Combinatorics HW 1.1

Student ID: 2019280373 Name: 费杰 Fedor Ivachev Score:

1. A large tournament has 569 entrants in total. If it is a single elimination tournament, how many matches have to be played out before the champion can be decided? (Please calculate the precise value)

Single elimination means that every match one of the participants is eliminated from the tournament and does not participate in the next matches. Assuming that there are 569 entrants in the tournament and only one champion left, it means that 568 participants will be eliminated. At the same time the participant may be eliminated only in one match. It means that to eliminate 568 participants, 568 matches have to be played.
Answer: 568.

2. The figure below shows a partial 4X4 matrix， is there some way of filling up the rest of the omitted entries to produce a magic square of size 4?

$$\begin{bmatrix} 2 & 3 & & \\ 4 & & & \\ & & & \\ & & & \end{bmatrix}$$

Let's put variables in several cells:

2	3	a	b
4		f	
c	e		
d			

Magic sum is 4 * (4^2+1) / 2 = 34.
What we know is that:
2+3+a+b=34, 2+4+c+d=34
Means that a+b=29 and c+d=28
Assuming that a,b,c,d are all different integers and each of them is >=1 and <= 16, we can get only four variants:

12	c	c	d	d
14	a	b	a	b
15	b	a	b	a
16	d	d	c	c

_____

*Other variants are not correct, because:

1. If a <= 12, then b >= 17 (coll)
2. If a == 13, then b = 16. If c <= 11, then d >= 17 (coll); if c = 12, then d = 16 (coll), if c == 14, then d = 14 (coll); if c == 15 then d == 13 (coll)
3. If a == 16, then b = 13. -||-
4. Other variants are shown in the table above.

_____

Assuming that numbers 2,3,4 are already used, e+f>=6 (taking 1 and 5).
Variants 1 and 2 are not right because d+e+f+b=34, d+b >= 30
It means e+f <= 4 (coll).

Variant 3:

2	3	14	15
4		f	
16	e		
12			

e+f = 34-12-15; e+f = 7
Let e = 1, f = 6.
Then the sum of two not filled cells of the second column is 30, which can't be reached using the left numbers.
Let e = 6, f = 1.
Then the sum of two not filled cells of the second row is 29, which can't be reached using the left numbers.
Because 2,3,4 are already in use, e != 2,3,4,5

Variant 4:

2	3	15	14
4		f	
16	e		
12			

e+f = 34-12-14; e+f = 8
Let e = 1, f = 7.
Then the sum of two not filled cells of the second column is 30, which can't be reached using the left numbers.
Let e = 7, f = 1.
Then the sum of two not filled cells of the second row is 29, which can't be reached using the left numbers.

Because 2,3,4 are already in use, e != 2,3,4,5,6

We have tried every option, but still magic square can’t be filled.
Answer: Can’t be filled.