Combinatorics HW 1.2

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1. How many odd numbers between 1000 and 9999 whose digits are distinct with each other?

Let the number have form of abcd.

Digit d may be one of the follows: 1,3,5,7,9 (5 options); if c is 1 to 9, then 8 options for c; if b is 1 to 9, then 7 options for b; a is 1 to 9 by default, so 6 options for a.

If c is 0, then 8 options left for b and 7 options left for a;

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Sum up: $5 * 8 * 7 * 6 + 5 * 1 * 8 * 7 + 5 * 8 * 1 * 7 = 2240$

2. How many 7-digit numbers are there such that the digits are distinct integers taken from {1, 2, …, 9} and such that the digits 5 and 6 do not appear consecutively in either order?

Number of 7-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} not containing 5 and 6 is $7 * 6 * 5 * 4 * 3 * 2 * 1 = 7!$

Number of 6-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} not containing 5 and 6 is $7 * 6 * 5 * 4 * 3 * 2 = 7!$ We can put 5 or 6 to 7 different positions to get the number of 7-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} not containing either 5 or 6, which is $2 * 7 * 7!$

Number of 5-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} and not containing 5 and 6 is $7 * 6 * 5 * 4 * 3$.

56 or 65 (5 next to 6) can be put on either of the 6 left positions (total 12 variants) to appear consecutively; 5 and 6 can be put on either of the $6 * 7 = 42$ positions (total 42 variants), if they can appear consecutively.

Number of 7-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} and containing 5 and 6 but not appearing consecutively is $7 * 6 * 5 * 4 * 3 * 42 - 7 * 6 * 5 * 4 * 3 * 12 = 7 * 6 * 5 * 4 * 3 * 30$.

Number of 7-digit numbers such that the digits are distinct integers taken from {1, 2, …, 9} and such that the digits 5 and 6 do not appear consecutively in either order is $7! + 14 * 7! + 30 * 7! / 2! = 15 * 7! + 7! * 15 = 151200$

Answer: 151200

3. How many different lattice paths from (-1,1) to (5,4)?

Let $x1 = x + 1$; $y1 = y - 1$. Then the question is to count how many different lattice paths from (0,0) to (6,3). Use formula, get answer $C(6+3, 6) = 9!/(3!*6!) = 9 * 8 * 7 / 6 = 84$

(x - go up, y - go right. The path has form of: xxy…yyx.

We need to choose the 6 'x' from the path of length 6+3)