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SESSION ID: CRYP-F01

Fast Secure Comparison for Medium-Sized Integers and Its Application in Binarized Neural Networks

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Motivation: A Scenario Involving a Trusted Third Party

Running Example: Second-Price Sealed-Bid Auction

- Bidders convey their bids in private to the Auctioneer
- Auctioneer sells item to the highest bidder for the price of the second-highest bid

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- Bidders convey their bids in private to the Auctioneer
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Problems

Privacy Auctioneer could leak bids to other bidders

Correctness Auctioneer could cheat, e.g. by increasing second-highest bid

Secure Multiparty Computation (MPC)

- Bidders jointly emulate the Auctioneer,
  by means of a distributed computation
- Bidders input their bids into their own computer
- Bids are distributed using secret sharing
- Computers execute a protocol (local computations + message exchange)
  to compute winning bidder and second price

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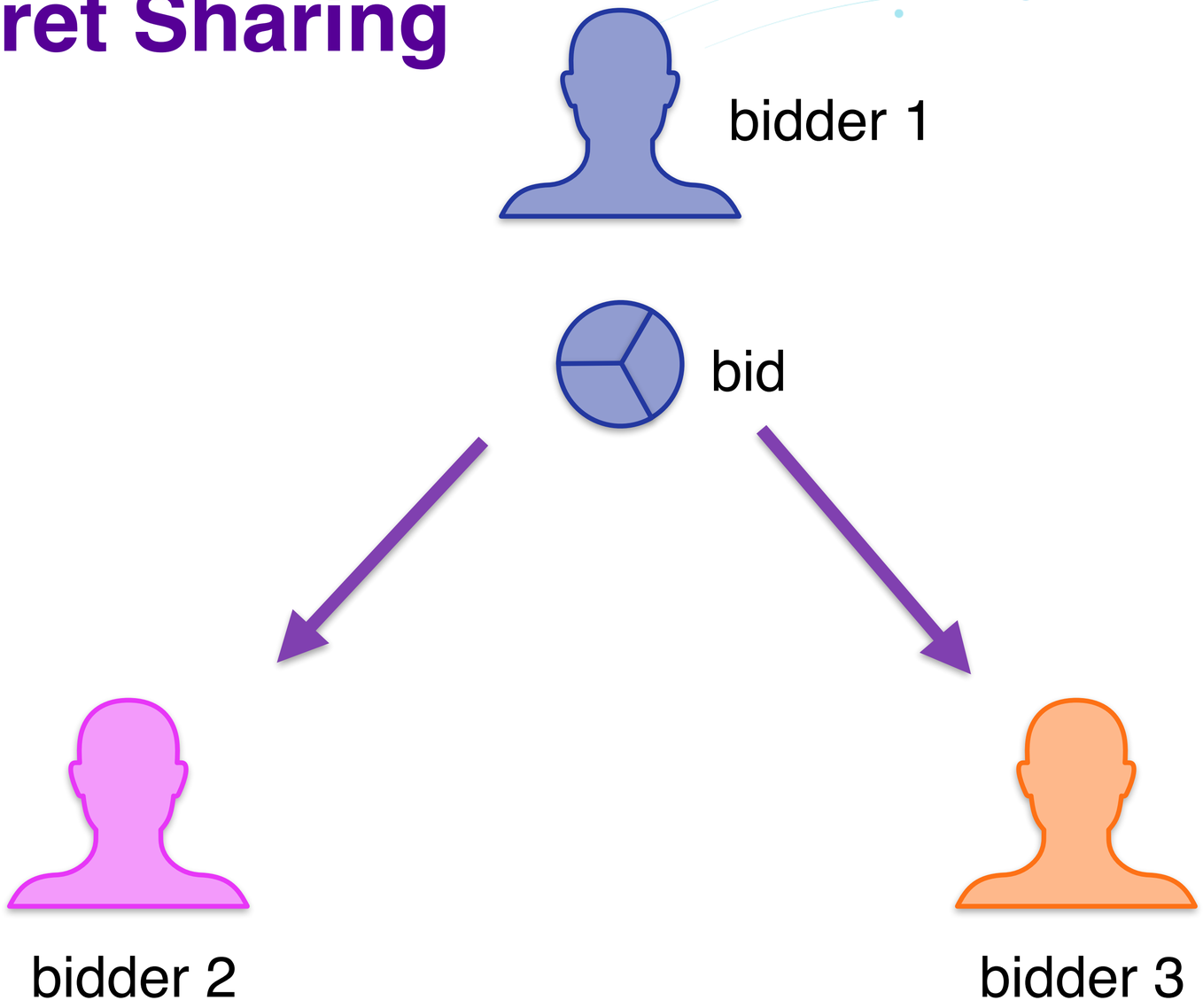
…Achieve privacy and correctness through cryptography!

Secret Sharing



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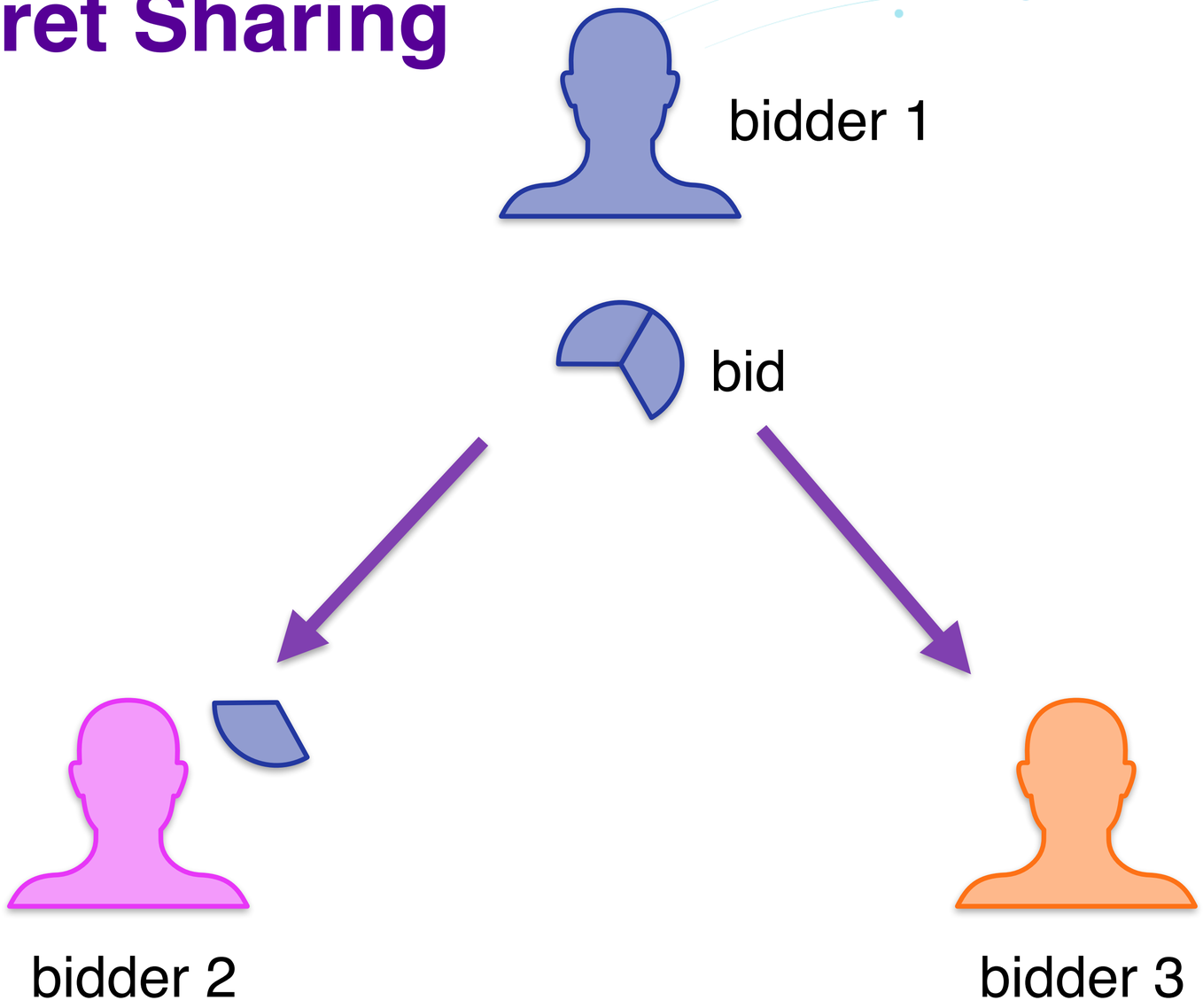

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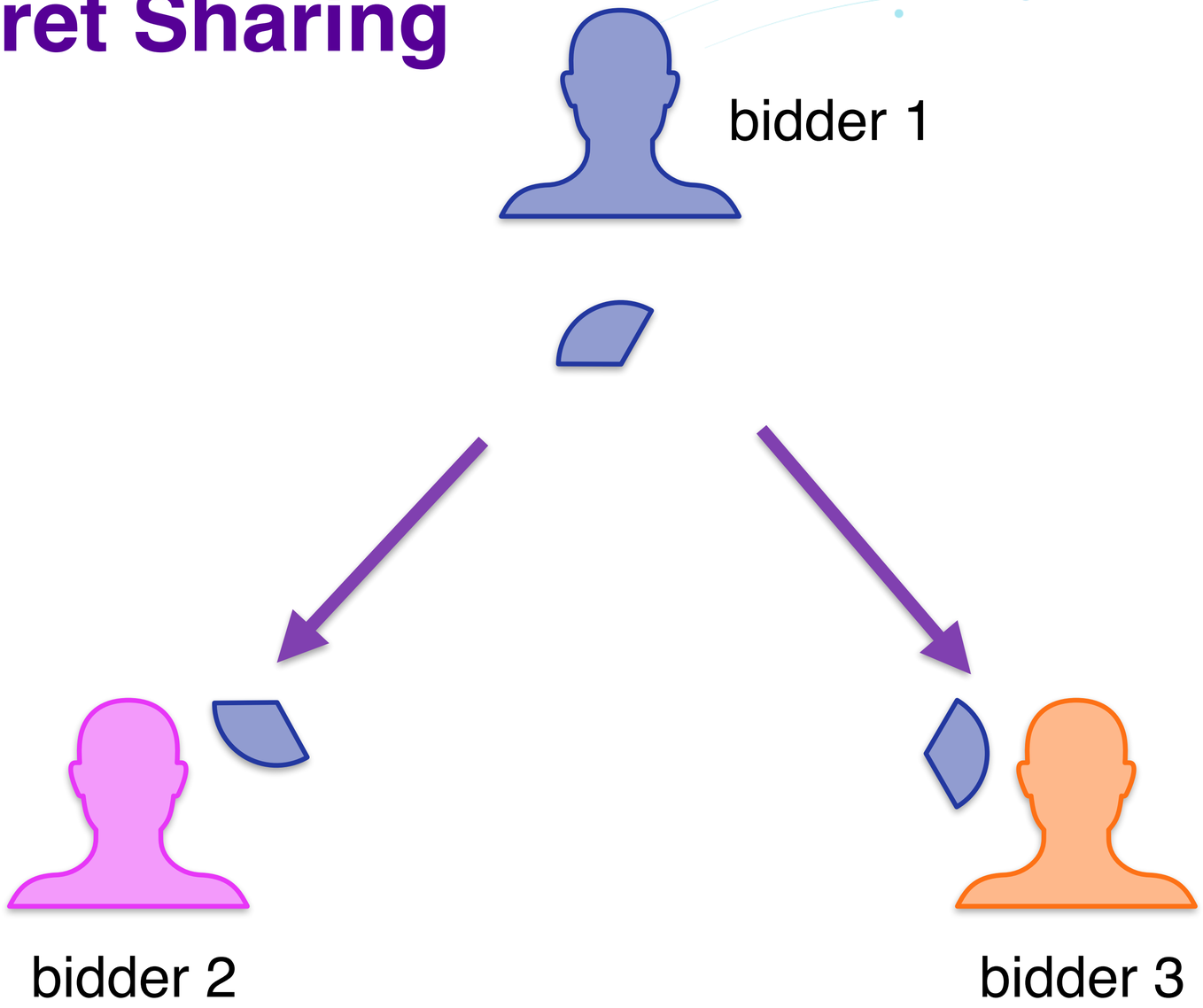

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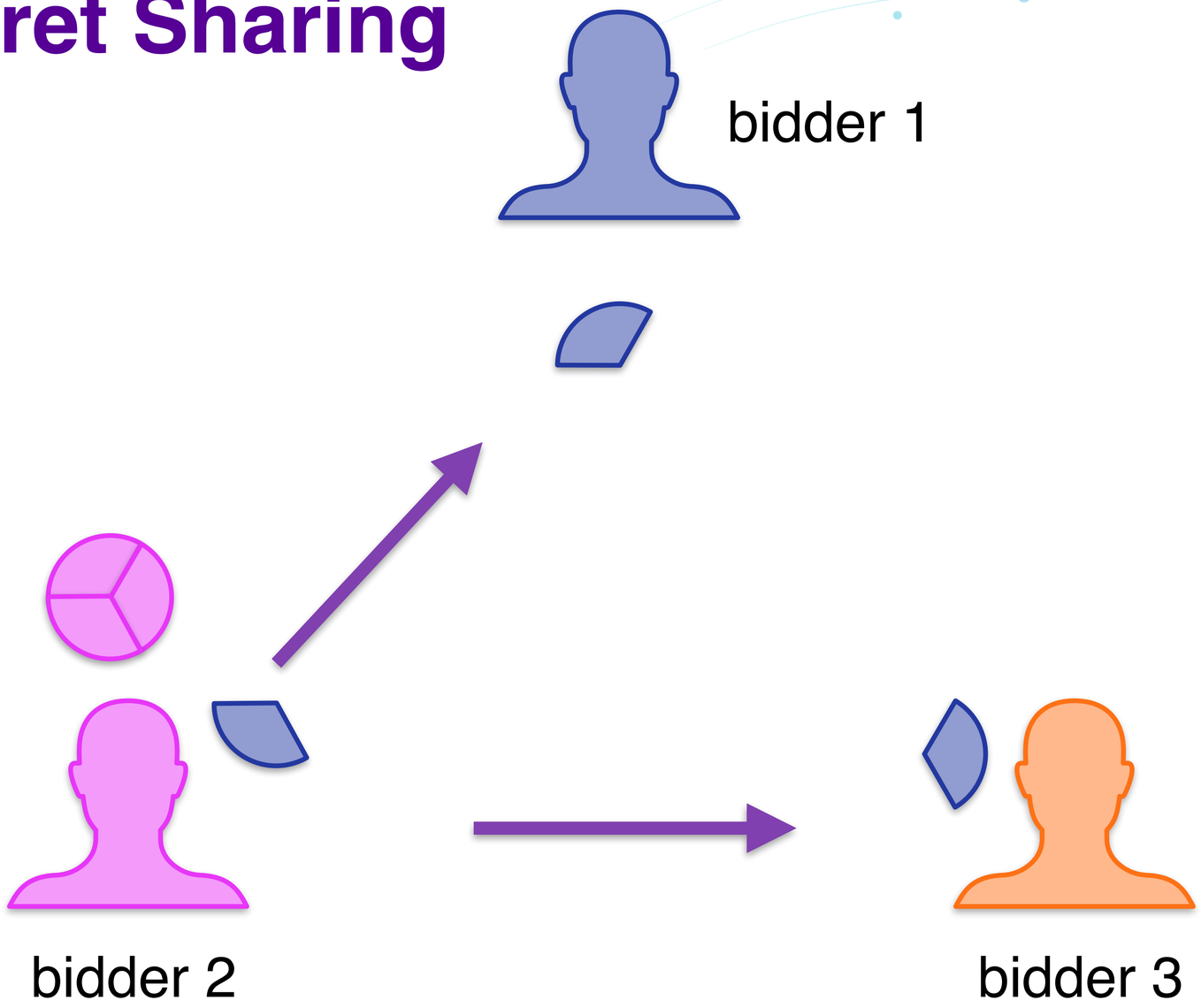

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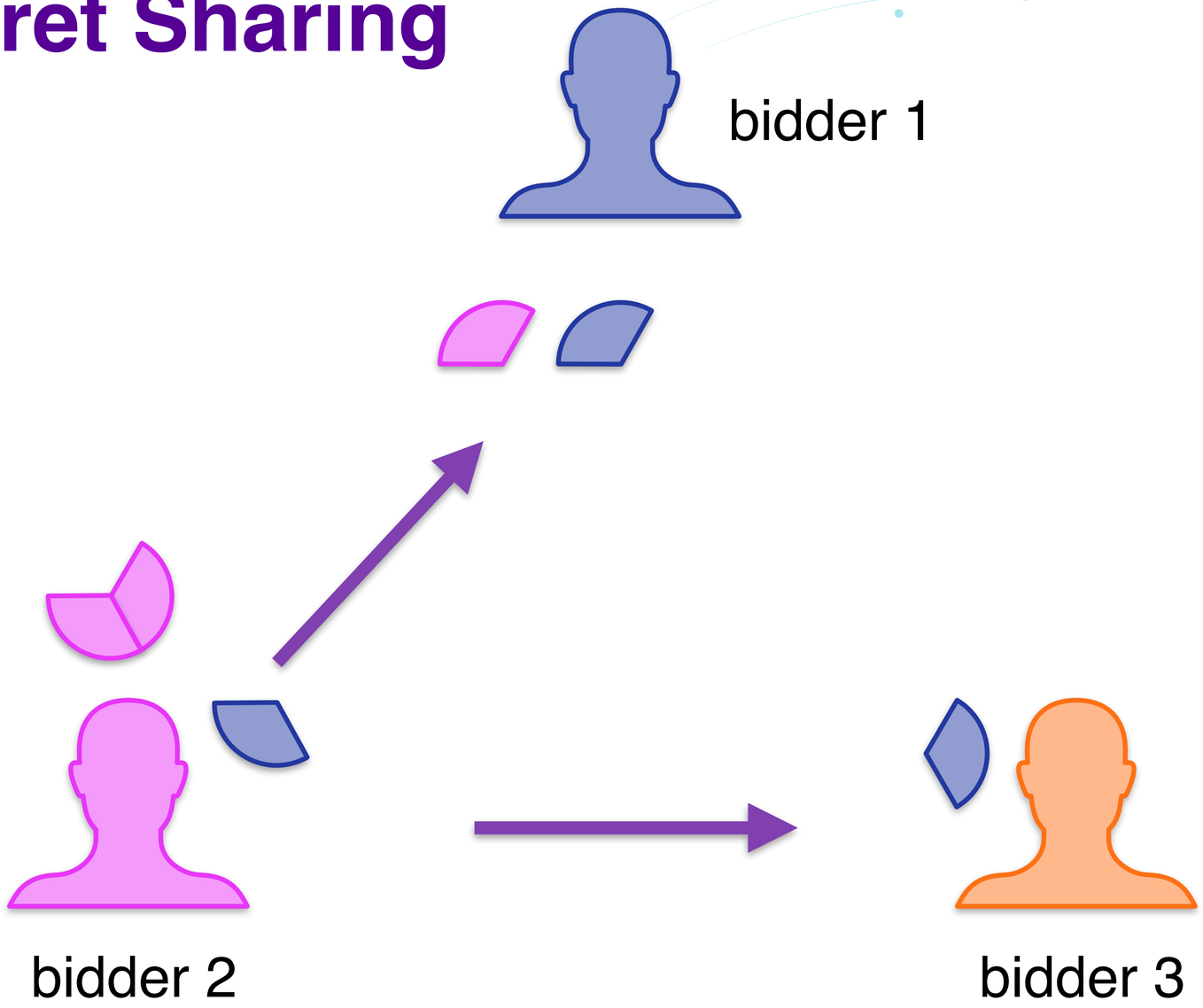

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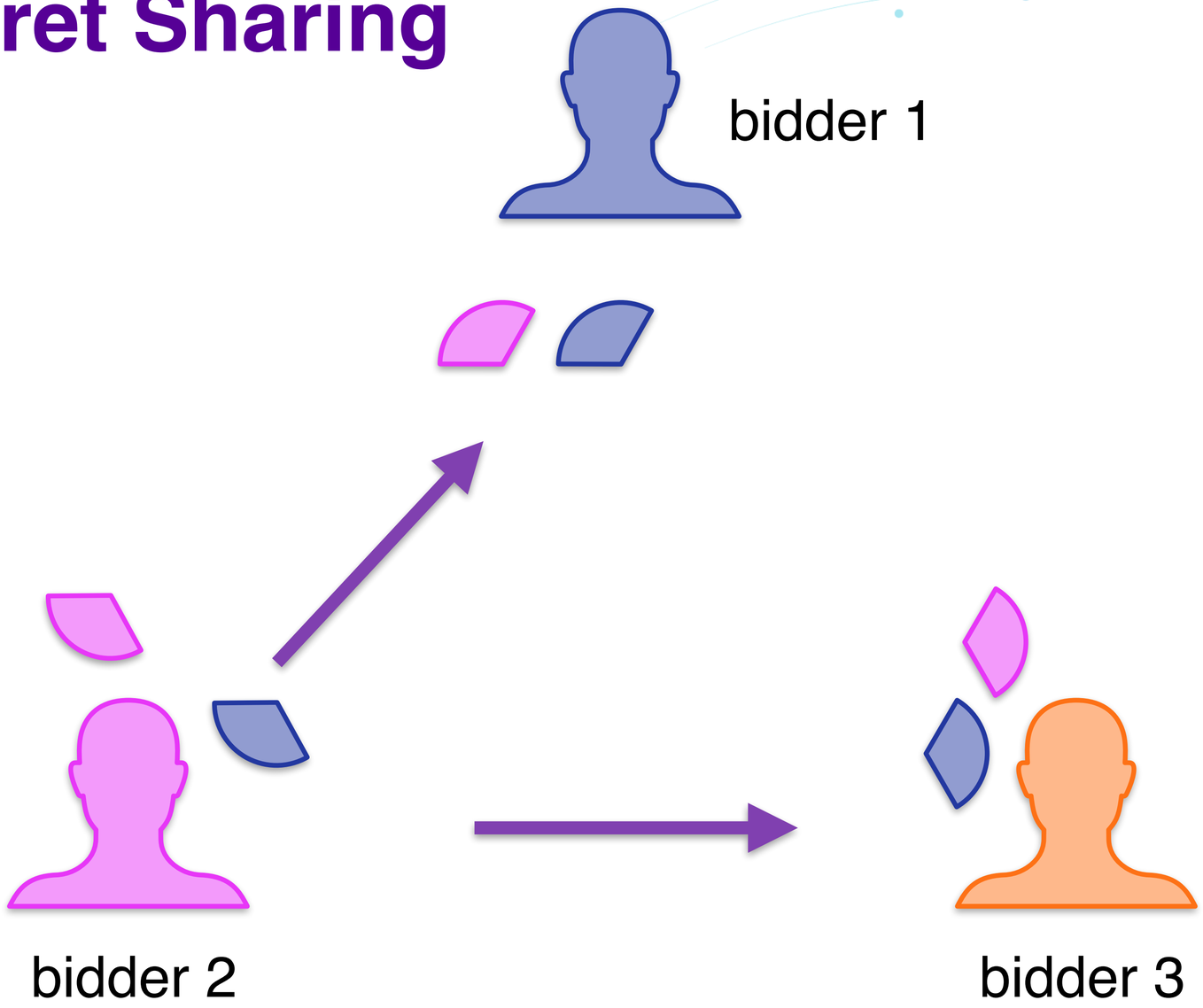

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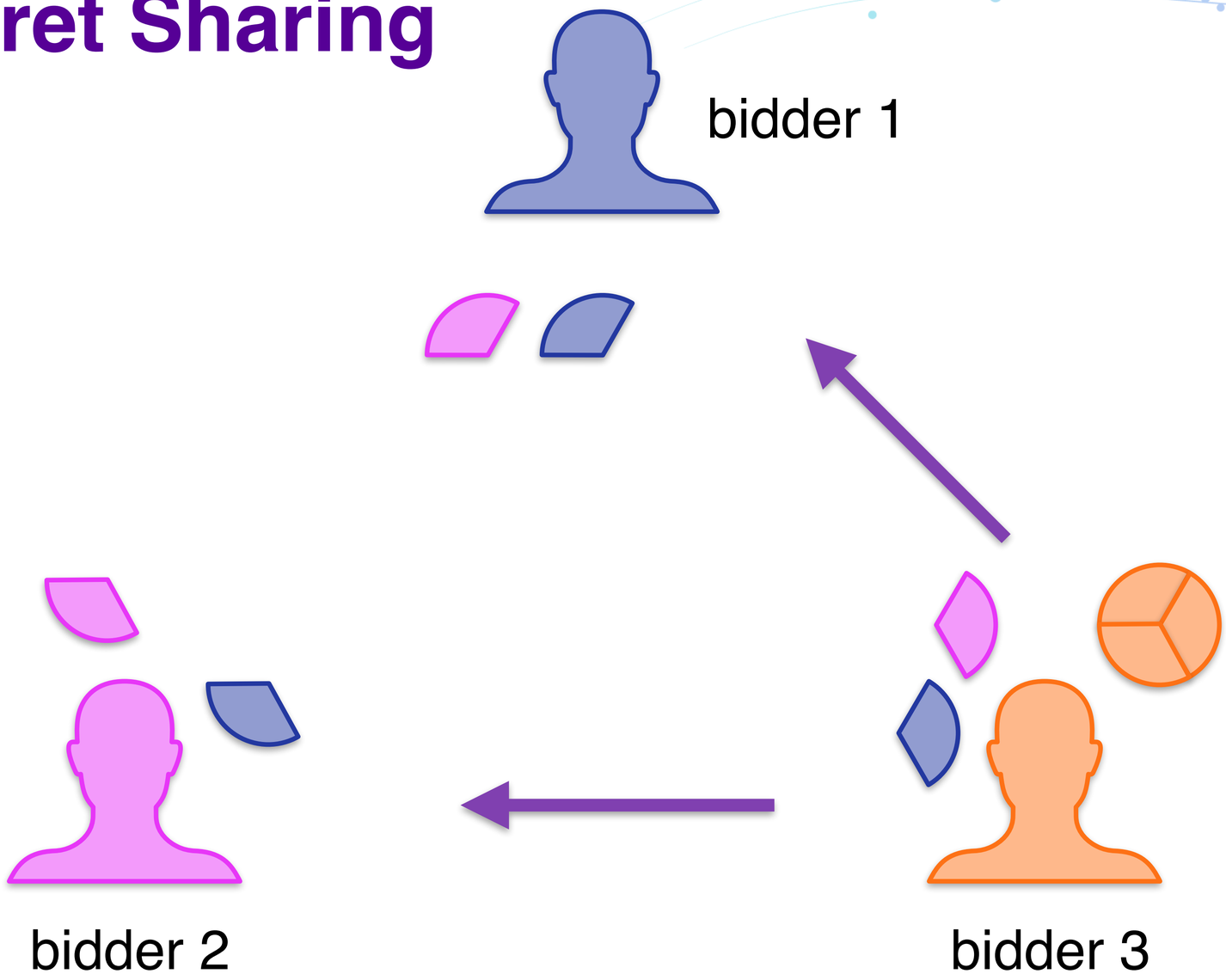

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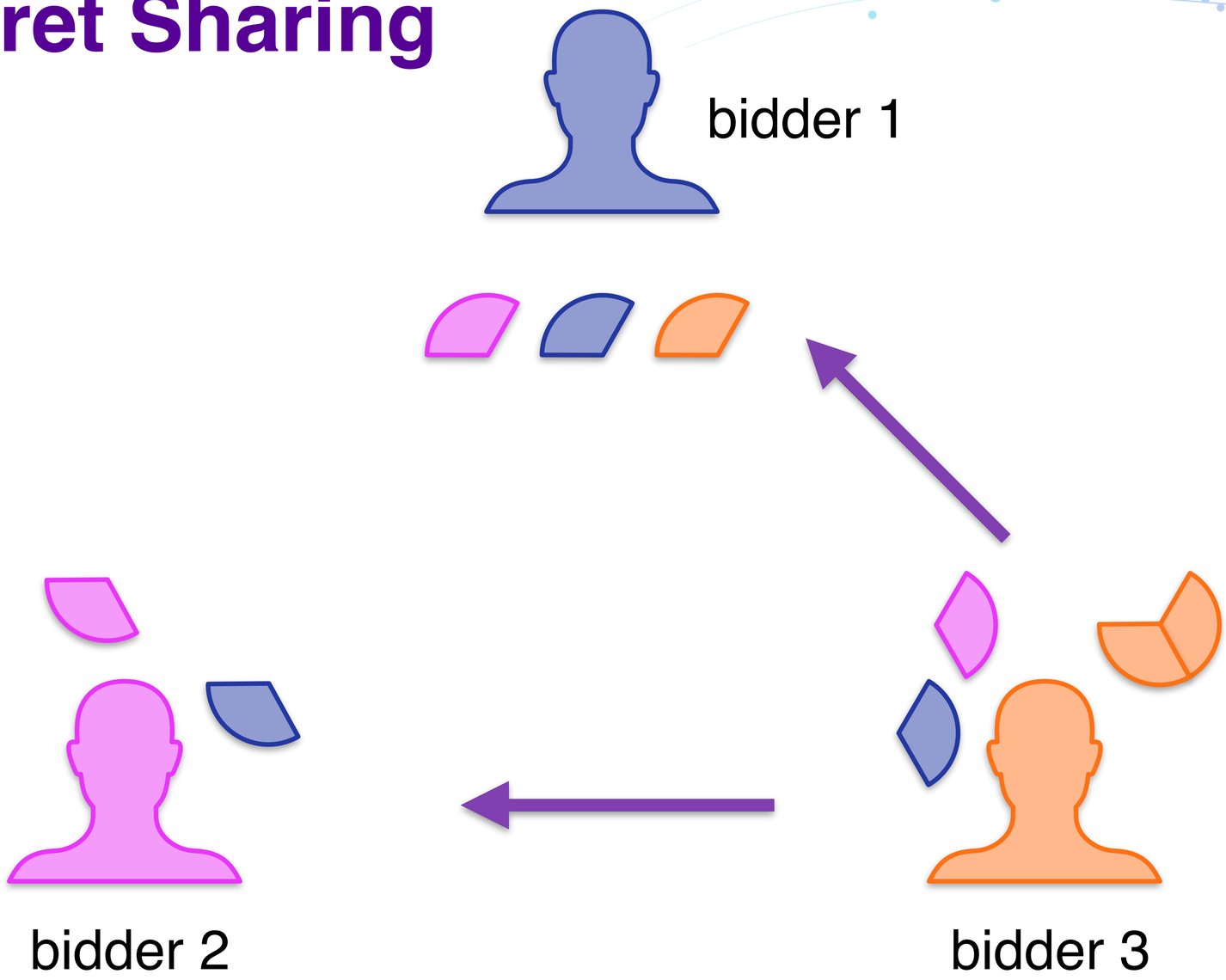

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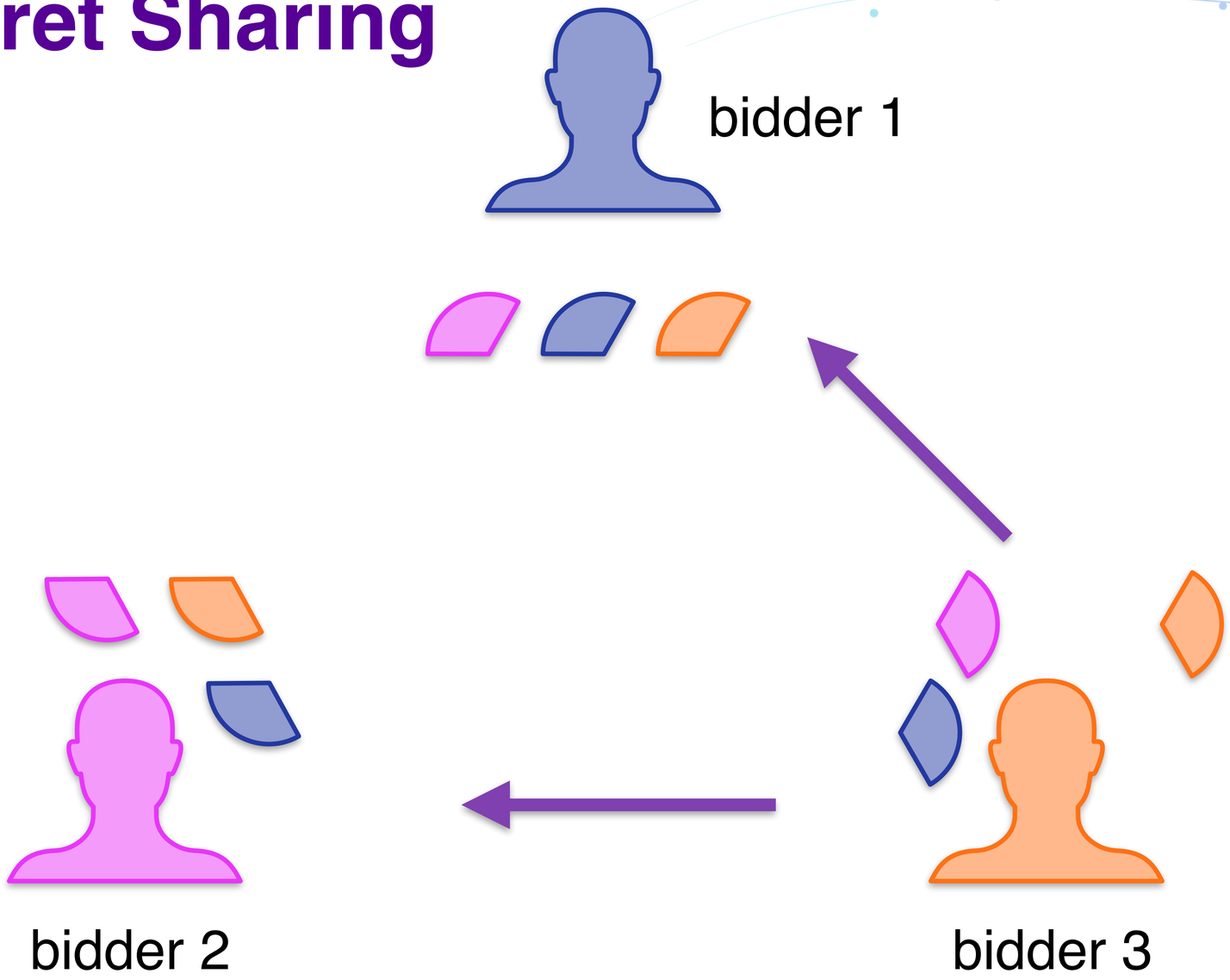

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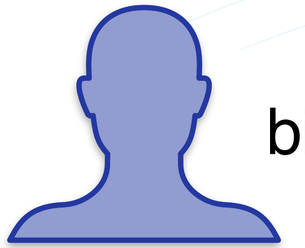
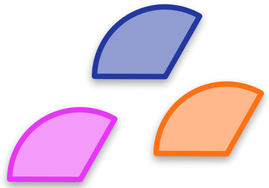
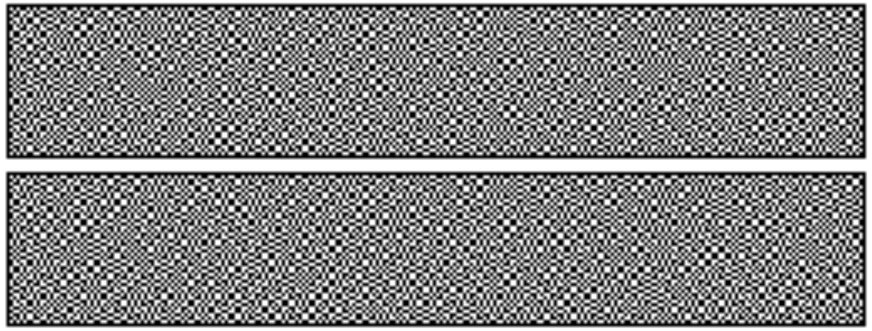
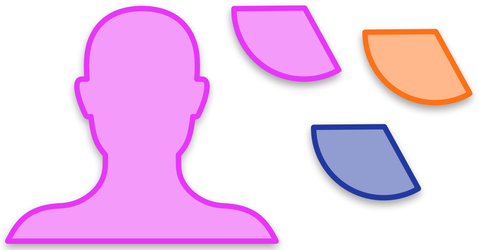


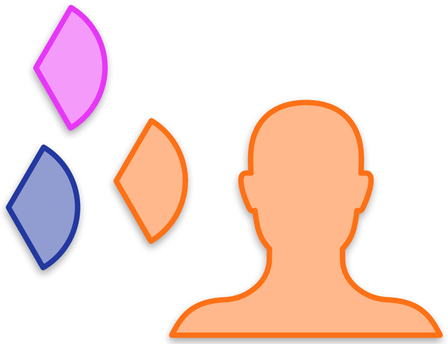
bidder 1







bidder 2



bidder 3

Secret Sharing

Let $s \in G$ be a secret input bid (e.g. $G = \mathbb{Z}_p$).

Pick random $r_1, \ldots, r_n \in G$ such that $s = r_1 + r_2 + \cdots + r_n$.

Send each r_i, **the *i*-th share**, to the i-th participant.

Here:

- n shares together reconstruct the secret.
- $n - 1$ shares or fewer give **zero information** about the secret.

Computing on Secret-Shared Data

We denote $[\![s]\!]$ for a secret-shared value.

- Addition: $[\![x + y]\!] = [\![x]\!] + [\![y]\!]$ (local operation)
- Multiplication $[\![x \cdot y]\!]$ requires interaction (exchange of messages)

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This work:

Comparison $$[\![x \leq y]\!] = \begin{cases} [\![\ \ 1]\!] & \text{if } [\![x]\!] \leq [\![y]\!] \\ [\![-1]\!] & \text{if } [\![x]\!] > [\![y]\!] \end{cases}$$

Secure Comparison in MPC

$$[\![x]\!] := [\![a]\!] - [\![b]\!]$$

- Compute $\mathrm{sgn}(x)$ or $\mathrm{bsgn}(x)$

$$\mathrm{sgn}(z) := \begin{cases} 1 & \text{if } z > 0, \\ 0 & \text{if } z = 0, \\ -1 & \text{if } z < 0. \end{cases} \qquad \mathrm{bsgn}(z) := \begin{cases} 1 & \text{if } z \geq 0, \\ -1 & \text{if } z < 0. \end{cases}$$

Using the Legendre Symbol to Compute the Sign

For infinitely many primes p, there exists $d = \Omega(\log p)$ such that

$$\left(x \mid p\right) = \operatorname{sgn}(x) \qquad \forall x \in [-d, d]$$

Idea goes back to [Feige et al., 1994], developed in [Yu, 2011].

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Example: $p = 311$, $\lceil \log_2 p \rceil = 9$, $d = 10$

x	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1
$\left(x \mid p\right)$	1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	...
$\left(x \mid p\right)$	0	1	1	1	1	1	1	1	1	1	1	-1	1	1	

Securely Computing the Legendre Symbol

Assume *arithmetic black box* $[\![\cdot]\!]$ over $\mathbb{Z}_p$

The Legendre symbol is completely multiplicative:
$(a \mid p)(b \mid p) = (ab \mid p)$ for any $a, b \in \mathbb{Z}$.

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Protocol Legendre$[\![x]\!]$

Given pre-processed pair $([\![r]\!], [\![s]\!])$ where $r \leftarrow_R \mathbb{Z}_p^*$ and $s = (r \mid p)$

1: $c \leftarrow [\![x]\!] \cdot [\![r]\!]$
2: $[\![z]\!] \leftarrow (c \mid p) \cdot [\![s]\!]$
3: **return** $[\![z]\!]$

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Cannot securely evaluate $(0 \mid p)$! (But we ignore this in this talk)

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Legendre-based comparison requires a single round in the online phase!

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- Caveat: Comparison range limited to *d*

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Can we increase d while keeping p fixed?

New Idea: Correcting “Errors” via Majority Vote

- Let’s take again $p = 311$

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	…
$(x \mid p)$	0	1	1	1	1	1	1	1	1	1	1	-1	1	1	1	1	1	-1	1	-1	1	…

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$(x \mid p)$	0	1	1	1	1	1	1	1	1	1	1	-1	1	1	1	1	1	-1	1	-1	1	…

- Inspect neighboring Legendre symbols, compute sign of

$$y(x) = (x-1 \mid p) + (x \mid p) + (x+1 \mid p) \in [-3, +3] \subset \mathbb{Z}$$

- We can compute sign of $y(x)$ using a low-degree polynomial

In General (Window Length $2k+1$, Arbitrary k)

Definition

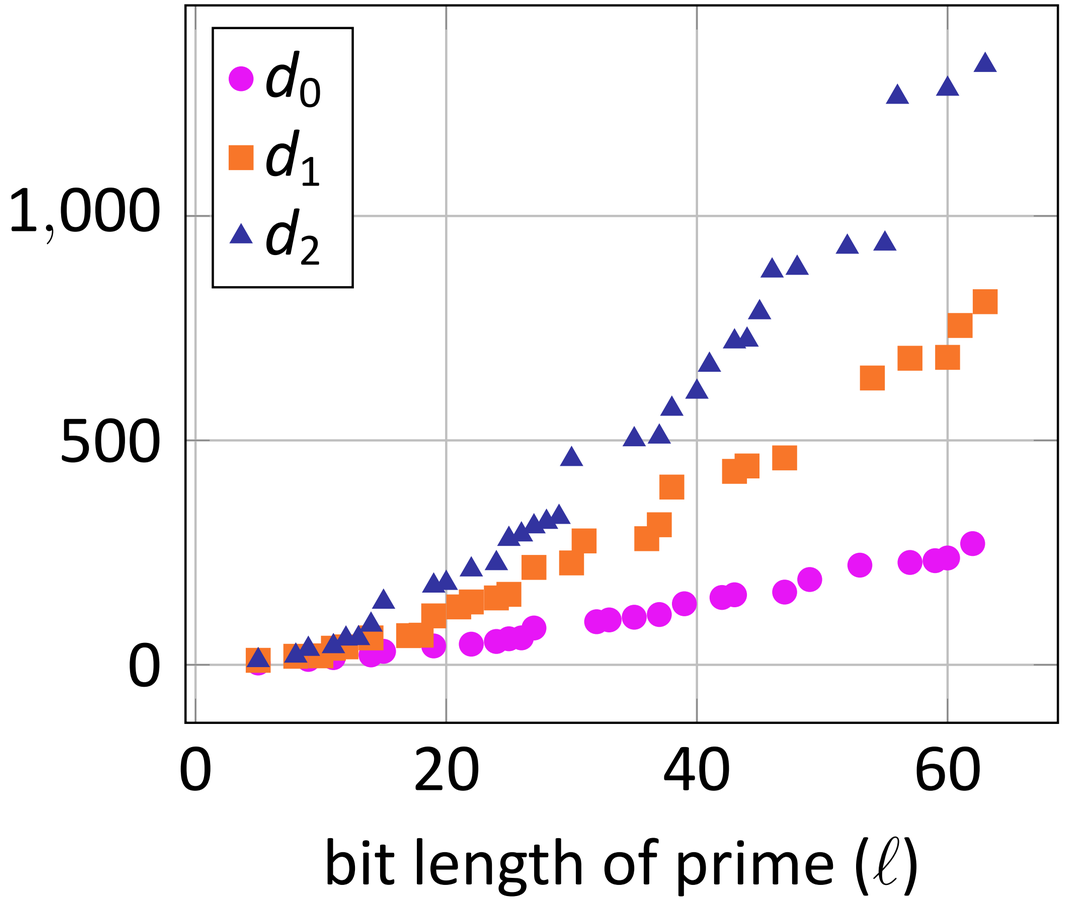
Let $k \in \mathbb{Z}_{\geq 0}$, and let $p > 2k+1$ be a Blum prime.

The *k-range of p*, denoted $d_k(p)$, is the largest integer d such that

$$\sum_{i=-k}^{k} (x+i \mid p) > 0 \quad \text{holds for all } x \text{ with } 1 \leq x \leq d, \tag{1}$$

and we set $d_k(p) := 0$ if no such d exists.

$k > 0$ Increases the Range of the Comparison



Finding Primes p with High $d_0(p)$

Naive search: try all primes $p = 2, 3, 5, 7, 11, \ldots$ and calculate $d_0(p)$.

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For $k > 0$: more involved. Details in paper, code on Github.

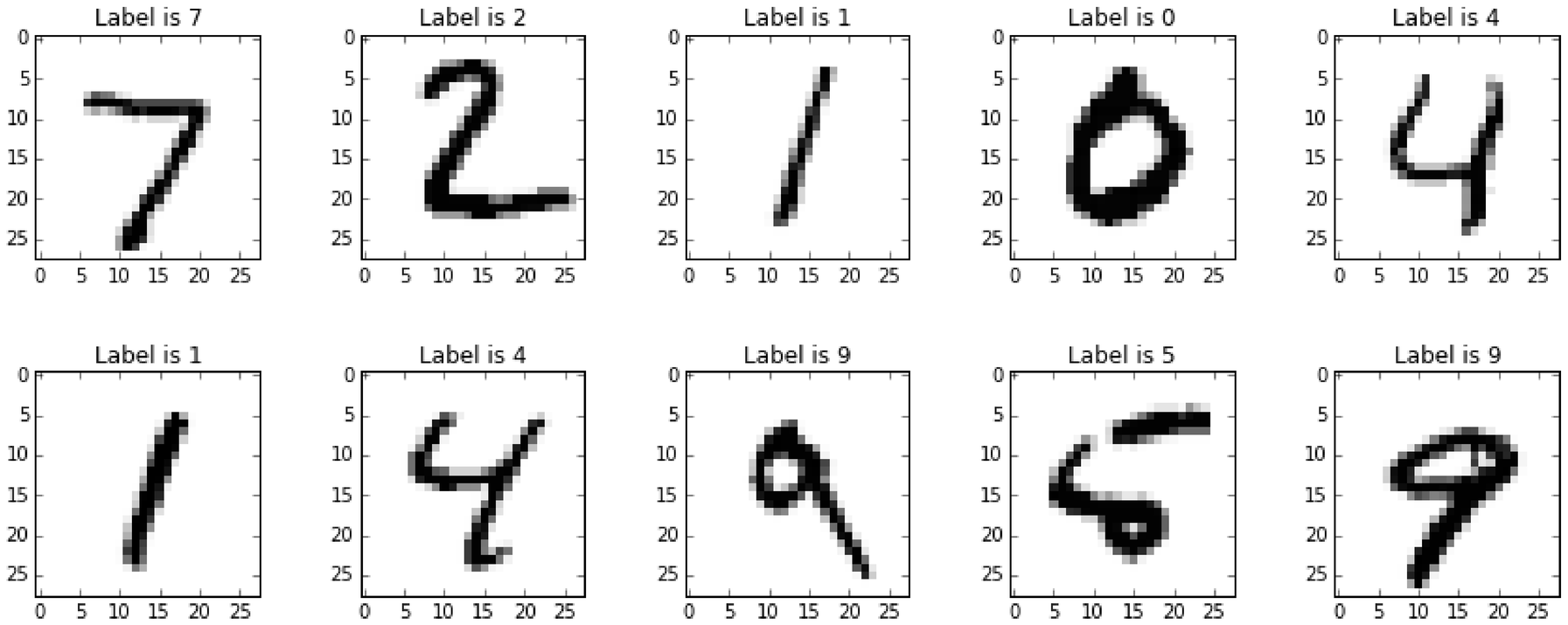
Results

- Our protocol for $k = 1$: single-round online phase, 3 secure multiplications in parallel.
- For $k = 1, 2$, we have searched all[1] primes up to 64 bits.

k	best d_k	range improvement (w.r.t. d_0)
0	270	
1	809	3.0×
2	1336	4.9×

[1] With conditions $d_0(p) \geq 64$, and for $k = 2$ also $d_1(p) \geq 400$.

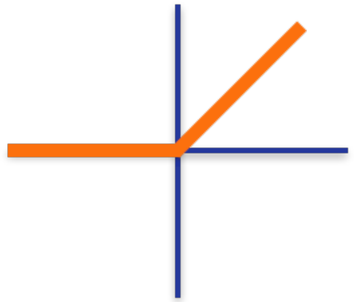
Application: Secure Neural Network Evaluation



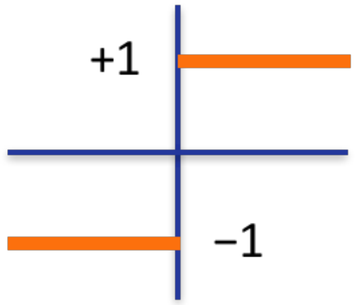
Application: Secure Neural Network Evaluation

- Handwritten digit recognition “MNIST”
- Binarized multi-layer perceptron
  - Neuron outputs are in $\{-1, +1\}$
  - Uses bsgn as activation function
- Implementation in the MPyC framework

ReLU Activation Function



bsgn Activation Function



Questions?

References

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