1 a 2:

1) $A + B^T$

$$A\begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & -2 \end{pmatrix} + \begin{pmatrix} 3 & 1 & 2 \\ -1 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 1 & 3 \\ 0 & 5 & -1 \end{pmatrix}$$

$B \cdot A - C$

$$\begin{pmatrix} 3 & -1 \\ 1 & 2 \\ 2 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 & 1 \\ 1 & 3 & -2 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 5 & -3 & 5 \\ 4 & 6 & -3 \\ 5 & 3 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 2 & 0 \\ 2 & 2 & 3 \\ 4 & 1 & 2 \end{pmatrix} = \begin{pmatrix} 4 & -5 & 5 \\ 2 & 4 & -6 \\ 1 & 2 & -2 \end{pmatrix}$$

2)

a) $\det(A) = -14$

$$|A| \begin{pmatrix} 3 & 2 & -2 \\ 1 & 2 & 3 \\ 1 & 4 & 5 \end{pmatrix} = 30 + 6 - 8 + 4 - 10 - 36 = -14$$

b) $\det(B) = -68$

$$|B| \begin{pmatrix} 1 & 2 & 2 & 5 \\ 1 & 3 & 2 & 4 \\ 1 & 0 & 3 & 2 \\ 2 & 1 & 6 & 3 \end{pmatrix} = -1 \cdot \begin{pmatrix} 2 & 2 & 5 \\ 3 & 2 & 4 \\ 1 & 6 & 3 \end{pmatrix} + 3 \begin{pmatrix} 1 & 2 & 5 \\ 1 & 3 & 4 \\ 2 & 1 & 3 \end{pmatrix} + (-2) \begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 2 \\ 2 & 1 & 6 \end{pmatrix} =$$

$$= -1(12 + 8 + 90 - 10 - 18 - 48) + 3(9 + 16 + 5 - 30 - 6 - 4) + (-2)(18 + 8 + 2 - 12 - 12 - 2)$$

$$= -34 - 30 - 4 = -68$$

~~$\det(C) = -3a - 42$~~

$$|C| \begin{pmatrix} 3 & -3 & 1 \\ 1 & -2 & 4 \\ 2 & 2 & a \end{pmatrix} = -6a - 24 + 2 + 4 + 3a - 24 = -3a - 42$$

$$-3a - 42 = 0$$

$$3a = -42$$

$$a = -14$$

c) Matice je singulární pro $a = -14$

3

3) a) $h(A) = 3$

$$A\begin{pmatrix} 1 & -2 & -3 & 1 & 2 \\ 4 & 3 & 2 & 0 & 2 \\ 3 & 2 & 1 & 2 & 2 \\ 2 & -1 & -2 & -1 & 2 \end{pmatrix} (-4)\ (-3)\ (-2) \Rightarrow \begin{pmatrix} 1 & -2 & -3 & 1 & 2 \\ 0 & 11 & 14 & -4 & -6 \\ 0 & 8 & 10 & -1 & -4 \\ 0 & 3 & 4 & -3 & -2 \end{pmatrix} \begin{matrix} \\ \\ \cdot(11) \\ \cdot(11) \end{matrix} \Rightarrow \begin{pmatrix} 1 & -2 & -3 & 1 & 2 \\ 0 & 11 & 14 & -4 & -6 \\ 0 & -88 & -110 & +11 & +44 \\ 0 & -33 & -44 & +33 & +22 \end{pmatrix} (8)\ (+3)$$

$$\begin{pmatrix} 1 & -2 & -3 & 1 & 2 \\ 0 & 11 & 14 & -4 & -6 \\ 0 & 0 & 2 & -21 & -4 \\ 0 & 0 & -2 & 21 & 4 \end{pmatrix} (1) \Rightarrow \begin{pmatrix} 1 & -2 & -3 & 1 & 2 \\ 0 & 11 & 14 & -4 & -6 \\ 0 & 0 & 2 & -21 & -4 \end{pmatrix}$$

b) Vektory jsou lineárně nezávislé

$$\cancel{\begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{pmatrix} (-2) \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -4 \\ 0 & 1 & 5 \end{pmatrix} (-3) \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 0 & -19 \\ 0 & 1 & 5 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & -19 \end{pmatrix}} \quad \ddot\smile$$

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & 3 & 0 \\ 0 & 1 & 5 \end{pmatrix} (-2) \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 3 & -4 \\ 0 & 1 & 5 \end{pmatrix} (-3) \sim \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & -19 \end{pmatrix}$$

4

4)

a) $$\left(\begin{array}{ccc|ccc} -1 & 2 & 2 & 1 & 0 & 0 \\ 1 & -2 & -3 & 0 & 1 & 0 \\ 1 & 2 & -1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} -1 & 2 & 2 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 4 & 1 & 1 & 0 & 1 \end{array}\right) \cdot 2 \sim \left(\begin{array}{ccc|ccc} -2 & 4 & 4 & 2 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 4 & 1 & 1 & 0 & 1 \end{array}\right) (-1)$$

$$\sim \left(\begin{array}{ccc|ccc} -2 & 0 & 3 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 4 & 1 & 1 & 0 & 1 \end{array}\right) \begin{matrix} 3 \\ 1 \end{matrix} \sim \left(\begin{array}{ccc|ccc} -2 & 0 & 0 & 4 & 3 & -1 \\ 0 & 0 & -1 & 1 & 1 & 0 \\ 0 & 4 & 0 & 2 & 1 & 1 \end{array}\right) \begin{matrix} :(-2) \\ \cdot(-1) \\ :4 \end{matrix} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -\frac{3}{2} & \frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 1 & -1 & -1 & 0 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -2 & -3/2 & 1/2 \\ 1/2 & 1/4 & 1/4 \\ -1 & -1 & 0 \end{pmatrix}$$

b) $$A^{-1} = \frac{1}{\det(A)} \cdot A^* \qquad \det(A) = -2-6+4+4+2-6 = -4$$

$$A^* \begin{pmatrix} \begin{vmatrix} -2 & -3 \\ 2 & -1 \end{vmatrix} & -\begin{vmatrix} +1 & -3 \\ 1 & -1 \end{vmatrix} & \begin{vmatrix} 1 & -2 \\ 1 & 2 \end{vmatrix} \\ -\begin{vmatrix} 2 & 2 \\ 2 & -1 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & -1 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 1 & 2 \end{vmatrix} \\ \begin{vmatrix} 2 & 2 \\ -2 & -3 \end{vmatrix} & -\begin{vmatrix} -1 & 2 \\ 1 & -3 \end{vmatrix} & \begin{vmatrix} -1 & 2 \\ 1 & -2 \end{vmatrix} \end{pmatrix}^T = \begin{pmatrix} 8 & -2 & 4 \\ 6 & -1 & 4 \\ -2 & -1 & 0 \end{pmatrix}^T = \begin{pmatrix} 8 & 6 & -2 \\ -2 & -1 & -1 \\ 4 & 4 & 0 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-4} \cdot \begin{pmatrix} 8 & 6 & -2 \\ -2 & -1 & -1 \\ 4 & 4 & 0 \end{pmatrix} = \begin{pmatrix} -2 & -\frac{3}{2} & \frac{1}{2} \\ 1/2 & 1/4 & 1/4 \\ -1 & -1 & 0 \end{pmatrix}$$

V zadání máte čtvrtého úkolu máte chybu. Místo adjungované matice tam máte napsané cramerovo pravidlo.

4. Inversní matice

Je-li dána matice $\mathbf{A}$:

$$\mathbf{A} = \begin{pmatrix} -1 & 2 & 2 \\ 1 & -2 & -3 \\ 1 & 2 & -1 \end{pmatrix}$$

Nalezněte:

a) A^{-1} pomocí Gaussovy metody,
b) A^{-1} pomocí Cramerova pravidla.