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Report for Control of Energy Systems

Modeling and Nonlinear Control of Fuel Cell / Supercapacitor Hybrid Energy Storage System for Electric Vehicles

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CHAPTER 1

Introduction

The project is based on the simulation of paper [2]. This paper deals with the problem of controlling hybrid energy storage system (HESS) for electric vehicles. The storage system has two power sources:

- The main one: Fuel Cell (FC)
- An auxiliary one: Super Capacitor (SC)

Furthermore, the HESS also contains two converters:

- A boost converter connected with the main source (FC)
- A boost-buck converter connected with the auxiliary source.

They are both connected to the same DC bus which is connected to the traction motor through an inverter.

The two converters have to be controlled in order to meet the following requirements:

1. Tight DC bus voltage regulation
2. Perfect tracking of SC current to its reference
3. Asymptotic stability of the closed loop system

For these purposes, a nonlinear controller has been used.

The above system description can be represented as follows:

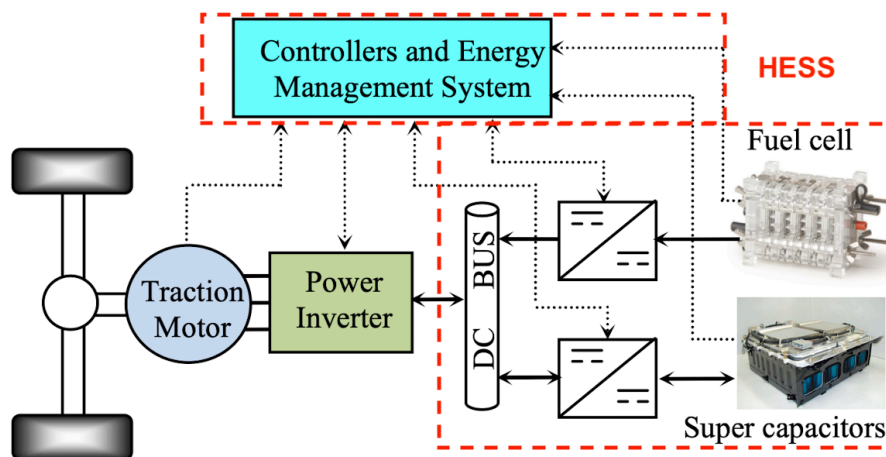


Figure 1.1: Power circuit of an hybrid vehicle.

The HESS circuit structure can be represented in this way:

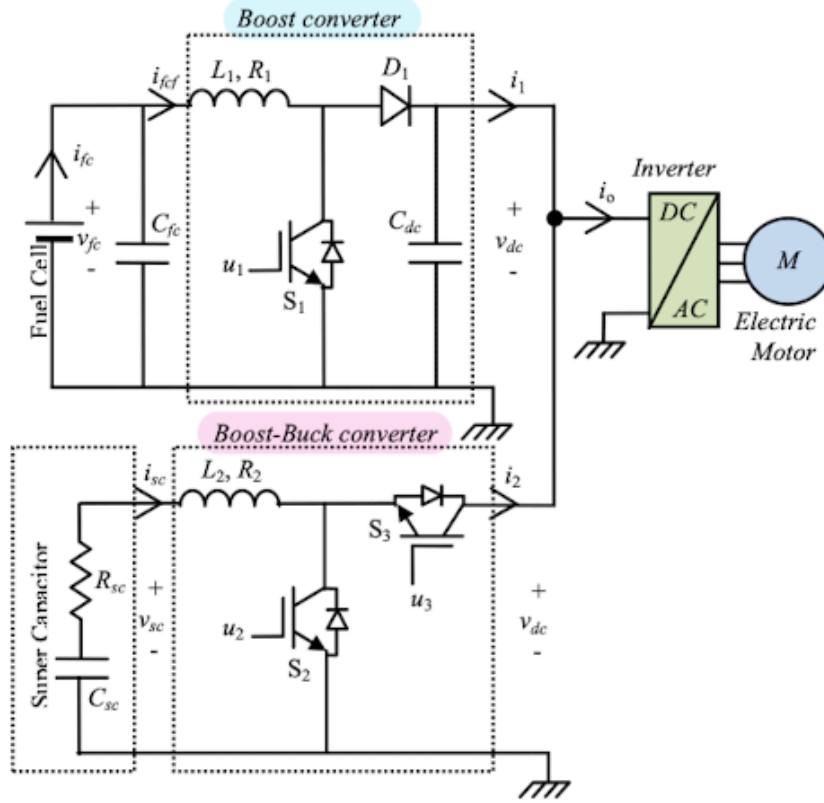


Figure 1.2: HESS circuit.

In the HESS the fuel cell has been used through a current nonreversible dc/dc boost converter, a SC bank used as an auxiliary source which is connected to the dc link through a current reversible dc/dc boost-buck converter, and the load (inverter + motor). The function of the FC is to supply mean power to the load, whereas the SC is used as a power source that supplies transient power demand and peak current required during acceleration and deceleration stages.

1.1 Fuel Cell and Boost Converter

Proton exchange membrane fuel cell (PEMFC) is an electrochemical device that combines hydrogen fuel and oxygen to produce electricity, heat and water. It has captured worldwide attention as a clean power source for various applications, including one of the most important: vehicles.

In our specific case of study, since the FC is not current reversible, the boost power converter is used to adapt the low dc voltage delivered by the FC at rated dc bus.

The power converter is composed of a high frequency inductor L_1 , an output filtering capacitor C_{dc} , a diode D_1 and a main IGBT (insulated-gate bipolar transistor) switch S_1 controlled by a binary input signal u_1 . The input capacitor C_{fc} is used to protect the FC against overvoltage power demand of the load.

1.2 SuperCapacitor and Boost-Buck Converter

A supercapacitor (SC), also called an ultracapacitor [7], is a high-capacity capacitor with a capacitance value much higher than other capacitors, but with lower voltage limits, that bridges the gap between electrolytic capacitors and rechargeable batteries. It typically stores 10 to 100 times more energy per unit volume or mass than electrolytic capacitors, can accept and deliver charge much faster than batteries, and tolerates many more charge and discharge cycles than rechargeable batteries.

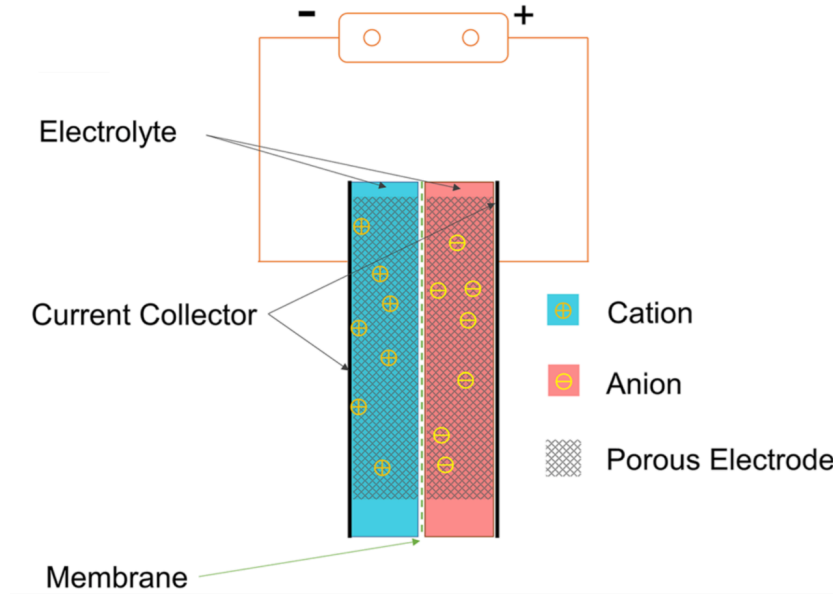


Figure 1.3: Schematic illustration of a SC.

In the HESS, the SC is connected to the dc bus by means of a two-quadrant dc/dc converter, also called boost-buck converter. The SC current, flowing across the storage device, can be positive or negative allowing energy to be transferred in both directions. The inductor L_2 is used for energy transfer and filtering. Classically, the inductor size is defined by switching frequency and current ripple. The converter is driven by means of binary input signals u_2 and u_3 applied on the gates of the two IGBTs S_2 and S_3 , respectively.

CHAPTER 2

System Modeling

2.1 Fuel Cell Modeling

To find a fuel cell model, we have referred to [6], [4] and in particular to [3].

Fuel Cell System Modeling: A fuel cell is composed of N cells in series. The fuel cell behavior is highly nonlinear and is dependent on several variables such as current density, stack temperature, membrane humidity, and reactant partial pressures. In our case we consider an ideal behavior for the operation of the Fuel Cell.

Fuel Cell Stack Voltage. A single fuel cell operating voltage can be modeled as:

$$V_{fc} = E - V_{act} - V_{ohm} - V_{conc} \quad (2.1)$$

Where E is the open circuit voltage and V_{act} , V_{ohm} , and V_{conc} present the activation loss, ohmic loss, and concentration loss, respectively.

The open circuit voltage E is expressed as:

$$E = 1.229 - 0.85 \cdot 10^{-3} (T_{fc} - 298.15) + 4.3085 \cdot 10^{-5} T_{fc} \left[\ln(p_{H_2}) + \frac{1}{2} \ln(p_{O_2}) \right] \quad (2.2)$$

where T_{fc} is the fuel cell stack temperature and P_{H_2} , P_{O_2} are the partial pressures of hydrogen and oxygen, respectively

The activation loss V_{act} , ohmic loss V_{ohm} , and concentration loss V_{conc} are expressed as follows:

1. $V_{act} = V_0 + V_a(1 - e^{-b_1 i})$ is due to the difference between the velocity of the reactions in the anode and cathode, i is the current density, V_0 (Volts) is the voltage drop at zero current density and V_a (Volts) and b_1 are constants that depend on the temperature and the oxygen partial pressure.
2. $V_{ohm} = iR_{ohm}$ is due to the electrical resistance of the electrodes, and the resistance to the flow of ions through the electrolyte. R_{ohm} represents the fuel cell internal electrical resistance.
3. $V_{conc} = i(b_3(i/i_{max}))^{b_4}$ results from the drop concentration of the reactants due to the consumption in the reaction. b_3 , b_4 and i_{max} are constants that depend on the

temperature and the reactant partial pressures. i_{max} is the current density that generates the abrupt voltage drop.

2.1.1 Issues

In our specific work we encountered some problems to find the fuel cell parameters. In particular, we know from [2] the kind of FC: Ballard FC 1020ACS, of which we have found the datasheet:



Figure 2.1: FC datasheet.

From the datasheet we can retrieve some details, but not the ones needed to correctly model the device.

For this reason, we could not analyze in simulation the Fuel Cell. This aspect is left as future work.

2.2 Boost converter modeling

The inspection of the circuit shown in Fig.1.2 leads to the following bilinear switching model:

$$\frac{di_{fcf}}{dt} = -(1 - u_1) \frac{v_{dc}}{L_1} - \frac{R_1}{L_1} i_{fcf} + \frac{v_{fc}}{L_1} \quad (2.3)$$

$$\frac{dv_{dc}}{dt} = (1 - u_1) \frac{i_{fcf}}{C_{dc}} - \frac{1}{C_{dc}} i_1 \quad (2.4)$$

where:

- i_{fcf} is the inductor input current
- i_1 is the output current of the boost converter
- v_{fc} is the FC voltage
- v_{dc} is the dc bus voltage

For the v_{fc} expression, we have already explained that we couldn't find the parameters, thus it is a missing data. Instead, for v_{sc} , we refer to [5] and the expression is:

$$\frac{dv_{sc}}{dt} = -\frac{1}{R_{sc}C_{sc}}(v_{sc} + R_{sc}i_{sc}) \quad (2.5)$$

where:

- v_{sc} is the SC voltage
- i_{sc} is the SC current

2.3 Boost-buck converter modeling

This converter operates as a boost converter or a buck converter. Indeed, in discharging mode ($i_{sc} > 0$) the converter operates as a boost converter, and in charging mode ($i_{sc} < 0$) it operates as a buck converter.

Since the goal is to enforce i_{sc} to track its reference i_{scref} (provided by the energy management system), a binary variable k is defined as follows:

$$k = \begin{cases} 1 & \text{if } i_{scref} > 0 \quad (\text{Boost mode}) \\ 0 & \text{if } i_{scref} < 0 \quad (\text{Buck mode}) \end{cases} \quad (2.6)$$

Thus, the boost-buck converter model can be obtained:

$$\frac{di_{sc}}{dt} = -[k(1 - u_2) + (1 - k)u_3] \frac{v_{dc}}{L_2} - \frac{R_2}{L_2} i_{sc} + \frac{v_{sc}}{L_2} \quad (2.7)$$

$$i_2 = [k(1 - u_2) + (1 - k)u_3] i_{sc} \quad (2.8)$$

2.4 Global system modeling

Using (2.8) and Fig.1.2, the following bilinear switched model of the global system is obtained:

$$\frac{di_{fcf}}{dt} = -(1 - u_1) \frac{v_{dc}}{L_1} - \frac{R_1}{L_1} i_{fcf} + \frac{v_{fc}}{L_1} \quad (2.9)$$

$$\frac{di_{sc}}{dt} = -u_{23} \frac{v_{dc}}{L_2} - \frac{R_2}{L_2} i_{sc} + \frac{v_{sc}}{L_2} \quad (2.10)$$

$$\frac{dv_{dc}}{dt} = (1 - u_1) \frac{i_{fcf}}{C_{dc}} + u_{23} \frac{i_{sc}}{C_{dc}} - \frac{i_o}{C_{dc}} \quad (2.11)$$

where i_o is the load current and u_{23} stands as a "virtual" control input variable of the boost-buck converter, it is defined as follows:

$$u_{23} = k(1 - u_2) + (1 - k)u_3 \quad (2.12)$$

Note that, in eq.(2.8), there are the two control inputs u_2 and u_3 which can be easily obtained from u_{23} . More details regard how to get them will be explained later. For control design purpose, it is more convenient to consider the following averaged model, obtained by averaging the above-described model over the switching periods:

$$\frac{dx_1}{dt} = -(1 - \mu_1) \frac{x_3}{L_1} - \frac{R_1}{L_1} x_1 + \frac{v_{fc}}{L_1} \quad (2.13)$$

$$\frac{dx_2}{dt} = -\mu_{23} \frac{x_3}{L_2} - \frac{R_2}{L_2} x_2 + \frac{v_{sc}}{L_2} \quad (2.14)$$

$$\frac{dx_3}{dt} = (1 - \mu_1) \frac{x_1}{C_{dc}} + \mu_{23} \frac{x_2}{C_{dc}} - \frac{i_o}{C_{dc}} \quad (2.15)$$

Where x_1 represents the average value of the current i_{fcf} ($x_1 = \langle i_{fcf} \rangle$), x_2 the average value of the SC current ($x_2 = \langle i_{sc} \rangle$), x_3 the average value of the dc bus voltage v_{dc} ($x_3 = \langle v_{dc} \rangle$). μ_1 and μ_{23} are the duty cycles, i.e. the average values of the binary control inputs u_1 and u_{23} ($\mu_1 = \langle u_1 \rangle$, $\mu_{23} = \langle u_{23} \rangle$).

By definition, the duty cycles take their values in the interval $[0, 1]$. Notice that the nonlinear model (2.13, 2.14, 2.15) is a multi-input multi-output (MIMO) system, which increases the complexity of the control problem.

2.4.1 Issues

As mentioned before, we had some problems regarding the Fuel Cell voltage (v_{fc}), so that we have decided to focus our project on the Super Capacitor dynamics.

More details about our work will be shown later. (Chp.4)

CHAPTER 3

Controller design

This chapter is devoted to show the controller, based on the model (2.13, 2.14, 2.15).
Controller objectives:

1. ensuring tight dc bus voltage regulation under load variations,
2. enforcing the SC current i_{sc} to track well its reference i_{scref} ,
3. and guaranteeing asymptotic stability of the whole energy system.

3.1 Control design

For the first control objective, the authors refer to I_{fcref} . Thus, the goal is translated into $i_{fcf} = I_{fcref}$, in this way we will obtain $v_{dc} = V_{dcref}$. It follows from power conservation considerations, also called PIPO (Power Input equals Power Output), that I_{fcref} is related to V_{dcref} by means of the following relationship:

$$I_{fcref} = \lambda \left(\frac{V_{dcref} i_o - v_{sc} I_{scref}}{v_{fc}} \right) \quad (3.1)$$

where $\lambda \geq 1$ is an ideality factor introduced to take into account all losses: switching losses in the converters and the losses in the inductances ESR (R_1 and R_2). To carry out the first control objective, the following error is defined:

$$e_1 = x_1 - I_{fcref} \quad (3.2)$$

Deriving the above equation, we can get:

$$\dot{e}_1 = - (1 - \mu_1) \frac{x_3}{L_1} - \frac{R_1 x_1}{L_1} + \frac{v_{fc}}{L_1} - \dot{I}_{fcref} \quad (3.3)$$

To make e_1 exponentially decrease, they enforce \dot{e}_1 to behave as follows:

$$\dot{e}_1 = -c_1 e_1 + e_3 \quad (3.4)$$

where $c_1 \geq 0$ is a design parameter and

$$e_3 = x_3 - x_{3d} \quad (3.5)$$

is the error between the dc bus voltage x_3 and x_{3d} is its desired value to be defined later. Comparing (3.4) and (3.3) one gets the control law of the boost converted control signal:

$$\mu_1 = 1 - \frac{L_1}{x_3} \left\{ c_1 e_1 - e_3 + \frac{v_{fc} - R_1 x_1}{L_1} - \dot{I}_{fcref} \right\} \quad (3.6)$$

In the above equation, e_3 is a damping term introduced in the control law to adjust the output response. Its dynamic will be investigated later.

The next step is to elaborate a control law for the boost-buck converter input signal μ_{23} , bearing in mind the second control objective. To this end, the following error is introduced

$$\dot{e}_2 = x_2 - I_{scref} \quad (3.7)$$

The time-derivation of the above equation, leads to:

$$\dot{e}_2 = -\frac{\mu_{23}x_3}{L_2} - \frac{R_2x_2}{L_2} + \frac{v_{sc}}{L_2} - \dot{I}_{scref} \quad (3.8)$$

The achievement of the tracking objective regarding the SC current i_{sc} amounts to enforcing the error e_2 to decreases, if possible exponentially. One possible way is to let e_2 undergo following differential equation:

$$\dot{e}_2 = -c_2 e_2 \quad (3.9)$$

where $c_2 > 0$ is a design parameter.

Finally, from (3.9) and (3.8) the control law μ_{23} can be obtained as follows:

$$\mu_{23} = \frac{L_2}{x_3} \left\{ c_2 e_2 + \frac{v_{sc} - R_2 x_2}{L_2} - \dot{I}_{scref} \right\} \quad (3.10)$$

Note that the two control laws guarantee the closed loop stability. For our purposes, the proof is not necessary, the authors did it in their article [2].

With the stability analysis, they have defined these two equations

$$\dot{e}_3 = -c_3 e_3 - e_1 \quad (3.11)$$

$$x_{3d} = \frac{1}{s} \left\{ \frac{1}{C_{dc}} [(1 - \mu_1) x_1 + \mu_{23} x_2 - i_o] + c_3 e_3 + e_1 \right\} \quad (3.12)$$

that are necessary to guarantee the stability.

CHAPTER 4

SC current control

As we reported in subsections (2.1.1) and (2.4.1) we encountered several issues regarding the fuel cell, for this reason we have focused our simulation only on the super capacitor. So that we have used the following model:

$$\frac{di_{sc}}{dt} = -\mu_{23} \frac{v_{dc}}{L_2} - \frac{R_2}{L_2} i_{sc} + \frac{v_{sc}}{L_2} \quad (4.1)$$

$$\frac{dv_{sc}}{dt} = -\frac{1}{R_{sc}C_{sc}}(v_{sc} + R_{sc}i_{sc}) \quad (4.2)$$

where the first one is the eq.(2.14) and the second one is the eq.(2.5). Thus, clearly, they are referred to the average model. Moreover, the control effect concerns only the current.

In the eq.(4.1) appears v_{dc} that is a state variable related to the fuel cell behaviours, for this reason we choose to set it to its reference value. Indeed, $v_{dc} = 400$.

The control objective defined in the paper is the tracking of the SC current. To achieve this goal, we have used the controller defined in the paper:

$$\mu_{23} = \frac{L_2}{v_{dc}} \left\{ c_2 e_2 + \frac{v_{sc} - R_2 i_{sc}}{L_2} - \dot{i}_{scref} \right\}^1 \quad (4.3)$$

4.1 Simulations setup

For simulations, we have used the parameters values defined in [2]. They are the following:

Parameter	Value
Inductance L_2	3.3mH
Resistance R_2	20 mΩ
Supercapacitor, C_{sc}	21.27 F
Supercapacitor ESR, R_{sc}	66 mΩ

Moreover, the control parameter is $c_2 = 10^3$.

¹Note that this expression is a typical gain expression ($\frac{V_{out}}{V_{in}}$). In our case v_{dc} is V_{in} and all the numerator is the output voltage of the buck-boost converter. More details about this will be explained later.

In briefly, our system is this one:

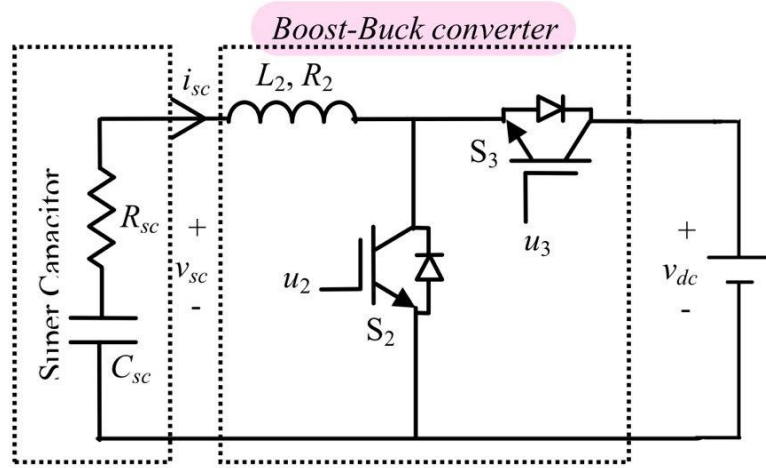


Figure 4.1: SC with buck-boost converter.

where u_2 and u_3 can be obtained from μ_{23} as follows:

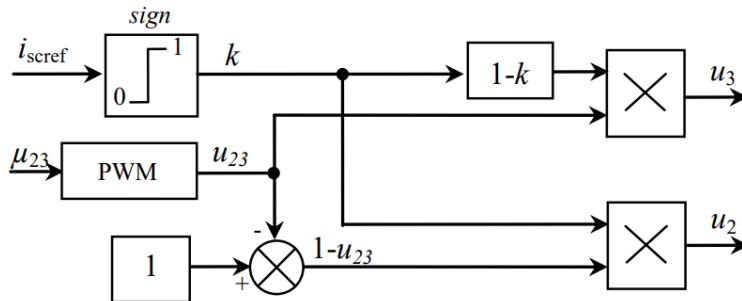


Figure 4.2: u_2 and u_3 from μ_{23} .

4.2 Figures

In this subsection, we are going to present the results, in particular we have focused on two different cases.

A list to schematize them is reported:

1. $\frac{di_{scref}}{dt}$ analytic calculation
2. $\frac{di_{scref}}{dt}$ numerical calculation

The Simulink solver has been set as follows:

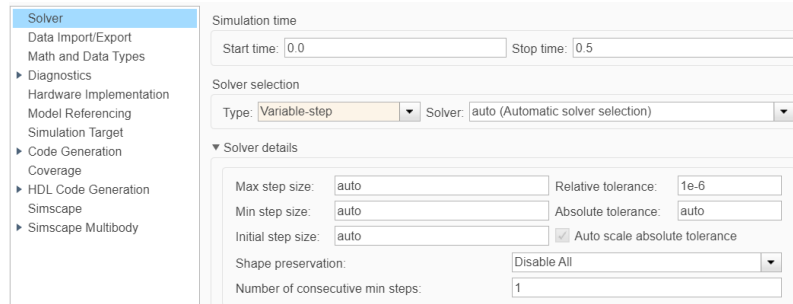


Figure 4.3: Continuous solver.

In the first case, where the $\frac{di_{scref}}{dt}$ is always equal to 0, these are the results:

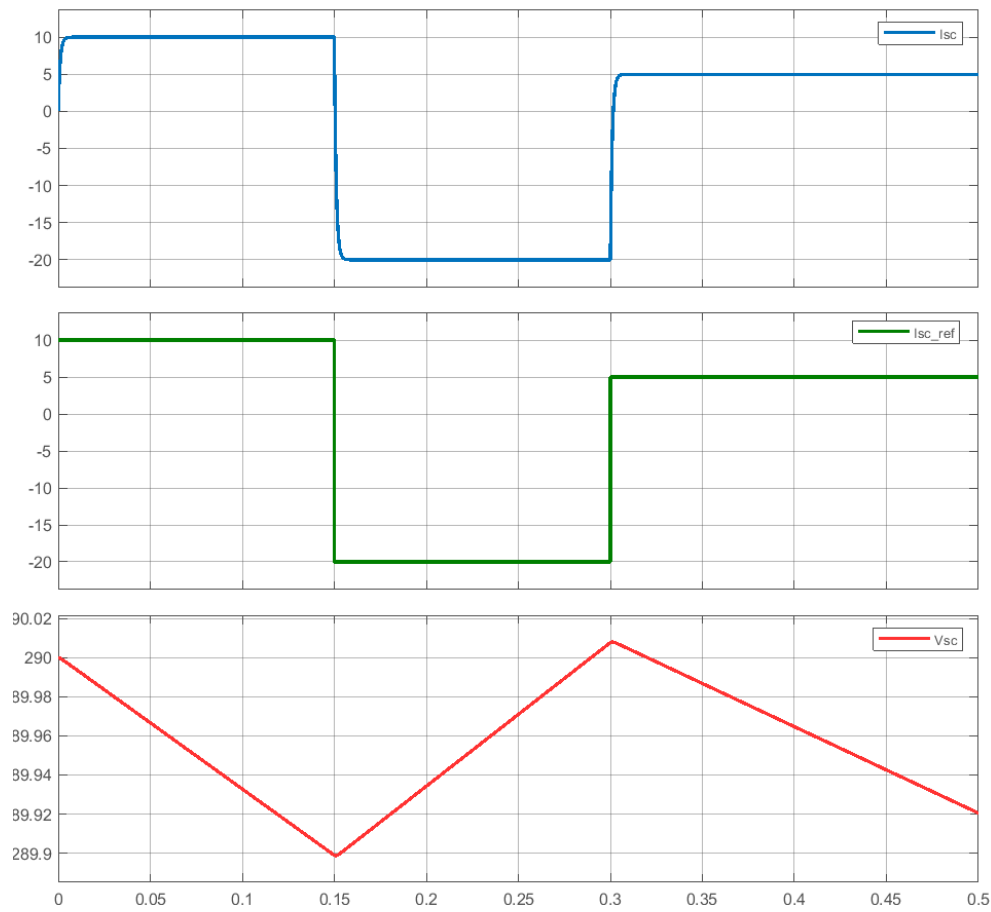


Figure 4.4: i_{sc} , i_{scref} , v_{sc} .

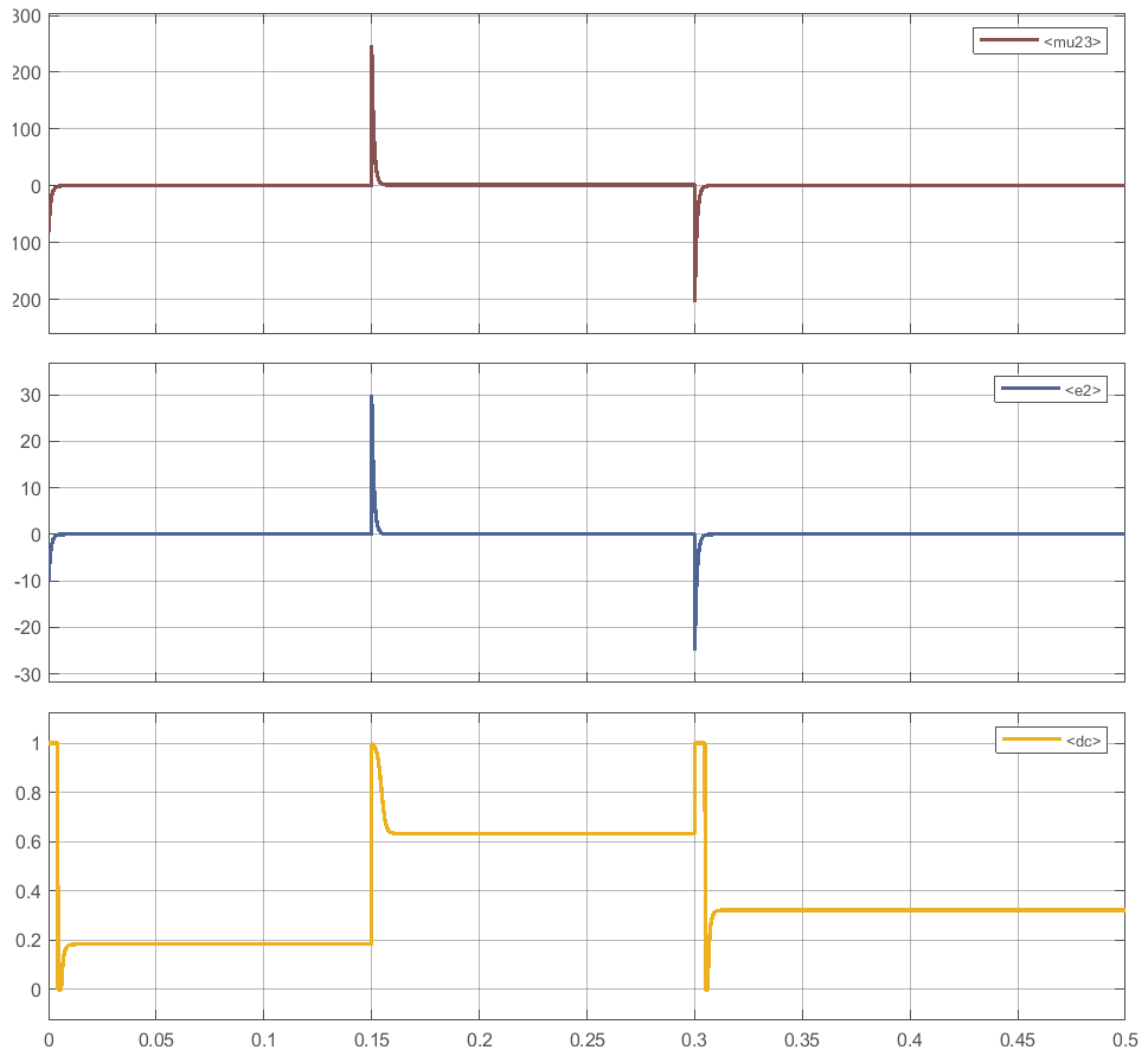


Figure 4.5: μ_{23} , e_2 , Duty cycle.

In the second case, we have used the "Derivative" block of Simulink and these are the results:

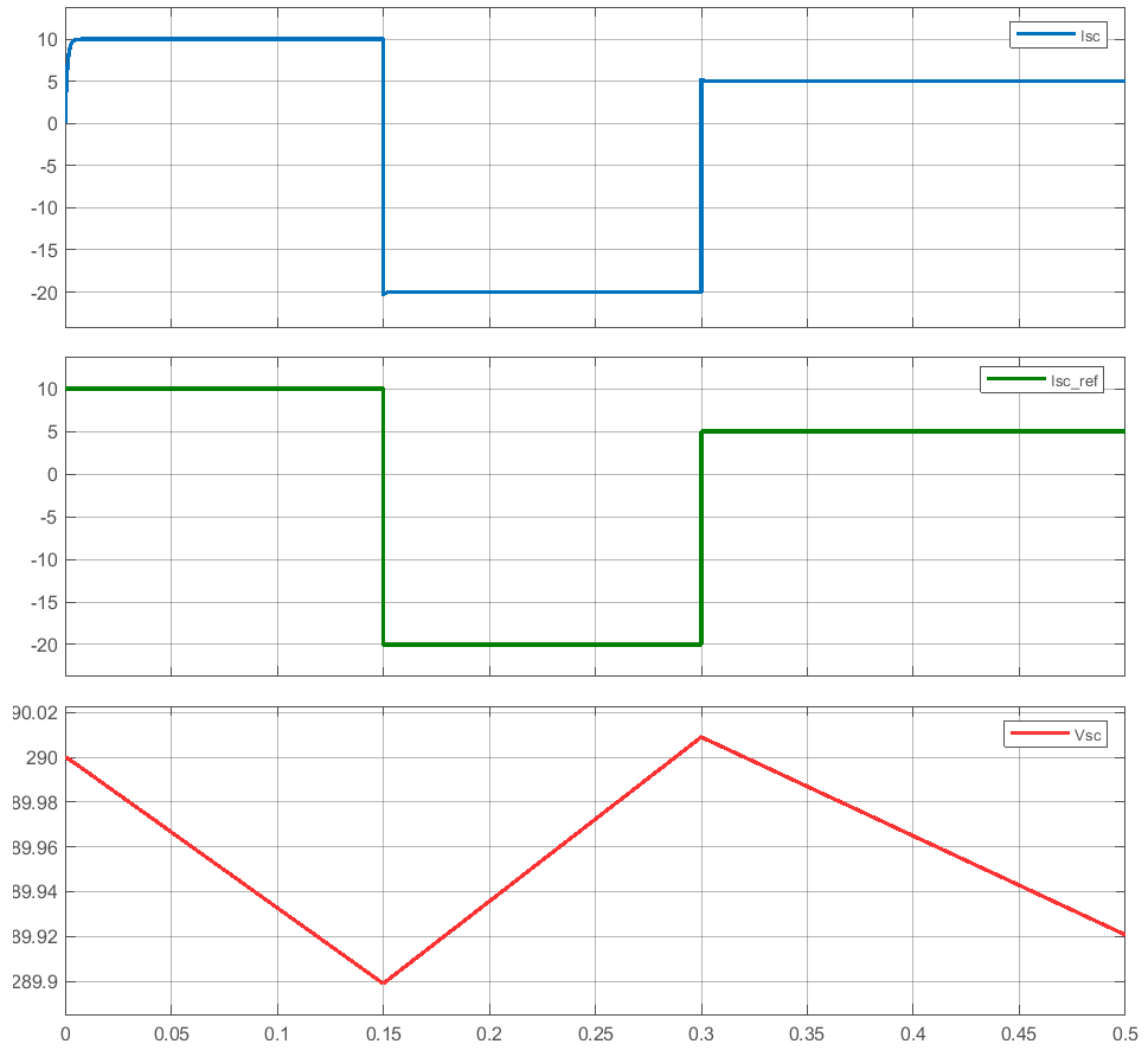


Figure 4.6: i_{sc} , i_{scref} , v_{sc} .

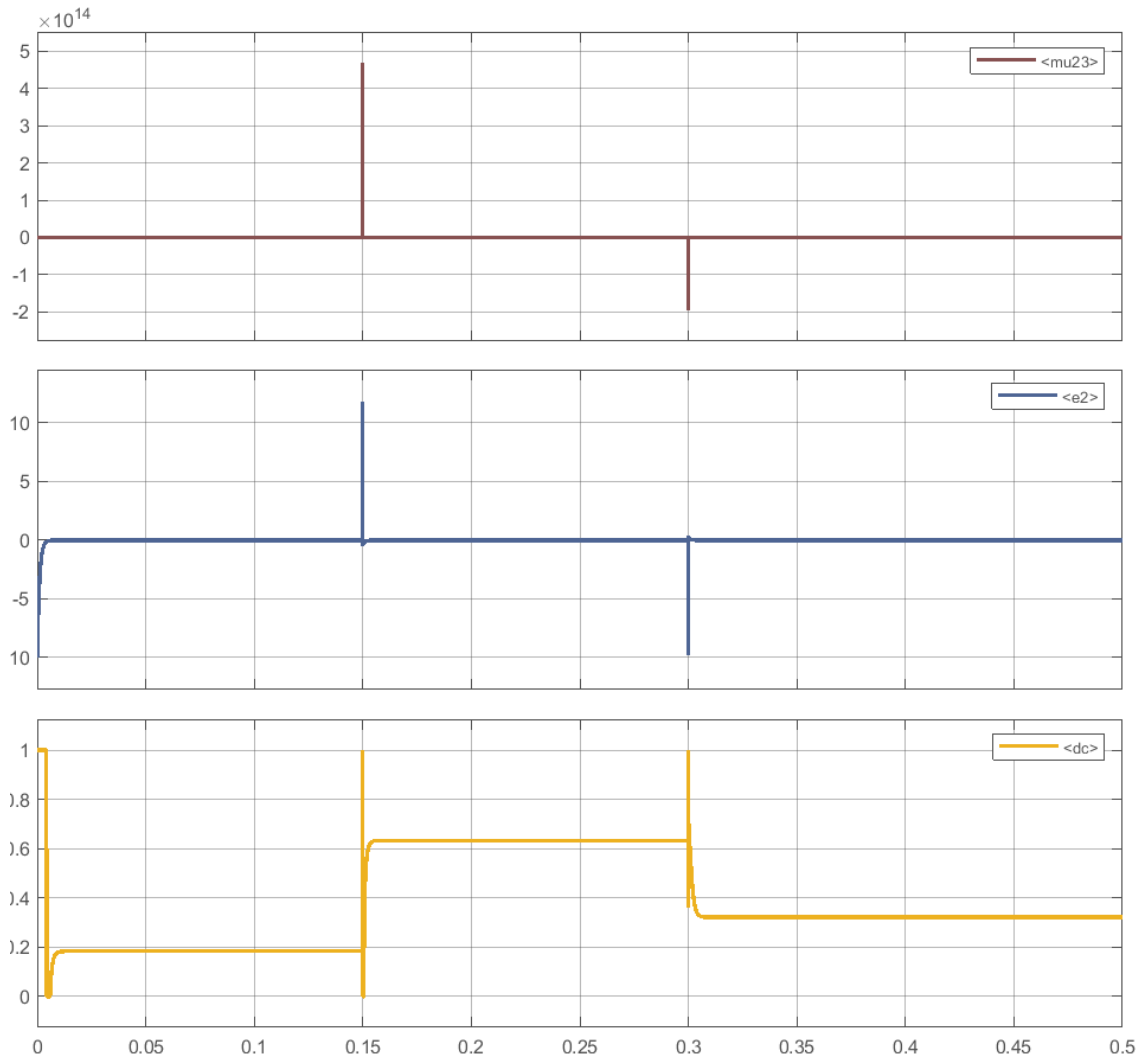


Figure 4.7: μ_{23} , e_2 , Duty cycle.

4.2.1 Comments

The above-reported graphs are satisfactory. Several aspects can be underlined:

- **First of all: the control objective**

It has been successfully achieved, the current tracks its reference.

- **μ_{23} and duty cycle**

The authors of [2] say that μ_{23} is a duty cycle, but, since we have obtained values outside the interval $[0, 1]$, we have supposed that it is the Buck-Boost converter gain. According to the equation (4.3), it has the typical expression of a gain. Furthermore, from [1], it is well known that from a Buck-Boost gain, we can retrieve the duty cycle: $dc = \mu_{23}/(1 + \mu_{23})$.

- **Spikes**

As we can see from the error and control signal plots, we have spikes due to instantaneous changes in the i_{sc} current reference (chosen by us), when it drops, the error is the result of the state variable (i_{sc}) value minus the desired value (i_{scref}) and it becomes very high. As a consequence, the control signal, that depends on the reference derivative, has a peak too. This is in accord with both the cases we have shown, indeed, when the derivative is calculated by Matlab, there are larger peaks (Fig.4.7) with respect to the other case in which the derivative is always 0 (Fig. 4.5).

4.3 Conclusions

It has been a very useful project, for several reasons. Just to write some of them, we had the possibility to work in a different way from a canonical exam. In addition, we had the chance to train ourselves for finding various solutions for various problems.

Moreover, this project makes us aware of the practical issues that come up when you have to deal with research articles and stuff like that.

Even though our reference article contains some hidden parts, like the fuel cell parameters or the duty cycle calculation, this can be considered as a positive thing from the learning point of view because it gives us the possibility to touch the complexity of these fields of study.

Lastly, we would like to encourage future students to continue this project, perhaps finding the right missing parameters. It would be great, and we will be happy to help them if they need.

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