene un volume

$$\Phi(\vec{E}) = \oint_{\substack{S \\ \text{sfera}}} \vec{E}\cdot\hat{n}\,ds = \oint_S E\,ds = E\oint_S ds = E\cdot 4\pi R^2$$

$$E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{R^2} \rightarrow \frac{1}{\cancel{4\pi}\varepsilon_0}\cdot\frac{q}{\cancel{R^2}}\,\cancel{4\pi R^2} \rightarrow \vec{E} = \frac{q}{\varepsilon_0}$$

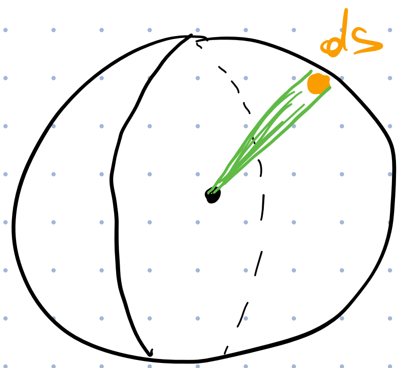
q carica che vive all'interno della sfera

DIMOSTRAZIONE GAUS (I MAXWELL)



$$d\theta = \frac{dl}{R}$$

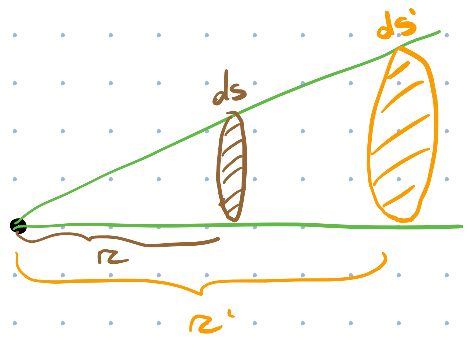
ANGOLO SOLIDO:



CONO → racchiude una porzione di spazio

$$d\Omega = \frac{ds}{R^2}$$

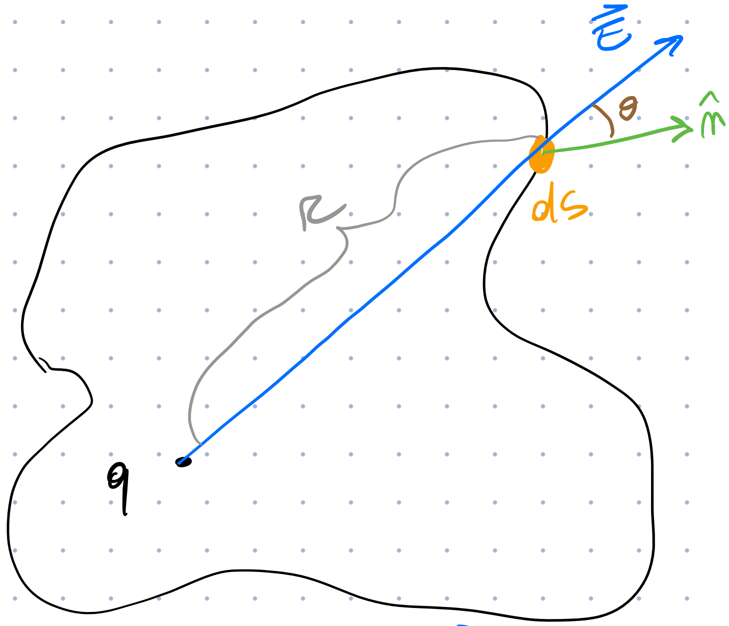
→ ANGOLO SOLIDO



$$ds' > ds$$

se $d\Omega = cost \Rightarrow \frac{ds}{r^2} = \frac{ds'}{r'^2}$ $\quad \Omega = [\text{STERADIANTI}]$

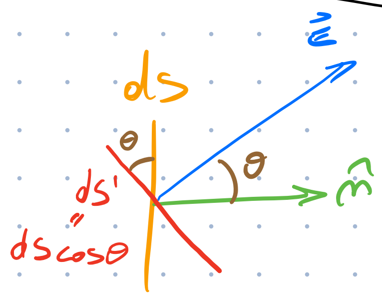
$$\Omega = \int_{sfera} d\Omega = \int_{sfera} \frac{ds}{R^2} = \frac{1}{R^2} \oint_{sfera} ds = \frac{1}{\cancel{R^2}} \cdot 4\pi \cancel{R^2} = \underline{\underline{4\pi}}$$



SUPERFICIE CHIUSA

$$d\Phi(\vec{E}) = \vec{E}\cdot\hat{n}\, dS = E\underbrace{\cos\theta\, dS}_{dS'}$$

$\cos\theta$ fa diventare la sup di una sup. non sferica in una sup. sferica



→ $\vec{E}$ ha la stessa direzione di $\hat{n}$

$$d\Phi(\vec{E}) = E R^2 \cdot d\Omega$$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{\cancel{R^2}}\hat{r}\cdot\cancel{R^2}\, d\Omega$$

$$\Downarrow$$

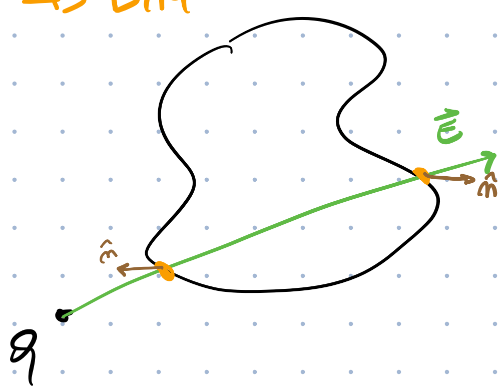
$$d\Phi(\vec{E}) = \frac{q}{4\pi\varepsilon_0}\, d\Omega$$

$d\Phi$ NON dipende dalla distanza

→ $$\Phi = \oint d\Phi = \oint \frac{q}{4\pi\varepsilon_0}\, d\Omega = \frac{q}{4\pi\varepsilon_0}\oint d\Omega = \frac{q}{4\pi\varepsilon_0}\cdot 4\pi = \frac{q}{\varepsilon_0}$$

INT ?

→ DIM



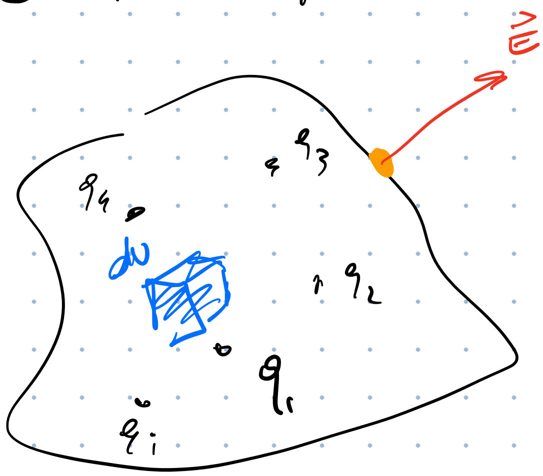
$d\Phi(\vec{E}) = 0$

! VERSORI 1 OPPOSTO E 1 CONCORDE

$\Downarrow$

SI ANNULLANO

SE HO TANTE CARICHE?



$$\vec{E}_{TOT} = \sum_{i=1}^{n} \vec{E}_i$$

$$\Phi(E) = \oint_S \vec{E}\,\hat{n}\,ds$$

$$= \oint \sum_{i=1}^{n} \vec{E}_i\,\hat{n}\,ds = \sum_{i=1}^{n} \oint \vec{E}_i \cdot \hat{n} \cdot ds$$

$$= \sum_{i=1}^{n} \frac{q_i^{(int)}}{\varepsilon_0} = \Phi = \frac{Q^{int}}{\varepsilon_0}$$

$$\Phi(\vec{E}) = \frac{1}{\varepsilon_0} \cdot \sum q_i^{(int)}$$

Distribuzione continua nel volume

$$\Phi(\vec{E}) = \frac{1}{\varepsilon_0} Q^{(int)}$$

I LEGGE DI MAXWELL

$$Q = \int dq = \int_V \rho\,dv \rightarrow \Phi(\vec{E}) = \frac{1}{\varepsilon_0} \int_V \rho\,dv$$