Below is a **step-by-step, scenario-style illustration** of how **AgACI** operates, without relying on an actual image. Imagine a **table** of "experts" and their interval predictions, and then how the aggregator merges them at each time step.

A Fictional Day-by-Day Example (ASCII/Step-by-Step Style)

Suppose we have **3 experts**, each corresponding to a **different** γ_k ($\gamma_1 = 0.01,\ \gamma_2 = 0.03,\ \gamma_3 = 0.07$). We have a time series $\{x_t, y_t\}$ for $t = 1, 2, \ldots$. Each day t:

1. **Each Expert (i.e., each** γ_k**)** runs ACI.
   - They have *their own* adapted $\alpha_{t,k}$.
   - They produce *their own* prediction interval $\hat{C}_{t,k}$ = $[\hat{b}^{(\ell)}_{t,k},\ \hat{b}^{(u)}_{t,k}]$.
2. **We Observe** y_t after the intervals are formed.
3. **We Compute a Loss** (pinball or similar) for each expert k to see how well their interval performed (did it miss? was it too large, etc.).
4. **We Combine** these intervals by weighting each expert's lower/upper bound using an online "expert aggregator."

ASCII Representation

Let's imagine a *table* at each day t:

```
 Day t | Expert k=1    | Expert k=2    | Expert k=3    | Aggregator
       | (γ=0.01)      | (γ=0.03)      | (γ=0.07)      | (Combined)
---------------------------------------------------------------
Lower  | L_{t,1}       | L_{t,2}       | L_{t,3}       | Weighted average of L_{t,k}
Upper  | U_{t,1}       | U_{t,2}       | U_{t,3}       | Weighted average of U_{t,k}
(We see y_t, compute losses, update weights, produce final [L, U])
```

More Detail:

1. **Before** day t:
   - Each expert k has a *running* coverage error record that yields a current $\alpha_{t,k}$.
   - They produce the day's interval $\hat{C}_{t,k} = [\hat{b}^{(\ell)}_{t,k}, \hat{b}^{(u)}_{t,k}]$.
2. **Day** t **Observes** y_t:
   - Suppose the aggregator sees these intervals:
     - Expert 1: $\hat{C}_{t,1} = [2.5,\ 6.0]$
     - Expert 2: $\hat{C}_{t,2} = [1.2,\ 8.1]$
     - Expert 3: $\hat{C}_{t,3} = [3.4,\ 4.9]$
   - Then the actual label is $y_t = 5.5$.
   - Some intervals might be infinite if a certain γ_k had $\alpha_{t,k} < 0$. In that case, say Expert 3 gives an upper bound of ∞, we clamp it to some large M.
3. **Loss Calculation**:
   - Each expert gets a pinball loss for its lower bound + upper bound w.r.t. y_t.
   - For example, if Expert 3's upper was only 4.9, but the true label is 5.5, Expert 3 missed coverage on the upper side → incurring a *large* pinball loss for the upper bound.
4. **Weights Update**:

- The aggregator keeps track of each expert's cumulative performance from previous days.
- If an expert's interval is "just right" (not too big, not missing coverage), it receives a smaller loss → the aggregator will *increase* that expert's weight.
- If an expert is frequently infinite or missing coverage, the aggregator *decreases* that expert's weight.

5. **Aggregated Bounds**:
   - Let's say the aggregator's new weights for each expert's **lower** side are $\omega_{t,1}^{(\ell)} = 0.2,\ \omega_{t,2}^{(\ell)} = 0.5,\ \omega_{t,3}^{(\ell)} = 0.3.$
   - Then the aggregator's final **lower** bound is

$$\tilde{b}_t^{(\ell)} = \frac{0.2 \times 2.5 + 0.5 \times 1.2 + 0.3 \times 3.4}{0.2 + 0.5 + 0.3} \approx 2.0.$$

   - Similarly for the upper side with different weights.
   - So day t's final aggregator interval might be $\tilde{C}_t = [2.0, 6.7]$, a weighted average of the three experts' bounds.
6. **Day $t+1$**:
   - Each expert γ_k now updates $\alpha_{t+1,k}$ based on whether it personally missed coverage at day t.
   - The aggregator also updates each $\omega_{t+1,k}^{(\ell,u)}$ (or uses a standard formula for online weighting) based on how each bound performed.
   - Next day, we do it all again.

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2. Summary of the Process

At each day t:

1. **Multiple ACIs**: We run K different ACI procedures, each with a different γ_k.
2. **Each Expert's Interval**: We gather $\hat{C}_{t,k}$.
3. **Clamping** if infinite: If an expert's interval is infinite, we clamp it to a max size M.
4. **Pinball Loss**: The aggregator checks how each bound did on day t.
5. **Weights**: The aggregator updates each expert's weighting for lower vs. upper bounds.
6. **Aggregate**: A final interval emerges as a weighted average of lower and upper bounds from all experts.

Hence, you get a single final "aggregated" interval at each time t, pulling from whichever γ_k is performing well, in a smoother way than picking a single best γ_k or flipping among them.