HW W6-2:

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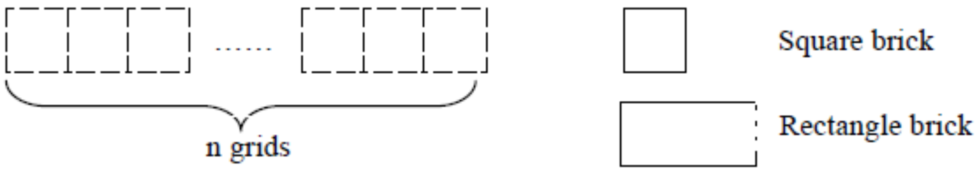
1. A worker is tiling a road with n square grids, there are two different kinds of bricks available, one is the square brick and the other is the rectangle brick which covers 2 grids. Please count the total different ways the road can be tiled.



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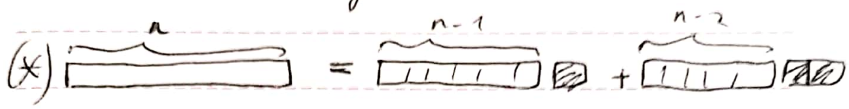
① □ = 1 way

□□ = 2 ways

□□□ = 3 ways



A_n - the number of ways to put grids.

Then, according to (*),

$A_n = A_{n-1} + A_{n-2}$ (because we can put either a single square brick to the end of n-1 length road or we can put double brick to the end of n-2 length road. Since the road can ~~be~~ have only single or double brick in the end, the total amount of different ways is summed up from these two).

We can see that this is a Fibonacci sequence.

So the answer is F_n, where $F_0 = 1$.

For this problem: $F_n = \dfrac{(\frac{1+\sqrt{5}}{2})^{n+1} - (\frac{1-\sqrt{5}}{2})^{n+1}}{\sqrt{5}}$

2. How many different ways to color n grids in a line with red, white or blue colors but no two adjacent grids are colored with red?

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③ Let A_n be the number of ways to color n grids in a line with red, white or blue colors, but no two adjacent grids are colored with red;

B_n is the same, but the last grid is blue or white;

R_n is the same, but the last grid is red.

Then $B_{n+1} = B_n \cdot 2 + R_n \cdot 2$, b/c we can put blue or white to the last grid

$R_{n+1} = B_n$, because ~~we~~ the $(n-1)^{st}$ grid must not be red.

$$A_n = B_n + R_n = B_n + B_{n-1}$$

Also $B_{n+1} = 2B_n + 2R_n = 2B_n + 2B_{n-1}$

In other words $x^2 = 2x + 2 \iff x^2 - 2x - 2 = 0$

roots are: $x_1 = 1+\sqrt{3}$; $x_2 = 1-\sqrt{3}$

Let $B_n = A(1+\sqrt{3})^n + B(1-\sqrt{3})^n$

then
$$\begin{cases} A(1+\sqrt{3})^1 + B(1-\sqrt{3})^1 = 2 & , \quad B_1 = 2 \\ A(1+\sqrt{3})^2 + B(1-\sqrt{3})^2 = 6 & , \quad B_2 = 6 \end{cases}$$

$$\Updownarrow$$

$$\begin{cases} A = (3-\sqrt{3})^{-1} \\ B = (2-\sqrt{3})(3-\sqrt{3})^{-1} \end{cases}$$

$$B_n = (3-\sqrt{3})^{-1}(1+\sqrt{3})^n + (2-\sqrt{3})(3-\sqrt{3})^{-1}(1-\sqrt{3})^n$$

Answer: $A_n = \dfrac{2+\sqrt{3}}{3-\sqrt{3}}(1+\sqrt{3})^{n-1} + \dfrac{(2-\sqrt{3})^2}{(3-\sqrt{3})}(1-\sqrt{3})^{n-1}$