If you **see** a kamer ~~something,~~ **say** something® (to me)

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PLEASE I AM BEGGING U

Rooms in Delft

REPORT ~~SUSPICIOUS ACTIVITY~~

TO ME PLEASE YES ME

or ~~911 in case of emergency~~

feeble bridges

AE1110-I

Introduction to Aerospace Engineering Lecture Notes

Feeblebridges

November 19, 2024

Contents

1 General Intro Lecture 1

1.1. Hour 1

1.2. Hour 2

1.2.1. General takes

Can't scale designs up since weight (volume) scales with power of three while lift (area) scales with power of two.
Early designs for flying systems didn't work because they were using flapping wings (biomimicry) instead of fixed wing designs.

1.2.2. History

KongMing Lantern was the first hot air balloon, used military communications and/or surveillance (not for manned flight!! They only built small ones).
Balloon flight was the first manned flight. Back then they believed hot air was a special gas. They tested with duck, rooster, and sheep (in that order). First manned flight on Nov. 21, 1783. This was the only way to fly for more than a century.
Airships were used for a while. One accident ruined the image of airships. It was the Hindenburg Zeppelin, that was filled with flammable hydrogen gas and painted in highly flammable paint.

Why aren't airships and balloons used more often nowadays? Today's airships and balloons still look very similar. Their big benefits are that they have very efficient (essentially free) lift and they can go very high. Their disadvantages are that they have very high drag and using helium is not sustainable.

1.2.3. Actual math!!

$$p \cdot V = nRT$$

Note:-

$$\rho = \frac{m}{V} \quad V = \frac{m}{V} \quad n = \frac{m}{M}$$

$$M = 28.97 g/mol = 0.02897\ kg/mol$$

$$\mathcal{R} = 8.314462175\ J/mol$$

$$R = R_{air} = \frac{\mathcal{R}}{M} = \frac{8.314 \ldots}{0.02897 \ldots} = 287.00\ J/(kg\ K)$$

$$\frac{p}{\rho} = \frac{1}{M}\mathcal{R}T \Rightarrow p = \rho \frac{\mathcal{R}}{M} T$$

$$p = \rho R T$$

Archimedes said that if you replace a bubble of air with something else, the pressures that would usually be providing the lifting forces would still be there, so the lift force can be described as

$$L = W_{air} - W_{gas}$$

$$L = m_{air}g - m_{gas}g$$

$$L = \rho_{air}Vg - \rho_{gas}Vg$$

$$L = \rho_{air}Vg\left(1 - \frac{\rho_{gas}}{\rho_{air}}\right)$$

$$\frac{\rho_1}{\rho_2} = \frac{\cancel{p_1}M_1\cancel{\mathscr{R}}T_2}{\cancel{p_2}M_2\cancel{\mathscr{R}}T_1}$$

$$L_{gas} = \rho Vg\left(1 - \frac{M_{gas}}{M_{air}}\right)$$

$$L = \rho Vg\left(1 - \frac{T_{air}}{T_{air,hot}}\right)$$

$$L = \rho Vg\left(\frac{T + \Delta T}{T + \Delta T} - \frac{T}{T + \Delta T}\right)$$

$$= \rho Vg\frac{\Delta T}{T + \Delta T}$$

1.3. Homework

$$V = 2500m^3$$

$$T_{max} = 120C$$

$$\rho_{atm} = 1.225\frac{kg}{m^3}$$

$$T = 15C$$

$$g = 9.81\frac{m}{s^2}$$

Find the total weight of the balloon, basket and payload for such a balloon

$$\sum \vec{F}_{z\uparrow+} = 0 = L_{max} - m_{max}g$$

Remember that

$$L = \rho Vg\frac{\Delta T}{T + \Delta T}$$

$$m_{max}g = L_{max}$$

$$m_{max}g = \rho Vg\frac{\Delta T}{T + \Delta T}$$

$$\boxed{m_{max} = \rho V\frac{\Delta T}{T + \Delta T}}$$

$$m_{max} = 1.225\frac{kg}{m^3} \cdot 2500m^3 \cdot \frac{120 - 15}{120 + 273}$$

$$m_{max} = 818.225 \cdot kg \approx 818kg$$

□What was the temperature of the balloon?

$$p = \rho RT$$
$$\rho = \frac{p}{RT}$$

$$L = \rho V g \frac{\Delta T}{T + \Delta T}$$
$$m_{max}\cancel{g} = \rho V \cancel{g} \frac{\Delta T}{T + \Delta T}$$
$$m_{max} = \frac{p}{RT} \frac{\Delta T}{T + \Delta T}$$

□How many 14L helium balloons to lift 80kg?

$$L = \rho V g \left(1 - \frac{M_{gas}}{M_{air}}\right)$$
$$m\cancel{g} = \rho V \cancel{g} \left(1 - \frac{M_{gas}}{M_{air}}\right)$$
$$V = \frac{m}{\rho} \frac{1}{1 - \frac{M_{gas}}{M_{air}}}$$
$$\boxed{n_{balloons} = \frac{m}{\rho} \frac{1}{1 - \frac{M_{gas}}{M_{air}}} \frac{1}{V_{balloon}}}$$

□Anderson 2.1 Air pressure and temperature are 1.2 atm and 300K, respectively. Find the density and specific volume?

$$\rho = \frac{p}{RT}$$
$$\rho = \frac{1.2 \cdot 101325}{287 \cdot 300} = 1.412195 \ldots$$
$$\boxed{\rho \approx 1.412 \frac{kg}{m^3}}$$
$$v_{spec} = \frac{1}{\rho} = \frac{1}{1.412195 \ldots} = 0.708117 \ldots$$
$$\boxed{v_{spec} \approx \frac{0.708 m^3}{kg}}$$

□Anderson 2.7 Air temperature is $-10^\circ C$ and pressure is $1.7 \times 10^4 \frac{N}{m^2}$. Find the density at this point.

$$\rho = \frac{p}{RT}$$
$$\rho = \frac{1.7 \times 10^4}{287 \cdot (288.15 - 10)} = 0.212955 \ldots$$
$$\boxed{\rho \approx 0.213 \frac{kg}{m^3}}$$

□Anderson 2.11 Consider a helium-filled balloon with a volume of 2.2 ft^3. Assuming the balloon is at sea level, calculate the maximum weight that can be lifted by the balloon. Note: $M_{air} = 28.8$ and $M_{He} = 4$. Archimedes' principle states that

$$L_{gas} = \rho V g \left(1 - \frac{M_{gas}}{M_{air}}\right)$$

$$W_{max} = L_{gas} = 1.225 \cdot 2.2 \cdot 0.3048^3 \cdot 9.80665 \left(1 - \frac{4}{28.8}\right)$$

$$W_{max} = 0.644441539 \ldots [N]$$

$$\boxed{W_{max} \approx 0.64N}$$

Anderson 2.12 Instant combustion case. At the start of the combustion process, the values for pressure and temperatures are: $\rho_{start} = 11.3\frac{kg}{m^3}$ and $T_{start} = 625K$. At the end, the temperature has increased to $T_{end} = 4000K$. Find the pressure p_{end} at the end of the constant-volume combustion.

$$\rho = \frac{p}{RT} \Rightarrow p = \rho RT$$

Density stays constant so

$$p_{end} = \rho RT_{end} = 11.3 \cdot 287 \cdot 4000 = 12972400\frac{N}{m^2}$$

$$\boxed{p_{end} \approx 13.0MPa}$$

Anderson 2.13 For the conditions of Anderson 2.12, calculate the force exerted on the top of the piston by the gas at (a) the beginning of combustion, and (b) the end of combustion. The diameter of the circular piston face is 9cm.

$$p = \rho RT$$

$$F = pA = p\pi\frac{d^2}{4} = \rho RT\pi\frac{d^2}{4}$$

$$a) : F_{start} = 11.3 \cdot 287 \cdot 625 \cdot \pi\frac{0.09^2}{4}$$

$$= 12760.03017 \ldots$$

$$\boxed{F_{start} \approx 12.8kN}$$

$$b) : F_{end} = 11.3 \cdot 287 \cdot 4000 \cdot \pi\frac{0.09^2}{4}$$

$$= 81664.19307 \ldots$$

$$\boxed{F_{end} \approx 81.7kN}$$

Anderson 2.14 Consider the constant pressure combustion process of a gas turbine jet engine. The gas pressure and temperature entering the combustor are $4 \times 10^6\frac{N}{m^2}$ and $900K$ respectively, and at the exit the gas temperature is $1500K$. Find the gas density at (a) the inlet to the combustor, and (b) the exit of the combustor.

$$\rho = \frac{p}{RT}$$

$$a) : \rho_{start} = \frac{4 \times 10^6}{287 \cdot 900} = 15.48586914 \ldots$$

$$\boxed{\rho_{start} \approx 15.5\frac{kg}{m^3}}$$

$$b) : \rho_{end} = \frac{4 \times 10^6}{287 \cdot 1500} = 9.291521487 \ldots$$

$$\boxed{\rho_{end} \approx 9.29\frac{kg}{m^3}}$$

2

General Intro Lecture 2

2.1. Hour 1

I was 15 min late :V Thanks Keolis :)

2.1.1. Standard atmosphere

The standard atmosphere is used so performance specifications can be given. Safety is also a concern since aircraft using different sea-level pressures could crash into each other even when flying at differing pressure altitudes.

$$p = \rho RT \text{ with } R = 287.00\frac{J}{kgK}$$

$$\begin{aligned}\sum \vec{F}_{z\uparrow+} = 0 = pA &= (p + \Delta p)A + F_g \\ \cancel{pA} &= \cancel{pA} + \Delta pA + \rho V g \\ \Delta p &= -\rho g\, \Delta h \\ dp &= -\rho g\, dh\end{aligned}$$

Standard Sea level values

$$\begin{aligned}p &= 101325.0Pa \\ T &= 15^{\circ}C = 288.15K \\ \rho &= 1.225\frac{kg}{m^3}\end{aligned}$$

The pressure, however, is considered as a resultant of the temperature and pressure.

International Standard Atmosphere (ISA) describes the atmosphere. The atmosphere is split into 8 layers and described using linear equations. Temperatures are assumed to either be constant OR have a constant lapse rate $a\left[\frac{^{\circ}C}{km}\right]$. Toussaint's formula:

$$T = T_0 + a(h - h_0)$$

$$a = \frac{\Delta T}{\Delta h} = \frac{dT}{dh}$$

2.2. Hour 2

2.2.1. Derivation of pressure and temperature changes in ISA

$$dp = -\frac{p}{RT} g\, dh$$
$$dp = -\frac{p}{RT} g \frac{dT}{a}$$
$$\frac{1}{p} dp = -\frac{g}{aR} \frac{1}{T} dT$$
$$\int_{p_0}^{p_0} \frac{1}{p} dp = \frac{g}{aR} \int_{T_0}^{T_1} \frac{1}{T} dT$$
$$\ln p_1 - \ln p_0 = -\frac{g}{aR} (\ln T_1 - \ln T_0)$$
$$e^{\ln p_1 - \ln p_0} = e^{-\frac{g}{aR}(\ln T_1 - \ln T_0)}$$
$$\boxed{\frac{p_1}{p_0} = \frac{T_1}{T_0}^{-\frac{g}{aR}}}$$

Note:-

The above formula only works for regions of the atmosphere where there is a temperature gradient ($a \neq 0$)

Geometric altitude formula:

$$h = h_g \frac{R_e}{R_e + h_g}$$

where R_e is the radius of the Earth, h_g is the geometric altitude, and h is the geopotential (default) altitude. Geopotential altitude is the default altitude in aviation. If it's geometric altitude, it will be mentioned specifically.

$$dp = -\frac{p}{RT} g\, dh$$
$$\frac{1}{p} dp = -\frac{g}{RT} dh$$
$$\ln p_1 - \ln p_0 = -\frac{g}{RT} (h_1 - h_0)$$
$$e^{\ln p_1 - \ln p_0} = e^{-\frac{g}{RT}(h_1 - h_0)}$$
$$\boxed{\frac{p_1}{p_0} = e^{-\frac{g}{RT}(h_1 - h_0)}}$$

Note:-

The above equation only works for regions of the atmosphere where the temperature is constant ($a = 0$) also known as **isothermal** layers

Example 2.2.1 (Calculating atmospheric properties)

0 → 11km

h = 11000m

$$T_1 = 288.15 - 0.0065(11000 - 0) = 216.65K$$

$$p_1 = p_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g}{aR}}$$
$$p_1 = 101325 \cdot \left(\frac{216.65}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}}$$
$$p_1 = 20587.697 \cdots \approx 20588 \frac{N}{m^2}$$

2.3. Homework

Anderson chapter 3 problems: 3.1, 3.3, 3.5, 3.6, 3.8, 3.9, 3.11 Use $R_e = 6371km.$

□Anderson 3.1Calculate the ISA values of pressure, density, and temperature at an altitude of 18km given that

$$p_{12km} = 1.9399 \cdot 10^4 \frac{N}{m^2},\ \rho_{12km} = 3.1194 \cdot 10^{-1} \frac{kg}{m^3},\ T_{12km} = 216.66K$$

Altitude is between 11 and 20km so the temperature is constant (isothermal)

$$\text{isothermal} \Rightarrow T_{18km} = T_{12km} = 216.66K$$

Now to calculate the pressure we can use

$$\frac{p_1}{p_0} = e^{-\frac{g}{RT}(h_1 - h_0)}$$
$$p_1 = p_0 e^{-\frac{g}{RT}(h_1 - h_0)}$$
$$= 1.9399 \cdot 10^4 \cdot e^{-\frac{9.80665}{287 \cdot 216.66}}(18000 - 12000)$$
$$= 7530.487323 \ldots$$
$$\boxed{p_1 \approx 7530 \frac{N}{m^2}}$$

The density can then be calculated using

$$\rho_1 = pRT = \frac{7530.487 \ldots}{287 \cdot 216.66} = 0.121105104$$
$$\boxed{\rho_1 \approx 0.121 \frac{kg}{m^3}}$$

□Anderson 3.3 Find the ambient air pressure at a standard altitude of 35000 ft $35000 \cdot 0.3048 = 10668m < 11000m$ so the plane is still in the troposphere where $a = -6.5 \frac{°C}{km}$. The pressure at that altitude can be calculated using

$$\frac{p_1}{p_0} = \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}}$$
$$T_0 = 288.15\ [K]$$
$$T_1 = T_0 + a(h_1 - h_0)$$
$$T_1 = 288.15 - 6.5 \times 10^{-3}(10668 - 0)\ [K]$$
$$p_1 = p_0 \cdot \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} = 101325 \cdot \left(\frac{288.15 - 6.5 \times 10^{-3}(10668)}{288.15}\right)^{-\frac{9.80665}{-6.5 \times 10^{-3} \cdot 287}}$$
$$p_1 = 23835.91892 \ldots$$
$$\boxed{p_1 \approx 23.8\, kPa}$$

□Anderson 3.6 Using Toussaint's formule, calculate the pressure at a geopotential altitude of 5km

$$
\begin{aligned}
T_1 &= T_0 + a(h_1 - h_0) \\
\frac{p_1}{p_0} &= \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} \\
p_1 &= p_0 \cdot \left(\frac{T_0 + a(h_1 - h_0)}{T_0}\right)^{-\frac{g_0}{aR}} \\
&= 101325 \cdot \left(\frac{288.15 - 0.0065 \cdot 5000}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}} \\
&= 54013.62935 \ldots \\
&\boxed{p_1 \approx 54.0\, kPa}
\end{aligned}
$$

□Anderson 3.8 A plane is climbing through a standard altitude of 25000ft with a rate-of-climb of $500 \frac{ft}{s}$. Find the time rate of change of ambient pressure for this airplane.

$$
\begin{aligned}
\frac{p_1}{p_0} &= \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} \\
T_1 &= T_0 + a(h_1 - h_0) \\
p_1 &= p_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} = \frac{p_0}{T_0^{-\frac{g_0}{aR}}} T_1^{-\frac{g_0}{aR}} \\
\frac{dp_1}{dT_1} &= -\frac{g_0}{aR} \frac{p_0}{T_0^{-\frac{g_0}{aR}}} T_1^{-\frac{g_0}{aR}-1} \\
\frac{dp_1}{dt} &= \frac{dp_1}{dT_1} \underbrace{\frac{dT_1}{dh_1}}_{=a} \frac{dh_1}{dt} \\
\frac{dp_1}{dt} &= -\frac{g_0}{aR} \frac{p_0}{T_0^{-\frac{g_0}{aR}}} T_1^{-\frac{g_0}{aR}-1} \cdot a \cdot ROC \\
&= -\frac{9.80665}{-0.0065 \cdot 287} \frac{101325}{288.15^{-\frac{9.80665}{-0.0065 \cdot 287}}} [288.15 - 0.0065 \cdot 0.3048 \cdot 25000]^{-\frac{9.80665}{-0.0065 \cdot 287}} \\
&\quad \cdot -0.0065 \cdot 500 \cdot 0.3048 \\
\frac{dp_1}{dt} &= 828.2041465 \ldots \cdot -0.0065 \cdot 500 \cdot 0.3048 \\
\frac{dp_1}{dt} &= -820.4190275 \ldots \\
&\boxed{\frac{dp_1}{dt} \approx -820 \frac{Pa}{s}}
\end{aligned}
$$

□Anderson 3.9 Find the upward speed that corresponds to a 1 percent decrease in pressure per minute at sea level

$$\begin{aligned}
\frac{1}{p}\,dp &= -\frac{g_0}{RT}\,dh \\
\frac{1}{p}\frac{dp}{dt} &= -\frac{g_0}{RT}\frac{dh}{dt} \\
\frac{dp}{dt} &= -p\frac{g_0}{RT}v_{vertical} = -\frac{1}{60}\frac{p}{100} \\
v_{vertical} &= \frac{RT}{60\cdot 100 g_0} \\
&= \frac{287\cdot 288.15}{60\cdot 100\cdot 9.80665} = 1.405492701\ldots \\
\boxed{v_{vertical} \approx 1.41\frac{m}{s}}
\end{aligned}$$

□Anderson 3.11 Consider a square tank of water open to the atmosphere. It is 10ft deep with 30ft long walls. The tank is located at sea level. Calculate the force exerted on the side of the walls.

$$\begin{aligned}
dp &= -\rho g_0\,dh \\
\int dp &= \int -\rho g_0\,dh \\
p_1 - p_0 &= -\rho g_0(h_1 - h_0) \\
p_1 &= -\rho g_0(h_1 - h_0) \\
p &= -\rho g_0 h
\end{aligned}$$

Where h_0 is the top of the tank. Then the force on the walls per unit length of the wall can be described as

$$\begin{aligned}
\frac{F}{L} &= \int_{-10\cdot 0.3048}^{0} -\rho g_0 h\,dh \\
&= \left[-\frac{1}{2}\rho g_0 h^2\right]_{-10\cdot 0.3048}^{0} \\
&= \cancelto{0}{\left(\frac{1}{2}\cdot 999.55\cdot 9.80665\cdot 0\right)} - \left(-\frac{1}{2}\cdot 999.55\cdot 9.80665\cdot(-10\cdot 0.3048)^2\right) \\
&= 45532.88084\ldots \\
\frac{F}{L} &\approx 45.5\frac{kN}{m}
\end{aligned}$$

Every wall is 30ft long so the force on one of the walls is

$$\begin{aligned}
F &= \frac{F}{L}\cdot L = 45532.88084\ldots\cdot 30\cdot 0.3048 \\
&= 416352.6624\ldots \\
&\boxed{F \approx 416.4kN} \\
m_{equivalent} &= \frac{F}{g_0} = \frac{416352.6624\ldots}{9.80665} = 42456.15602\ldots \\
&\boxed{m_{equivalent} = 42.5t}
\end{aligned}$$

3
General Intro Lecture 3

3.1. Idk what's hour 1 or 2, im taking these notes from the slides

3.1.1. History

In the 19th century, Otto Lilienthal made more than 2000 flights on gliders. He built more than a dozen himself. Hang gliders are basically derivatives of Lilienthal's gliders.

The early 20th century brought some new players out on the field, including Samuel Langley and the Wright Brothers. The Wright Brothers used a lot of wind tunnel testing. This helped them build very efficient propeller and wing setups.

3.1.2. Lift equation

For straight, horizontal, steady flight (kinda comphet if you ask me), the four forces Weight W, Lift L, Drag D, and Thrust T are assumed to be in balance:

$$L = W$$
$$D = T$$

The formula for lift is

$$L = \underbrace{\frac{1}{2}\rho V^2}_{q} \; C_L S \tag{3.1}$$

L = Lift [N]
ρ = Density of the air [kg/m^3]
V = Air speed [m/s]
C_L = Lift coefficient [-]
S = Wing area [m^2]
q = Dynamic pressure [Pa] $= \frac{1}{2}\rho V^2$

The C_L depends on the shape of the aircraft and wing, and on the angle of attack.

3.1.3. Bernoulli's principle

A derivation of this equation can be found in Section 3.2

Bernoulli's principle states that the sum of the static pressure and dynamic pressure stays constant along a streamline:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2$$
$$p_{static} + p_{dynamic} = p_{total}$$
$$p_{static} + q = \text{constant along streamline}$$

A derivation of this equation can be found in section 3.2 This principle holds only for constant ρ, so for incompressible flow ($\Rightarrow$ low speeds, complete flow subsonic)

Lift depends on:

- $C_L \Rightarrow$ angle of attack α, airfoil/wing profile
- $\rho \Rightarrow$ altitude & temperature (atmosphere)
- V and S, but these are design parameters.

3.1.4. Airfoils & Wings

Airfoils are like a 2D cross-section of an airplane wing, so they allow you to see the shape of a wing. Since airfoils can be scaled, they allow for comparison between different designs. Analysis of airfoils also removes the effects of having a finite wing from the comparison.

NACA did a lot of research into airfoils and wing profiles in the early 20th century. They came up with a standardised naming method using names in the format of NACAxxxx. The first number is the maximum camber (2 means 2

3.1.5. Back to Bernoulli

Bernoulli's principle states that the sum of static and dynamic pressure remains constant. This means that when air flows faster, there is a lower local pressure. This pressure difference is how a wing generates its lift.

Reynolds number Re is used to describe the airflow, which is useful for comparing different wings and airfoils to each other. The Reynolds number is described as

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu} \tag{3.2}$$

where

Re = Reynolds number
ρ = Density of the air [kg/m^3]
V = Airspeed [m/s]
L = Typical length (= chord c for airfoils) [m]
μ = Dynamic viscosity (Ns/m^2)
for air: $\mu = 1.7894 \cdot 10^{-5}$ kg/ms
ν = Kinematic viscosity = $\frac{\mu}{\rho}$

The lift provided by a wing is consists of a part where it increases linearly with angle of attack α, followed by a part where it departs from this linear relationship to a lower slant until it reaches a maximum, quickly followed by a sharp decrease in lift called the stall.

The drag equation is very similar to the lift equation. The only difference is the usage of the Drag coefficient C_D instead of the Lift coefficient C_L.

$$D = C_D \frac{1}{2} \rho V^2 \tag{3.3}$$

Drag coefficient consists of:

- Parasitic drag (profile drag)
- Induced drag (from lift)

Drag can be split into three categories: Friction drag, Pressure drag, and Wave drag. A different division can be made using Zero lift drag C_{D_0} and Lift induced drag C_{D_i}. Drag coefficient: $C_D = C_{D_0} + C_{D_i}$

$$C_D = C_{D_0} + kC_L^2 = \underbrace{C_{D_0}}_{\text{Zero lift drag}} + \underbrace{\frac{C_L^2}{\pi A e}}_{\text{Lift-induced drag}} \tag{3.4}$$

where $A = \frac{span}{chord} = \frac{b}{c} = \frac{S}{c^2}$ and e is the Oswald efficiency factor.

$\frac{C_L}{C_D}$ is an important design parameter since a high lift-drag ratio means a lower fuel consumption.

3.2. Deriving Bernoulli's principle

$$A = \Delta y\,\Delta z$$
$$V = \Delta x\,\Delta y\,\Delta z$$

Using $F = m \cdot a$ in x-direction (equilibrium in others)

$$\begin{aligned}
F_x &= m \cdot a = \rho\,\Delta x \Delta y \Delta z \cdot \frac{\Delta V}{\Delta t} \\
F_x &= pA - (p + \Delta p)A \\
&= p \cdot \Delta y \Delta z - (p + \Delta p)\Delta y \Delta z \\
F_x &= -\Delta p \Delta y \Delta z \\
\rho \Delta x \cancel{\Delta y} \cancel{\Delta z} \cdot \frac{\Delta V}{\Delta t} &= -\Delta p \cancel{\Delta y} \cancel{\Delta z} \\
\rho \frac{\Delta x}{\Delta t} \cdot \Delta V &= -\Delta p \\
\Delta p + \rho \frac{\Delta x}{\Delta t} \Delta v &= 0 \\
dp + \rho V\,dV &= 0 \quad \text{Assume } \rho = \text{constant} \\
&\Rightarrow \text{incompressible flow} \Rightarrow \text{low speeds} \\
\int dp + \rho \int V\,dV &= \text{constant} \\
\boxed{p + \frac{1}{2}\rho V^2 = \text{constant}}
\end{aligned}$$

3.3. Homework

Anderson problems: 5.2 - 5.6, 5.21, 5.22 □Anderson 5.2 Consider NACA1412 with a chord length of 3ft. The airfoil is at an angle of 5°in a 100ft/s airflow at standard sea-level conditions. Find the lift, drag, and moment about the quarter-chord per unit span (so L, D, and M)

$$\begin{aligned}
c &= 3ft = 3 \cdot 0.3048 \\
Re = \frac{\rho V c}{\mu} &= \frac{1.225 \cdot 100 * 0.3048 \cdot 3 \cdot 0.3048}{1.7894 \cdot 10^{-5}} \\
&= 1908006.43791 \ldots \\
Re &\approx 1.9 \times 10^{6} \\
C_l &= 0.7 \\
C_d &= 0.007 \\
C_m &= -0.025
\end{aligned}$$

$$L = \frac{1}{2}\rho V^2 C_L S$$
$$D = \frac{1}{2}\rho V^2 C_D S$$
$$M = C_m \frac{1}{2}\rho V^2 S c$$
$$L = \frac{1}{2} \cdot 1.225 \cdot (100 \cdot 0.3048)^2 \cdot 0.7 \cdot 3 \cdot 0.3048$$
$$L = 364.2254393 \dots$$
$$\boxed{L \approx 364.2N}$$
$$D = \frac{1}{2} \cdot 1.225 \cdot (100 \cdot 0.3048)^2 \cdot 0.007 \cdot 3 \cdot 0.3048$$
$$D = 3.642254393 \dots$$
$$\boxed{D \approx 3.64N}$$
$$M = -0.025 \cdot \frac{1}{2} \cdot 1.225 \cdot (100 \cdot 0.3048)^2 \cdot (3 \cdot 0.3048)^2$$
$$M = -11.8945622 \dots$$
$$\boxed{M \approx -11.9Nm}$$

Anderson 5.3 Consider a rectangular wing in a low-speed subsonic wind tunnel. It has a NACA23012 airfoil section and a chord of 0.3m. Find the lift, drag, and moment about the quarter-chord per unit span when the airflow pressure, temperature, and velocity are 1atm, 303K, and 42m/s respectively. The angle of attack is 8°.

$$p = 101325Pa$$
$$T = 303K$$
$$V = 42\text{m/s}$$
$$\alpha = 8^\circ$$
$$\rho = \frac{p}{RT} = \frac{101325}{287 \cdot 303}$$
$$Re = \frac{\rho V c}{\mu} = \frac{\frac{101325}{287 \cdot 303} \cdot 42 \cdot 0.3}{1.7894 \cdot 10^{-5}}$$
$$Re \approx 820 \times 10^3$$
$$C_l = 1$$
$$C_d = 0.009$$
$$C_m = -0.0125$$

$$q = \frac{1}{2}\rho v^2 = \frac{1}{2} \cdot \frac{101325}{287 \cdot 303} \cdot 42^2$$
$$q = 1027.686549 \dots [N]$$
$$L = qSC_l = 1027.68 \cdots \cdot 0.3 \cdot 1$$
$$\boxed{L \approx 308.3N}$$
$$D = qSC_d = 1027.68 \cdots \cdot 0.3 \cdot 0.009$$
$$\boxed{D \approx 2.77N}$$
$$M = C_m qSc = -0.0125 \cdot 1027.68 \cdots \cdot 0.3^2$$
$$\boxed{M \approx -1.12Nm}$$

□Anderson 5.4 The wing from the previous question is pitched to a new angle of attack. The lift of the entire wing is now 200N and it has a wingspan of 2m. Find the angle of attack for this wing.

$$L = \frac{1}{2}\rho V^2 S C_l$$
$$C_l = \frac{2L}{\rho V^2 S} = \frac{2 \cdot 200}{\frac{101325}{287 \cdot 303} \cdot 42^2 \cdot 2 \cdot 0.3}$$
$$C_l = 0.3243531149 \ldots$$
$$\boxed{C_l \approx 0.32}$$

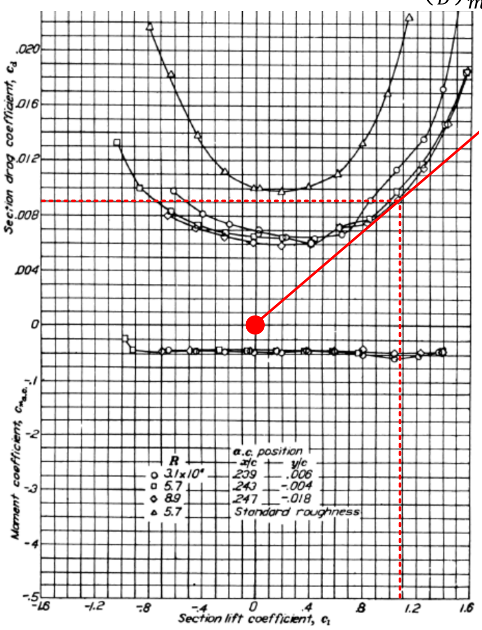
Reading from the chart of NACA23012 shows a corresponding angle of attack of $\alpha = 2°$

□Anderson 5.5 Rectangular wing with NACA0009 airfoil. Airflow conditions are that of standard sea level and velocity is 120mi/h. Angle of attack is 4°and the lift is measured at 29.5lb. Find the area of the wing.

$$V = 5280 \cdot 0.3048 \cdot 120 \cdot 3600 = 53.6448\frac{m}{s}$$
$$\rho = 1.225\frac{kg}{m^3}$$
$$p = 101325Pa$$
$$C_l = 0.4$$

$$L = \frac{1}{2}\rho V^2 S C_l$$
$$S = \frac{2L}{\rho V^2 C_l} = \frac{2 \cdot 9.80665 \cdot 453.59237}{1.225 \cdot 53.6448^2 \cdot 0.4}$$
$$= 6.3090660024266 \ldots$$
$$\boxed{S \approx 6.31m^2}$$

□Anderson 5.6 Consider an infinite wing with an NACA2412 airfoil. Estimate $\left(\frac{L}{D}\right)_{max}$,



assuming a Reynolds number of 9×10^6.

For $\left(\frac{L}{D}\right)_{max}$, $C_d = 0.009$ and $C_l = 1.1$, giving a Lift-drag coefficient of

$$\left(\frac{L}{D}\right)_{max} = \frac{1.1}{0.009} = 122.2 \cdots \approx 122$$

□Anderson 5.21 Cessna! $S = 16.2m^2$! $AR = 7.31$! omg! $e = 0.62$! heck yeah!! Standard sea-level conditions? You guessed it!! $V = 251\frac{km}{h}$(that's even faster than Dalton!!!) If Dalton would weight 9800N, what is his induced drag? From equation 3.4,

$$C_D = C_{D_0} + kC_L^2 = C_{D_0} + \frac{C_L^2}{\pi Ae}$$

$$C_L = \frac{2W}{\rho V^2 S}$$

$$C_{D_i} = \frac{C_L^2}{\pi Ae} = \frac{\left(\frac{2 \cdot 9800}{1.225 \cdot \frac{251}{3.6} \cdot 16.2}\right)^2}{\pi \cdot 7.31 \cdot 0.62}$$

$$= 0.01426933 \ldots$$

$$\boxed{C_{D_i} \approx 0.014}$$

$$D_i = qC_{D_i} = \frac{1}{2}\rho V^2 C_{D_i}$$

$$= \frac{1}{2} \cdot 1.225 \cdot \frac{251}{3.6}^2 \cdot \frac{\left(\frac{2 \cdot 9800}{1.225 \cdot \frac{251}{3.6} \cdot 16.2}\right)^2}{\pi \cdot 7.31 \cdot 0.62}$$

$$= 42.48661409 \ldots$$

$$\boxed{D_i \approx 42.5N}$$

□Anderson 5.22 Same Cessna!!! But now it's stalling!! uh oh!!! $V = 85.5\frac{km}{h}$...

From Anderson 5.21:

$$C_L = \frac{2W}{\rho V^2 S}$$

$$C_{D_i} = \frac{C_L^2}{\pi Ae}$$

$$= \frac{\frac{2W}{\rho V^2 S}^2}{\pi Ae}$$

$$= \frac{\left(\frac{2 \cdot 9800}{1.225 \cdot \frac{85.5}{3.6}^2 \cdot 16.2}\right)^2}{\pi \cdot 7.31 \cdot 0.62}$$

$$= 0.215326034 \ldots$$

$$\boxed{C_{D_i} \approx 0.215}$$

$$D_i = qC_{D_i} = \frac{1}{2}\rho V^2 \frac{\left(\frac{2 \cdot 9800}{1.225 \cdot \frac{251}{3.6} \cdot 16.2}\right)^2}{\pi \cdot 7.31 \cdot 0.62}$$

$$= 74.39262137 \ldots$$

$$\boxed{D_i \approx 74.4N}$$

4

General Intro Lecture 4

4.1. Hour 1

There is a point, the *aerodynamic center*, which is usually around $\frac{1}{4}$th of the chord length where the moment stays constant.

4 control *surfaces*: elevator, rudder, ailerons, throttle ⇒ 4 instantaneous degrees of freedom. 6 total degrees of freedom, which is achieved over time.

Fly by wire Flight Control System (FCS)
First used in military jets for increased agility (allows less stable aircraft), later also in commercial airplanes because of weight savings.

Body axes: Forward is x-axis, to the right w.r.t. x is y-axis, which leaves the z-axis to be pointing downwards. Moments are defined $\mathscr{L}$ when it's around x-axis, $\mathscr{M}$ when it's around y-axis, and $\mathscr{N}$ when it's around the z-axis.

$$L \sin\phi = F = \frac{W}{g}\frac{V^2}{R_t}$$

$$L\cos\phi = W$$

Load factor n

$$n = \frac{1}{\cos\phi}$$

$$\tan\phi = \frac{V^2}{g \cdot R_t} \text{(Turn radius bank angle)}$$

γ is climb angle
α is angle of attack
θ is pitch angle
ψ is heading
β is sideslip
χ is course

Positive stability is when the reaction to a deviation is opposite of the deviation: $\Delta\alpha \implies \Delta M < 0$ so

$$\frac{\Delta M}{\Delta \alpha} < 0 \tag{4.1}$$

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S c}$$

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 S c}$$

$$\frac{dC_m}{d\alpha} < 0 \quad \text{Holy grail :3}$$

Static stability vs Dynamic stability

4.2. Hour 2

4.2.1. Mathematical derivation (big fan)

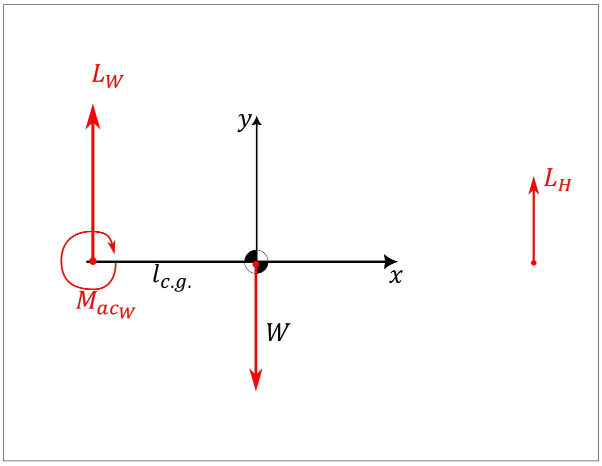
$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S}$$
$$C_{L_H} = \frac{L_H}{\frac{1}{2}\rho V^2 S_H}$$
$$C_m = \frac{M}{\frac{1}{2}\rho V^2 S\bar{c}}$$

$$L_H = C_{L_H}\frac{1}{2}\rho v^2 S_H$$

$$\alpha_H = \alpha - \epsilon + \underbrace{i_H}_{\text{Trim}}$$

ϵ is downwash angle. i_H is trim angle or angle of incidence

$$M \to C_m \to \underbrace{C_{m_\alpha}}_{\frac{dC_m}{d\alpha}} < 0$$



Sum of moments around centre of gravity

Note:-

There is a M_{ac_W} but no M_{ac_H} because the airfoils used for tailplanes are usually symmetrical.

$$\frac{dC_L}{d\alpha} = C_{L_\alpha} = a$$
$$\left(\frac{dC_L}{d\alpha}\right)_H = \frac{dC_{L_H}}{d\alpha_H} = C_{L_{\alpha_H}} = a_t$$
$$\frac{dC_m}{d\alpha} = C_{m_\alpha}$$

$$
\begin{aligned}
\sum \vec{M}_{c.g.} \circlearrowright + &= M_{ac_{wb}} + L_{wb} \cdot l_{c.g.} - L_H \cdot (l_H - l_{c.g.}) \\
&= M_{ac_{wb}} + L_{wb} \cdot l_{c.g.} - L_H \cdot l_H + L_H \cdot l_{c.g.} \\
&= M_{ac_{wb}} + (L_{wb} + L_H) \cdot l_{c.g.} - L_H \cdot l_H \\
M &= M_{ac_{wb}} + L \cdot l_{c.g.} - L_H \cdot l_H \\
\frac{M}{\frac{1}{2}\rho V^2 Sc} &= \frac{M_{ac_{wb}}}{\frac{1}{2}\rho V^2 Sc} + \frac{L \cdot l_{cg}}{\frac{1}{2}\rho V^2 Sc} - \frac{L_H \cdot l_H}{\frac{1}{2}\rho V^2 Sc}
\end{aligned}
$$

$$\boxed{V_H = \frac{S_H l_H}{Sc}}$$

$$
\begin{aligned}
C_m &= C_{m_{ac_{wb}}} + C_L \cdot \frac{L_{cg}}{c} - \frac{C_L S_H l_H}{Sc} \\
C_m &= C_{m_{ac_{wb}}} + C_L \frac{l_{c.g.}}{c} - C_{L_H} V_H \\
\frac{dC_m}{d\alpha} &= 0 + \frac{dC_L}{d\alpha}\frac{l_{c.g.}}{c} - \frac{dC_{L_H}}{d\alpha} V_H \\
\frac{dC_{L_H}}{d\alpha} &= \frac{dC_{L_H}}{d\alpha_H} \cdot \frac{d\alpha_H}{d\alpha} \\
\frac{dC_{L_H}}{d\alpha} &= 1 - \frac{d\epsilon}{d\alpha} \\
C_{m_\alpha} &= C_{L_\alpha} \cdot \frac{l_{c.g.}}{c} - C_{L_{\alpha_H}} \left(1 - \frac{d\epsilon}{d\alpha}\right) V_H \\
C_{m_\alpha} &= a \cdot \frac{l_{c.g.}}{c} - a_t \left(1 - \frac{d\epsilon}{d\alpha}\right) V_H < 0 \\
a \frac{l_{c.g.}}{c} &< a_t \left(1 - \frac{d\epsilon}{d\alpha}\right) V_H \\
\frac{l_{c.g.}}{c} &< \underbrace{\frac{a_t}{a} \left(1 - \frac{d\epsilon}{d\alpha}\right) V_H}_{\text{neutral point } \frac{l_{n.p.}}{c}} \\
\frac{l_{c.g.}}{c} &< \frac{l_{n.p}}{c}
\end{aligned}
$$

$l_{n.p.}$ is the center of gravity location where $C_{m_\alpha} = 0$

Distance between $l_{c.g.}$ and $l_{n.p.}$ is called the static margin.

4.3. Homework

□Anderson 7.1 For a given wing-body combination, the aerodynamic center lies 0.03 chord length ahead of the center of gravity. The moment coefficient about the center of gravity is 0.0050, and the lift coefficient is 0.50. Find the moment coefficient about the aerodynamic center.

$$M = M_{ac} + L \cdot l_{c.g.}$$
$$C_m = C_{m_{ac}} + C_L \cdot \frac{l_{c.g.}}{c}$$
$$C_{m_{ac}} = C_m - C_L \cdot \frac{l_{c.g.}}{c}$$
$$= 0.0050 - 0.50 \cdot 0.03$$
$$\boxed{C_{m_{ac}} \approx -0.010}$$

□Anderson 7.2 Consider a wing-body shape in a wind tunnel under standard sea-level flow conditions. The velocity is 100m/s. $A = 1.5m^2$, $c = 0.45m$. Moment about the center of gravity when the lift is zero is -12.4Nm. At another angle of attack, the lift and moment about the center of gravity become 3675N and 20.67Nm respectively. Find the value of the moment coefficient about the aerodynamic center and the location of the aerodynamic center.

$$M = M_{ac} + L \cdot l_{c.g.}$$
$$M = M_{ac} + \cancelto{0}{L \cdot l_{c.g.}}$$
$$M_{ac} = M$$
$$C_{m_{ac}} = \frac{M}{qSc} = \frac{2M}{\rho V^2 Sc}$$

$$= \frac{2 \cdot -12.4}{1.225 \cdot 100^2 \cdot 1.5 \cdot 0.45} = -0.002999244 \ldots$$
$$\boxed{C_{m_{ac}} \approx -0.0030}$$

$$M = M_{ac} + L \cdot l_{c.g.}$$
$$l_{c.g.} = \frac{M - M_{ac}}{L}$$
$$= \frac{20.67 - (-12.4)}{3675}$$
$$= 0.008998639 \ldots$$
$$\boxed{l_{c.g.} \approx 0.9cm}$$

□Anderson 7.3 Consider the model from Anderson 7.2. If mass is added to the rear of the model so that the center of gravity is shifted rearward by a length equal to 20% of the chord, calculate the moment about the center of gravity when the lift is 4000N.

$$M = M_{ac} + L \cdot l_{c.g.}$$
$$= M_{ac} + L \cdot \left(l_{c.g._0} + \frac{c}{5}\right)$$
$$= -12.4 + 4000 \cdot \left(0.0089986 \cdots + \frac{0.45}{5}\right)$$
$$= 383.5945578 \ldots$$
$$\boxed{M \approx 383.6Nm}$$

□Anderson 7.4 Consider the model from Anderson 7.2. Assume that a horizontal tail without elevator is added. The distance between the center of gravity of the plane to the tail's aerodynamic center is 1.0m. The area of the tail is $0.4m^2$ and the tail-setting angle (= trim angle) is 2.0°. The lift slope of the tail is 0.12 per degree. From measurements, $\epsilon_0 = 0$ and $\frac{\partial\epsilon}{\partial\alpha} = 0.42$. If the absolute angle of attack of the model is 5°and the lift at this angle of attack is 4134N, calculate the moment about the center of gravity.

$$L_H = 1.0m \qquad \epsilon_0 = 0$$
$$A_H = 0.4m^2 \qquad \frac{d\epsilon}{d\alpha} = 0.42$$
$$i_H = 2^\circ \qquad \frac{dC_L}{d\alpha_H} = 0.12$$
$$\alpha = 5^\circ \qquad \frac{dC_L}{d\alpha_H} = 0.12$$
$$L = 4134$$

$$\begin{aligned}
\epsilon &= \epsilon_0 + \frac{d\epsilon}{d\alpha}\alpha = 0.42\alpha \\
\alpha_H &= \alpha - \epsilon + i_H \\
&= 5 - 0.42 \cdot 5 + 2 \\
\alpha_H &= 4.9^\circ \\
C_{L_H} &= \frac{dC_{L_H}}{d\alpha_H}\alpha_H \\
&= 0.12 \cdot 4.9 = 0.588 \\
L_H &= \frac{1}{2}\rho V^2 A_H C_{L_H} \\
&= \frac{1}{2} \cdot 1.225 \cdot 100^2 \cdot 0.4 \cdot 0.588 \\
L_H &= 1440.6N \\
M &= M_{ac_{wb}} + L_{wb} l_{c.g.} - L_H(l_H - l_{c.g.}) \\
&= M_{ac_{wb}} + L \cdot l_{c.g.} - L_H(l_H - l_{c.g.}) \\
&= -12.4 + (4134) \cdot 0.008996 \cdots - 1440.6 \cdot (1.0 - (-0.09)) \\
&= -1428.763064 \ldots
\end{aligned}$$

$$\boxed{M \approx -1429Nm}$$

Anderson 7.5 Consider the wing-body-tail model from Anderson 7.4. Does this model have longitudinal static stability and balance? It would have longitudinal stability if $\frac{dC_m}{d\alpha} < 0$.

$$\begin{aligned}
\frac{dC_m}{d\alpha} &= C_{m_\alpha} = C_{L_\alpha} \cdot \frac{l_{c.g.}}{c} - V_H \cdot C_{L_{\alpha_H}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \\
C_{L_\alpha} &= \frac{L}{qS\Delta\alpha} \\
V_H &= \frac{S_H l_H}{Sc} \\
C_{m_\alpha} &= \frac{L}{qS\Delta\alpha} \cdot \frac{l_{c.g.}}{c} - \frac{S_H l_H}{Sc} \cdot C_{L_{\alpha_H}}\left(1 - \frac{d\epsilon}{d\alpha}\right) \\
&= \frac{4134}{\frac{1}{2} \cdot 1.225 \cdot 100^2 \cdot 1.5 \cdot 5} \cdot \frac{0.008998639 \ldots}{0.45} - \frac{0.4 \cdot 1.0}{1.5 \cdot 0.45} \cdot 0.12 \cdot (1 - 0.42) \\
&= -0.03944488 \ldots
\end{aligned}$$

$\boxed{C_{m_\alpha} \approx -0.0394}$ < 0 so it's stable

Anderson 7.6 For the configuration of 7.4, calculate the neutral point and static margin if $\frac{l_{c.g.}}{c} = 0.26$

$$
\begin{aligned}
\frac{l_{c.g.}}{c} &= 0.26 \implies l_{c.g.} = 0.26c \\
\frac{l_{c.g.}}{\cancel{c}} &= \frac{a_t}{a}\left(1 - \frac{d\epsilon}{d\alpha}\right)\frac{S_H l_H}{S\cancel{c}} \\
l_{n.p.} &= \frac{0.12 \cdot 1.225 \cdot 100^2 \cdot 1.5 \cdot 5}{2 \cdot 4134}(1 - 0.42)\frac{0.4 \cdot 1.0}{1.5} \\
&= 0.206240929 \ldots \\
l_{n.p.} &\approx 0.206m \\
\text{static margin} &= l_{c.g.} - l_{n.p.} = 0.206240929 \cdots - 0.45 \cdot 0.26 \\
&= 0.089240929 \ldots \\
&\approx 8.9cm
\end{aligned}
$$

5

General Intro Lecture 5

5.1. Hour 1

When deriving the moments formula, always make a drawing. The signs can change depending on the setup of the aircraft.

Spent some time reviewing the previous lecture

 Do the derivation of the moments formula for a canard setup (you can assume that the canard doesn't create a downwash as it is a symmetrical airfoil and doesn't really need to create lift (*other than for balancing purposes*))

One advantage of a canard setup for civilian aircraft is the stall behaviour. When a canard setup stalls, the main wing will push the nose down again to exit the stall. With a conventional setup, you will lose more control.

5.2. Hour 2: Propulsion

High school math and physics:

$$F = ma = \frac{dmV}{dt}$$
$$W = F\,\Delta x$$
$$P = \frac{dW}{dt} = \frac{dU}{dt}$$
$$U_{kin} = \frac{1}{2}mv^2$$
$$pV = nRT$$

5.2.1. Jet propulsion

V_0 is the speed at which the air enters the engine (due to the speed the aircraft flies at).

$$T = \dot{m}(V_j - V_0)$$
$$\dot{m} = \rho A_{inlet} V_0 \text{ (mass flow rate [kg/s])}$$
$$F = ma = \dot{m}\Delta t \cdot \frac{\Delta V}{\Delta t} = \dot{m}(V_j - V_0)$$

5.2.2. Propeller

Angular velocity is constant, but air velocity varies with radius. Because of this, the propeller is twisted to achieve optimal angle of attack for the local airspeed (higher α near center, lower α toward outer side).

$$W = T \cdot \Delta s$$
$$P_a = \frac{T \cdot \Delta s}{\Delta t} = TV$$

Power you put in using engine is P_{br} Propulsive efficiency $\eta = \frac{P_a}{P_{br}}$

$$
\begin{aligned}
W &= F\Delta x = p \cdot \underbrace{A \cdot \Delta x}_{\text{Volume}} \\
&= p\Delta Vol \\
W &= pdV \\
pV &= nRT \\
p &= \frac{nRT}{V}
\end{aligned}
$$

Jet engine continuous combustion: Intake → compression → work → exhaust

Pure jet engine has no bypassing air. Most modern civilian engines are turbofan engines, where the turbine of the

$$
\begin{aligned}
\eta_{jet} &= \frac{TV_0}{\frac{1}{2}\dot{m}V_j^2 - \frac{1}{2}\dot{m}V_0^2} \\
&= \frac{\cancel{\dot{m}}(V_j - V_0)V_0}{\frac{1}{2}\cancel{\dot{m}}(V_j^2 - V_0^2)} \\
&= \frac{2\cancel{(V_j - V_0)}V_0}{\cancel{(V_j - V_0)}(V_j + V_0)} \\
&= \frac{2V_0}{V_j + V_0} \\
\eta_{jet} &= \frac{2}{\frac{V_j}{V_0} + 1}
\end{aligned}
$$

By including bypass air, you reduce the V_j and therefore increase the efficiency.

5.3. Homework

Problem 1a Calculate the air density at FL350 to show that it equals 0.3796 kg/m^3.

$$T_1 = T_0 + a(h - h_0) \tag{5.1}$$

$$p_1 = p_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} \tag{5.2}$$

$$\rho_1 = \frac{p_1}{RT_1} \tag{5.3}$$

$$\boxed{\rho_1 = \frac{p_0\left(\frac{T_0 + a(h-h_0)}{T_0}\right)^{-\frac{g_0}{aR}}}{R(T_0 + a(h - h_0))} \quad \left[\frac{kg}{m^3}\right]} \tag{5.4}$$

Problem 1b Calculate the required available power per engine for the cruise at the given cruise speed and altitude. For steady, horizontal flight $T = D$, therefore

6

General Intro Lecture 6

6.1. Hour 1

Equation for drag polar (Eq. 3.4) and ISA will be given.

6.1.1. Jacco Hoekstra's favourite aerospace movies

This WILL be on the exam

- Flight of the Intruder (Vietnam, A-6 from aircraft carrier)
- Chevaliers du Ciel (skyfighters, mirage 2000)
- The right stuff (USAF test pilots)
- From the earth to the moon
- Bat*21 (Cessna Skymaster over Vietnam)

Analog flight instruments were replaced by electronic flight instruments in order to save weight. 747-400 cockpit is usually used as a reference cockpit.

- Top panel with the many buttons is circuit breakers that control the electronics of the plane. If some electronic system doesn't work you can switch if on/off here.
- Overhead control panel controls things like APU and oxygen.
- Mode control panel controls the autopilot settings.
- Display control panel contains
- Standby instruments are backup instruments in case the main instruments fail.
- Primary flight display contains most of the important flight data.
- Navigation Display pretty self explanatory.
- Upper EICAS and Lower EICAS shows status like fuel and errors.
- Command & Display Unit - Flight management system that allows you to select routes (can use these in mode control panel)

Newer airplanes also have Head-Up Displays (HUDs) that show most of the essential flight information.

Modern fighter cockpits use Helmet-Mounted Displays (HMDs)

6.1.2. Units in aviation

- Circumference of earth is 40000km
- 1nm = 1 minumte = 40000/360/60 = 1852m
- 1kts = 1nm/hr = 1852/3600 = 0.51444...m/s
- $M = \frac{V_{TAS}}{a}$ with speed of sound $a = \sqrt{\gamma RT}$ and γ=1.40
- 1ft = 0.3048m
- 1ft/min = 0.3048/60 =

Angle of the shockwave caused by supersonic flight is called the Mach angle:

$$\sin\mu = \frac{a\cancel{\Delta t}}{V\cancel{\Delta t}} = \frac{a}{V} = \frac{1}{M}$$

$$M = \frac{1}{\sin\mu}$$

6.2. Hour 2

$$p_{static} + \frac{1}{2}\rho V^2 = p_{static_{V=0}}$$
$$p_{st} + q = p_{tot}$$
$$q = p_{tot} - p_{st}$$
$$\frac{1}{2}\rho V^2 = q$$
$$V = \sqrt{\frac{2q}{\rho}} = \sqrt{\frac{2(p_{tot} - p_{st})}{\rho}}$$

Equivalent airspeed: V_{EAS} is kind of like a fake airspeed.

$$\frac{1}{2}\rho_0 V_{EAS}^2 = \frac{1}{2}\rho V_{TAS}^2$$
$$C_L\frac{1}{2}\rho_0 V_{EAS}^2 S = C_L\frac{1}{2}\rho V_{TAS}^2 S$$

Equivalent airspeed is more useful for pilots than the true airspeed, since for the equivalent airspeed all of the airplane's performance data is independent of altitude (because ρ_0 is used).

Note:-
For low speed, EAS = CAS =IAS

$$\boxed{q = \frac{1}{2}\rho_0 V_{EAS}^2 = \frac{1}{2}\rho V_{TAS}^2}$$
$$V_{EAS} = \sqrt{\frac{\rho}{\rho_0}}V_{TAS}$$
$$V_{TAS} = \sqrt{\frac{\rho_0}{\rho}}V_{EAS}$$

Flight level is always relative to 101325Pa
QNH is (local) real pressure at sea level
Altitude in ft is altitude adjusted for QNH

Control loops are used to model the control chain for flight control. In manual mode, the pilot controls the flight control systems (FCS) which control the aircraft. In simple autopilot mode, the pilot controls the mode selector and the mode selector controls the FCS which control the aircraft. It is also possible for the pilot to select a route in the FMS, which makes the airplane select and execute everything by itself, all the way to the runway at the destination.

6.2.1. Propulsion 2

$$\rho \cdot T \text{ at FL130} = 13000ft \cdot 0.3048 = h \approx 3962.4m$$

$$T = T_0 + a(h_1 - h_0) = 288.15 - 0.0065h \approx 262.3944K$$

$$p_1 = p_0\left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}} = p_0\left(\frac{T_0 + a(h_1 - h_0)}{T_0}\right)^{-\frac{g_0}{aR}}$$

$$p_1 = 101325\left(\frac{288.15 - 0.0065 \cdot 13000 \cdot 0.3048}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}}$$

$$\approx 61937.2358Pa$$

$$p_1 = \rho_1 R T_1 \implies \rho_1 = \frac{p_1}{RT_1}$$

$$\rho_1 = \frac{p_0\left(\frac{T_0 + a(h_1 - h_0)}{T_0}\right)^{-\frac{g_0}{aR}}}{287(T_0 + a(h_1 - h_0))} \approx 0.822461\frac{kg}{m^3}$$

$$a = \sqrt{\gamma RT} = \sqrt{\gamma R(T_0 + a(h_1 - h_0))}$$

$$C_L = \frac{mg}{\frac{1}{2}\rho V^2 S}$$

$$= \frac{m \cdot g_0}{\frac{1}{2} \cdot \frac{p_0\left(\frac{T_0 + a(h_1 - h_0)}{T_0}\right)^{-\frac{g_0}{aR}}}{287(T_0 + a(h_1 - h_0))} \cdot V^2 \cdot S}$$

$$\approx 0.44729$$

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e}$$

$$\approx 0.0227$$

7

Structures Lecture 1

7.1. Hour 1

What is an aircraft structure? The ”stressed” part of the aircraft. Kind of like a skeleton. It’s functions are to carry loads, protect the interior, maintain aerodynamic shape, and connect subsystems.

Five types of load Tension, compression, torsion, shear, bending

Requirements of an aircraft structure Strength, stiffness, lightweight, durable, cost-effective, maintainable. Strength describes the force the structure can take before breaking, whereas stiffness is a structure’s resistance to bending.

For a truss, the structure has to be made out of pin joints, and the forces can only be applied at the pins. Cables or rods are used to prevent the truss from sliding. Newer wings usually use beam structures, where caps resist the bending and the web resists shearing.

7.2. Hour 2

> **Note:-**
> Aluminum makes a funny noise when shaken

Compression forces cause local buckling → shear buckling (≠ failure)
Plastic buckling = permanent deformation

> **Note:-**
> Airplanes are delicate and fragile, but they’re also cute and pretty and precious and they deserve the world.

7.2.1. Principal Structural Elements

An airframe is the skeleton of an aircraft (including the skin).
A primary structure or Principal Structural Element (PSE) is a critical load-bearing structure. Failure is catastrophic. A secondary structure or Non-Principal Structural Element is a structure whose failure is not catastrophic.

8

Studio Classroom 1

8.1. Exercises

Altitudes:

- 2475m
- 13025m
- 22700m

$$T_1 = T_0 + a(h_1 - h_0)$$

$$T_1 = 288.15 - 0.0065 \cdot 2475$$

$$\boxed{T_1 = 272.0625K}$$

$$p_1 = p_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g_0}{aR}}$$

$$p_1 = 101325 \cdot \left(\frac{288.15 - 0.0065 \cdot 2475}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}}$$

$$p_1 = 74913.23859 \ldots$$

$$\boxed{p_1 \approx 74913.2Pa}$$

$$\rho_1 = \frac{p}{RT}$$

$$\rho_1 = \frac{101325 \cdot \left(\frac{288.15 - 0.0065 \cdot 2475}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}}}{287 \cdot 288.15 - 0.0065 \cdot 2475}$$

$$\rho_1 = 0.959418285 \ldots$$

$$\boxed{\rho_1 \approx 0.959 \frac{kg}{m^3}}$$

$$
\begin{aligned}
T_{start,tropo} &= T_0 + a(h_1 - h_0) \\
T_{start,tropo} &= 288.15 - 0.0065 \cdot 11000 \\
T_{start,tropo} &= 216.65K \\
T_2 = T_{start,tropo} &= 216.65K \text{ (isothermal)} \\
p_{start,tropo} &= p_1 \left(\frac{T_{start,tropo}}{T_0}\right)^{-\frac{g_0}{aR}} \\
&= 101325 \cdot \left(\frac{216.65}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}} \\
p_{start,tropo} &= 22625.79149 \ldots \\
&\approx 22626Pa \\
\rho_{start,tropo} &= \frac{p_{start,tropo}}{RT_{start,tropo}} \\
&= \frac{22625.79149 \ldots}{287 \cdot 216.65} \\
\rho_{start,tropo} &= 0.363884193 \ldots \\
\rho_{start,tropo} &\approx 0.364 \frac{kg}{m^3}
\end{aligned}
$$

$$\boxed{T_2 = 216.65K}$$

$$
\begin{aligned}
p_2 &= 101325 \cdot \left(\frac{216.65}{288.15}\right)^{-\frac{9.80665}{-0.0065 \cdot 287}} \cdot e^{-\frac{9.80665}{287 \cdot 216.65}(13025-11000)} \\
&= 16439.91775 \ldots
\end{aligned}
$$

$$\boxed{p_2 \approx 16440Pa}$$

$$
\begin{aligned}
\rho_2 &= \frac{p_2}{RT_2} \\
&= \frac{16439.91775 \ldots}{287 \cdot 216.65} = 0.264398539 \ldots
\end{aligned}
$$

$$\boxed{\rho_2 \approx 0.264 \frac{kg}{m^3}}$$

$$
\begin{aligned}
T_{start,strato} &= T_2 = 288.15K \\
p_{start,strato} &= p_{start,tropo} \cdot e^{-\frac{g_0}{RT_2}(h_{start,strato} - h_{start,tropo})} \\
p_{start,strato} &= 22625.79149 \ldots \cdot e^{-\frac{9.80665}{287 \cdot 216.65}(20000-11000)} \\
p_{start,strato} &= 5471.935072 \ldots \\
p_{start,strato} &\approx 5472Pa \\
\rho_{start,strato} &= \frac{p_{start,strato}}{RT_{start,strato}} \\
&= \frac{5471.935072 \ldots}{287 \cdot 216.65} = 0.088003581 \ldots \\
\rho_{start,strato} &\approx 0.088 \frac{kg}{m^3}
\end{aligned}
$$

$$
\begin{aligned}
T_3 &= T_{start,strato} + a \cdot (h_3 - h_{start,strato}) \\
&= 216.65 + 0.001 \cdot (22700 - 20000) \\
&= 219.35
\end{aligned}
$$

$$\boxed{T_3 \approx 219.35K}$$

$$
\begin{aligned}
p_3 &= p_{start,strato} \left(\frac{T_3}{T_{start,strato}} \right)^{-\frac{g_0}{aR}} \\
&= 5471.935072 \ldots \cdot \left(\frac{219.35}{216.65} \right)^{-\frac{9.80665}{0.001 \cdot 287}} \\
&= 3583.810158 \ldots
\end{aligned}
$$

$$\boxed{p_3 \approx 3584Pa}$$

$$
\begin{aligned}
\rho_3 &= \frac{p_3}{RT_3} \\
&= \frac{3583.810158 \ldots}{287 \cdot 219.35} \\
&= 0.056927939 \ldots
\end{aligned}
$$

$$\boxed{\rho_3 \approx 0.0569 \frac{kg}{m^3}}$$

$$
\begin{aligned}
H &= \frac{R_e}{R_e + H_G} H_G \\
H_G &= \frac{H \cdot R_e}{R_e - H} \\
&= \frac{22700 \cdot 6371 \times 10^3}{6371 \times 10^3 - 22700} \\
&= 22781.16976 \ldots
\end{aligned}
$$

$$\boxed{H_G \approx 22781m}$$

9

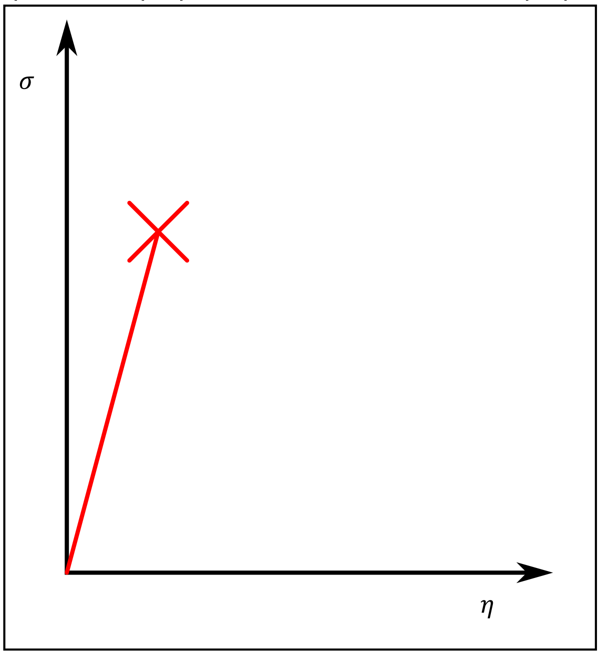
Structures Lecture 2

9.1. Hour 1

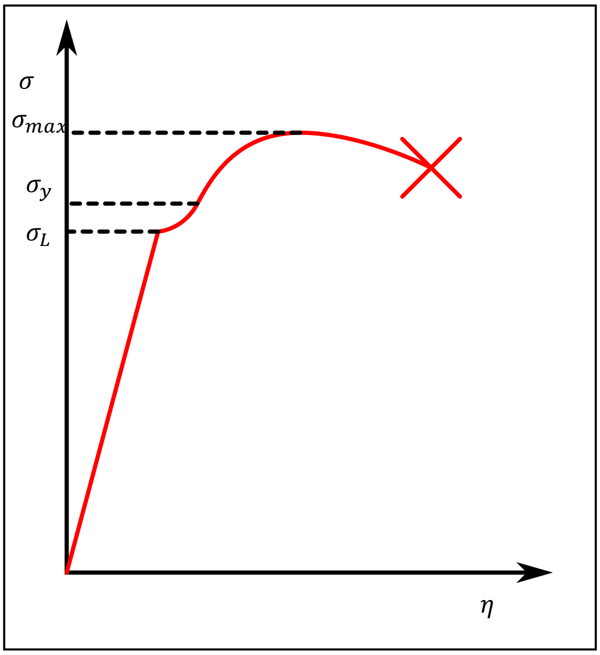
Flight envelope correlates speed to max loading at that speed (V,n-graph)
Limit load is a once-in-a-lifetime load which is not allowed to cause residual damage.
Ultimate load is the limit load × safety factor, and failure is allowed after 3 seconds.

9.1.1. Stress and strain

Stress and strain separate the properties of a material from the properties of its geometry.



For design, typically keep strains below 30% of ultimate strength. This is to account for a non-uniform stress distribution.



Note:-
Stress is **NOT** a force. You cannot add up stresses to obtain equilibrium.

Horizontal component of P on dA:

$$p \sin\theta \; dA \implies R_p = \int_0^{180^\circ} (bRd\theta)$$
$$= p(2Rb)$$

$$\sum \vec{F}_{x \overset{+}{\leftarrow}} : 0 = R_p - \sigma_{long}(2\pi r t)$$
$$R_p = pA_{projected} = p\pi r^2$$
$$\boxed{\sigma_{long} = \frac{pr}{2t}}$$

9.2. Hour 2

Multi-bubble design uses two different radii, but you can use the same equations for the σ_{hoop} and σ_{long}. It's used to minimize wasted space you get when scaling up the diameter.

Main material classifications:

- **Metals**: Malleable and ductile
- **Ceramics**: Brittle, high-strength
- **Polymers**: Low strength and stiffness
- **Composites**: Fake material but it's fine

9.2.1. Metals

Isotropic (same properties in all directions)
Can be strengtened
Plastic behaviour; can melt
Good processability
Often low cost
Can be reused/recycled

9.2.2. Ceramics

Usually too brittle to be used in loadings other than compression. Concrete (beton) can be cool though, like, reinforced concrete is kinda funky.

9.2.3. Polymers

Two classifications: Thermoplastics (pasta), and thermosets (organised). The thermoplastics have separated chains that are piled on top of each other, whereas thermosets have interconnected chains. The separated chains of thermoplastics causes a lower stiffness, as you don't have to break bonds.

Thermoplastics can melt, thermosets can't melt. Isotropic
Low strength, stiffness
Huge variety
Plastic flow and melt-able (thermoplastics)
Good processability
Low costs (often)

Polymers have two solid states: one at low temperatures called the glassy state, and one at higher temperature called the rubbery state.

9.2.4. Composites

Anisotropic (different properties in different directions)
Layered structure
(Relatively) expensive
Elastic-brittle deformation
Future: recyclable

10

Aerodynamics Lecture 1

10.1. Hour 1

Fundamental properties of flow are properties of the fluid (pressure, density, temperature) and properties of the motion (velocity).

Pressure
At a macroscopic level, pressure is the limit of $\frac{F}{A}$ when A goes to 0.
At a microscopic level, pressure is the result of molecules impacting upon the surface.

$$p = \frac{N \cdot m \cdot v_n^2}{3V}$$

Density
Macroscopic: Density is the limit of the $\frac{m}{V}$ as V approaches 0.
Microscopic: Density is rather straight forward.

$$\rho = \frac{N \cdot m}{V}$$

Specific volume
Volume V is just good old standard volume.
Specific volume v is the volume occupied by a unit mass of matter

$$v = \frac{V}{m} = \frac{1}{\frac{m}{V}} = \frac{1}{\rho}$$

Temperature
Macroscopic: Temperature is a measure of gas internal energy e

$$e = c_v T$$

Microscopic: Temperature is the kinetic energy of gas molecules random motion

$$KE = \frac{3}{2}kT$$

10.1.1. Equation of state

A **perfect gas** is a gas where intermolecular forces are negligible.

$$p = \rho RT \tag{10.1}$$
$$pv = RT \tag{10.2}$$

10.2. Hour 2

$$R = \frac{\mathcal{R}}{M}$$

Standard conditions for air:

$$p_s = 1.01325 \times 10^5 \frac{N}{m^2}$$
$$\rho_s = 1.225 \frac{kg}{m^3}$$
$$T_s = 288.15K = 15°C$$

10.2.1. Velocity and streamlines

When you look at the velocity, you look at an infinitesimally small cube of air that is still big enough to comply with the continuum hypothesis.
Streamlines show the path that particles move through. In steady flow, particles cannot cross these streamlines.
At the front of a wing, there is a point called the *stagnation point* where the velocity reaches zero ($V = 0$).

Note:-
All aerodynamic forces originate from pressure and wall shear.

10.2.2. Flow equations

Continuity equation
Conservation of mass → mass flow remains constant across a stream-tube.

$$\dot{m} = \text{mass flow (kg/s)} = \rho V A$$

For steady fluid flow:

$$\dot{m}_{in} = \dot{m}_{out} \implies \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

For incompressible flow: ρ = constant therefore

$$VA = \text{constant}$$

$\frac{V_2}{V_1}$ is called contraction number.

Euler Equation
Newton's Second Law applied to a moving fluid particle

$$\sum \vec{F}_x \overset{+}{\rightarrow} : F = pA - (pA + dpA) \tag{10.3}$$
$$F = p\,dy\,dz - (p + dp)\,dy\,dz \tag{10.4}$$
$$dp = \frac{dp}{dx} dx \tag{10.5}$$
$$F = p\,dy\,dz - \left(p + \frac{dp}{dx} dx\right) dy\,dz \tag{10.6}$$
$$F = -\frac{dp}{dx} dx\,dy\,dz \tag{10.7}$$

$$m = \rho V = \rho\,dx\,dy\,dz \tag{10.8}$$
$$a = \frac{dV}{dt} = \frac{dV}{dx}\frac{dx}{dt} = \frac{dV}{dx} V \tag{10.9}$$

$$-\frac{dp}{dx}\cancel{(dxdydzC)} = \rho\cancel{(dxdydz)}V\frac{dV}{dx} \tag{10.10}$$

$$-\frac{dp}{dx} = \rho V\frac{dV}{dx} \tag{10.11}$$

$$dp = -\rho V\,dV \tag{10.12}$$

Assumptions made: steady flow, inviscid flow, gravity neglected.

Note:-

Gravity is fair to neglect when dealing with air, since there the variation in pressure due to gravity is smaller. When dealing with other (heavier) liquids, ignoring gravity can become an issue.

10.3. Homework

□Problem 2.1 Low-speed flight of shuttle close to landing. Air pressure and temperature at nose are 1.2 atm and 300K, respectively. Find the density and specific volume. From equation of state (Eq. 10.1):

$$p = \rho RT \tag{10.13}$$

$$\rho = \frac{p}{RT} \tag{10.14}$$

$$v = \frac{1}{\rho} = \frac{RT}{p} \tag{10.15}$$

$$\boxed{\rho = \frac{1.2\times10^5}{287\cdot300}\left[\frac{kg}{m^3}\right]} \tag{10.16}$$

$$\boxed{v = \frac{287\cdot300}{1.2\times10^5}\left[\frac{m^3}{kg}\right]} \tag{10.17}$$

□Problem 2.2 Consider 1kg of helium at 500K. Assuming that the total internal energy of helium is due to the mean kinetic energy of each atom summed over all the atoms, calculate the internal energy e of this gas.

$$E_{kin,molecule} = \frac{3}{2}kT \tag{10.18}$$

$$e = \frac{n_{avogadro}}{M_{He}}\cdot E_{kin,molecule} \tag{10.19}$$

$$e = \frac{n_{avogadro}}{M_{He}}\cdot\frac{3}{2}kT \tag{10.20}$$

$$e = \frac{3}{2}\frac{6.02\times\cancel{10^{23}}}{4}\cdot1.38\times\cancel{10^{-23}}\cdot500 \tag{10.21}$$

$$\boxed{e = \frac{3\cdot500\cdot6.02\cdot1.38}{8}\left[\frac{J}{kg}\right]} \tag{10.22}$$

□Problem 2.7 Assume that, at a point on the wing of the Concorde, the air temperature is $-10°C$ and the pressure is 1.7×10^4 N/m^2. Calculate the density at this point. From the Equation of State (Eq. 10.1):

$$\rho = \frac{p}{RT} \tag{10.23}$$

$$\boxed{\rho = \frac{1.7\times10^4}{287\cdot(-10+273.15)}} \tag{10.24}$$

□Problem 2.8 At a point in the test section of a supersonci wind tunnel, the air pressure and temperature are 0.5×10^5 N/m^2 and 240K, respectively. Calculate the specific volume. From the Equation of state (Eq. 10.1):

$$v = \frac{RT}{p} \tag{10.25}$$

$$\boxed{v = \frac{287 \cdot 240}{0.5 \times 10^5} \left[\frac{m^3}{kg}\right]} \tag{10.26}$$

□Problem 4.3 Consider an airplane flying with a velocity of 60 m/s at a standard altitude of 3km. At a point on the wing, the airflow velocity is 70 m/s. Calculate the pressure at this point. Assume incompressible flow (well duh, it's M<0.3 xddxdxdxdxdxd)

$$T = T_0 + a(h - h_0) \tag{10.27}$$

$$T = 288.15 - 0.0065(3000)\ [K] \tag{10.28}$$

$$p = p_0 \left(\frac{T}{T_0}\right)^{-\frac{g}{aR}} \tag{10.29}$$

$$\rho = \frac{p}{RT} \tag{10.30}$$

$$p_1 + \frac{1}{2}\rho_1 V_1^2 = p_2 + \frac{1}{2}\rho_2 V_2^2 \tag{10.31}$$

$$\boxed{p_2 = p_1 + \frac{1}{2}\rho(V_1^2 - V_2^2)} \tag{10.32}$$

$$p_2 = p_0 \left(\frac{T_0 + a(h - h_0)}{T_0}\right)^{-\frac{g}{aR}} + \frac{1}{2}\frac{p_0\left(\frac{T_0 + a(h-h_0)}{T_0}\right)^{-\frac{g}{aR}}}{R(T_0 + a(h - h_0))}(V_1^2 - V_2^2) \tag{10.33}$$

The last result is too much work to calculate and, I mean, if we're assuming incompressible flow anyway who really cares. Just use the boxed one and throw some rounded numbers at it. In the end it's all about having fun anyway so yeah. Just follow your heart and everything, stay true to yourself. Never change who you are because of someone else.

□Problem 4.5 Flow through convergent-divergent duct. Inlet, throat, and exit areas are 3, 1.5, and 2m^2, respectively. Inlet and exit pressures are 1.02×10^5 and 1.00×10^5N/m^2, respectively. Find the flow velocity at the throat.

$$A_1 = 3 \qquad A_2 = 1.5 \qquad A_3 = 2\ [m^2] \tag{10.34}$$

$$p_1 = 1.02 \times 10^5 \qquad p_3 = 1.00 \times 10^5 \left[\frac{N}{m^2}\right] \tag{10.35}$$

$$\rho = 1.225$$

$$A_1\rho_1V_1 = A_2\rho_2V_2 = A_3\rho_3V_3 \tag{10.36}$$

(10.37)

$$V_3 = \frac{A_1}{A_3}V_1 \tag{10.38}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_3 + \frac{1}{2}\rho V_3^2 \tag{10.39}$$

$$p_1 - p_3 = \frac{1}{2}\rho(V_3^2 - V_1^2) \tag{10.40}$$

$$\frac{2(p_1 - p_3)}{\rho} = \left(\frac{A_1}{A_3}V_1\right)^2 - V_1^2 \tag{10.41}$$

$$V_1^2\left(\left(\frac{A_1}{A_3}\right)^2 - 1\right) = \frac{2(p_1 - p_3)}{\rho} \tag{10.42}$$

$$V_1 = \sqrt{\frac{2(p_1 - p_3)}{\rho\left(\left(\frac{A_1}{A_3}\right)^2 - 1\right)}} \tag{10.43}$$

$$V_2 = \frac{A_1}{A_2}V_1 \tag{10.44}$$

$$\boxed{V_2 = \frac{A_1}{A_2}\sqrt{\frac{2(p_1 - p_3)}{\rho\left(\left(\frac{A_1}{A_3}\right)^2 - 1\right)}}} \tag{10.45}$$

$$V_2 = 102.2202504\ldots \tag{10.46}$$

$$V_2 \approx 102.2\ \left[\frac{m}{s}\right] \tag{10.47}$$

11 Aerodynamics Lecture 2

11.1. Hour 1

Note on fans

For an open fan, the streamlines before the fan are spread out wider than the diameter of the fan, so the velocity can go up as the streamlines get more narrow after they pass the fan. For a closed fan, the area is fixed, so the velocity before and after the fan is equal.

Euler Equation (cont.)

$$dp = -\rho V dV \tag{11.1}$$

Integrating the Euler equation (eq. 11.1) gives:

$$\int_{p_1}^{p_2} dp + \int_{V_1}^{V_2} \rho V \, dV = 0 \tag{11.2}$$

$$(p_2 - p_1) + \rho\left(\frac{1}{2}V_2^2 - \frac{1}{2}V_1^2\right) = 0 \tag{11.3}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{11.4}$$

$$\underbrace{p_1}_{\text{Static pressure}} + \underbrace{\frac{1}{2}\rho V^2}_{\text{Dynamic pressure}} = \underbrace{p_t}_{\text{Total pressure}} \tag{11.5}$$

11.1.1. Pitot-static tube

Pitot tube stops air and measures total pressure at stagnation point.
Static port measures atmospheric pressure.
The difference between these two pressures is the dynamic pressure

$$p_t - p_s = \frac{1}{2}\rho V_0^2 \tag{11.6}$$

$$V_0 = \sqrt{\frac{2(p_t - p_s)}{\rho}} \tag{11.7}$$

11.2. Hour 2

11.2.1. Compressibility

Go fast $\implies$ hot
Don't use Bernouilli for fast flow. Don't use perfect gas law either.

11.2.2. Elementary Thermodynamics

Internal energy
e is internal energy of the system (specific energy [J/kg]).
Energy of the system can be changed through work δw or heat δq.

$$\delta q + \delta w = de \tag{11.8}$$

Work on the system

$$W = F \cdot s \tag{11.9}$$
$$W = p\,dA \cdot s \tag{11.10}$$
$$\delta w = \int_A pdAs = p \underbrace{\int_A s\,dA}_{\text{specific volume } dv} \tag{11.11}$$
$$\delta w = -p\,dv \tag{11.12}$$

$$de = \delta q - p\ dv \tag{11.13}$$

Enthalpy
Enthalpy is internal energy + boundary work:

$$h = e + pv$$

$$dh = de + d(pv) \tag{11.14}$$
$$dh = de + p\,dv + v\,dp \tag{11.15}$$
$$dh = \delta q + v\,dp \tag{11.16}$$

11.2.3. Thermodynamic Processes

Constant volume heating describes the process of heating a fluid while keeping the volume constant. This is different from a constant pressure heating process, where the fluid is allowed to expand to keep the pressure at the same level.

$$c_v = \left(\frac{\delta q}{dT}\right)_{v=\text{const}} \tag{11.17}$$
$$c_p = \left(\frac{\delta q}{dT}\right)_{p=\text{const}} \tag{11.18}$$

11.3. Homework

□Problem 2.12 Very long question. Consider constant volume combustion process. Gas density and temperature at the start are 11.3kg/m^3 and 625K, respectively. At the end the temperature is 4000K. Find the gas pressure at the end of the combustion process. From Equation of State (Eq. 10.1):

$$p = \rho RT \tag{11.19}$$
$$p = 11.3 \cdot 287 \cdot 4000\ [Pa] \tag{11.20}$$

□Problem 2.13

$$p = \rho RT \tag{11.21}$$
$$F = pA \tag{11.22}$$
$$F_0 = 11.3 \cdot 287 \cdot 625 \cdot \frac{0.09^2\pi}{4}\ [N] \tag{11.23}$$
$$F_1 = 11.3 \cdot 287 \cdot 4000 \cdot \frac{0.09^2\pi}{4}\ [N] \tag{11.24}$$

□Problem 2.14 Consider the constant pressure heating process inside the combustion chamber of a gas turbine. The gas pressure and temperature at the inlet are 4×10^6 N/m^2 and 900K, respectively.

At the exit the temperature is 1500K. Calculate the gas density at (a) the inlet, and (b) the exit.

$$\rho = \frac{p}{RT} \tag{11.25}$$

$$a) \boxed{\rho_{inlet} = \frac{4 \times 10^6}{287 \cdot 900} \left[\frac{kg}{m^3}\right]} \tag{11.26}$$

$$b) \boxed{\rho_{exit} = \frac{4 \times 10^6}{287 \cdot 1500} \left[\frac{kg}{m^3}\right]} \tag{11.27}$$

□Problem 4.7 You use a venturi tube with a 1.3 : 1 area ratio to measure airspeed in an airplane with a maximum velocity of 90m/s at sea level. What is the maximum pressure difference you expect the gauge to experience? Apply continuity equation assuming incompressible flow:

$$A_1V_1 = A_2V_2 \tag{11.28}$$

$$V_2 = V_1\frac{A_1}{A_2} \tag{11.29}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{11.30}$$

$$p_1 - p_2 = \frac{1}{2}\rho(V_2^2 - V_1^2) \tag{11.31}$$

$$p_1 - p_2 = \frac{1}{2}\rho\left(V_1^2\left(\frac{A_1}{A_2}^2\right) - V_1^2\right) \tag{11.32}$$

$$p_1 - p_2 = \frac{1}{2}\rho V_1^2\left(\left(\frac{A_1}{A_2}\right)^2 - 1\right) \tag{11.33}$$

$$\boxed{p_1 - p_2 = \frac{1}{2} \cdot 1.225 \cdot 90^2 \cdot (1.3^2 - 1)\ [Pa]} \tag{11.34}$$

□Problem 4.8 Consider a supersonic nozzle. At the reservoir, the pressure and temperature are 10 atm and 300 K, respectively. At the nozzle exit, the pressure is 1 atm. Calculate the temperature and density at the flow of the exit. Assume isentropic flow. For isentropic, compressible flow, use the relations obtained in Eq. 12.23:

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^\gamma = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{11.35}$$

$$T_2 = T_1\left(\frac{p_2}{p_1}\right)^{\frac{\gamma-1}{\gamma}} \tag{11.36}$$

$$\boxed{T_2 = 300 \cdot 10^{\frac{1.4-1}{1.4}}\ [K]} \tag{11.37}$$

$$\rho_1 = \frac{p_1}{RT_1} \tag{11.38}$$

$$\rho_2 = \rho_1\left(\frac{p_2}{p_1}\right)^{\frac{1}{\gamma}} \tag{11.39}$$

$$\boxed{\rho_2 = \frac{10 \cdot 101325}{287 \cdot 300} \cdot 10^{\frac{1}{1.4}} \left[\frac{kg}{m^3}\right]} \tag{11.40}$$

□Problem 4.10 Consider an airplane flying at an altitude of 5km with a velocity of 270 m/s. At a point on the wing, the velocity is 330 m/s. Calculate the pressure at this point.

$$T_1 = T_0 + a(h_1 - h_0) \tag{11.41}$$

$$p_1 = p_0 \left(\frac{T_1}{T_0}\right)^{-\frac{g}{aR}} \tag{11.42}$$

$$c_p = \frac{\gamma R}{\gamma - 1} \tag{11.43}$$

$$c_p T_1 + \frac{1}{2}V_1^2 = c_p T_2 + \frac{1}{2}V_2^2 \tag{11.44}$$

$$c_p T_2 = c_p T_1 + \frac{1}{2}(V_1^2 - V_2^2) \tag{11.45}$$

$$T_2 = T_1 + \frac{1}{2c_p}(V_1^2 - V_2^2) \tag{11.46}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{11.47}$$

$$\boxed{p_2 = p_1 \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \quad [Pa]} \tag{11.48}$$

$$p_2 = p_0 \left(\frac{T_0 + a(h_1 - h_0)}{T_0}\right)^{-\frac{g}{aR}} \left(\frac{T_0 + a(h_1 - h_0) + \frac{1}{2c_p}(V_1^2 - V_2^2)}{T_0 + a(h_1 - h_0)}\right)^{\frac{\gamma}{\gamma-1}} \tag{11.49}$$

□Problem 4.15 A Boeing 747 is cruising at a velocity of 250 m/s ata standard altitude of 13km. What is its Mach number? Recall Eq. 12.46:

$$a = \sqrt{\gamma R T} \tag{11.50}$$

$$T_{13km} = T_{11km} = T_0 + a(h - h_0) \tag{11.51}$$

$$T_{11km} = 288.15 - 0.0065 \cdot 11000 = 216.65K \tag{11.52}$$

$$a = \sqrt{1.4 \cdot 287 \cdot 216.65} = 295.0423189\ldots \tag{11.53}$$

$$M = \frac{V}{a} = \frac{270}{295.0423189\ldots} \tag{11.54}$$

$$= 0.9151229590\ldots \tag{11.55}$$

$$\boxed{M \approx 0.91} \tag{11.56}$$

□Problem 4.18 Incomplete question wtf slasdhf sad :(

□Problem 4.26 A high-performance F-16 fighter is flying at Mach 0.96 at sea level. What is the air temperature at the stagnation point at the leading edge of the wing? For stagnation point, the velocity is 0.

$$a = \sqrt{\gamma R T} \tag{11.57}$$

$$c_p T_{stag} + \frac{1}{2}\cancelto{0}{V_{stag}^2} = c_p T_\infty + \frac{1}{2}V_\infty^2 \tag{11.58}$$

$$c_p T_{stag} = c_p T_\infty + \frac{1}{2}\sqrt{\gamma R T_\infty}^2 \tag{11.59}$$

$$\frac{\gamma R}{\gamma - 1} T_{stag} = \frac{\gamma R}{\gamma - 1} T_\infty + \frac{1}{2}\gamma R T_\infty \tag{11.60}$$

$$T_{stag} = T_\infty + \frac{1}{2}\frac{\gamma - 1}{\cancel{\gamma R}}\cancel{\gamma R} T_\infty \tag{11.61}$$

$$T_{stag} = T_\infty + \frac{1}{2}T_\infty(\gamma - 1) \tag{11.62}$$

$$T_{stag} = T_\infty\left(1 + \frac{\gamma - 1}{2}\right) \tag{11.63}$$

$$\boxed{T_{stag} = T_\infty \frac{1 + \gamma}{2}} \tag{11.64}$$

$$T_{stag} = 288.15 \cdot \frac{1 + 1.4}{2} \tag{11.65}$$

$$T_{stag} = 345.78 K \tag{11.66}$$

12

Aerodynamics Lecture 3

12.1. idk what hour, taking these notes from the slides :)

Recall the derivatives of internal energy e and enthalpy h, equations 11.13 and 11.16, respectively. For constant volume heating, the equation for internal energy becomes:

$$de = \delta q - \cancelto{0}{p\,dv} \tag{12.1}$$
$$de = c_v\,dT \tag{12.2}$$
$$e = c_v T \tag{12.3}$$

And for constant pressure, the same equation for enthalpy becomes:

$$dh = \delta q + \cancelto{0}{v\,dp} \tag{12.4}$$
$$dh = c_p\,dT \tag{12.5}$$
$$h = c_p T \tag{12.6}$$

I forgot to mention it in my notes before but the specific heat ratio γ is defined as:

$$\gamma \equiv \frac{c_p}{c_v} \tag{12.7}$$

Plugging equations 12.2 and 12.5 into equation 12.7 gives:

$$\gamma \equiv \frac{c_p}{c_v} = \frac{\left(\frac{dh}{\cancel{dT}}\right)}{\left(\frac{de}{\cancel{dT}}\right)} = \frac{dh}{de} \tag{12.8}$$

Therefore, for an isentropic flow, equation 12.8 can be rewritten to:

$$\gamma = \frac{dh}{de} \tag{12.9}$$
$$\gamma de = dh \tag{12.10}$$
$$\gamma(\cancelto{0}{\delta q} - p\,dv) = \cancelto{0}{\delta q} + v\,dp \tag{12.11}$$
$$v\,dp = -p\gamma\,dv \tag{12.12}$$
$$\frac{dp}{p} = -\gamma\frac{dv}{v} \tag{12.13}$$

12.1.1. Isentropic flow relations

With this, we're able to derive some important relations between pressure, density, and temperature without assuming incompressible flow. From equation 12.13:

$$\frac{dp}{p} = -\gamma \frac{dv}{v} \tag{12.14}$$

$$\int \frac{dp}{p} = -\gamma \int \frac{dv}{v} \tag{12.15}$$

$$\ln p_2 - \ln p_1 = -\gamma(\ln v_2 - \ln v_1) \tag{12.16}$$

$$\ln\left(\frac{p_2}{p_1}\right) = -\gamma \ln\left(\frac{v_2}{v_1}\right) \tag{12.17}$$

$$\frac{p_2}{p_1} = \left(\frac{v_2}{v_1}\right)^{-\gamma} \tag{12.18}$$

$$\frac{p_2}{p_1} = \left(\frac{\rho_1}{\rho_2}\right)^{-\gamma} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{12.19}$$

$$\frac{p_2}{p_1} = \left(\frac{\frac{p_2}{\cancel{R}T_2}}{\frac{p_1}{\cancel{R}T_1}}\right)^{\gamma} = \left(\frac{p_2}{p_1}\frac{T_1}{T_2}\right)^{\gamma} \tag{12.20}$$

$$\left(\frac{p_2}{p_1}\right)^{1-\gamma} = \left(\frac{T_1}{T_2}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{-\gamma} \tag{12.21}$$

$$\frac{p_2}{p_1} = \left(\frac{T_2}{T_1}\right)^{\frac{-\gamma}{1-\gamma}} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \tag{12.22}$$

To summarize, we have derived the following equations that relate the different flow properties to each other for isentropic flow:

$$\boxed{\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}} \tag{12.23}$$

In order to be able to calculate the motion properties as well, we return to Euler's equation (eq. 11.1) and combine it the derivative for enthalpy (eq. 11.16):

$$dh = \delta q + v\ dp \tag{12.24}$$

$$\delta q = dh - v\ dp \tag{12.25}$$

$$\text{From Euler: } dp = -\rho V\ dV \tag{12.26}$$

$$\delta q = dh - \cancel{v}(-\cancel{\rho}V\ dV) \tag{12.27}$$

$$\text{Adiabatic: } \cancelto{0}{\delta q} = dh + V\ dV = 0 \tag{12.28}$$

$$\int dh + \int V\ dV = \text{Constant} \tag{12.29}$$

$$h_1 + \frac{1}{2}V_1^2 = h_2 + \frac{1}{2}V_2^2 \tag{12.30}$$

$$\boxed{c_p T_1 + \frac{1}{2}V_1^2 = c_p T_2 + \frac{1}{2}V_2^2} \tag{12.31}$$

12.1.2. Speed of sound

Set up continuity equation across sound wave:

$$\rho_1 V_1 \cancel{A_1} = \rho_2 V_2 \cancel{A_2} \tag{12.32}$$

$$\rho a = (\rho + d\rho)(a + da) \tag{12.33}$$

$$\cancel{\rho a} = \cancel{\rho a} + a\ d\rho + \rho\ da + \underbrace{\cancelto{0}{d\rho\ da}}_{\text{negligible}} \tag{12.34}$$

$$a = -\rho\ \frac{da}{d\rho} \tag{12.35}$$

Applying Euler equation:

$$dp = -\rho V\ dV \implies dp = -\rho a\ da \tag{12.36}$$

$$da = -\frac{1}{\rho a} dp \tag{12.37}$$

Combining Eq. 12.37 into Eq. 12.35 gives:

$$a = -\cancel{\rho}\frac{-\frac{1}{\cancel{\rho} a} dp}{d\rho} \tag{12.38}$$

$$a^2 = \frac{dp}{d\rho} \tag{12.39}$$

$$a = \sqrt{\frac{dp}{d\rho}} \tag{12.40}$$

By deriving Eq. 12.19 we get

$$\frac{p_2}{p_1} = \left(\frac{\rho_2}{\rho_1}\right)^{\gamma} \tag{12.41}$$

$$\frac{dp}{p} = \gamma \frac{d\rho}{\rho} \tag{12.42}$$

$$\frac{dp}{d\rho} = \gamma \frac{p}{\rho} \tag{12.43}$$

Now we can combine Eq. 12.40, Eq. 12.43, and the Ideal Gas Law (Eq. 10.1)

$$a = \sqrt{\frac{dp}{d\rho}}, \qquad \frac{dp}{d\rho} = \gamma \frac{p}{\rho}, \qquad \frac{p}{\rho} = RT \tag{12.44}$$

$$a = \sqrt{\gamma \frac{p}{\rho}} \tag{12.45}$$

$$\boxed{a = \sqrt{\gamma R T}} \tag{12.46}$$

12.2. Homework

Problem 4.3Consider an airplane flying with a velocity of 60 m/s at a standard altitude of 3 km. At a point on the wing, the airfl ow velocity is 70 m/s. Calculate the pressure at this point. Assume incompressible flow. Incompressible flow means we can use Bernoulli's equation:

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{12.47}$$

$$\tag{12.48}$$

$$T_1 = T_0 + a(h_1 - h_0) \tag{12.49}$$

$$p_1 = p_0 \left(\frac{T_0 + a(h_1 - h_0)}{T_0} \right)^{-\frac{g_0}{aR}} \tag{12.50}$$

$$p_2 = p_1 + \frac{1}{2}\rho V_1^2 - \frac{1}{2}\rho V_2^2 \tag{12.51}$$

$$p_2 = p_1 + \frac{\rho}{2}(V_2^2 - V_1^2) \tag{12.52}$$

$$\boxed{p_2 = p_0 \left(\frac{T_0 + a(h_1 - h_0)}{T_0} \right)^{-\frac{g_0}{aR}} + \frac{\rho}{2}(V_2^2 - V_1^2)} \tag{12.53}$$

$$p_2 = 70900.02017 \ldots [Pa] \tag{12.54}$$

Problem 4.5Consider the flow of air through a convergent–divergent duct, such as the venturi tube described in Prob. 4.4 . The inlet, throat, and exit areas are 3, 1.5, and 2 m2, respectively. The inlet and exit pressures are 1.02 × 105 and 1.00 × 105 N/m2, respectively. Calculate the flow velocity at the throat. Assume incompressible flow with standard sea-level density.

$$A_1\rho_1 V_1 = A_* \rho_* V_* = A_2 \rho_2 V_2 \tag{12.55}$$

$$V_2 = V_1 \frac{A_1}{A_2} \tag{12.56}$$

$$p_1 + \frac{1}{2}\rho V_1^2 = p_2 + \frac{1}{2}\rho V_2^2 \tag{12.57}$$

$$p_1 - p_2 = \frac{\rho}{2}(V_1^2 - V_2^2) \tag{12.58}$$

$$\frac{2}{\rho}(p_1 - p_2) = V_1^2 - V_2^2 \tag{12.59}$$

$$\frac{2}{\rho}(p_1 - p_2) = V_1^2 - \left(V_1 \frac{A_1}{A_2} \right)^2 = V_1^2 \left(1 - \left(\frac{A_1}{A_2} \right)^2 \right) \tag{12.60}$$

$$V_1^2 = \frac{2}{\rho}(p_1 - p_2) \frac{1}{1 - \frac{A_1^2}{A_2^2}} = \frac{2}{\rho}(p_1 - p_2) \frac{A_2^2}{A_2^2 - A_1^2} \tag{12.61}$$

$$V_1 = \sqrt{\frac{2}{\rho}(p_1 - p_2) \frac{A_2^2}{A_2^2 - A_1^2}} \tag{12.62}$$

$$V_* = V_1 \frac{A_1}{A_*} \tag{12.63}$$

$$V_* = \frac{A_1}{A_*} \sqrt{\frac{2}{\rho}(p_1 - p_2) \frac{A_2^2}{A_2^2 - A_1^2}} \tag{12.64}$$

13

Aerodynamics Lecture 5

13.1. Hour 1: Viscous flows in the laminar and turbulent regimes

Inviscid flow results in zero drag as well as zero lift since the airflow is symmetrical both vertically and horizontally. In real flows (those with friction), there is finite drag and (sometimes) an area of separation behind the shape.

13.1.1. Viscous flow

Air at the surface adheres to the surface due to friction → no-slip condition ($V_{Surface} = 0$). Therefore there exists a velocity gradient near the surface.
The boundary layer gets larger over time (i.e. it increases with distance from leading edge). The thickness of the boundary layer is denoted as δ.

$$\tau_w = \mu \left(\frac{du}{dy}\right)_{y=0} \tag{13.1}$$

and

$$\tau \propto \left(\frac{du}{dy}\right) \tag{13.2}$$

Where μ is the dynamic viscosity. Pressure is a normal force. Boundary layer causes tangential stress (friction). The pressure remains constant throughout the boundary layer. Equilibrium of forces along x **inside the boundary layer**

$$\sum \vec{F}_{x \overset{+}{\rightarrow}} : F = Ma \tag{13.3}$$

$$F = -A\ dp + A\ d\tau \tag{13.4}$$

13.2. Hour 2

13.2.1. Reynolds number

$$Re = \frac{\rho_\infty V_\infty L}{\mu_\infty} = \frac{V_\infty L}{\nu_\infty} = \frac{\text{inertia forces}}{\text{viscous forces}} \tag{13.5}$$

$$Re_x = \frac{\rho_\infty V_\infty x}{\mu_\infty} \tag{13.6}$$

$$\delta = \frac{5.2x}{\sqrt{Re_x}} \tag{13.7}$$

$$\text{Total force} = \text{total pressure force} + \text{total friction force} \tag{13.8}$$

$$F_w = \tau_w(x)\ dx \cdot 1 = \tau_w(x)\ dx \tag{13.9}$$

$$c_{f_x} = \frac{\tau_w}{\frac{1}{2}\rho_\infty V_\infty^2} = \frac{\tau_w}{q_\infty} \tag{13.10}$$

$$c_{f_x} = \frac{0.664}{\sqrt{Re_x}} \tag{13.11}$$

Thus c_{f_x} and τ_w decrease as $\sqrt{x}$ increases
Another way of looking at this is that the friction is the highest at the leading edge since the velocity has to go from V_∞ to 0 in a smaller space, so the $\left(\frac{du}{dy}\right)$ is larger.

$$D_f = \int_0^L \tau_w \, dx \tag{13.12}$$

$$= \int_0^L c_{f_x} q_\infty \, dx \tag{13.13}$$

$$= 0.664 q_\infty \int_0^L \frac{dx}{\sqrt{Re_x}} \tag{13.14}$$

$$\sqrt{Re_x} = \sqrt{\frac{V_\infty}{\nu}} \cdot \sqrt{x} \tag{13.15}$$

$$D_f = \frac{0.664 q_\infty}{\sqrt{\frac{V_\infty}{\nu_\infty}}} \int_0^L \frac{dx}{\sqrt{x}} \tag{13.16}$$

$$D_f = \frac{0.664 q_\infty}{\sqrt{\frac{V_\infty}{\nu_\infty}}} 2\sqrt{L} \tag{13.17}$$

$$D_f = \frac{1.328 q_\infty L}{\sqrt{\frac{V_\infty L}{\nu_\infty}}} \tag{13.18}$$

$$C_f = \frac{D_f}{q_\infty S} \tag{13.19}$$

$$C_f = \frac{1.328}{\sqrt{Re_L}} \tag{13.20}$$

$$D_{f_w} = 2D_f \quad \text{wings have two sides xd} \tag{13.21}$$

Turbulent boundary layer mixes more effectively, so it is thicker.

$$\delta = \frac{0.37x}{Re_x^{0.2}} \tag{13.22}$$

$$C_f = \frac{0.074}{Re_L^{0.2}} \tag{13.23}$$

14
Flight Mechanics Lecture 1

14.1. Hour 1

14.1.1. Equations of Motion

$$\vec{F} = m\vec{a} \tag{14.1}$$

Earth frame of reference (i.e. the regular frame of reference): x_g-axis is pointing forward.
z_g-axis is pointing downwards.
y_g-axis is pointing to the right w.r.t. the x_g-axis, but we won't use this axis.
Flat Earth assumption: Assume the aircraft is flying in a straight line.

Coordinate frame	Origin position	Axis	Direction
Earth frame of reference	Fixed position on Earth	x_g	Horizontal w.r.t. Earth's surface
		z_g	Vertical w.r.t. Earth's surface
		y_g	To the right w.r.t. the x_g-axis, but not used in this course
Moving Earth axis frame	Fixed to Centre of Gravity	x_E	Parallel to x_g ($x_E \parallel x_g$)
		z_E	Parallel to z_g ($z_E \parallel z_g$)
Body axis system	Fixed to Centre of Gravity	x_B	Parallel to nose (or chord?) of the aircraft
		z_B	Perpendicular to nose (or chord?) of the aircraft
Air path axis system	Fixed to Centre of Gravity	x_A	Parallel to velocity vector ($x_A \parallel \vec{V}$)
		z_A	Perpendicular to velocity vector ($z_A \perp \vec{V}$)

Moving Earth axis frame is fixed to the centre of gravity and consists of the x_E-axis and z_E-axis.
Body axis system is fixed to the centre of gravity and consists of the x_B-axis that is parallel to the nose of the aircraft, and the z_B-axis that is perpendicular to the nose of the aircraft.
Air path axis system is fixed to the centre of gravity and consists of the x_A-axis that is parallel to the velocity vector $\vec{V}$, and the z_A-axis which is perpendicular to the velocity vector.
Air path angle, climb angle, descent angle γ is the angle between x_A and x_E.
Angle of attack α is the angle between x_B and x_A.
Pitch angle θ is the angle between x_B and x_E ($\theta = \gamma + \alpha$).

FBD:

$$W \parallel \vec{z}_E \tag{14.2}$$

$$L \perp \vec{V},\ L \perp \vec{x}_A \tag{14.3}$$

$$D \parallel \vec{x}_A \tag{14.4}$$

$$T \approx \parallel \vec{x}_B \tag{14.5}$$

$$\tag{14.6}$$

Angle between thrust T and velocity $\vec{V}$ is α_t or α_T.

Kinetic diagram:

$$a_t \parallel \vec{x}_A \tag{14.7}$$

$$a_n \perp \vec{x}_A \text{ (but "up")} \tag{14.8}$$

Note:-
We know a lot about circles (it's true!)

$$a_n = \frac{V^2}{R} \tag{14.9}$$
$$\omega R = V \tag{14.10}$$
$$\omega = \frac{d\gamma}{dt} \tag{14.11}$$

$$V = \frac{d\gamma}{dt}R \tag{14.12}$$
$$a_n = \frac{V \cdot V}{R} = \frac{V\frac{d\gamma}{dt}\cancel{R}}{\cancel{R}} = V\frac{d\gamma}{dt} \tag{14.13}$$

14.2. Hour 2

14.2.1. Equations of Motion in the Air Path axis system

$$F \parallel \vec{V}: \boxed{T\cos\alpha_T - D - W\sin\gamma = m\frac{dV}{dt}} \tag{14.14}$$
$$F \perp \vec{V}: -T\sin\alpha_T - L + W\cos\gamma = -mV\frac{d\gamma}{dt} \tag{14.15}$$
$$\boxed{T\sin\alpha_T + L - W\cos\gamma = mV\frac{d\gamma}{dt}} \tag{14.16}$$

t is an independent variable.
V, γ are state variables.
T, α_T, D, L, W are other variables.
Goal is to express the other variables in terms of state variables.

14.2.2. Aerodynamics

$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} \tag{14.17}$$

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2 \quad \text{(2-term drag polar)} \tag{14.18}$$
$$C_D = C_{D_0} + \frac{C_L^2}{\pi A e} \text{ or } C_{D_0} + kC_L^2 \quad \text{(1-term drag polar)} \tag{14.19}$$
(14.20)

$$D = f(V)$$

Assume $L = W$ and constant altitude $\Leftrightarrow$ ρ = constant

$$C_L \frac{1}{2}\rho V^2 S = W \;\Leftrightarrow\; C_L = \underbrace{\left(\frac{W}{S}\right)}_{\text{wing loading}} \frac{2}{\rho} \cdot \frac{1}{V^2} \tag{14.21}$$

$$C_D = C_{D_0} + k_1 C_L + k_2 C_L^2 \tag{14.22}$$

$$D = (C_{D_0} + k_1 C_L + k_2 C_L^2) \cdot \frac{1}{2}\rho V^2 S \tag{14.23}$$

$$D = \left(C_{D_0} + k_1\left(\frac{W}{S}\frac{2}{\rho}\frac{1}{V^2}\right) + k_2\left(\frac{W}{S}\frac{2}{\rho}\frac{1}{V^2}\right)^2\right) \cdot \frac{1}{2}\rho V^2 S \tag{14.24}$$

$$D = C_{D_0}\frac{1}{2}\rho V^2 S + k_1 \frac{W}{\cancel{S}}\frac{\cancel{2}}{\cancel{\rho}}\frac{1}{\cancel{V^2}} \cdot \frac{1}{\cancel{2}}\cancel{\rho}\cancel{V^2}\cancel{S} + k_2\left(\frac{W}{S}\frac{2}{\rho}\frac{1}{V^2}\right)^2 \frac{1}{2}\rho V^2 S \tag{14.25}$$

$$D = C_{D_0}\frac{1}{2}\rho V^2 S + k_1 W + k_2 \frac{W^2}{S}\frac{2}{\rho}\frac{1}{V^2} \tag{14.26}$$

$$D = f_1(V^2) + f_2\left(\frac{1}{V^2}\right) \tag{14.27}$$

Therefore one part of the drag polar increases with speed, while another part goes down with speed. There will be a speed at which the drag is minimal.

14.2.3. Propulsion

$$T = (\dot{m} + \dot{m}_f)V_j - \dot{m}V_0 \approx \dot{m}(V_j - V_0) \tag{14.28}$$

$$W = T\Delta x \tag{14.29}$$

$$P_a = TV \tag{14.30}$$

$$\dot{Q} = \dot{m}_f H \quad \text{where H is the amount of energy per unit of fuel} \tag{14.31}$$

$$\eta_t = \frac{P_a}{\dot{Q}} \tag{14.32}$$

$$P_j = \frac{1}{2}\dot{m}V^2 - \frac{1}{2}\dot{m}V_0^2 \tag{14.33}$$

$$\eta_t = \frac{P_a}{\dot{Q}} = \frac{P_a}{\dot{Q}} \cdot \frac{P_j}{P_j} \tag{14.34}$$

$$\eta_t = \eta_j \cdot \eta_{th} \tag{14.35}$$

$$T = \underbrace{\dot{m}}_{\text{goes up with V}} \underbrace{\left(\underbrace{V_j}_{\text{stays constant}} - \underbrace{V_0}_{\text{goes up with V}} \right)}_{\text{goes down with V}} \tag{14.36}$$

15

Flight Mechanics Lecture 3

15.1. Hour 1

15.1.1. Maximum range

$$\frac{V}{F} = \left[\frac{m/s}{kg/s}\right] = \left[\frac{m}{kg}\right] \tag{15.1}$$

$$\frac{V}{F} = \frac{V}{c_p \cdot P_{br}} \tag{15.2}$$

$$= \frac{\cancel{V}}{\frac{c_p}{\eta_j} \cdot D \cdot \cancel{V}} \tag{15.3}$$

$$= \frac{\eta_j}{c_p \cdot D} \propto \frac{1}{D} \tag{15.4}$$

c_p and η_j are constant.

$$D_{min} = D \cdot \frac{L}{L} = \left(\frac{C_D}{C_L}\right)_{min} \cdot W \tag{15.5}$$

So D_{min} occurs at $\left(\frac{C_D}{C_L}\right)_{min}$ or $\left(\frac{C_L}{C_D}\right)_{max}$.

$$\frac{d\left(\frac{C_D}{C_L}\right)}{dC_L} = 0 \tag{15.6}$$

$$C_D = f(C_L) = C_{D_0} + k_1 C_L + k_2 C_L^2 \tag{15.7}$$

$$\frac{d\left(\frac{C_D}{C_L}\right)}{dC_L} = \frac{d\left(\frac{C_{D_0}+k_1C_L+k_2C_L^2}{C_L}\right)}{dC_L} \tag{15.8}$$

$$= \frac{d\left(\frac{C_{D_0}}{C_L} + k_1 + k_2 C_L\right)}{dC_L} \tag{15.9}$$

$$0 = -\frac{C_{D_0}}{C_L} + k_2 \tag{15.10}$$

$$C_{L_{opt}} = \sqrt{\frac{C_{D_0}}{k_2}} \tag{15.11}$$

$$\Longrightarrow \; V_{opt} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{opt}}}} \tag{15.12}$$

15.1.2. Endurance

Objective is to minimize fuel flow F [kg/s].

$$F = c_p P_{br} \qquad P_a = \eta_j P_{br} \tag{15.13}$$

$$F = c_p P_{br} \qquad \Rightarrow \frac{c_p P_a}{\eta_j} \tag{15.14}$$

$$(F)_{min} \Leftrightarrow (P_a)_{min} \qquad \Rightarrow (P_r)_{min} \tag{15.15}$$

$$P_r = D \cdot V \tag{15.16}$$

$$= D \cdot \frac{L}{L} \cdot V \tag{15.17}$$

$$= \frac{C_D}{C_L} L \cdot V \tag{15.18}$$

$$= \frac{C_D}{C_L} W \cdot V \tag{15.19}$$

$$= \frac{C_D}{C_L} W \cdot \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \tag{15.20}$$

$$(P_r)_{min} = \left(\sqrt{\frac{W^3}{S} \cdot \frac{2}{\rho} \cdot \frac{C_D^2}{C_L^3}}\right)_{min} \tag{15.21}$$

Weight, area and density are constant, so $(P_r)_{min}$ at $\left(\frac{C_D^2}{C_L^3}\right)_{min}$ or $\left(\frac{C_L^3}{C_D^2}\right)_{max}$.

$$\left(\frac{C_L^3}{C_D^2}\right)_{max} \Rightarrow \frac{d\left(\frac{C_L^3}{C_D^2}\right)}{dC_L} = 0 \tag{15.22}$$

$$\frac{d\left(\frac{C_L^3}{C_D^2}\right)}{dC_L} = \frac{3C_L^2 C_D^2 - 2C_D C_L^3 \cdot \frac{dC_D}{dC_L}}{C_D^4} = 0 \tag{15.23}$$

$$\Rightarrow 3\cancel{C_L^2} C_D^{\cancel{2}} = 2\cancel{C_D} C_L^{\cancel{3}} \cdot \frac{dC_D}{dC_L} \tag{15.24}$$

$$3C_D = 2C_L \cdot \frac{dC_D}{dC_L} \tag{15.25}$$

$$3C_D + 3k_1 C_L + 3k_2 C_L^2 = 2C_L \cdot (k_1 + 2k_2 C_L) \tag{15.26}$$

$$0 = C_L^2(3k_2 - 4k_2) + C_L(3k_1 - 2k_1) + 3C_{D_0} \tag{15.27}$$

$$0 = -k_2 C_{L^2} + k_1 C_L + 3C_{D_0} \tag{15.28}$$

$$C_{L_{1,2}} = \frac{-k_1 \pm \sqrt{k_1^2 + 12k_2 C_{D_0}}}{-2k_2} \tag{15.29}$$

Airspeed for maximum endurance is **ALWAYS** lower than the airspeed for maximum range (for propeller aircraft).

15.2. Hour 2

$$\sum F_{\parallel V} : T \cancelto{1}{\cos\alpha_t} - D - W \cancelto{0}{\sin\gamma} = \frac{W}{g}\frac{dV}{dt} \tag{15.30}$$

$$\sum F_{\perp V} : L - W \cancelto{1}{\cos\gamma} + T \cancelto{0}{\sin\alpha_t} = \frac{W}{g} V \frac{d\gamma}{dt} \tag{15.31}$$

$$T - D = \frac{W}{g}\frac{dV}{dt} \tag{15.32}$$

Backside is the power curve is the part of the power curve that is at a lower speed than $V_{D_{min}}$.

15.3. Hour 2: Climbing/descending flight

Max climb angle is important for being able to avoid obstacles. Max climb rate is relevant for reaching cruise altitude quickly.
For descending flight, the angle of descend $\bar{\gamma} = -\gamma$ is used instead of the flight path angle γ.

Note:-

Angle of climb γ is **NOT** the same as the Rate of Climb (ROC).
Rate of Climb is **NOT** the same as Climb Speed.

$$ROC = V \sin\gamma$$

with ROC = Rate of Climb, V=Climb Speed, and γ=Flight path angle.

15.3.1. Maximum glide angle

$$\sum F_{\parallel V} : T \cancelto{1}{\cos\alpha_t} - D - W\sin\gamma = \frac{W}{g}\frac{dV}{dt} \tag{15.33}$$

$$\sum F_{\perp V} : L - W\cos\gamma + T \cancelto{0}{\sin\alpha_t} = \frac{W}{g} V \frac{d\gamma}{dt} \tag{15.34}$$

Assume γ is small but not zero (not more than 15°)

$$L = W \tag{15.35}$$

$$\sin\gamma = \frac{T - D}{W} \tag{15.36}$$

$$V\sin\gamma = \frac{P_a - P_r}{W} = ROC \tag{15.37}$$

$$\sin\gamma = \frac{T - D}{W} \tag{15.38}$$

$$\sin\gamma_{max} = \frac{(T - D)_{max}}{W} \tag{15.39}$$

So max climb angle γ_{max} at D_{min}.

15.3.2. Minimum descent angle (gliding flight)

$$\bar{\gamma}_{min} = \frac{D_{min}}{W} \tag{15.40}$$

16

Flight Mechanics Lecture 4

16.1. Hour 1: Recap

Minimum Rate of Descent (ROD)

$$\left(\frac{C_D^2}{C_L^3}\right)_{opt} \implies C_L = \frac{k_1 \pm \sqrt{k_1^2 + 12C_{D_0}k_2}}{2k_2} \tag{16.1}$$

16.2. Hour 1: Altitude effects on aircraft performance

Goal is to find the flight envelope of an aircraft, meaning a graph showing the max altitude and minimum altitude the aircraft is able to reach at a given airspeed.

16.2.1. Minimum airspeed

$$L = W = C_L \frac{1}{2}\rho V^2 S \tag{16.2}$$

$$\implies V_{min} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_{L_{max}}}} \tag{16.3}$$

$$\implies V_{min} \propto \sqrt{\frac{1}{\rho}} \tag{16.4}$$

Flight envelope is limited by stall limit for low velocities and thrust/power limit at higher altitudes and velocities.
A supercharger can be used to compress the air and delay the onset of thrust limited speed.

16.2.2. Maximum rate of climb

$$\frac{P_a - P_r}{W} = ROC \tag{16.5}$$

Maximum rate of climb (generally?) decreases with altitude since the available power P_a decreases faster than the P_r until they match. The altitude at which they match is called the theoretical or absolute ceiling. The service ceiling is a more practical metric: it is the altitude at which the ROC is 500ft/min reached.

16.2.3. Example: how to build your own U2

Engine: $\frac{T}{T_s} = \frac{\rho}{\rho_s}$ (s subscript means tropopause = 11km)

$C_D = C_{D_0} + k_2 C_L^2 = C_{D_0} + \frac{C_L^2}{\pi A e}$

L=W, T=D $\implies D = \frac{C_D}{C_L}W = T$

$$\frac{\rho}{\rho_s} = \frac{1}{T_s} \cdot \frac{C_D}{C_L} \cdot W \tag{16.6}$$

$$H_{max} \implies \rho_{min} \tag{16.7}$$

$$\rho_{min} \implies \frac{\rho_s}{T_s}\left(\frac{C_D}{C_L}\right)_{min} W \implies \left(\frac{C_L}{C_D}\right)_{max} \tag{16.8}$$

$$\implies C_L = \sqrt{\frac{C_{D_0}}{k_2}} \implies C_D = C_{D_0} + k_2\frac{C_{D_0}}{k_2} = 2C_{D_0} \tag{16.9}$$

$$\implies \rho_{min} = \frac{\rho_s}{T_s} \cdot \frac{2C_{D_0}}{\sqrt{C_{D_0}\pi Ae}}W = \frac{\rho_s}{T_s} \cdot 2\sqrt{\frac{C_{D_0}}{\pi Ae}}W \tag{16.10}$$

16.2.4. Operational limits

Load factor $n \equiv \frac{L}{W}$ or $L = nW$.
Flight manoeuvering envelope plots load factor n against equivalent airspeed V_{EAS}.

$$n = \frac{L}{W} \iff L = nW = C_L\frac{1}{2}\rho V^2 S \tag{16.11}$$

$$\implies V = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{1}{C_L}} \tag{16.12}$$

Since diving faster than 0g would overspeed the aircraft, a limit is set by connecting points $(V_D, 0)$ and $(V_C, -1)$ with a diagonal straight line.
Another limit is the flutter speed, which is higher than the V_{dive}, where flutter starts to occur, denoted by V_{Df}.
When there are gusts, direction and speed of the airflow changes. For a vertical gust, the airfoil will "see" a higher angle of attack.

$$\Delta n = \frac{\Delta L}{W} \tag{16.13}$$

$$\Delta L = \underbrace{\Delta C_L}_{\Delta\alpha=\arctan\left(\frac{U}{V}\right)\approx\frac{U}{V}} \frac{1}{2}\rho(V^2 + U^2)S \tag{16.14}$$

$$\Delta L = \frac{dC_L}{d\alpha}\Delta\alpha\frac{1}{2}\rho V^2 S \tag{16.15}$$

$$= \frac{dC_L}{d\alpha}\frac{U}{\cancel{V}}\frac{1}{2}\rho V^{\cancel{2}} S \tag{16.16}$$

$$= \frac{dC_L}{d\alpha}UV\frac{1}{2}\rho S \tag{16.17}$$

$$\Delta n = \frac{dC_L}{d\alpha}U \cdot V \cdot \frac{\rho S}{2W} \tag{16.18}$$

$$\Delta n = k \cdot U \cdot V \tag{16.19}$$

Total manoeuvre + gust envelope is called the V,n-diagram.
Maximum operating speed V_{MO} is published in the flight manual instead of the V_D. This V_{MO} is the V_D with a built-in safety factor.
Aircraft have a maximum Mach number M_{max}, which limits the speed to a decreasing value as altitude increases, up until 11km where the temperature stays constant. The maximum operating Mach number M_{MO} is published in the flight manual.
Pressurized cabins can only hold so much difference between inside and outside pressure, so there is an altitude limit because of that.

16.2.5. Flight envelope and instruments