Combinatorics HW 2.1

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1. How many different permutations for word"Combinatorics"? (Case sensitive)
Let S be a multiset of objects of k = 11 types, with repetition numbers: C:1, o:2, m:1, b:1, i:2, n:1, a:1, t:1, r:1, c:1, s:1. The size of S is 1+2+1+1+2+1+1+1+1+1+1=13, the number of permutations of the letters for this word equals
$$\frac{13!}{1!*2!*1!*1!*2!*1!*1!*1!*1!*1!*1!}=\frac{6227020800}{4}=1556755200$$

2. The coefficient number of $a^2b^2c^2$ in the expanded equation of $(2a+b+c)^6$ is ____。【Please calculate the exact number】
Using Pascal's triangle and Newton binomial formula:
$$(2a+b+c)^6=(2a)^6+\frac{6}{1!}(2a)^5(b+c)+\frac{6*5}{2!}(2a)^4(b+c)^2+\ldots+(b+c)^6$$
The coefficient number of a^2 is: $C_6^{6-4}*(2^2)*(b+c)^4$, because every other term contains a in a power different from 2.
Again, using Pascal's triangle and Newton binomial formula:
$$(b+c)^4=b^4+C_4^1*b^3*c+C_4^2*b^2*c^2+C_4^3*b*c^3+c^4$$
Thus the coefficient number of is $C_6^{6-4}*4*C_4^2=15*4*6=360$

3. For the case of giving fruits to 3 kids, in total there are 12 identical apples, each child may at least have one apple, how many different ways to give apples to 3 kids?
It is equivalent to find the solution of
$x_1+x_2+x_3=12, x_1\geq 1; x_2\geq 1; x_3\geq 1$
Introducing new variables:
$y_1=x_1-1; y_2=x_2-1; y_3=x_3-1$
$y_1+y_2+y_3=9$
The number of different ways is $C_{9+3-1}^{9}=55$

4. What is the number of integral solutions of the equation $x_1+x_2+x_3=30$, in which $x_1\geq 5$, $x_2\geq -8$, $x_3\geq 5$.
Introduce new variables:
$y_1=x_1-5; y_2=x_2+8; y_3=x_3-5$
$y_1+y_2+y_3=30-5+8-5=28; y_1\geq 0; y_2\geq 0; y_3\geq 0$
The number of nonnegative solutions for the equation above (same for

$x_1+x_2+x_3=30$) is $C_{28+3-1}^{28} = C_{30}^{28} = 435$