1 Intro to Semigroups

1.1 Basic Definitions

A **semigroup** is a set S together with an associative binary operaiton on S.

1.2 Examples

1. Empty Set with the Empty Function as the binary operator.
2. Groups are semigroups.
3. Any set with a constant map for it's binary operator forms a semigroup.
4. Singleton with only possible function.
5. The Cyclic group C_3.
6. The flip-flop monoid { "set", "reset", "do nothing"}
7. The set $\{-1, 0, 1\}$ under integer multiplication.
8. The **symmetric semigroup** - For any set X the mappings (or transformation)of X into X are the elements of a semigroup; the operation is composition of mappings.
9. The symmetric semigroup on a set X can be expanded as follows. A **partial mapping** of X into itself (usually called a **partial transformation** of X) is a mapping $\alpha : A \to X$ whose domain A is a subset of X. We generally write mappings on the left, but for partial transformations we use the right operator notation. When $\alpha : A \to X$ and $\beta : B \to X$ are partial transformations, the domain of $\alpha\beta$ is $D = \{x \in A; x\alpha \in B\}$; then $x(\alpha\beta) = (x\alpha)\beta$ for all $x \in D$.

1.2.1 Exercises - Demonstrate Examples

1. Verify that 1-4 and 8-9 above are examples of semigroups.
2. Given any sets I and Λ show that the operation $(i, \lambda)(j, \mu) = (i, \mu)$ on $I \times \Lambda$ is associative.