Combinatorics HW 5-2

Student ID: 2019280373 Name: 费杰 Score:

1. Integer composition: Integer 5 is partitioned into orderly partitions which are made up by numbers 1,2,3,4. Such as (1+1+3, or 1+3+1 or 2+3, 4+1,....) How many different ways are there?
   We can make 2,3,4,5-splittings. (Can't make 1-split because 4<5).
   N's orderly r-splitting numbers is C_{n-1}^{r-1}.
   The answer of the task is $C_4^1 + C_4^2 + C_4^3 + C_4^4 = 15$

2. Integer partition: How many ways to partition n into several numbers that the order between numbers is ignored. Please write the corresponding generating function.

   Let the numbers given be: $a_1, \ldots, a_k$. Then the generating function of partitioning n into the sum of these numbers is:
   $$\prod_{i=1}^{k} (1 + x^{a_i} + x^{2a_i} + \ldots + x^{\lfloor \frac{n}{a_i} \rfloor})$$
   The answer if the coefficient of x^n in the expanded equation.

3. Provide proof that the partition number for integer n using **different odd numbers** (ordering is ignored), equals to the partition number of n being partitioned into the self-conjugated Ferrers Diagrams. (1st row exchanged with 1st column, 2nd row exchanged with 2nd column, …, as image is rotated by the dotted line as axis shown in slices; is still Ferrers diagram. 2 Ferrers diagrams are known as a pair of conjugated Ferrers diagram. If both the conjugated Ferrers Diagram and it original diagram are the same, the diagram is called self-conjugated.)

MEMO NO: Combinatorics HW5-2

DATE: / /

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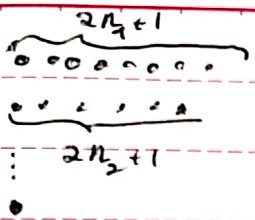
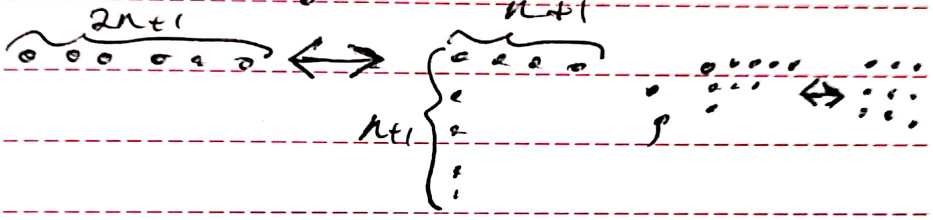
3. 

For the partition using different odd numbers, every line can be represented as a "corner":



Starting from the biggest odd number, we can form a self-conjucated Ferrers Diagram from a partition with different odd numbers, folding it one by one.

For every partition with different odd numbers exist only one self-conjucated diagram. Same for the reverse projection. It means that the partition numbers are the same.