HW w7-1

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1. Using 5 numbers 1， 2， 3， 4, 5 to fill in 1x n grids, each grid is filled with one digit. If there are odd number of grids that have 1 written on them, and an even number of grids with 2, please write the corresponding exponential generating function and figure out how many arrangements there for 1*6 grids?____

① $G(x)=\left(x+\frac{x^3}{3!}+\frac{x^5}{5!}\right)\left(1+\frac{x^2}{2!}+\frac{x^4}{4!}+\frac{x^6}{6!}\right)\left(\sum_{n=0}^{6}\frac{x^n}{n!}\right)^3$

Since we need to find the coef. of x^6, all of the terms can be put into $O(x^7)$;

since $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$, $e^{-x}=\sum_{n=0}^{\infty}(-1)^n\cdot\frac{x^n}{n!}$, we get:

$G(x)=\frac{1}{2}(e^x-e^{-x}+O(x^7))\frac{1}{2}(e^x+e^{-x}+O(x^7))(e^x+O(x^7))^3=$

$=\frac{1}{4}(e^{2x}-e^{-2x}+O(x^7))(e^{3x}+O(x^7))=$

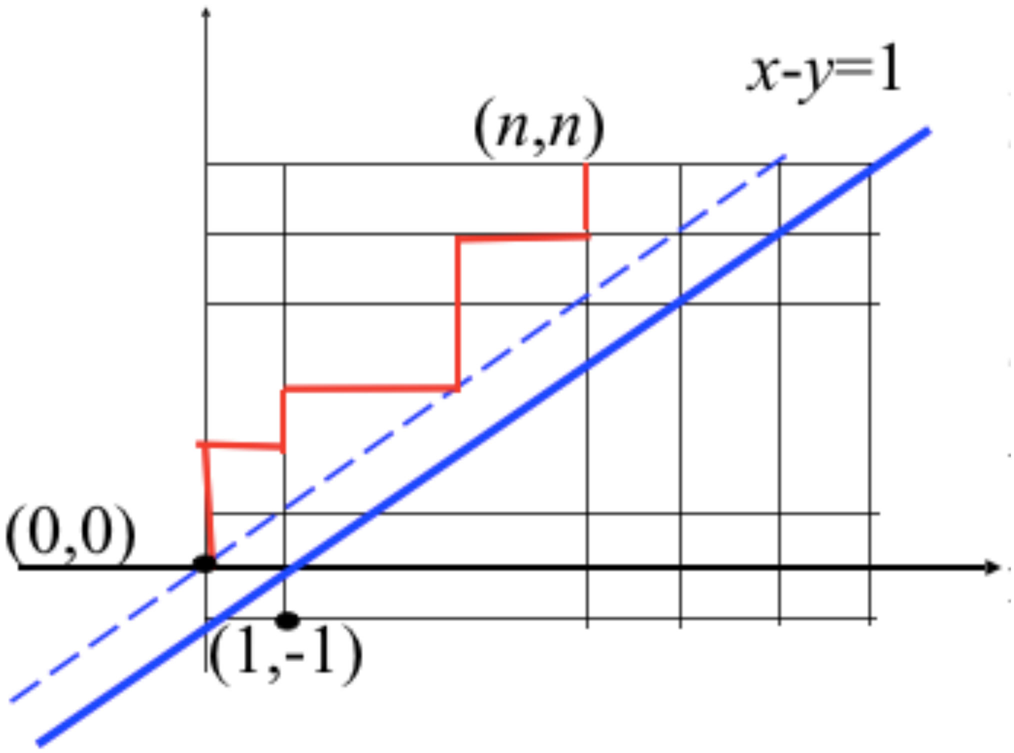
$=\frac{1}{4}(e^{5x}+e^x+O(x^7))=\frac{1}{4}\left(\sum_{n=0}^{6}\frac{x^n}{n!}(5^n-1)+O(x^7)\right)$

here $a_n=\frac{5^n-1}{4}$, $n=0,..,6$; $a_6=\frac{5^6-1}{4}=3906$

* here $O(x^n)$ does not mean that $\frac{O(x^n)}{x^n}\xrightarrow[n\to\infty]{}1$ (or ∞), but means that this function is a function $O(x^n)=\sum_{i=n}^{\infty}a_n x^n$.

2. There are six people in a library queuing up, three of them want to return the book “Interviewing Skills”, and 3 of them want to borrow the same book. If at the beginning, all the books of “Interviewing Skills” are out of stock in the library, how many ways can these people line up?____

Dyck Path:



Borrowing books (Units) is horizontal axis, returning boos (units) is vertical. The problem is finding the lattice path not touching the blue line.

The number of pathes is: $\frac{1}{n+1}C_{2n}^{n}$

For n = 3: $\frac{1}{4}C_6^3$.

Since the number of people is 3 returning and 3 borrowing, the total number of arrangements is $\frac{1}{4}C_6^3 * 3! * 3! = 180$

Answer: 180