Combinatorics HW Pigeon Hole Pricinple

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1. A basket of fruit is being arranged out of apples, bananas, and oranges. What is the smallest number of pieces of fruits that should be put in the basket in order to guarantee that either there are at least 8 apples or at least 6 bananas or at least 9 oranges?

Let's say we put fruits into different boxes, the first box contains at least 8 apples, second box contains at least 6 bananas, third box contains at least 9 oranges.

If we have 20 fruits, then it may happen that we got 7 apples in first box, 5 bananas in 2nd and 8 oranges in 3rd (still not enough).

We take one more fruit and then it will be enough, because the number of fruits in one of three boxes will become at least of the amount needed.

The total number of fruits in all boxes is no more then (8-1) + (6-1) + (9-1) = 20 before we distribute the last fruit.

Answer is 21.

2. Show that for any given 52 integers there exists two of them whose sum, or else whose difference, is divisible by 100.

Proof by contradiction

Lets write down all of the modulos by 100 of these numbers: mod(a_1,100), mod(a_2, 100), … , mod(a_{52}, 100)

Firstly, assuming that $0 \le mod(a_i,100) < 100$, for each of the numbers above, it means that there are only 100 different modulas for this set of numbers.

Secondly, if we have two different numbers a_i and a_j, $1 \le i, j < 52$, for which $mod(a_i,100) + mod(a_j,100) = 100$, we know that their sum is divisible by 100. Also, the integers must not have the same modulas, or their difference is divided by 100.

The pairs of modulas, for which the equation above is true are: 1 and 99, 2 and 98, …, 49 and 51. It means that if there are any of the two numbers from one of these pairs, the sum of them is divisible.

Numbers must not form any of these 49 pairs. It means that 49 numbers of the set of 52 numbers are in each of these pairs, and used only once.

The last three numbers can have only these two left modulas: 0, 50. Otherwise, the modula will be in one of those pairs.

If two of the numbers have modula 0, or two of them have modula 50, then their sum is divisible by 100. So they must be different. The last left integer's modula

is either 0, or 50, or be in one of the pairs written above.
By pigeonhole principle, the last integer will be in one of these “boxes”.
So for any given 52 integers there exists two of them whose sum, or else whose difference, is divisible by 100.