Combinatorics HW w6-1

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1. Please prove the following equation of fibonacci sequence F_i:

$$F_1 + F_3 + F_5 + \boxed{?} + F_{2n-1} = F_{2n}$$

Prove by induction ($\sum_{i=1}^{n} F_{2i-1} = F_{2n}$)

Base: $k=1$: $F_1 = F_2$ $(F_1 = F_2 = 1)$

$k=2$: $F_1 + F_3 = F_2 + F_3 = F_4$

Let $\sum_{i=1}^{n} F_{2i-1} = F_{2n}$ be correct for $n=k-1$.

For $n=k$: $\sum_{i=1}^{k} F_{2i-1} = \sum_{i=1}^{k-1} F_{2i-1} + F_{2k-1} =$

$= F_{2k-2} + F_{2k-1} = F_{2k}$

Means that $\sum_{i=1}^{n} F_{2i-1} = F_{2n}$

2. Please provide the corresponding characteristic equations for the following recurrence relation:

$$a_n = 2a_{n-1} + 4a_{n-2} - 5a_{n-3}$$

$x^3 - 2x^2 - 4x + 5 = 0$

3. Solve the recurrence relation $h_n=2h_{n-1}+8h_{n-2}$,n≥2, $h_1=1$, $h_2=10$

3. Solve $h_n = 2h_{n-1} + 8h_{n-2}$, $n \geq 2$, $h_1 = 1$
$h_2 = 10$

Characteristic equation: $\cancel{x^2 - 8x}$ $x^2 - 2x - 8 = 0$
roots are 4 and -2

$h_n = A(4)^n + B(-2)^n$

$h_1 = 1$; $h_2 = 10$

$h_1 = 1$: $4A - 2B = 1$ $\Rightarrow$ $A = 1/2$, $B = 1/2$

$h_2 = 10$: $16A + 4B = 10$

$h_n = 1/2\,(4^n) + 1/2\,(-2)^n$