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1\.

1) $$P_2(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

代入数据为 $$P_2(x) = \frac{(x-1.76)(x-1.78)}{0.0008}\times 0.9857 + \frac{(x-1.74)(x-1.78)}{0.0004}\times 0.9822 + \frac{(x-1.74)(x-1.76)}{0.0008}\times 0.9782$$

代入 $x=1.75$,

$P_2(1.75) = 0.9840$

2) $f(x_0) = 0.9857$

$$f(x_0,x_1) = \frac{f(x_0)-f(x_1)}{x_0-x_1} = -0.175$$

$$f(x_0,x_1,x_2) = \frac{f(x_0,x_1)-f(x_1,x_2)}{x_0-x_2} = -0.625$$

$$N_2(x) = f(x_0) + f(x_0,x_1)(x-x_0) + f(x_0,x_1,x_2)(x-x_0)(x-x_1)$$

$$= 0.9857 \cancel{+} -0.175(x-1.74) \cancel{*} -0.625(x-1.74)(x-1.76)$$

代入 $x=1.75$, $N_2(1.75) = 0.9840$

3) $f(x) = \sin x$, $f'(x) = \cos x$, $f''(x) = -\sin x$, $f'''(x) = -\cos x$

$$R(1.75) \cancel{\leq} = \left|\frac{f'''(\xi)}{3!}(x-x_0)(x-x_1)(x-x_2)\right|$$

$$\leq \left|\frac{-\cos 1.78}{3!}(1.75-1.74)(1.75-1.76)(1.75-1.78)\right| = 1.038\times 10^{-7}$$

3\. 1) $$P_3(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)}y_1 + \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)}y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}y_3$$

代入得 $$P_3(x) = \frac{(x-2)(x-4)(x-6)}{-15}\times 4 + \frac{(x-1)(x-4)(x-6)}{8}\times 1 + \frac{(x-1)(x-2)(x-4)}{40}\times 1$$

2) $f(x_0) = 4$

$$f(x_0,x_1) = \frac{f(x_0)}{x_0-x_1} + \frac{f(x_1)}{x_1-x_0} = -3$$

$$f(x_0,x_1,x_2) = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)} + \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)} + \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)} = \frac{5}{6}$$

$$f(x_0,x_1,x_2,x_3) = \frac{f(x_0)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} + \frac{f(x_1)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} + \frac{f(x_2)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} + \frac{f(x_3)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)}$$

$$= -\frac{7}{60}$$

$$N_3(x) = 4 - 3(x-x_0) + \frac{5}{6}(x-x_0)(x-x_1) \cancel{*} - \frac{7}{60}(x-x_0)(x-x_1)(x-x_2)$$

3) $$R(x) = \frac{f''''(\xi)}{4!}(x-x_0)(x-x_1)(x-x_2)(x-x_3) = \frac{f''''(\xi)}{24}(x-1)(x-2)(x-4)(x-6)$$

6.

$$p(x)=\begin{cases} x+2, & x\in[0,1) \\ x+2, & x\in[1,3] \\ -\frac{3}{2}x+19, & x\in[3,5) \end{cases}$$

$f(2.5)\approx p(2.5)=4.5$

7.

$$h_0(x)=\left(1+2\frac{x-x_0}{x_1-x_0}\right)\left(\frac{x-x_1}{x_0-x_1}\right)^2=(2x-1)(x-2)^2$$

$$h_1(x)=\left(1+2\frac{x-x_1}{x_0-x_1}\right)\left(\frac{x-x_0}{x_1-x_0}\right)^2=(-2x+5)(x-1)^2$$

$$H_0(x)=(x-x_0)\left(\frac{x-x_1}{x_0-x_1}\right)^2=(x-1)(x-2)^2$$

$$H_1(x)=(x-x_1)\left(\frac{x-x_0}{x_1-x_0}\right)^2=(x-2)(x-1)^2$$

$$H(x)=2(2x-1)(x-2)^2+3(-2x+5)(x-1)^2+(x-1)(x-2)^2-(x-2)(x-1)^2$$

$$H(1.5)=\frac{11}{4}$$

8.

$$p(x)=a_0+a_1x+a_2x^2+a_3x^3+a_4x^4$$

$$\begin{cases} a_0=0 \\ a_1=0 \\ a_2+a_3+a_4=1 \\ 2a_2+3a_3+4a_4=1 \\ 4a_2+8a_3+16a_4=1 \end{cases}$$

解得 $a_2=\frac{9}{4}$

$a_3=-\frac{3}{2}$

$a_4=\frac{1}{4}$

即 $p(x)=\frac{9}{4}x^2-\frac{3}{2}x^3+\frac{1}{4}x^4$

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