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Neutrino–electron scattering theory*

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Abstract
Standard model predictions for neutrino–electron scattering cross-sections, including effects of electroweak radiative corrections, are reviewed. The sensitivity of these quantities to neutrino dipole moments, Z' bosons and dynamical symmetry breaking is described. Neutrino indices of refraction in matter are also discussed. A perspective on future initiatives with intense neutrino sources, such as from stopped pion decays at a neutron spallation source, superbeams or neutrino factories, is given.

1. Introduction

Neutrino–electron scattering cross-sections are extremely small and correspondingly very difficult to measure. Nevertheless, because of heroic experimental efforts, they have played a crucial role in confirming the $SU(2)_L \times U(1)_Y$ structure of the standard model (SM) and in helping us to unravel subtle properties of neutrinos. In particular, the initial observation of $\bar{\nu}_\mu e \rightarrow \bar{\nu}_\mu e$ scattering at CERN [1] confirmed the existence of weak neutral currents. Subsequently, higher statistics studies [2] of $\nu_\mu e$ and $\bar{\nu}_\mu e$ scatterings provided a clean (purely leptonic) determination of the weak mixing angle, $\sin^2\theta_W$. On a separate front, low energy studies of $\nu e \rightarrow \nu e$ solar neutrino scattering by the Super K Collaboration [3] helped unveil the nature of neutrino mixing and oscillations by exploiting specific SM differences between $\nu_e e$ and $\nu_\ell e$ ($\ell = \mu$ or τ), scattering cross-sections.

Agreement between measured neutrino–electron and antineutrino–electron cross-sections and SM expectations has also been used to constrain physics beyond the SM, (i.e. 'new physics') effects. Bounds on neutrino dipole moments (magnetic, electric and transition) at about the 10^{-9}–$10^{-10} e/2m_e$ level have been given [4]. Constraints on Z' bosons, excitations of extra dimensions, dynamical symmetry breaking etc can also be extracted. However, currently, they are usually not as stringent as other precision electroweak tests. In the future, more precise studies of νe scattering could be made competitive by utilizing high intensity neutrino sources such as stopped pion decays at neutron spallation sources, superbeams or

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neutrino factories. The last one would allow for intense high energy ν_e, $\bar{\nu}_\mu$, $\bar{\nu}_e$ and ν_μ beams derived from $\mu^\pm$ decays in storage rings. The statistical figure of merit for cross-section measurements with such beams grows as E_μ^3; so, high energy is of prime importance. If constructed, such facilities would play a valuable role in the next generation of 'new physics' probes, with νe scattering providing a small part of their extensive physics programs [5][1].

Of course, to fully utilize precision νe cross-section measurements, electroweak radiative corrections must be included. Such effects have been fully computed at the one loop level [6] and if required, two loop calculations (albeit tedious and demanding) could be carried out. Theoretical efforts to push these calculations much further would require motivation from new experiments.

In this overview of νe scattering theory, our plan is as follows. First, we give in section 2 the SM predictions for such cross-sections. Then in section 3, the order α (one loop + bremsstrahlung) electroweak radiative corrections to those reactions are presented. Effects of 'new physics', including neutrino dipole moments, Z' bosons and dynamical symmetry breaking are discussed in section 4. A somewhat different, but related topic, neutrino indices of refraction in matter, is reviewed in section 5. This interesting phenomenon which finds application in terrestrial, solar and supernova studies stems from neutrino forward scattering amplitudes with the constituents of matter (which include electrons). Hence, we deem this subject appropriate for this paper. Finally, in section 6, we give a perspective for future neutrino–electron studies that would be made possible by intense neutrino sources such as stopped pion decays at neutron spallation sources, superbeams or neutrino factories.

2. Tree level cross-sections

Within the SM $SU(2)_L \times U(1)_Y$ framework of electroweak interactions, a variety of neutrino and antineutrino scattering cross-sections are possible. Here, we divide them into three categories. The first set, although kinematically suppressed, is conceptually simple. It corresponds to pure W exchange (charged current interactions). In the t-channel, one can have

$$\nu_\ell + e \to \ell + \nu_e \qquad (\ell = \mu \text{ or } \tau) \tag{1}$$

while in the s-channel

$$\bar{\nu}_e + e \to \ell + \bar{\nu}_\ell \qquad (\ell = \mu \text{ or } \tau). \tag{2}$$

These are sometimes referred to as inverse muon (or tau) decays. The threshold (for electrons at rest)

$$E_\nu \geqslant \frac{m_\ell^2 - m_e^2}{2m_e} \tag{3}$$

is quite high for $\ell = \mu$, i.e. $E_\nu \gtrsim 10.8$ GeV and essentially inaccessible for $\ell = \tau$, $E_\nu \gtrsim 3$ TeV. Nevertheless, for generality we leave ℓ arbitrary. Note also that for the s-channel in equation (2) semileptonic reactions $\bar{\nu}_e + e \to d + \bar{u}$, $s + \bar{u}$, $d + \bar{c}$ etc are also possible. However, these possibilities will not be discussed in this paper. Instead, we focus only on leptonic reactions.

Pure Z exchange in the t-channel (weak neutral currents) gives rise to a second more easily accessible set of reactions

$$\overset{(-)}{\nu}_\ell + e \to \overset{(-)}{\nu}_\ell + e \qquad (\ell = \mu \text{ or } \tau) \tag{4}$$

where the shorthand notation $\overset{(-)}{\nu}$ stands for ν or $\bar{\nu}$.

[1] These studies present a physics case for short baseline physics at a neutrino factory.

Finally, the third possibility

$$\overset{(-)}{\nu}_e + e \to \overset{(-)}{\nu}_e + e \tag{5}$$

proceeds through a combination of W and Z exchange amplitudes.

Before reviewing the tree level predictions for the above processes, we state our simplifying assumptions. Neutrino masses and mixing are neglected (except indirectly in section 5 when neutrino indices of refraction are discussed). We assume $|q^2| \ll m_W^2$ or m_Z^2; so, propagator effects can be ignored and effective four-fermion amplitudes employed. Because the electron target is at rest, that is a very good approximation. All neutrino scattering amplitudes are normalized in terms of the Fermi constant, G_μ, obtained from the muon decay rate (lifetime) [7]

$$G_\mu = \frac{g^2}{4\sqrt{2}m_W^2} = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}. \tag{6}$$

This quantity is very accurately determined and will prove useful in section 3 when electroweak radiative corrections are considered.

With these assumptions, charged current reactions result from the effective tree level amplitude

$$M_{cc} = -\mathrm{i}\frac{G_\mu}{\sqrt{2}}\bar{u}_\ell \gamma^\alpha (1-\gamma_5) u_{\nu_\ell} \bar{u}_{\nu_e} \gamma_\alpha (1-\gamma_5) u_e \tag{7}$$

where the u_f are four component spinors corresponding to their subscript fermions. To obtain the differential cross-sections, one squares the amplitude, $|M_{cc}|^2$, averages over the initial electron polarizations, sums over final state polarizations and integrates over the unobserved final state neutrino momentum. In this way, one finds in the electron rest frame (the lab system) for $\ell = \mu$ or τ

$$\frac{\mathrm{d}\sigma(\nu_\ell e \to \ell \nu_e)}{\mathrm{d}y} = \frac{G_\mu^2}{\pi}\big(2m_e E_\nu - \big(m_\ell^2 - m_e^2\big)\big) \tag{8}$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_e e \to \ell \bar{\nu}_\ell)}{\mathrm{d}y} = \frac{G_\mu^2}{\pi}\big(2m_e E_\nu (1-y)^2 - \big(m_\ell^2 - m_e^2\big)(1-y)\big) \tag{9}$$

where E_ν is the initial state neutrino energy and

$$y = \frac{E_\ell - \frac{m_\ell^2 + m_e^2}{2m_e}}{E_\nu} \tag{10}$$

with E_ℓ being the final state charged lepton energy. The range of y is

$$0 \leqslant y \leqslant y_{\max} = 1 - \frac{m_\ell^2}{2m_e E_\nu + m_e^2}. \tag{11}$$

Note that at threshold, $E_\nu = \frac{m_\ell^2 - m_e^2}{2m_e}$, the range of integration collapses to zero.

Integrating over y, one finds in the very high energy (extreme relativistic) limit $E_\nu \gg \frac{m_\ell^2 - m_e^2}{2m_e}$

$$\begin{aligned}\sigma(\nu_\ell e \to \ell \nu_e) \simeq 3\sigma(\bar{\nu}_e e \to \ell \bar{\nu}_\ell) &\simeq \frac{2G_\mu^2 m_e E_\nu}{\pi} \\ &\simeq 1.5 \times 10^{-41} (E_\nu/\mathrm{GeV})\,\mathrm{cm}^2. \end{aligned} \tag{12}$$

The pure neutral current reactions in equation (4) can be analysed in the same way. Their kinematics is simpler since $m_\ell \to m_e$, but a slight complication is a combination of left and right-handed electron couplings in the effective amplitude ($\ell = \mu$ or τ)

$$M_{\rm NC} = \mathrm{i}\frac{G_\mu}{\sqrt{2}}\bar{u}_{\nu_\ell}\gamma^\alpha(1-\gamma_5)u_{\nu_\ell}[\epsilon_-\bar{u}_e\gamma_\alpha(1-\gamma_5)u_e + \epsilon_+\bar{u}_e\gamma_\alpha(1+\gamma_5)u_e] \qquad (13)$$

where at the tree level [8]

$$\epsilon_- = \tfrac{1}{2} - \sin^2\theta_W \qquad \epsilon_+ = -\sin^2\theta_W. \qquad (14)$$

In terms of

$$y = \frac{E'_e - m_e}{E_\nu} \qquad 0 \leqslant y \leqslant y_{\max} = \frac{1}{1 + m_e/2E_\nu} \qquad (15)$$

with E'_e being the final state electron energy, the differential cross-sections for $\overset{(-)}{\nu}_\ell e \to \overset{(-)}{\nu}_\ell e$ are given by (for $\ell = \mu$ or τ)

$$\frac{\mathrm{d}\sigma(\nu_\ell e \to \nu_\ell e)}{\mathrm{d}y} = \frac{2G_\mu^2 m_e E_\nu}{\pi}\left[\epsilon_-^2 + \epsilon_+^2(1-y)^2 - \epsilon_-\epsilon_+\frac{m_e}{E_\nu}y\right] \qquad (16)$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_\ell e \to \bar{\nu}_\ell e)}{\mathrm{d}y} = \frac{2G_\mu^2 m_e E_\nu}{\pi}\left[\epsilon_+^2 + \epsilon_-^2(1-y)^2 - \epsilon_-\epsilon_+\frac{m_e}{E_\nu}y\right] \qquad (17)$$

where a small $\epsilon_-\epsilon_+$ interference term has been retained for low energy applications. Note that $\sigma(\nu_\ell e \to \nu_\ell e)$ and $\sigma(\bar{\nu}_\ell e \to \bar{\nu}_\ell e)$ are related by $\epsilon_- \leftrightarrow \epsilon_+$ interchange [6].

Neglecting terms of relative order m_e/E_ν, one finds for the integrated cross-sections

$$\sigma(\nu_\ell e \to \nu_\ell e) = \frac{G_\mu^2 m_e E_\nu}{2\pi}\left[1 - 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W\right] \qquad (18)$$

$$\sigma(\bar{\nu}_\ell e \to \bar{\nu}_\ell e) = \frac{G_\mu^2 m_e E_\nu}{2\pi}\left[\frac{1}{3} - \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W\right]. \qquad (19)$$

For $\sin^2\theta_W \simeq 0.23$, these cross-sections are very small $\sim 10^{-42}(E_\nu/\mathrm{GeV})\,\mathrm{cm}^2$. Nevertheless, they have been rather well measured (for $\ell = \mu$) [2], yielding $\sin^2\theta_W$ to about $\pm 3.5\%$. At that level, electroweak radiative corrections become important and must be applied in any serious study. Indeed, they must be considered just to define $\sin^2\theta_W$ in a meaningful way. These corrections will be discussed in section 3.

The final cross-sections to be considered are those in equation (5) that result from combined W and Z boson exchange. They are obtained at tree level from equations (16) and (17) under the replacements (for $\overset{(-)}{\nu}_\ell \to \overset{(-)}{\nu}_e$) [6]

$$\begin{aligned} \epsilon_- &\to \epsilon'_- = \epsilon_- - 1 = -\tfrac{1}{2} - \sin^2\theta_W \\ \epsilon_+ &\to \epsilon'_+ = \epsilon_+ = -\sin^2\theta_W. \end{aligned} \qquad (20)$$

Again ignoring m_e/E_ν effects, one finds

$$\begin{aligned} \sigma(\nu_e e \to \nu_e e) &= \frac{G_\mu^2 m_e E_\nu}{2\pi}\left[1 + 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W\right] \\ \sigma(\bar{\nu}_e e \to \bar{\nu}_e e) &= \frac{G_\mu^2 m_e E_\nu}{2\pi}\left[\frac{1}{3} + \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W\right]. \end{aligned} \qquad (21)$$

Table 1. Relative size of various tree level $\overset{(-)}{\nu} e$ cross-sections in units of $\frac{G_\mu^2 m_e E_\nu}{2\pi}$ for the limit $E_\nu \gg m_\mu^2/2m_e$ but $-q^2 \ll m_W^2$.

Reaction	$\sigma/(G_\mu^2 m_e E_\nu/2\pi)$	Relative size ($\sin^2\theta_W = 0.23$)
$\nu_\mu e \to \mu^- \nu_e$	4	4
$\bar\nu_e e \to \mu^- \bar\nu_\mu$	$\frac{4}{3}$	$\frac{4}{3}$
$\nu_\mu e \to \nu_\mu e$	$1 - 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W$	0.362
$\bar\nu_\mu e \to \bar\nu_\mu e$	$\frac{1}{3} - \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W$	0.309
$\nu_e e \to \nu_e e$	$1 + 4\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W$	2.2
$\bar\nu_e e \to \bar\nu_e e$	$\frac{1}{3} + \frac{4}{3}\sin^2\theta_W + \frac{16}{3}\sin^4\theta_W$	0.922

These cross-sections are roughly a factor of 7 and 3 respectively larger than those in equations (18) and (19). The significant difference between $\sigma(\nu_e e \to \nu_e e)$ and $\sigma(\nu_\ell e \to \nu_\ell e)$, $\ell = \mu$ or τ has played a key role in sorting out what fraction of solar neutrinos reach the earth as ν_e and what fraction arrive as ν_ℓ ($\ell = \mu$ or τ). This information is important for unfolding neutrino mixing and oscillations [3].

In table 1, we summarize the tree level predictions and relative sizes for the scattering cross-sections discussed in this section. Precise measurements of these cross-sections are generally limited by systematic uncertainties in the neutrino flux and spectrum. To help overcome that limitation, various ratios of cross-sections are often discussed. Two cases considered for low energy studies are [2]

$$R_1 \equiv \frac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar\nu_\mu e \to \bar\nu_\mu e)} \tag{22}$$

and

$$R_2 = \frac{\sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\nu_e e \to \nu_e e) + \sigma(\bar\nu_\mu e \to \bar\nu_\mu e)}. \tag{23}$$

The latter, R_2, would use low energy neutrinos stopped from π^+ and μ^+ decays in the chain

$$\pi^+ \to \mu^+ \nu_\mu, \qquad \mu^+ \to e^+ \nu_e \bar\nu_\mu \tag{24}$$

to normalize the flux in that ratio. It was proposed [9] for an experiment at LAMPF, but not carried out. It would be useful for neutrino physics at an intense neutron spallation facility where many π^+ are produced. If flux normalizations can be controlled, then one expects at tree level

$$R_1 = \frac{3 - 12\sin^2\theta_W + 16\sin^4\theta_W}{1 - 4\sin^2\theta_W + 16\sin^4\theta_W} \tag{25}$$

$$R_2 = \frac{3 - 12\sin^2\theta_W + 16\sin^4\theta_W}{4 + 8\sin^2\theta_W + 32\sin^4\theta_W}. \tag{26}$$

Neutrino factories [5] offer the best solution to flux normalization. The decay possibilities $\mu^- \to e^- \bar\nu_e \nu_\mu$ and $\mu^+ \to e^+ \nu_e \bar\nu_\mu$ from a long straight section at a muon storage ring would have very well-specified neutrino energy spectra. One possibility would be to run in both modes and measure (after weighting for the different spectra)

$$R_3 = \frac{\sigma(\bar\nu_e e \to \bar\nu_e e) + \sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\nu_e e \to \nu_e e) + \sigma(\bar\nu_\mu e \to \bar\nu_\mu e)} \tag{27}$$

which is predicted at tree level to be

$$R_3 = \frac{1 - 2\sin^2\theta_W + 8\sin^4\theta_W}{1 + 2\sin^2\theta_W + 8\sin^4\theta_W}. \tag{28}$$

At a very high energy muon storage ring, e.g. $E_\mu \simeq 50$ GeV, where the average neutrino energies are $\sim$40 GeV, one can use the reactions $\bar{\nu}_e e \to \mu^- \bar{\nu}_\mu$ and $\nu_\mu e \to \mu^- \nu_\mu$ from the μ^- decay neutrinos to normalize the flux. This possibility corresponds to measuring (after accounting for different spectra)

$$R_4 \equiv \frac{\sigma(\bar{\nu}_e e \to \bar{\nu}_e e) + \sigma(\nu_\mu e \to \nu_\mu e)}{\sigma(\bar{\nu}_e e \to \mu^- \bar{\nu}_\mu) + \sigma(\nu_\mu e \to \mu \nu_\mu)} \tag{29}$$

which is predicted at tree level to be (using the high energy limits in equation (12))

$$R_4 = \tfrac{1}{4}(1 - 2\sin^2\theta_W + 8\sin^4\theta_W). \tag{30}$$

Of course, to properly utilize such quantities either to extract $\sin^2\theta_W$ with high precision or to search for signs of 'new physics' requires inclusion of electroweak radiative corrections, a subject we now address.

3. Electroweak radiative corrections

The full $\mathcal{O}(\alpha)$ electroweak radiative corrections to neutrino–electron scattering in the standard model were computed [6] in 1983. Here, we provide a brief summary of the results, generally making the same simplifying kinematic assumptions as in section 2. Also, we now ignore effects of relative order $\alpha m_e/E_\nu$ (left–right interference). A detailed study of the latter for low energy neutrinos can be found in [10].

$\mathcal{O}(\alpha)$ corrections to $\overset{(-)}{\nu} e$ scattering cross-sections include the full one loop electroweak corrections of the SM as well as photon bremsstrahlung effects. The loops contain short-distance ultraviolet divergences which are cancelled by renormalization counterterms induced by replacing bare couplings and masses with renormalized quantities. The usual prescription [11] involves replacing $g_0^2/4\sqrt{2}m_W^{0^2}$ by G_μ and $\sin^2\theta_W^0$ by some appropriately defined renormalized weak mixing angle. Here, we employ a modified minimal subtraction ($\overline{MS}$) definition [12]

$$\sin^2\theta_W(\mu)_{\overline{MS}} \tag{31}$$

where μ is the 't Hooft unit of mass in dimensional regularization. We later specialize to $\mu = m_Z$. When the $\overset{(-)}{\nu} e$ cross-sections are expressed in terms of G_μ and $\sin^2\theta_W(\mu)$, the radiative corrections become finite and calculable. Of course, the results can be easily translated into other schemes, for example the on-shell definition [13] $\sin^2\theta_W \equiv 1 - m_W^2/m_Z^2$.

Specializing to the extreme relativistic limit where terms of relative order m_e/E_ν and m_e^2/q^2 can be ignored, one finds that the bulk of the loop corrections to the charged current reactions in equations (1) and (2) is absorbed in G_μ. The remaining loop effects plus bremsstrahlung modify the differential cross-sections in equations (8) and (9) by the overall factors [6]

$$1 + \frac{\alpha}{\pi} f_-(y)$$

$$f_-(y) = -\frac{2}{3}\ln\frac{2E_\nu}{m_e} + \left(\ln(1-y) - \frac{1}{2}\ln y + \frac{y}{2} + \frac{1}{4}\right)\ln\left(\frac{2m_e E_\nu}{m_\ell^2}\right) + \frac{1}{2}\left[L(y) + \frac{\pi^2}{6}\right]$$
$$- \frac{1}{2}\ln^2\left(\frac{1-y}{y}\right) + y\ln y - \left(\frac{23}{12} + \frac{y}{2}\right)\ln(1-y) - \frac{47}{36} - \frac{11}{12}y + \frac{y^2}{24} \tag{32}$$

for $\mathrm{d}\sigma(\nu_\ell e \to \ell \nu_e)/\mathrm{d}y$ and

$$1 + \frac{\alpha}{\pi} f_+(y)$$

$$\begin{aligned}(1-y)^2 f_+(y) = &-\frac{2}{3}(1-y)^2 \ln\frac{2E_\nu}{m_e} + \left[\left(y(1-y) - \frac{1}{2}\right)\ln y\right.\\ &\left.+ (1-y)^2\ln(1-y) - \frac{1-y}{2}\right]\ln\frac{2m_e E_\nu}{m_\ell^2} + \left(y(1-y) - \frac{1}{2}\right)\\ &\times\left(\ln^2 y - \frac{\pi^2}{6} - L(y)\right) + (1-y)^2\ln(1-y)\left[\ln y - \frac{1}{2}\ln(1-y)\right] + (\ln y)\\ &\times\left(-\frac{3}{4} + \frac{y}{2} + y^2\right) + \frac{1}{3}(1-y)\ln(1-y)\left(-\frac{7}{2} + 5y\right) - \frac{1-y}{72}(31-49y)\end{aligned} \tag{33}$$

for $\mathrm{d}\sigma(\bar{\nu}_e e \to \ell \bar{\nu}_\ell)$, where

$$L(y) = \int_0^y \mathrm{d}t \frac{\ln(1-t)}{t}. \tag{34}$$

Integrating these expressions over y (assuming $E_\nu \gg m_\mu^2/2m_e$), one finds

$$\begin{aligned}\sigma(\nu_\ell e \to \ell \nu_e) &\simeq \frac{2G_\mu^2 m_e E_\nu}{\pi}\left(1 + \frac{\alpha}{\pi}F_-\right)\\ F_- &= -\frac{2}{3}\ln\frac{2E_\nu}{m_e} - \frac{1}{6}\left(\pi^2 - \frac{19}{4}\right)\end{aligned} \tag{35}$$

and

$$\begin{aligned}\sigma(\bar{\nu}_e e \to \ell \bar{\nu}_\ell) &= \frac{2G_\mu^2 m_e E_\nu}{3\pi}\left(1 + \frac{\alpha}{\pi}F_+\right)\\ F_+ &= -\frac{2}{3}\ln\frac{2E_\nu}{m_e} - \frac{1}{6}\left(\pi^2 - \frac{43}{4}\right).\end{aligned} \tag{36}$$

Note, the $\ln\frac{2m_e E_\nu}{m_\ell^2}$ terms cancel as expected for total cross-sections.

For very high energy neutrinos, for example $E_\nu \simeq 40$ GeV, where the above results are applicable, these corrections decrease the cross-sections by 2.0% and 1.8%, respectively. They must be included in future high precision studies where $\nu_\mu e \to \mu^- \nu_e$ and/or $\bar{\nu}_e e \to \mu^- \bar{\nu}_\mu$ is used to normalize the flux at say 0.1% or better.

The radiative corrections to the neutral current and mixed reactions in equations (4) and (5) are somewhat more involved. One problem is that the loop corrections are q^2 dependent. However, their variation is not very significant for the range $0 < -q^2 < 2m_e E_\nu$, in the case of realistic E_ν; so, we will approximate them by an average $-q^2$ value. For a more thorough discussion see [6].

Radiative corrections to the total NC cross-sections in equations (18) and (19) are obtained in three steps [6]. First, G_μ is replaced by ρG_μ

$$\begin{aligned}G_\mu &\to \rho G_\mu\\ \rho &= 1 + \frac{\alpha}{4\pi}\left[\frac{3}{4s^4}\ln c^2 - \frac{7}{4s^2} + \frac{2}{c^2 s^2}\left(\frac{19}{8} - \frac{7}{2}s^2 + 3s^4\right)\right.\\ &\quad\left. + \frac{3}{4}\frac{\xi}{s^2}\left[\frac{\ln(c^2/\xi)}{c^2-\xi} + \frac{1}{c^2}\frac{\ln\xi}{1-\xi}\right] + \frac{3}{4s^2}\frac{m_t^2}{m_W^2}\right]\end{aligned} \tag{37}$$

where $s^2 \equiv \sin^2\theta_W(m_Z)_{\overline{MS}}$, $c^2 \equiv \cos^2\theta_W(m_Z)_{\overline{MS}}$, $\xi = m_H^2/m_Z^2$ and $m_t \simeq 175$ GeV is the top quark mass. For $s^2 = 0.231$ and a Higgs mass, $m_H = 130$ GeV, one finds

$$\rho \simeq 1.013 \tag{38}$$

which on its own would increase the NC cross-sections by 2.6%.

The second effect of radiative corrections is to replace $\sin^2\theta_W$ in the tree level ϵ_- and ϵ_+ by $\sin^2\theta_W(q^2)$ which when expressed in terms of the $\overline{MS}$ definition is given by (for $\overset{(-)}{\nu}_\ell e$ scattering)

$$\sin^2\theta_W(q^2) = \kappa_\ell(q^2)\sin^2\theta_W(m_Z)_{\overline{MS}} \tag{39}$$

where

$$\kappa_\ell(q^2) = 1 - \frac{\alpha}{2\pi s^2}\left\{2\sum_f \left(T_{3f}Q_f - 2s^2Q_f^2\right)J_f(q^2) - 2R_\ell(q^2) + \frac{c^2}{3} + \frac{1}{2} + \frac{1}{c^2}\left(\frac{19}{8} - \frac{17}{4}s^2 + 3s^4\right) - \left(\frac{7}{2}c^2 + \frac{1}{12}\right)\ln c^2\right\} \tag{40}$$

where

$$J_f(q^2) = \int_0^1 \mathrm{d}x\, x(1-x)\ln\frac{m_f^2 - q^2x(1-x)}{m_Z^2} \tag{41a}$$

$$R_\ell(q^2) = \int_0^1 \mathrm{d}x\, x(1-x)\ln\frac{m_\ell^2 - q^2x(1-x)}{m_W^2}. \tag{41b}$$

The sum in equation (40) is over all fermions, with $T_{3f} = \pm 1/2$ and $Q_f =$ electric charge. For $q^2 = 0$, the quark contributions have been evaluated using $e^+e^- \to$ hadrons data in a dispersion relation, one finds

$$\kappa_\mu(q^2 = 0) = 0.9970 \tag{42a}$$

$$\kappa_\tau(q^2 = 0) = 1.0064. \tag{42b}$$

The difference between these two values can be viewed as a measure of the different ν_μ and ν_τ charge radii [15] (or anapole moments for Majorana neutrinos). For $\langle q^2\rangle \simeq -0.02$ GeV^2, relevant for $E_\nu \simeq 40$ GeV, the κ in equation (42) are reduced by about 0.001. Overall, the effect of the radiative correction in equation (42a) is to decrease the $\overset{(-)}{\nu}_\mu e$ cross-sections by about 1%.

The final source of radiative corrections comes from QED, including bremsstrahlung. These corrections are basically the same as in $\nu_\ell e \to \ell\nu_e$ and $\bar{\nu}e \to \ell\bar{\nu}_\ell$ for left–left and left–right amplitudes, but with $m_\ell \to m_e$. In total one finds [6]

$$\sigma(\nu_\ell e \to \nu_\ell e) = \frac{\rho^2 G_\mu^2 m_e E_\nu}{2\pi}\left[(1 - 2\kappa_\ell(\bar{q}^2)s^2)^2\left(1 + \frac{\alpha}{\pi}F_-\right) + \frac{1}{3}(-2\kappa_\ell(\bar{q}^2)s^2)^2\left(1 + \frac{\alpha}{\pi}F_+\right)\right] \tag{43}$$

where F_- and F_+ are given in equations (35) and (36) and $\bar{q}^2$ represents an average q^2. For $\sigma(\bar{\nu}_\ell e \to \bar{\nu}_\ell e)$, one simply interchanges $1 - 2\kappa_\ell s^2$ and $-2\kappa_\ell s^2$. Overall, the radiative corrections tend to cancel and result in $\mathcal{O}(1\%)$ shifts in the cross-sections.

In the case of $\overset{(-)}{\nu}_e e \to \overset{(-)}{\nu}_e e$ cross-sections, the same procedure as above applies except $G_\mu \to \rho G_\mu$ for the NC amplitude while $G_\mu \to G_\mu$ for the CC amplitude. Also, $\kappa_e(0)$ is smaller than $\kappa_\mu(0)$ by $\frac{\alpha}{3\pi s^2}\ln\frac{m_e}{m_\mu} \simeq -0.0179$, because of the significantly larger ν_e charge radius [15]. This effect is more sensitive to q^2. In total, one finds [6]

$$\sigma(\nu_e e \to \nu_e e) = \frac{\rho^2 G_\mu^2 m_e E_\nu}{2\pi}\left[\left(1 - \frac{2}{\rho} - 2\kappa_e(\bar{q}^2)s^2\right)^2\left(1 + \frac{\alpha}{\pi}F_-\right) + \frac{1}{3}(-2\kappa_e(\bar{q}^2)s^2)^2\left(1 + \frac{\alpha}{\pi}F_+\right)\right] \tag{44}$$

and $\sigma(\bar{\nu}_e e \to \bar{\nu}_e e)$ is obtained by interchanging $1 - \frac{2}{\rho} - 2\kappa_e(\bar{q}^2)s^2$ and $-2\kappa_e(\bar{q}^2)s^2$. The effect of radiative corrections on $\overset{(-)}{\nu}_e e$ cross-sections is overall more significant than $\overset{(-)}{\nu}_\mu e$.

If future measurements of $\overset{(-)}{\nu} e$ cross-sections aim for $\pm 0.1\%$ precision (or even better), the above radiative corrections must be applied with care. Even some leading two loop effects should probably be included. But why push these measurements to such extreme precision? Some motivative, the search for 'new physics', will be given in section 4.

4. 'New physics' effects

Very precise measurements of $\overset{(-)}{\nu} e$ scattering can in principle test the SM at its quantum loop level and probe for 'new physics' effects. G_μ is extremely well determined via muon decay (see equation (6)) and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ has been determined to be better than $\pm 0.1\%$

$$\sin^2\theta_W(m_Z)_{\overline{MS}} = 0.2312 \pm 0.0002. \tag{45}$$

Future efforts could further improve both these quantities by as much as an order of magnitude. Employing these values and the explicit electroweak radiative corrections outlined in section 3 leads to very precise predictions. For example, one finds the following SM predictions

$$\frac{\mathrm{d}\sigma(\nu_\mu e \to \nu_\mu e)^{\mathrm{SM}}}{\mathrm{d}y} = 0.2995\left[1 + \frac{\alpha}{\pi}f_-(y) + 0.7243(1-y)^2\left(1 + \frac{\alpha}{\pi}f_+(y)\right)\right]\sigma(E_\nu) \tag{46a}$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)^{\mathrm{SM}}}{\mathrm{d}y} = 0.2169\left[1 + \frac{\alpha}{\pi}f_-(y) + 1.380(1-y)^2\left(1 + \frac{\alpha}{\pi}f_+(y)\right)\right]\sigma(E_\nu) \tag{46b}$$

$$\frac{\mathrm{d}\sigma(\nu_e e \to \nu_e e)^{\mathrm{SM}}}{\mathrm{d}y} = 2.087\left[1 + \frac{\alpha}{\pi}f_-(y) + 0.1003(1-y)^2\left(1 + \frac{\alpha}{\pi}f_+(y)\right)\right]\sigma(E_\nu) \tag{46c}$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_e e \to \bar{\nu}_e e)^{\mathrm{SM}}}{\mathrm{d}y} = 0.2093\left[1 + \frac{\alpha}{\pi}f_-(y) + 9.969(1-y)^2\left(1 + \frac{\alpha}{\pi}f_+(y)\right)\right]\sigma(E_\nu) \tag{46d}$$

$$\sigma(E_\nu) \equiv \frac{G_\mu^2 m_e E_\nu}{2\pi} \tag{46e}$$

where the functions $f_-(y)$ and $f_+(y)$ were presented in equations (32) and (33) (here m_l is replaced by m_e). These functions give rise to large corrections to the final state electron spectrum see [6]; however, they integrate only to about an overall 2% decrease. Note, that we have given predictions to about 0.1% to emphasize their precision; but the numerical coefficients will actually change somewhat for better specified experimental conditions (e.g. q^2 and E_ν).

If measured $\overset{(-)}{\nu} e$ scattering cross-sections disagree with the SM predictions in equation (46), it would be evidence for 'new physics' in $\overset{(-)}{\nu} e$ scattering or G_μ and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ determinations. Some examples of 'new physics' that could cause such deviations will be discussed in this section. To illustrate the potential sensitivity of $\overset{(-)}{\nu} e$ scattering, we will sometimes assume that a future uncertainty of ±0.1–0.5% in these cross-sections is achievable. Such a goal is very challenging. Its attainability will be further discussed in section 6.

4.1. Neutrino dipole moments

We now believe that neutrinos have small masses and large mixing with one another. If they are Dirac particles, they will have, albeit tiny, magnetic dipole moments (and even smaller electric dipole moments). In terms of electron Bohr magnetons, $e/2m_e$, one finds [16] the SM prediction

$$\vec{\mu}_{\nu_i} = \frac{e}{m_e}\kappa_{\nu_i}\,\vec{S} \qquad \kappa_{\nu_i} = \frac{3}{4}\frac{G_\mu m_e m_{\nu_i}}{\sqrt{2}\pi^2} \simeq 3\times 10^{-19}(m_{\nu_i}/\mathrm{eV}). \tag{47}$$

Both Dirac and Majorana neutrinos can have transition moments that link distinct mass eigenstates, giving rise to $\nu_2 \to \nu_1 + \gamma$, for example.

Since m_{ν_i} are expected to be < 0.05 eV, neutrino dipole moments appear to be unobservable in the SM. However, in some left–right symmetric models or extended Higgs models, it is possible to have much larger dipole moments. It is, therefore, of interest to ask what direct experimental bounds can be placed on neutrino dipole moments (magnetic, electric, or transition), independent of theory?

Astrophysics considerations give the best constraints on $\kappa\frac{e}{2m_e}$

$$\kappa \leqslant 10^{-12} \quad \text{(Astrophysics)} \tag{48}$$

but they depend on assumptions. It is useful also to obtain bounds from neutrino scattering experiments, since they are direct.

The existence of any neutrino dipole moment (magnetic, electric or transition) of magnitude $\kappa e/2m_e$ will increase $\overset{(-)}{\nu} e$ cross-sections by [4, 18]

$$\frac{\Delta \mathrm{d}\sigma(\nu e)}{\mathrm{d}y} = |\kappa|^2\frac{\pi\alpha^2}{m_e^2}\left(\frac{1}{y}-1\right). \tag{49}$$

A larger than expected cross-section, particularly one exhibiting a departure following the distinctive $1/y$ dependence of equation (49) could be taken as evidence for a non-vanishing κ. Consistency of current $\bar{\nu}_e e$ and $\nu_\mu e$ cross-sections with SM expectations gives the bounds [4]

$$|\kappa_{\nu_e}| < 4\times 10^{-10} \qquad |\kappa_{\nu_\mu}| < 10^{-9}. \tag{50}$$

These bounds could be translated to mass eigenstates by including explicit mixing, but we do not carry out that exercise here.

It is quite difficult to do much better than the bounds in equation (50) because of the $|\kappa|^2$ factor in equation (49). Low energy neutrino beams are favoured along with y dependence studies. For total cross-sections (with $y_{\min} = 0.01$), equation (49) leads to a fractional change in $\sigma(\nu_\mu e \to \nu_\mu e)$ by a factor

$$1 + 0.6\left|\frac{\kappa}{10^{-9}}\right|^2 \frac{1\ \mathrm{GeV}}{E_\nu}. \tag{51}$$

Reaching $\kappa \simeq 10^{-10}$ sensitivity would require 0.006 GeV$/E_\nu$ precision. For high energy neutrinos $\gtrsim$10 GeV, this might be difficult but perhaps not impossible. In the case of lower energies, such as neutrinos from stopped pion decays, e.g. $E_\nu \sim 0.03$–1 GeV, it seems more straightforward. In fact, at lower energies, such as neutrinos from stopped pion decays, one might aim for a few $\times 10^{-11}$ sensitivity, particularly if the y dependence is measured. Ultimately reaching $|\kappa| \simeq 10^{-11}$ is extremely challenging, but provides a worthwhile goal. We should add that evidence for any neutrino dipole moment significantly larger than the tiny SM predictions for such quantities (e.g. equation (47)) would have important implications, particularly for supernova physics [19]. Of course, it would also inspire theoretical explanation.

4.2. Extra Z' bosons

Many extensions of the SM predict the existence of additional neutral Z' bosons [20, 21]. They occur in $SO(10)$, E_6 and other grand unified theories as well as in some superstring models. Evidence for their existence would have profound implications for 'new physics'.

Such bosons would lead to additional four-fermion operators at low energies whereas they would have little if any effect on Z pole properties. Currently, direct searches for Z' bosons at the Tevatron $p\bar{p}$ collider reach ∼350–700 GeV [14, 21], depending on couplings. Low energy experiments such as atomic parity violation, neutrino scattering, polarized e^-e^- scattering etc probe similar mass scales. The LHC will push those searches to the multi-TeV region. Future e^+e^- colliders could indirectly probe even higher mass scales via interference effects in cross-sections and asymmetries. Here, to roughly illustrate the sensitivity of $\overset{(-)}{\nu}$ scattering to Z' bosons, we consider the concrete example of the $SO(10)$ Z_χ boson. The couplings for this case are fully specified and give rise to an additional $\overset{(-)}{\nu} e$ (flavour independent) amplitude

$$M_{Z_\chi} = -\mathrm{i}\frac{G_\mu}{\sqrt{2}}\frac{3}{4}\sin^2\theta_W \bar{u}_\nu\gamma^\alpha(1-\gamma_5)u_\nu\left[\bar{u}_e\gamma_\alpha(1-\gamma_5)u_e + \frac{1}{3}\bar{u}_e\gamma_\alpha(1+\gamma_5)u_e\right]\frac{m_Z^2}{m_{Z_\chi}^2}. \tag{52}$$

In this way, the ϵ_- and ϵ_+ of equation (14) effectively become (using $s^2 \equiv \sin^2\theta_W$)

$$\epsilon_- \to \frac{1}{2} - s^2 - \frac{3}{4}s^2\frac{m_Z^2}{m_{Z_\chi}^2} \qquad \epsilon_+ \to -s^2 - \frac{1}{4}s^2\frac{m_Z^2}{m_{Z_\chi}^2} \tag{53}$$

for pure neutral current scattering. One then finds for $s^2 = 0.2312$ and $m_Z^2/m_{Z_\chi}^2 \ll 1$

$$\frac{\mathrm{d}\sigma(\nu_\mu e\to\nu_\mu e)}{\mathrm{d}y} = \frac{\mathrm{d}\sigma(\nu_\mu e\to\nu_\mu e)^{\mathrm{SM}}}{\mathrm{d}y} + \sigma(E_\nu)(-0.37 + 0.11(1-y)^2)\frac{m_Z^2}{m_{Z_\chi}^2} \tag{54a}$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_\mu e\to\bar{\nu}_\mu e)}{\mathrm{d}y} = \frac{\mathrm{d}\sigma(\bar{\nu}_\mu e\to\bar{\nu}_\mu e)^{\mathrm{SM}}}{\mathrm{d}y} + \sigma(E_\nu)(0.11 - 0.37(1-y)^2)\frac{m_Z^2}{m_{Z_\chi}^2} \tag{54b}$$

$$\frac{\mathrm{d}\sigma(\nu_e e\to\nu_e e)}{\mathrm{d}y} = \frac{\mathrm{d}\sigma(\nu_e e\to\nu_e e)^{\mathrm{SM}}}{\mathrm{d}y} + \sigma(E_\nu)(1.0 + 0.11(1-y)^2)\frac{m_Z^2}{m_{Z_\chi}^2} \tag{54c}$$

$$\frac{\mathrm{d}\sigma(\bar{\nu}_e e\to\bar{\nu}_e e)}{\mathrm{d}y} = \frac{\mathrm{d}\sigma(\bar{\nu}_e e\to\bar{\nu}_e e)^{\mathrm{SM}}}{\mathrm{d}y} + \sigma(E_\nu)(0.11 + 1.0(1-y)^2)\frac{m_Z^2}{m_{Z_\chi}^2}. \tag{54d}$$

Comparison with equation (46) indicates that Z_χ has the largest impact on $\nu_\mu e$ scattering. Indeed, its integrated effect on $\bar{\nu}_\mu e$ scattering is nearly zero. Of course, other Z' models, extra dimensions, compositeness etc can have quite different influences.

A $\pm 0.1\%$ measurement of $\sigma(\nu_\mu e\to\nu_\mu e)$ would probe m_χ of 2–3 TeV. This appears to be a good benchmark for a next generation experiment. Such precision might be possible at a neutrino factory where high statistics are possible. However, if the neutrinos stem from μ^- decays, one actually measures a spectrum flux weighted combination of $\mathrm{d}\sigma(\nu_\mu e)/\mathrm{d}y + \mathrm{d}\sigma(\bar{\nu}_e e)/\mathrm{d}y$ at such a facility. The weighting of each will depend on the muon polarization and the region of y explored. Therefore, good electron energy resolution will be useful. Note that the simple integrated sum of equations (54*a*) and (54*d*) shows little Z_χ sensitivity.

At more conventional neutrino sources, i.e. horn focused beams or stopped pions at an intense neutron spallation source, ± 0.5–1% measurements of $\sigma(\nu_\mu e \to \nu_\mu e)$ are more realistic expectations. At that level, m_{Z_χ} of order 800–1100 GeV would be explored. This is similar to the capability of current precision measurements in atomic parity violation, polarized e^-e^- scattering etc.

4.3. Dynamical symmetry breaking

Ideas such as technicolour provide interesting alternatives to the elementary Higgs scalar mechanism. In these dynamical scenarios new fermion condensates break $SU(2)_L \times U(1)_Y \to U(1)_{em}$. The basic premise of dynamical symmetry breaking is very appealing, but no simple phenomenologically viable model currently exists.

A rather generic prediction of dynamical models is the existence of new quantum loop effects due to effectively heavy fermions in gauge boson self energies. A nice formalism for studying such loop effects is the S, T and U parametrization [24], which is sometimes expanded to include additional V, W, X and Y corrections [25] due to loop changes between $|q^2| = 0$ and m_Z^2. Here, we will not review that formalism. Instead, we focus on S and T, the most interesting of these parameters, and show how non-zero values for these quantities would affect $\overset{(-)}{\nu} e$ scattering. We should note that generically one expects $S \simeq \frac{2}{3\pi} \simeq 0.2$ for a very heavy fourth generation of fermions and similar or larger effects in dynamical symmetry breaking scenarios. Also, currently precision electroweak measurements already provide the constraints (from global fits) [26] (for $m_H \simeq 300$ GeV)

$$S = -0.11 \pm 0.11 \qquad T = -0.07 \pm 0.13. \tag{55}$$

These bounds are already quite restrictive. The constraint on S severely limits dynamical symmetry breaking scenarios and seems to imply that a heavy fourth generation of fermions is very unlikely. Continuing to search for non-vanishing S and T is strongly warranted, but individual next generation experiments should aim for ± 0.1 or better S and T sensitivity to be competitive.

Within the framework where α, G_μ and m_Z are fixed by their experimental values, the SM plus new heavy fermion loops predicts modifications in $\sigma(\overset{(-)}{\nu} e)$ resulting from the shifts [22]

$$\begin{aligned} \rho &= \rho^{\mathrm{SM}}(1 + 0.0078T) \\ \sin^2\theta_W(m_Z)_{\overline{MS}} &= 0.2312 + 0.00365S - 0.0026T. \end{aligned} \tag{56}$$

One sees that roughly speaking, measuring individual $\sigma(\overset{(-)}{\nu}_\mu e)$ and $\sin^2\theta_W(m_Z)_{\overline{MS}}$ to about $\pm 0.1\%$ would determine S and T to better than ± 0.1. More specifically, we consider the shift in $\overset{(-)}{\nu} e$ cross-sections due to S and T

$$\Delta \frac{\mathrm{d}\sigma}{\mathrm{d}y} = \frac{\mathrm{d}\sigma}{\mathrm{d}y} - \frac{\mathrm{d}\sigma^{\mathrm{SM}}}{\mathrm{d}y}. \tag{57}$$

One finds

$$\frac{\Delta\mathrm{d}\sigma(\nu_\mu e\to\nu_\mu e)}{\mathrm{d}y}\simeq[(0.01T-0.008S)+(1-y)^2(-0.0015T+0.007S)]\sigma(E_\nu) \tag{58a}$$

$$\frac{\Delta\mathrm{d}\sigma(\bar\nu_\mu e\to\bar\nu_\mu e)}{\mathrm{d}y}\simeq[(-0.0015T+0.007S)+(1-y)^2(0.01T-0.008S)]\sigma(E_\nu) \tag{58b}$$

$$\frac{\Delta\mathrm{d}\sigma(\nu_e e\to\nu_e e)}{\mathrm{d}y}\simeq[(-0.027T+0.02S)+(1-y)^2(-0.0015T+0.007S)]\sigma(E_\nu) \tag{58c}$$

$$\frac{\Delta\mathrm{d}\sigma(\bar\nu_e e\to\bar\nu_e e)}{\mathrm{d}y}\simeq[(-0.0015T+0.007S)+(1-y)^2(-0.027T+0.02S)]\sigma(E_\nu). \tag{58d}$$

An interesting possibility at a neutrino factory is to use a combination of $\sigma(\nu_\mu e\to\nu_\mu e)+\sigma(\bar\nu_e e\to\bar\nu_e e)$ with each weighted by spectral flux functions. This combination can be normalized using the inverse muon decay reactions $\sigma(\nu_\mu e\to\mu^-\nu_e)+\sigma(\bar\nu_e e\to\mu^-\nu_\mu)$ which are S and T independent in the above formalism. Note that R_4 obtained using this normalization when fully integrated over y has reduced S and T dependence. Detailed studies of neutrino factory capabilities [5] suggest that $\pm0.1\%$ determinations of $\sin^2\theta_W(m_Z)_{\overline{MS}}$ may be possible. At that level, S and T sensitivity will be about ±0.1. So, we conclude that $\overset{(-)}{\nu}e$ scattering measurements can potentially be competitive next generation probes of S and T, but experiments must be capable of roughly $\pm0.1\%$ sensitivity for $\sigma(\nu e)$ and $\sin^2\theta_W(m_Z)_{\overline{MS}}$. Also, good final state electron energy resolution will be very useful in unfolding the y dependence. The quantity R_3 in equation (28) will be a particularly sensitive probe if it can be measured with small systematic uncertainties.

5. Neutrino indices of refraction

Neutrino propagation through matter can be sensitive to neutrino indices of refraction. These indices result from forward scattering amplitudes for neutrino scattering with the constituents of matter, including electrons [27]. This makes this topic appropriate for a paper on neutrino–electron scattering.

The amplitude for low energy ν_ℓ scattering off a fermion f, where ℓ now stands for e, μ or τ is given by

$$M(\nu_\ell f+\nu_\ell f)=-\mathrm{i}\frac{G_\mu}{\sqrt{2}}\bar\nu_\ell\gamma^\alpha(1-\gamma_5)\nu_\ell\bar f\gamma_\alpha\left(C^V_{\nu_\ell f}+C^A_{\nu_\ell f}\gamma_5\right)f. \tag{59}$$

For an unpolarized medium, the coherent forward scattering of neutrinos with momentum p_ν can be described by an index of refraction n_{ν_ℓ} given by [28]

$$p_\nu(n_{\nu_\ell}-1)=-\sqrt{2}G_\mu\sum_{f=e,u,d}C^V_{\nu_\ell f}N_f \tag{60}$$

where N_f is the fermion number density and at tree level in the SM

$$\begin{aligned}C^V_{\nu_\ell f}&=T_{3f}-2Q_f\sin^2\theta_W \qquad f\neq\ell\\ C^V_{\nu_\ell\ell}&=1+T_{3\ell}-2Q_\ell\sin^2\theta_W\end{aligned} \tag{61}$$

with $T_{3f}=\pm1/2$. The difference between $\overset{(-)}{\nu}_e e$ and $\overset{(-)}{\nu}_\mu e$, $\overset{(-)}{\nu}_\tau e$ scattering cross-sections due to CC interactions in the former gives rise to a difference in the indices of refraction

$$-p_\nu\left(n_{\nu_e}-n_{\nu_\mu}\right)=-p_\nu\left(n_{\nu_e}-n_{\nu_\tau}\right)=\sqrt{2}G_\mu N_e. \tag{62}$$

This difference can significantly impact neutrino oscillations in matter [27].

To gain insight into the origin of equation (62), it is useful to consider $-\mathrm{i}M(\nu_\ell f \to \nu_\ell f)$ as an effective Lagrangian. Averaging that Lagrangian over the background matter medium, one finds [28]

$$\left\langle C^V_{\nu_\ell f} \bar{f}\gamma_0 f\right\rangle = C^V_{\nu_\ell f} N_f \tag{63}$$

whereas (for an unpolarized medium) all other currents average to zero. So, the medium can be interpreted as providing an external potential

$$V = \sqrt{2} G_\mu \sum_f C^V_{\nu_\ell f} N_f \tag{64}$$

experienced by propagating neutrinos. With such a potential

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t} \to \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t} - V \tag{65}$$

in the equation of motion (for antineutrinos $V = -V$). Although the potential is small

$$|V| \sim 4 \times 10^{-14}\,\mathrm{eV}\,(N_f/6 \times 10^{23}\,\mathrm{cm}^{-3}) \tag{66}$$

it can have truly remarkable consequences when it interferes with other very small effects such as neutrino energy differences $m_i^2 - m_j^2 \big/ 2E_\nu$, neutrino dipole moment precession in magnetic fields [19, 29], neutrino decay in matter etc. We will not review these interesting topics. Instead, we conclude this discussion by reviewing radiative corrections to indices of refraction.

It was shown in [30] that the radiative corrections to

$$p_\nu\left(n_{\nu_e} - n_{\nu_\mu}\right) = -\sqrt{2} G_\mu N_e \tag{67}$$

are negligible, $\mathcal{O}\left(\alpha m_\mu^2 \big/ m_W^2\right)$. They do, however, give rise to an interesting one loop induced $p_\nu\left(n_{\nu_\tau} - n_{\nu_\mu}\right)$ of order $\alpha m_\tau^2 \big/ m_W^2$. For an unpolarized medium and with $N_n = N_p = N_e$, one finds [30]

$$n_{\nu_\tau} - n_{\nu_\mu} \simeq 5 \times 10^{-5} (n_{\nu_e} - n_{\nu_\mu}). \tag{68}$$

Although small, this loop induced difference can be of some importance in supernova studies where extremely high densities are possible.

Of course, if 'new physics' exists which differentiates ν_e, ν_μ and ν_τ interactions at the tree level, it can have dramatic effects on neutrino oscillations in matter. Future studies of neutrino oscillations over very long terrestrial baselines will provide interesting probes of such interactions.

6. Outlook

Currently, there are no precision $\overset{(-)}{\nu} e$ scattering studies underway. The only use of this set of reactions is for solar neutrino flux measurements by Super K. However, the discovery of oscillations has invigorated neutrino physics. Future efforts to measure neutrino mixing and mass parameters with high precision and search for new phenomena will demand high intensity neutrino sources. One can envision using these facilities to carry out precision studies of other neutrino properties at short baselines where neutrino oscillations may not be operational, but other phenomena can be explored. Here, we will not comment on neutrino oscillations or the many other very interesting phenomena that can be studied with new intense neutrino sources, although they will provide the primary motivation for such facilities. Instead, we conclude this paper with a brief perspective on the utility of low, medium and high energy intense neutrino facilities for studying $\overset{(-)}{\nu} e$ scattering.

6.1. Low energy

Very intense spallation neutron sources are copious sources of pions. The decay chain $\pi^+ \to \mu^+\nu_\mu$, $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ for stopped π^+ would provide equal fluxes of low energy ν_μ, ν_e and $\bar{\nu}_\mu$ with very well-predicted energy spectra. The latter two are time separated from the ν_μ by the muon lifetime. One can, therefore, contemplate the measurement of R_2 in equation (23) with high precision. An unfulfilled LAMPF proposal [9] would have measured that ratio to $\pm 1.7\%$. One can imagine using low energy neutrinos from future (several megawatt) spallation neutron facilities to push that goal to about $\pm 0.5\%$ (perhaps further if systematics can be controlled). At that level $m_{Z'}$ of $\mathcal{O}(1\ \text{TeV})$ would be probed and a sensitivity of ± 0.2 in S and T could be achieved. Because of the small neutrino energy, bounds on ν_e and ν_μ dipole moments of about $2 \times 10^{-11} e/m_e$ would also be possible.

The goal of 0.5% for R_2 is difficult but not unrealistic. The neutron spallation sources will exist and the detector requirements are not that demanding. Such studies are not so costly and well worth the effort.

6.2. Superbeams

Using conventional horn focused pions from intense proton sources (1 MW or more), ν_μ or $\bar{\nu}_\mu$ beams of high intensity (sometimes called superbeams) are possible. Their average energy (if spawned by protons of energy ~28 GeV) will be ~1–2 GeV. They could be used to statistically measure $\sigma(\nu_\mu e \to \nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$ to $\pm 0.5\%$ or better. A systematic limitation will be flux spectrum normalization. Achieving $\pm 0.5\%$ or better will be very challenging.

If a $\pm 0.5\%$ determination of $\sigma(\nu_\mu e \to \nu_\mu e)$ and $\sigma(\bar{\nu}_\mu e \to \bar{\nu}_\mu e)$ is possible, it will explore $m_{Z'}$ of $\mathcal{O}(1\ \text{TeV})$ and $S, T \sim \pm 0.2$. To do well on neutrino dipole moments, it must map out the y dependence of those cross-sections. A detailed study is necessary before one can confidently assess the capabilities of (conventional) superbeams for experiments other than very long baseline neutrino oscillations, an area where they are extremely well motivated [31–33].

6.3. Neutrino factories

High energy neutrinos (~40 GeV) from muon storage rings are particularly interesting for studying cross-sections (at short baselines). Their flux scales as E_μ^2 and cross-sections grow with E_ν making high statistics very straightforward. An attractive case for measuring structure functions, QCD effects, CKM elements etc has been made [5].

Even for $\overset{(-)}{\nu} e \to \overset{(-)}{\nu} e$ scattering, such a facility is potentially very powerful. With integrated luminosities of $\sim 10^{47}\ \text{cm}^{-2}$ and $\overset{(-)}{\nu} e$ cross-sections of $\sim 10^{-39}$–$10^{-40}\ \text{cm}^2$, one can envision statistics of 10^7–10^8 events at such a facility in each of the $\overset{(-)}{\nu} e$ scattering modes [34]. The inverse muon decay cross-sections $\sigma(\nu_\mu e \to \mu^-\nu_e) + \sigma(\bar{\nu}_e \to \mu\bar{\nu}_\mu)$ can be used (at least for μ^- decay neutrinos) to normalize the ν_μ and $\bar{\nu}_e$ spectra to $\pm 0.1\%$ or better, a rather unique capability. Neutrinos originating from μ^+ decays will be harder to normalize, but their relative cross-sections will carry much information.

If $\pm 0.1\%$ precision on the various $\overset{(-)}{\nu} e \to \overset{(-)}{\nu} e$ cross-sections can be achieved [5], it will represent a major advance. Within the SM, that will allow $\Delta \sin^2\theta_W \simeq \pm 0.1\%$. In terms of the 'new physics' capabilities $m_{Z'}$ of several TeV will be explored and ΔS, ΔT approaching ± 0.05 may be possible. These are impressive capabilities, particularly since they represent a small part of the intended full program. Of course, the overall cost of a high energy neutrino factory is prohibitive. It will be interesting to see if the strong physics case is enough to justify the facility.

Neutrino physics is difficult, but well worth pursuing. Its pursuit will not only push forward physics frontiers, but will demand technological innovation. Intellectual curiosity is truly the mother of invention and catalyst for the ascent of science.

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