HW w8-1

Name： 费杰 Student ID: 2019280373

1. Solving the following linear program using simplex method

$$\max Z = 3x_1 + 6x_2 + 2x_3$$

$$\begin{cases} 3x_1 + 4x_2 + x_3 \le 2 \\ x_1 + 3x_2 + 2x_3 \le 1 \\ x_1 \ge 0, x_2 \ge 0, x_3 \ge 0 \end{cases}$$

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1. $L = 3x_1 + 6x_2 + 2x_3$

$$\begin{cases} 3x_1 + 4x_2 + x_3 + x_4 = 2 \\ x_1 + 3x_2 + 2x_3 + x_5 = 1 \\ x_i \ge 0, \ i = \overline{1,5} \end{cases}$$

Basis	B	x_1	x_2	x_3	x_4	x_5
x_4	2	3	4	1	1	0
x_5	1	1	3	2	0	1
$L(x)$	0	-3	(-6)	-2	0	0

We have negative coef. in 3rd (row) line (non optimal)

⇒ take 3rd column (|-6| - max)

min (2/4, 1/3) = 1/3 ⇒ take 2nd row as leading.

Get new simplex table, take x_2 instead of x_5 into Basis

Basis	B	x_1	x_2	x_3	x_4	x_5	min
x_4	$2 - \frac{4}{3} = \frac{2}{3}$	$3 - \frac{4}{3} = \frac{2}{3}$	$4 - \frac{3 \cdot 4}{3} = 0$	$1 - \frac{2 \cdot 4}{3} = -\frac{5}{3}$	$1 - \frac{0 \cdot 4}{3} = 1$	$0 - \frac{1 \cdot 4}{3} = -\frac{4}{3}$	2/5
x_5	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{3}{3} = 1$	$\frac{2}{3}$	$\frac{0}{3} = 0$	$\frac{1}{3}$	1
$L(x)$	$0 - \frac{-6}{3} = 2$	$-3 - \frac{1 \cdot -6}{3} = -1$	$-6 - \frac{3 \cdot -6}{3} = 0$	$-2 - \frac{2 \cdot -6}{3} = 2$	$0 - \frac{0 \cdot -6}{3} = 0$	$0 - \frac{1 \cdot -6}{3} = 2$	

Again, neg. coef. in 3rd (row) line (non optimal)

⇒ take 2nd column (|-1| - max)

$\min\left(\frac{2}{3}/\frac{5}{3}, \frac{1}{3}/\frac{1}{3}\right) = \frac{2}{5} \Rightarrow 1^{st}$ row is leading

Count new simplex table, now x_1 in basis instead of x_4

Basis	b	x_1	x_2	x_3	x_4	x_5
x_1	$\frac{2}{3}/\frac{5}{3} = \frac{2}{5}$	$\frac{5}{3}/\frac{5}{3} = 1$	$\frac{0}{5/3} = 0$	$-\frac{5}{3}/\frac{5}{3} = -1$	$\frac{1}{5} = \frac{3}{5}$	$-\frac{4}{3}/\frac{5}{3} = -\frac{4}{5}$
x_2	$\frac{1}{3} - \frac{\frac{2}{3}\cdot\frac{1}{3}}{5/3} = \frac{1}{5}$	$\frac{1}{3} - \frac{\frac{5}{3}\cdot\frac{1}{3}}{5/3} = 0$	$1 - \frac{0\cdot\frac{1}{3}}{5/3} = 1$	$\frac{4}{3} - \frac{-\frac{5}{3}\cdot\frac{1}{3}}{5/3} = 1$	$0 - \frac{\frac{1}{3}\cdot\frac{1}{5}}{5/3} = -\frac{1}{5}$	$\frac{1}{3} - \frac{-\frac{4}{3}\cdot\frac{1}{3}}{5/3} = \frac{3}{5}$
$L(X)$	$2 - \frac{\frac{2}{3}\cdot -1}{5/3} = \frac{12}{5}$	$-1 - \frac{-\frac{5}{3}\cdot -1}{5/3} = 0$	$0 - \frac{0\cdot -1}{5/3} = 0$	$2 - \left(\frac{-\frac{5}{3}\cdot -1}{5/3}\right) = 1$	$0 - \frac{1\cdot -1}{5/3} = \frac{3}{5}$	$2 - \frac{-\frac{4}{3}\cdot -1}{5/3} = \frac{6}{5}$

no negative values in 3rd row

$\Rightarrow$ answer $x_1 = \frac{2}{5}$, $x_2 = \frac{1}{5}$, $x_3 = 0$

$\max Z = 3\cdot\frac{2}{5} + 6\cdot\frac{1}{5} + 2\cdot 0 = \frac{12}{5}$