4)1)

a) $f(x) = x \cdot e^{-\frac{x}{2}}$ $x_0 = 2$ $dx = 0{,}05$

$f'(x) = e^{-\frac{x}{2}} + x \cdot e^{-\frac{x}{2}} \cdot \left(-\frac{1}{2}\right) = \frac{1}{e^{\frac{x}{2}}} - x \cdot \frac{1}{2e^{\frac{x}{2}}} = \frac{2-x}{2e^{\frac{x}{2}}}$

$\frac{1}{e} \neq 2\frac{1}{2e}$

$f'(x_0) = 0$

$df(x_0) = f'(x_0) \cdot dx$

$df(x_0) = 0$

b) $f(x) = \frac{\ln(x^2)}{x}$ $x_0 = 1$ $dx = 0{,}05$

$f'(x) = \frac{2x \cdot \frac{1}{x^2} \cdot x - \ln(x^2)}{x^2} = \frac{2 - \ln(x^2)}{x^2}$

$f'(x_0) = \frac{2}{1} = 2$

$df(x_0) = 2 \cdot 0{,}05 = \underline{\underline{0{,}01}}$

c) $f(x) = \operatorname{tg} 3x$ $x_0 = \frac{\pi}{12}$

$f'(x) = 3 \cdot \frac{1}{\cos^2 x} = \frac{3}{\cos^2 x}$

$f'(x_0) \doteq 3$

$df(x_0) = 3 \cdot dx$

d) $f(x) = \operatorname{arctg}(x^2+1)$

$f'(x) = 2x \cdot \frac{1}{x^2+1} = \frac{2x}{x^2+1}$

$f'(x_0) = \frac{2x_0}{x_0^2+1}$

$df(x_0) = \frac{2x_0}{x_0^2+1} \cdot dx$

2) a) $\sqrt[3]{0{,}98}$

$f(x)=\sqrt[3]{x}$ $\quad \bar{x}=0{,}98$ $\quad x_0=1$ $\quad dx=-0{,}2=-\frac{1}{5}$

$f'(x)=\frac{d_x}{x}\, x^{\frac{1}{3}}=\frac{1}{3}x^{-\frac{2}{3}}=\frac{1}{3}\cdot\frac{1}{x^{\frac{2}{3}}}\doteq\frac{1}{3x^{\frac{2}{3}}}\doteq\frac{1}{3\sqrt[3]{x^2}}$

$f'(x_0)=\frac{1}{3}$

$df(x_0)=\frac{1}{3}\cdot\left(-\frac{1}{5}\right)$

$df(x_0)\doteq-\frac{1}{15}$ $\qquad f(x_0)+df(x_0)=1-\frac{1}{15}\doteq\frac{14}{15}$

b) $\cos(0{,}07)$ $\quad \bar{x}=0{,}07$ $\quad x_0=0$ $\quad dx=0{,}07$

$f\,x=\cos x$

$f'\,x=-\sin x$

$f'(x_0)=0=dx\; df(x_0)$

$\cos(0{,}07)=\cos(0)+df(x_0)\doteq 1$

a) $f(x) = \sin 2x \quad x_0 = \frac{7\pi}{6} \quad n = 4 \qquad f(x_0) = \sin \frac{14}{6}\pi$

$f'(x) = 2\cos 2x \qquad f'(x_0) = 2 \cdot \cos \frac{14}{6}\pi$

$f''(x) = -4\sin 2x \qquad f''(x_0) = -4 \cdot \sin \frac{14}{6}\pi$

$f^3(x) = -8\cos 2x \qquad f^3(x_0) = -8\cos \frac{14}{6}\pi$

$f^4(x) = 16\sin 2x \qquad f^4(x_0) = 16\sin \frac{14}{6}\pi$

$$T_4(x) = \sin \frac{14}{6}\pi + 2\cos \frac{14\pi}{6}\left(x - \frac{7\pi}{6}\right) - \frac{4\sin \frac{14\pi}{6}}{2!} \cdot \left(x - \frac{7\pi}{6}\right)^2 + \frac{-8\cos \frac{14\pi}{6}}{3!} \cdot \left(x - \frac{7\pi}{6}\right)^3 + \frac{16\sin \frac{14}{6}\pi}{4!} \cdot \left(x - \frac{7}{6}\pi\right)^4$$

b) $f(x) = \operatorname{arctg} x \quad x_0 = 1 \quad n = 3 \qquad f(x_0) = \operatorname{arctg} 1$

$f'(x) = \frac{1}{x^2+1} \qquad f'(x_0) = \frac{1}{2}$

$f''(x) = \frac{-2x}{(x^2+1)^2} = \frac{-2x}{x^4+1} \qquad f''(x_0) = \frac{-2}{2} = -1$

$f'''(x) = \frac{-2(x^4+1) + 2x \cdot 4x^3}{x^8+1} = \frac{-2x^4 - 2 + \cancel{2x \cdot 4x^3}\, 8x^4}{(x^8+1)} = \frac{6x^4 - 2}{x^8+1} \qquad f^{III}(x) = \frac{4}{2} = 2$

$$T_3 = \operatorname{arctg} 1 + \frac{1}{2}(x-1) - \frac{1}{2} \cdot (x-1)^2 + \frac{1}{3} \cdot (x-1)^3$$

c)

$$f(x) = \arcsin x \quad x_0 = 0 \quad n = 2$$

$$f(x) = 0$$

$$f'(x) = \frac{1}{\sqrt{1-x^2}} = \frac{1}{(1-x^2)^{1/2}} = (1-x^2)^{-1/2}$$

$$f'(x) = 1^{-1/2} = \frac{1}{\sqrt{1}}$$

$$f''(x) = -\frac{1}{2}(1-x^2)^{-3/2} \cdot (-2x)$$

$$f''(x) = -\frac{1}{2} \cdot \frac{1}{\sqrt{1^3}} = -\frac{1}{2\sqrt{1}}$$

$$T_2 = \frac{1}{\sqrt{1}} x - \frac{1}{4\sqrt{1}} x^2$$