$\boxed{N 1}$

$\boxed{a}$ $\tilde{\theta}_1 = 2\overline{x}$

$\tilde{\theta}_2 = x_{min}$

$\tilde{\theta}_3 = x_{max}$

$\theta_4 = x_1 + \dfrac{\sum_{k=2}^{n} x_k}{n-1}$

$\xi \sim R(0, \theta)$

$\theta = const$

$M\xi = \dfrac{\theta}{2}$

$M\xi^2 = \dfrac{\theta^2}{3}$

$D\xi = \dfrac{\theta^2}{12}$

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① $\tilde{\theta}_1 = 2\overline{x}$ ✓

- Несмещённость; x_i – независ. случ. величины; $x_i \sim R(0, \theta)$

$$\forall \theta > 0 \quad M\tilde{\theta}_1 = M\left(2 \cdot \frac{1}{n} \sum x_i\right) =$$

$$= \frac{2}{n} \sum M x_i = \frac{2}{n} \cdot n \cdot M\xi = \theta \Rightarrow$$

$\Rightarrow$ $\tilde{\theta}_1$ несмещённая

- Состоятельность ✓

$$D\tilde{\theta}_1 = D\left(2 \cdot \frac{1}{n} \sum x_i\right) = \frac{4}{n^2} \sum D x_i =$$

$$= \frac{4}{n^2} \cdot n \cdot D\xi = \frac{\theta^2}{3n} \xrightarrow[n \to \infty]{} 0 \text{ при } \forall \theta > 0$$

Достат. условие выполнены $\Rightarrow$

$\Rightarrow$ $\tilde{\theta}_1$ состоятельна

② $\tilde{\Theta}_2 = X_{min}$

$\xi \sim F(x)$

?почему $\xi_{min} \sim 1 - \underbrace{(1-F(x))^n}_{\Phi(x)}$

?поч. $\Phi(x)$

$\varphi(x) = \Phi'(x) = n(1-F(x))^{n-1} \cdot F'(x) =$

$= n\left(1 - \frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta} \cdot \{[0, \Theta]\}$

$M x_{min} = \ldots = \frac{\Theta}{n+1} \Rightarrow$ смещ.

Поправим $\tilde{\Theta}_2' = (n+1) X_{min}$

$M \tilde{\Theta}_2' = (n+1) M x_{min} = \Theta \Rightarrow$ несмещён.

Применим дост. условие

$D \tilde{\Theta}_2' = D((n+1) X_{min}) = (n+1)^2 D x_{min}$

$M x_{min}^2 = \ldots = n\Theta^2 \left(\frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}\right)$

$D x_{min} = \ldots = \frac{n\Theta^2}{(n+1)^2(n+2)}$

$D \tilde{\Theta}_2' \underset{n\to\infty}{\not\to} 0$

$\Rightarrow$ дост. усл. не работ.

Пытаемся по определению

$\forall \Theta \quad \forall \varepsilon > 0 \quad P(|\tilde{\Theta}_2^{?} - \Theta| \geq \varepsilon) \xrightarrow[n \to \infty]{} 0?$

$P(|\tilde{\Theta}_2^{?} - \Theta| \geq \varepsilon) \geq$

$\geq P(\tilde{\Theta}_2^{?} \geq \Theta + \varepsilon) =$

$= P((n+1) X_{min} \geq \Theta + \varepsilon) =$

$= P\left(X_{min} \geq \frac{\Theta + \varepsilon}{n+1}\right) =$

$= 1 - \Phi\left(\frac{\Theta + \varepsilon}{n+1}\right) = \ldots$

$\ldots = \left(1 - \frac{\Theta + \varepsilon}{\Theta (n+1)}\right)^n \xrightarrow[n \to \infty]{} e^{\left(-\frac{\Theta + \varepsilon}{\Theta}\right)} > 0$

$\Longrightarrow \tilde{\Theta}_2^{?}$ не явл. сост.

• А может $\hat{\Theta}_2 = x_{min}$ будет сост.? ? this part

$P((x_{min} - \Theta) \geq \varepsilon) = P(x_{min} \leq \Theta - \varepsilon) =$

$= \Phi(\Theta - \varepsilon) = \ldots =$

$= 1 - (1 - F(\Theta - \varepsilon))^n = 1 - \left(1 - \frac{\Theta - \varepsilon}{\Theta}\right)^n =$

$= 1 - \left(\frac{\varepsilon}{\Theta}\right)^n \xrightarrow[n \to \infty]{} 1$

$\frac{\varepsilon}{\Theta} < 1$

Значит, x_{min} не явл. сост.

③ $\tilde{\hat{\Theta}}_3 = x_{max}$

$M\tilde{\hat{\Theta}}_3 = Mx_{max}$

? поч такая ф-ла

$x_{max} \sim \underbrace{(F(x))^n}_{\Psi(x)}$

$$\varphi(x) = \Psi'(x) = n \cdot \left(\frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta} \{(0, \Theta)\}$$

Считаем числовые характеристики

$Mx_{max} = \ldots = \frac{n}{n+1}\Theta \Rightarrow$ ~~смещ~~.

$\Rightarrow \tilde{\hat{\Theta}}_3$ смещ.

Возьмём $\tilde{\hat{\Theta}}_3' = \left(\frac{n+1}{n}\right) \cdot x_{max}$

$$M\tilde{\hat{\Theta}}_3' = \left(\frac{n+1}{n}\right) \cdot Mx_{max} = \Theta \Rightarrow \tilde{\hat{\Theta}}_3' \text{ несмещ.}$$

$$D\tilde{\hat{\Theta}}_3' = \left(\frac{n+1}{n}\right)^2 Dx_{max}$$

$$Mx_{max}^2 = \int_0^{\Theta} x^2 \cdot n \cdot \left(\frac{x}{\Theta}\right)^{n-1} \cdot \frac{1}{\Theta}\, dx =$$

$$= \frac{n}{\Theta^n} \cdot \frac{\Theta^{n+2}}{n+2} = \Theta^2 \cdot \frac{n}{n+2}$$

$$Dx_{max} = \ldots = \frac{n\Theta^2}{(n+2)(n+1)^2}$$

$$D\tilde{\hat{\Theta}}_3' = \left(\frac{n+1}{n}\right)^2 \cdot Dx_{max} = \frac{\Theta^2}{n(n+2)} \xrightarrow[n \to \infty]{} 0$$

$\Rightarrow \tilde{\hat{\Theta}}_3'$ состоят.

(1) $\tilde{\theta}_n = x_1 + \frac{1}{n-1} \sum_{i=2}^{n} x_i$

$M\tilde{\theta}_n = Mx_1 + \frac{1}{n-1} \cdot \sum_{i=2}^{n} Mx_i =$

$= M\xi + \frac{1}{n+1} \sum_{i=2}^{n} M\xi = \frac{\theta}{2} + \frac{\theta}{2} = \theta \Rightarrow$

$\Rightarrow$ $\tilde{\theta}_n$ несмещ.

$D\tilde{\theta}_n = Dx_1 + \frac{1}{(n-1)^2} \sum_{i=2}^{n} Dx_i =$

$= \frac{\theta^2}{12} + \frac{1}{n-1} \cdot \frac{\theta^2}{12} \underset{n\to\infty}{\not\longrightarrow} 0$

не выполн. ся дост. усл. $\rightarrow$

$\rightarrow$ делаем по опр.

$\tilde{\theta}_n \xrightarrow{P} \theta$

$x_1 + \frac{1}{n+1} \sum_{i=2}^{n} x_i \xrightarrow{\theta}$ . . . .

?

$x_1 \xrightarrow{P} \xi$

ЗБЧ Хинчина:

ξ_n — независимы;

одинаково распределённые,

$\exists M\xi_n$

$\rightarrow \frac{1}{n}\sum_{i=1}^{n}\xi_n \xrightarrow{P} M\xi_1$

?

$x_1 + \frac{1}{n+1}\sum_{i=2}^{n} x_i \xrightarrow{P} \xi + \frac{\theta}{2}$

$\Downarrow$

$M\xi = \frac{\theta}{2}$

$\hat{\theta}_4$ не явл. сост.

$\boxed{б}$

$\tilde{\hat{\theta}}_3 = \frac{n+1}{n} x_{max}$

$\tilde{\hat{\theta}}_1 = 2\bar{x}$

две сост. оценки

$D\hat{\theta}_1 = \frac{\theta^2}{3n}$

$D\tilde{\hat{\theta}}_3 = \frac{\theta^2}{(n+2)n}$

сравнить эффективности

$D\hat{\theta}_1 \vee D\hat{\theta}_3$

$\frac{1}{3n} \vee \frac{1}{n(n+2)}$

$n^2+2n > 3n$

$n > 1$

$\Rightarrow \tilde{\hat{\theta}}_3$ более эффективна, чем $\tilde{\hat{\theta}}_1$

? см опр. 2)

теор: $\geqslant \backslash > - ?$