RSAConference2019
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SESSION ID: CRYP-W02

Doubly half-injective PRGs for incompressible white-box cryptography

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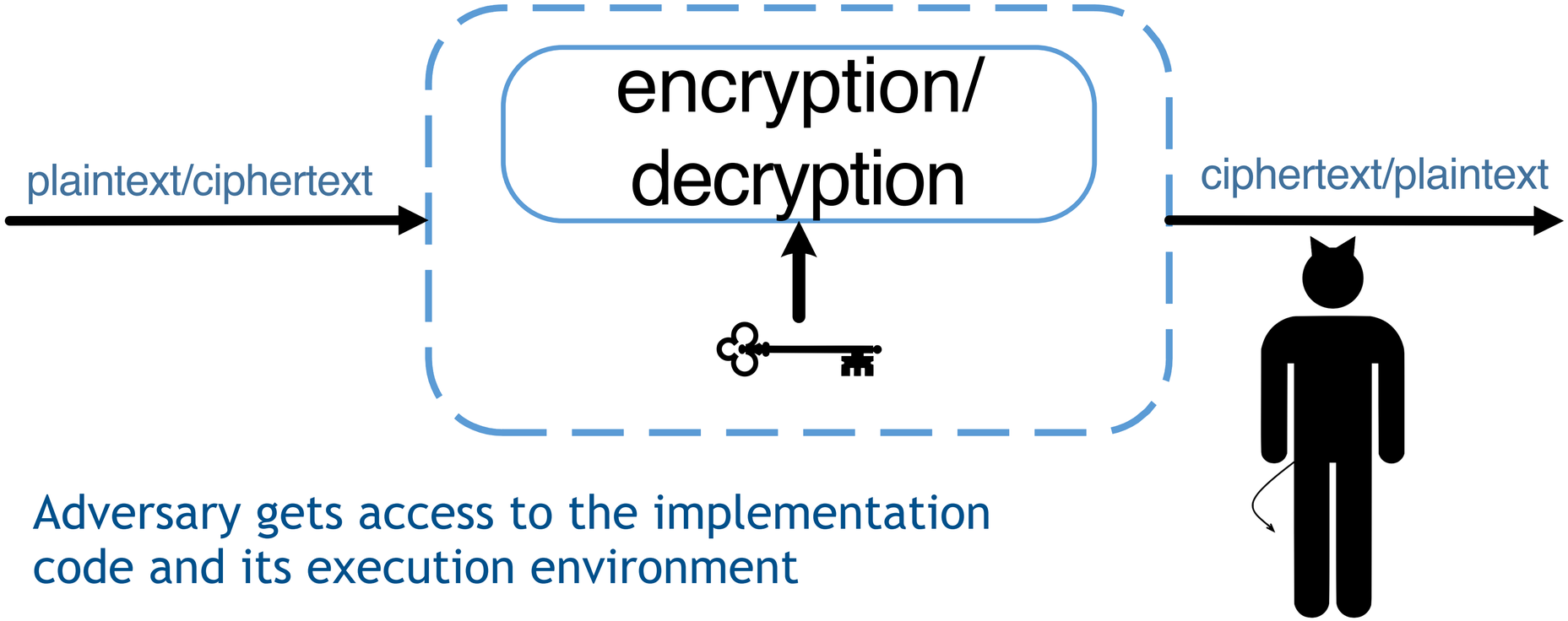
White-box attack scenario



Adversary gets access to the implementation code and its execution environment

→ WB Cryptography aims to maintain a program secure even when subject to this attack model

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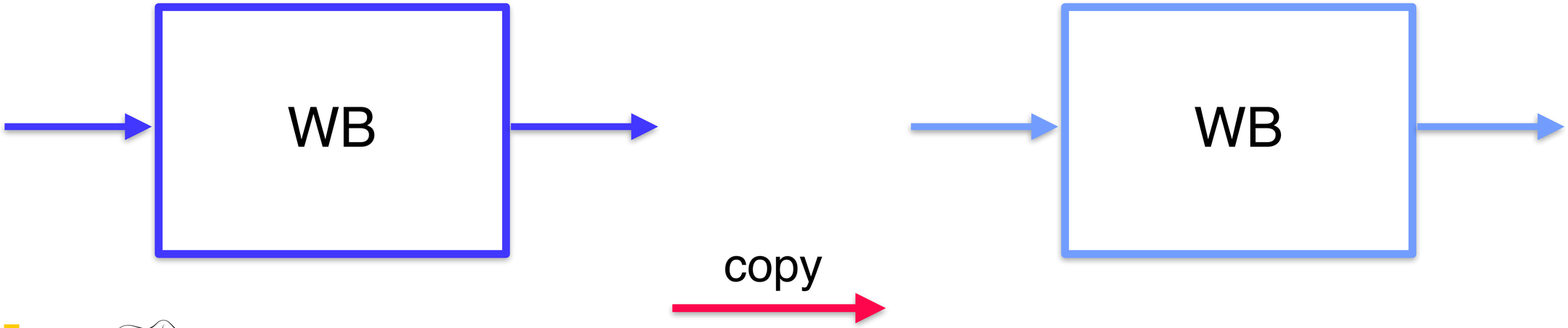
Incompressibility for white-box cryptography

Adversarial capabilities

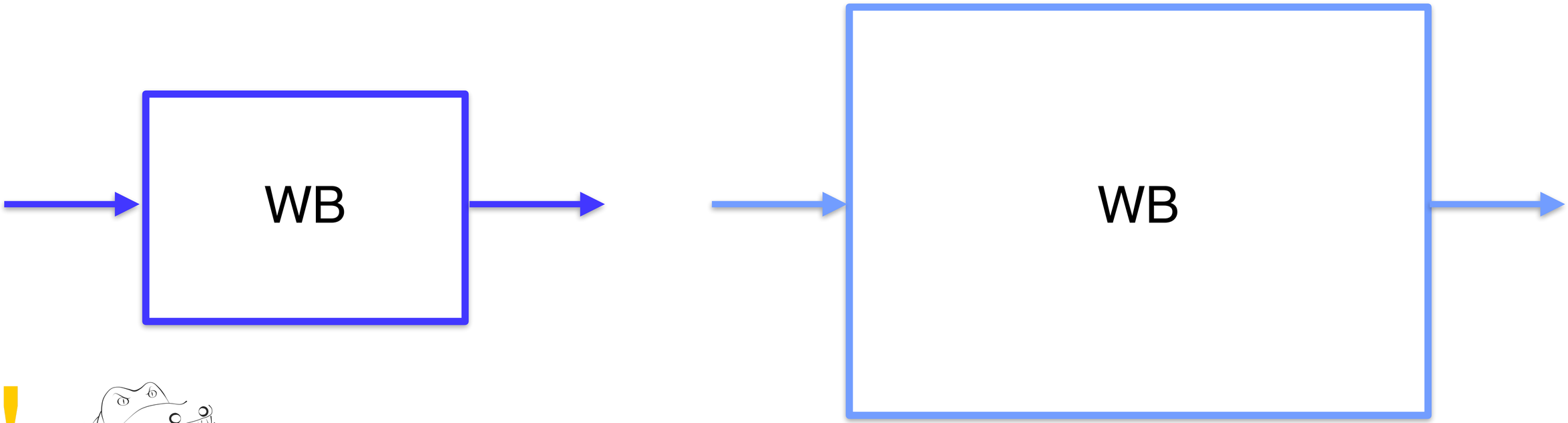
- The adversary gets access to the program code of an implementation
- He could extract keys, but also copy the program and its functionality
- Threat of code-lifting attacks



Methods for mitigating code-lifting attacks

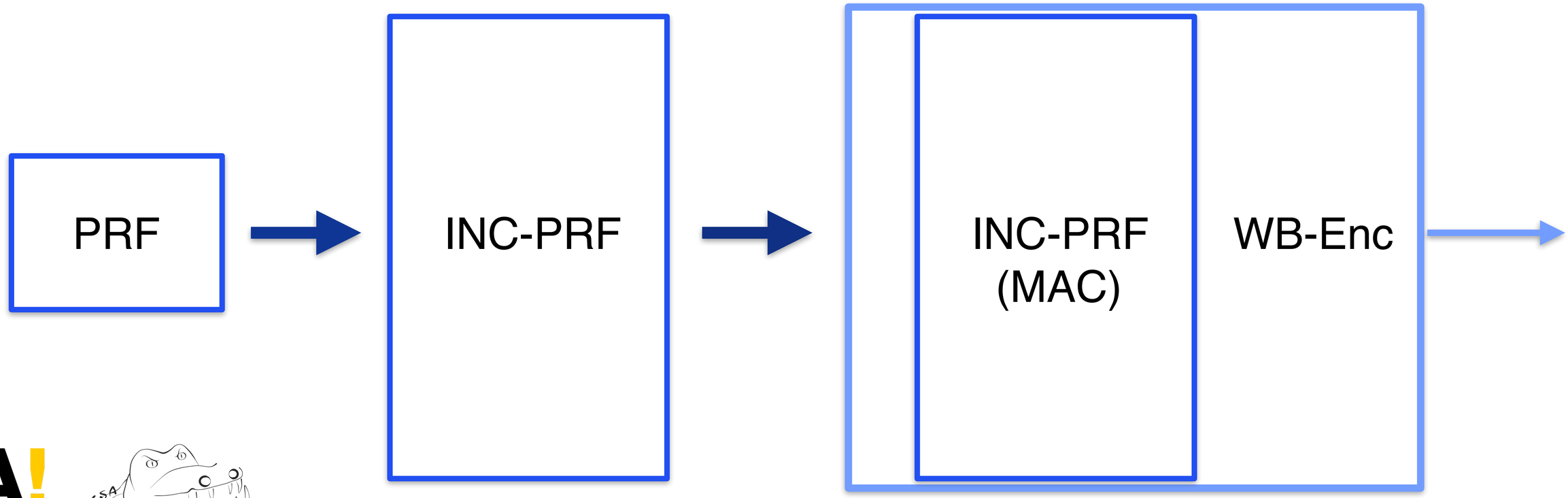
- Incompressibility

  Delerablée, Lepoint, Paillier, Rivain: *White-box security notions for symmetric encryption schemes*
  Fouque, Karpman, Kirchner, Minaud: *Efficient and provable white-box primitives*



In this work

- We build an incompressible wb-encryption scheme
- Our construction is based on standard assumptions, such as pseudorandom generators and pseudorandom functions



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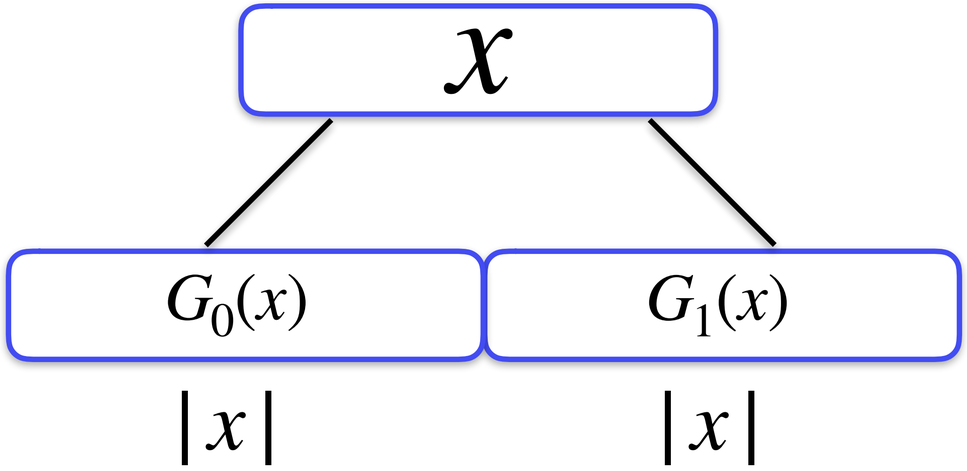
PRGs, PRFs and the GGM tree

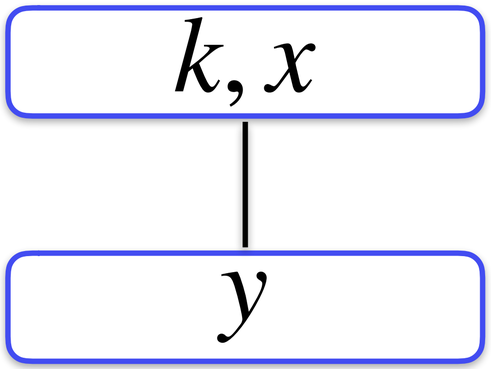
Pseudorandom generators

- Deterministic, polynomial time computable function satisfying:
  - **Length-expansion**: for all $x \in \{0,1\}^* \, |PRG(x)| = 2|x|$
  - **Pseudorandomness**: the output from the PRG should be indistinguishable from random



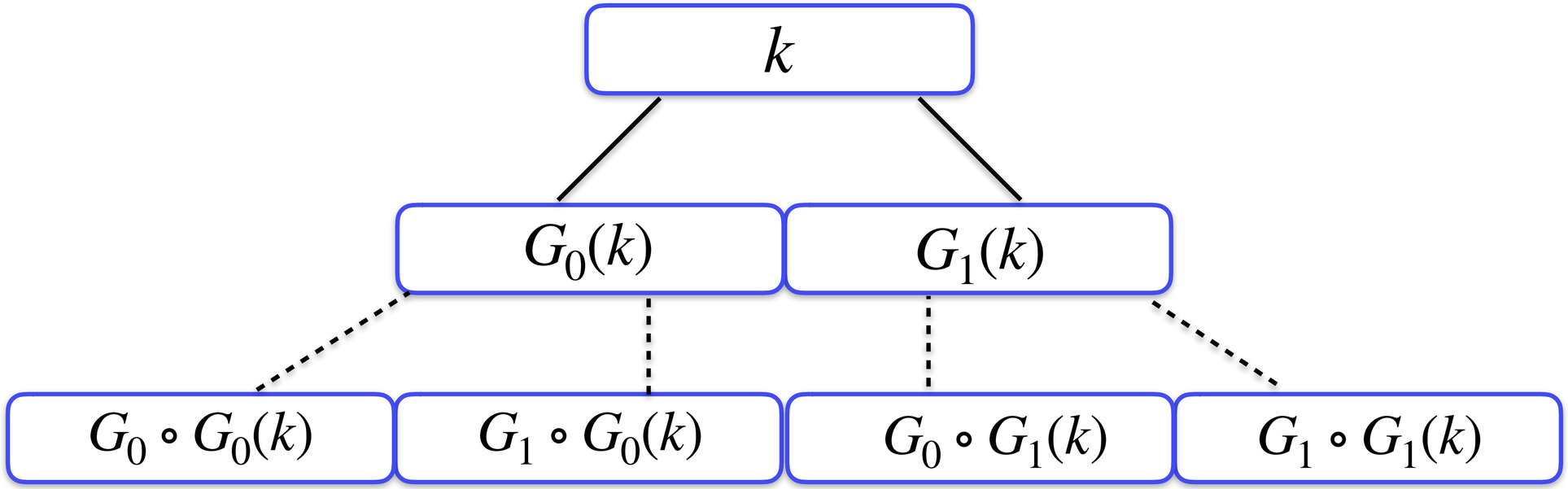
Pseudorandom functions

- Deterministic, polynomial time computable function satisfying:
  - **Length-condition**: for all $n \in \mathbb{N}, k, x \in \{0,1\}^n, \; |PRF(k,x)| = |y|$
  - **Pseudorandomness**: the output from the PRF should be indistinguishable from random



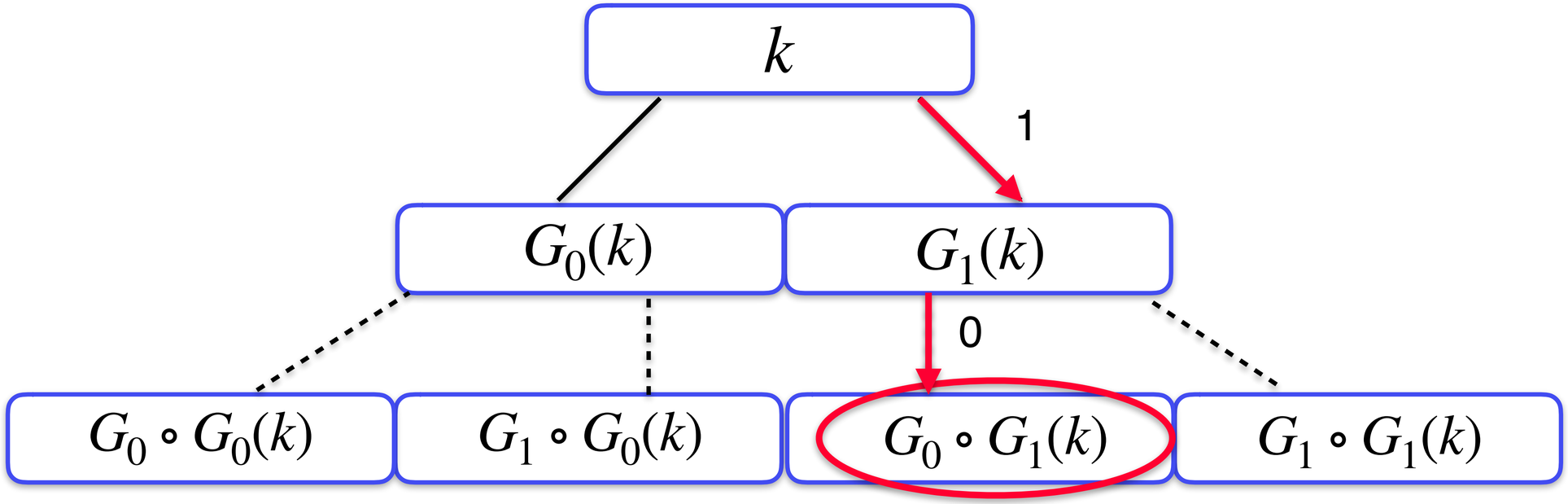
GGM tree: building a PRF from a PRG

- Introduced by Goldreich, Goldwasser and Micali
- Input x of the PRF(k,x) represents the binary address of the binary tree



GGM tree

- E.g. x= 10
- PRF(k,x)= GGM(k,m)= $G_0 \circ G_1(k)$



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An incompressible white-box pseudorandom function

(Incompressible) PRF implementation

- Build a PRF which uses a large, incompressible key
  - **Key expansion**

$$k \in \{0,1\}^*$$

$$K = Comp_{PRF}(k), \text{with} \, |K| >> |k|$$

  - **Functionality preservation**:

$$\forall k, x \in \{0,1\}^*, f(k,x) = F(K,x)$$

Construction (1) - PRF

$$\underline{f(k, x)}$$

$$y \leftarrow \texttt{GGM}(k, x)$$

$$\textbf{return} \quad y$$

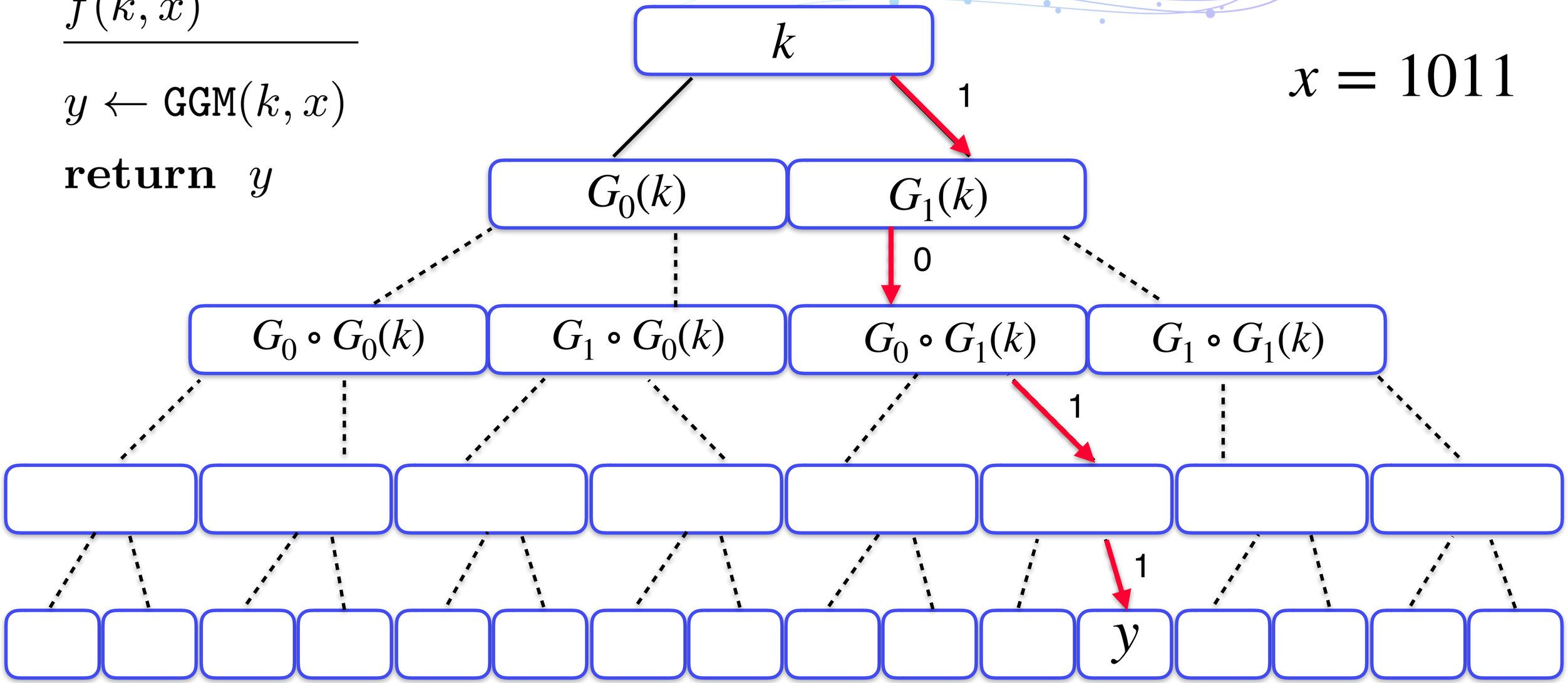
- Standard PRF based on the GGM tree

$f(k, x)$

$y \leftarrow \texttt{GGM}(k, x)$

return y

$x = 1011$



Construction (2) - Compiler

$\underline{\mathtt{Comp}_{\mathtt{PRF}}(k)}$

for j **from** 0 **to** $2^{\ell} - 1$

$\quad k_j := \mathtt{GGM}(k, < j >)$

$K \leftarrow k_0 || ... || k_{2^{\ell}-1}$

return K

- Iterate the GGM on key *k* and all possible values of length ℓ

$\texttt{Comp}_{\texttt{PRF}}(k)$

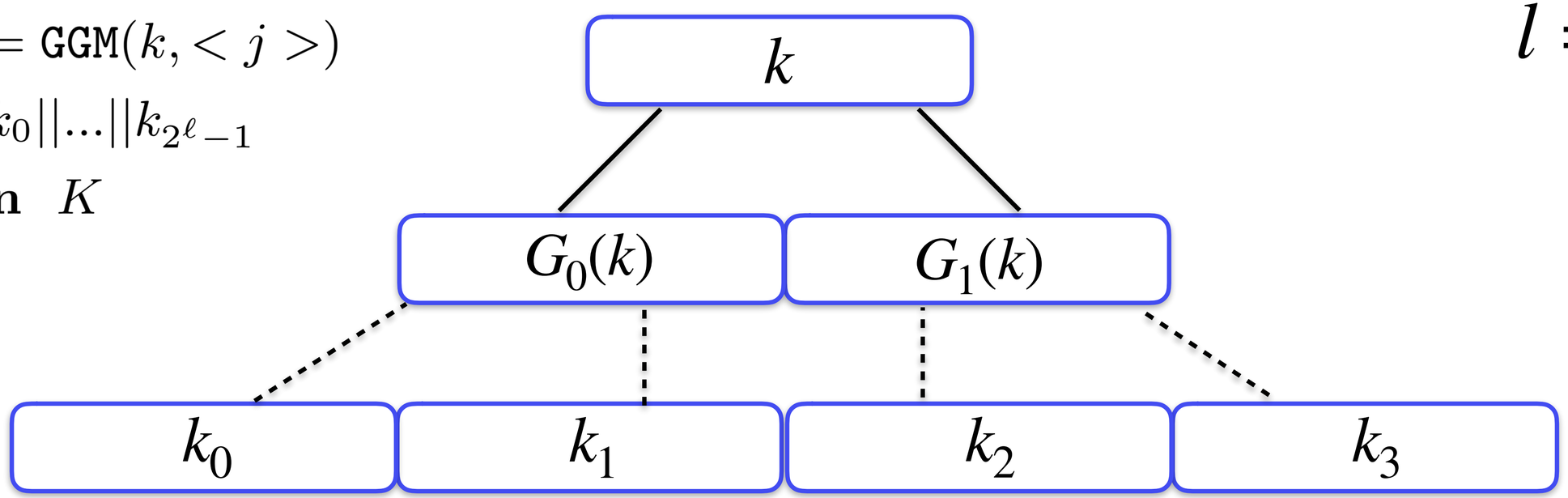
for j **from** 0 **to** $2^{\ell} - 1$

$k_j := \texttt{GGM}(k, < j >)$

$K \leftarrow k_0 || ... || k_{2^{\ell}-1}$

return K

$l = 2$



$$K = k_0 || k_1 || k_2 || k_3$$

Construction (3) - Incompressible PRF

$F(K, x)$

$(x[1...\ell], x[\ell+1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

$y \leftarrow \mathtt{GGM}(k_j, x[\ell+1...|x|])$

$\mathbf{return}\ \ y.$

- F takes as input the long key K. Input x is split in two.

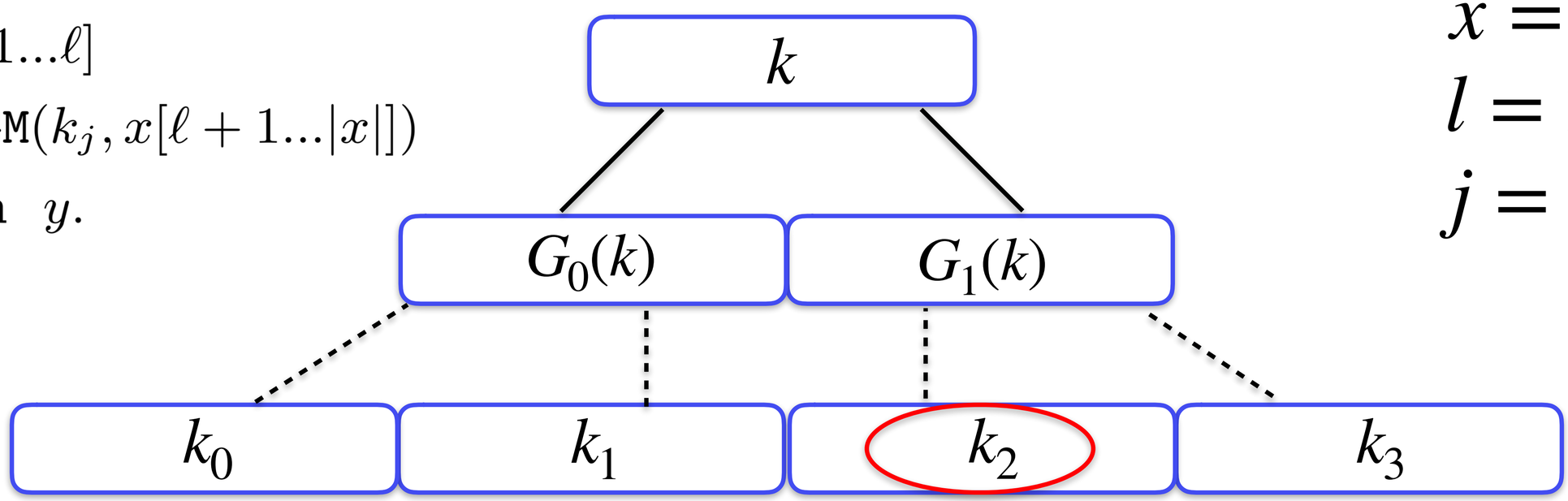
$F(K, x)$

$(x[1...\ell], x[\ell+1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

$y \leftarrow \mathtt{GGM}(k_j, x[\ell+1...|x|])$

return y.



$x = 1011$

$l = 2$

$j = 10$

GGM(k_2,11)

$F(K, x)$

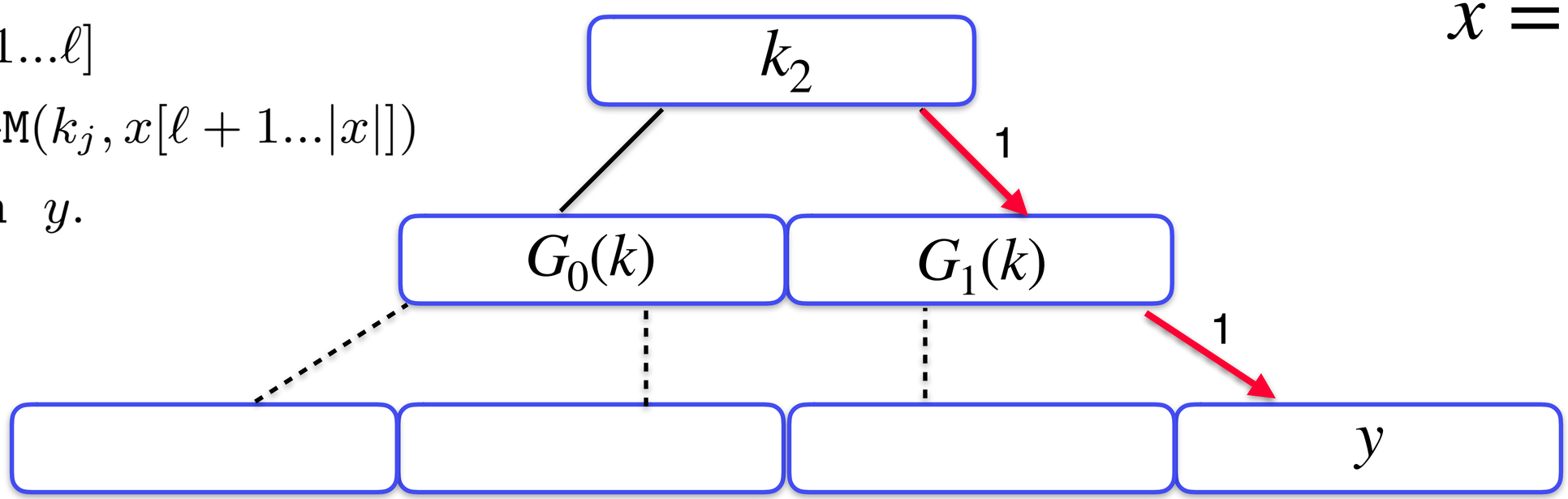
$(x[1...\ell], x[\ell+1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

$y \leftarrow \texttt{GGM}(k_j, x[\ell+1...|x|])$

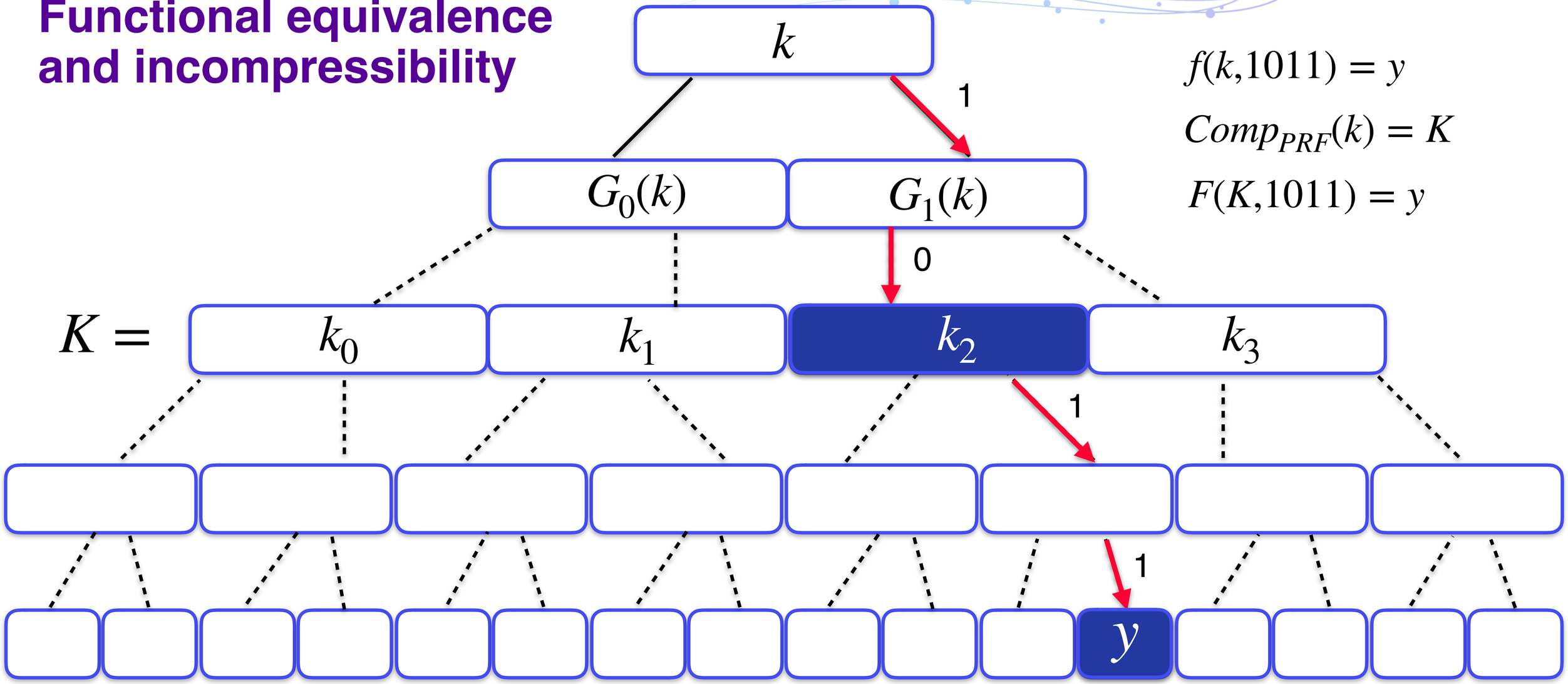
return y.

$x = 1011$



$y \leftarrow \text{GGM}(k_2, 11)$

Functional equivalence and incompressibility

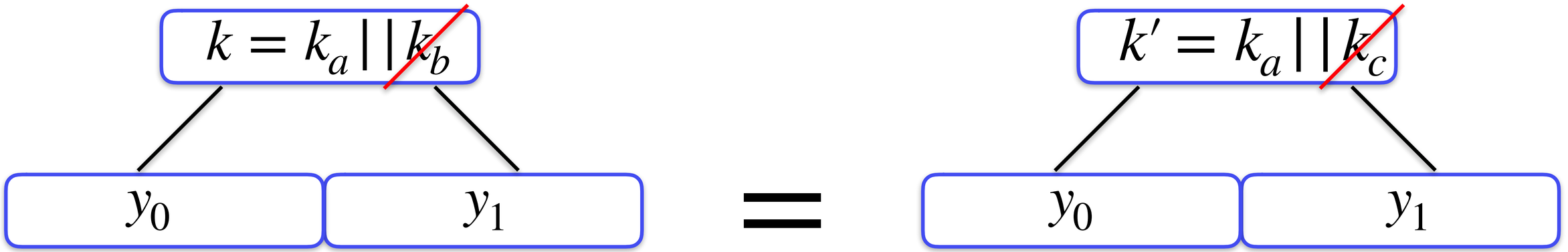


$$f(k,1011) = y$$

$$Comp_{PRF}(k) = K$$

$$F(K,1011) = y$$

Possible collisions



For our incompressibility property to hold, we need injectivity

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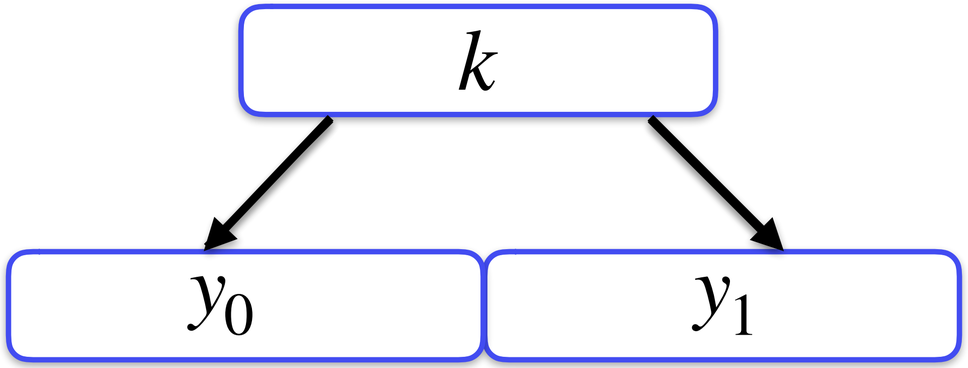
Doubly-half injective PRGs

PRG with double injectivity



We want injectivity from

L to the set Y, with $k \in L$ and $y_0, y_1 \in Y$.

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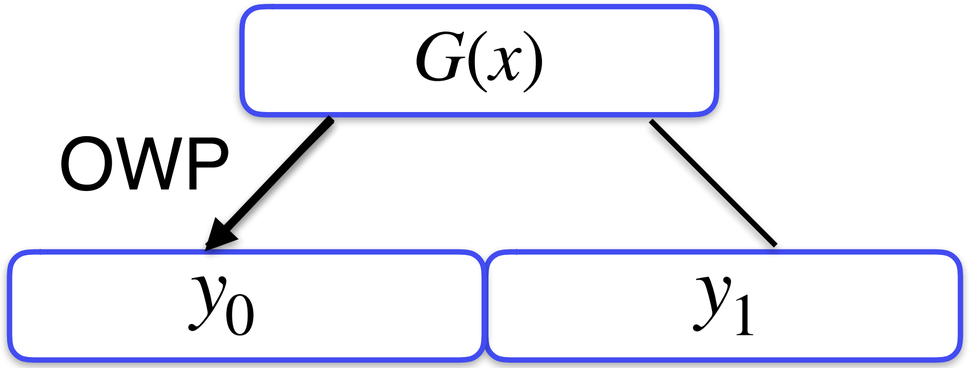
Left-half-injective PRG

- Construction by Garg, Pandey, Srinivasan and Zhandry: use a one-way permutation to construct a left-half injective PRG

  Breaking the sub-exponential barrier in obfustopia



$$G(x) := \mathsf{OWP}^{|x|}(x)||B(x)||B(\mathsf{OWP}(x))||\ldots||B(\mathsf{OWP}^{|x|-1}(x)),$$

with $B =$ hardcore bit

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Doubly-half injective PRG

- Assuming a left-half injective, length doubling PRG

$$G = G_0 \,||\, G_1$$

OWP-*Injective*

$$g(x_0 \,||\, x_1) := G_0(x_0) \,||\, G_1(x_0) \oplus G_0(x_1) \,||\, G_0(x_1) \,||\, G_1(x_1) \oplus G_0(x_0)$$

Injective *Injective*

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Doubly-half injective PRG

$$g(x_0||x_1) := G_0(x_0)||G_1(x_0) \oplus G_0(x_x)||G_0(x_1)||G_1(x_1) \oplus G_0(x_0)$$

- Left half is injective,

Let $w_0||w_1$, s.t. $g_0(w_0||w_1) = g_0(x_0||x_1)$

G_0 is a permutation $\rightarrow x_0 = w_0$

$G_1(w_0) \oplus G_0(w_1) = G_1(x_0) \oplus G_0(x_1)$

G_0 is a permutation $\rightarrow x_1 = w_1$

The injectivity of the right half follows analogously

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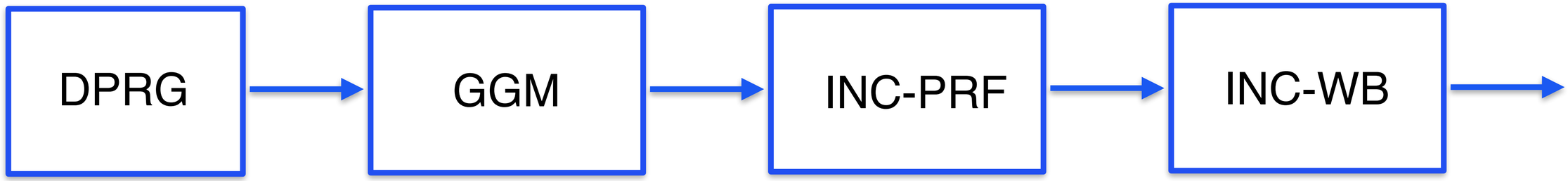
Conclusions

Overview of our construction



- Provide an incompressible (big key) white-box encryption scheme
- Results based on standard crypto-assumptions
- Construct a new type of PRG

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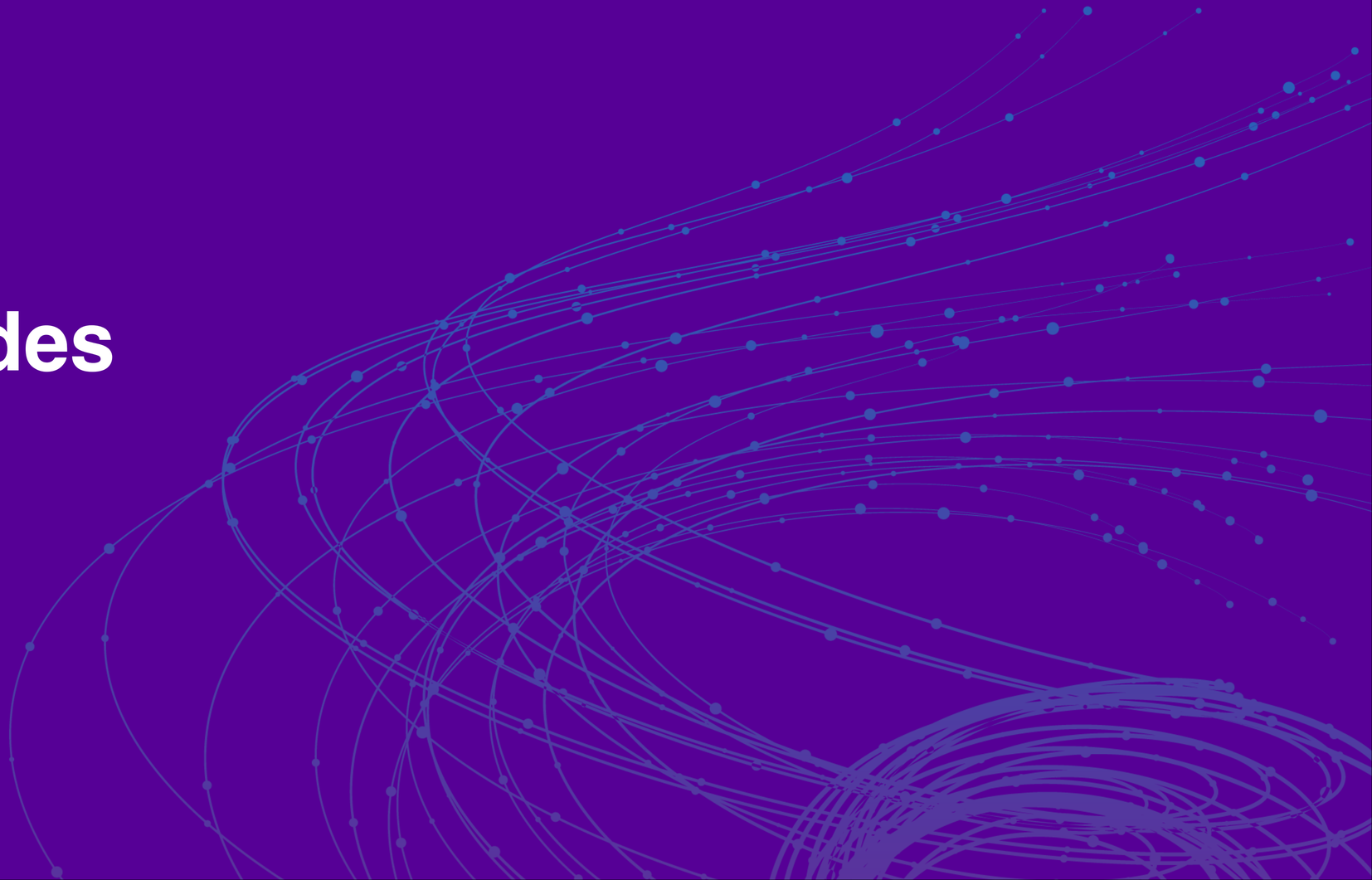
Backup slides



Conclusions

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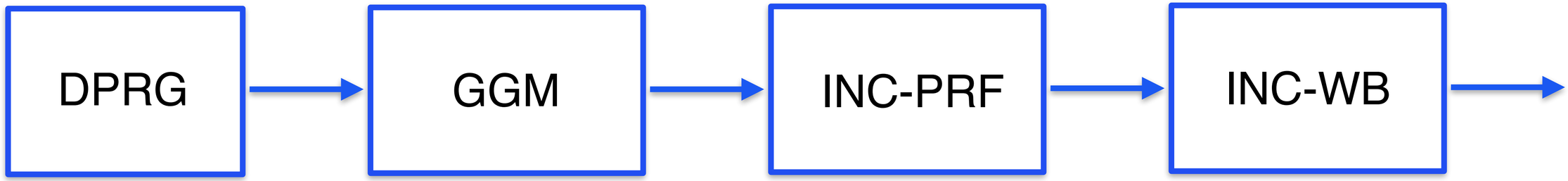

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Alternative desirable properties

- Making a program traceable (*traceability*)
- Binding the WB to a precise hardware device (*hardware binding*)
- Making the functionality of the WB dependent of a set of inputs (*input binding/application binding*)

Why is F incompressible?

$F(K, x)$

$(x[1...\ell], x[\ell+1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

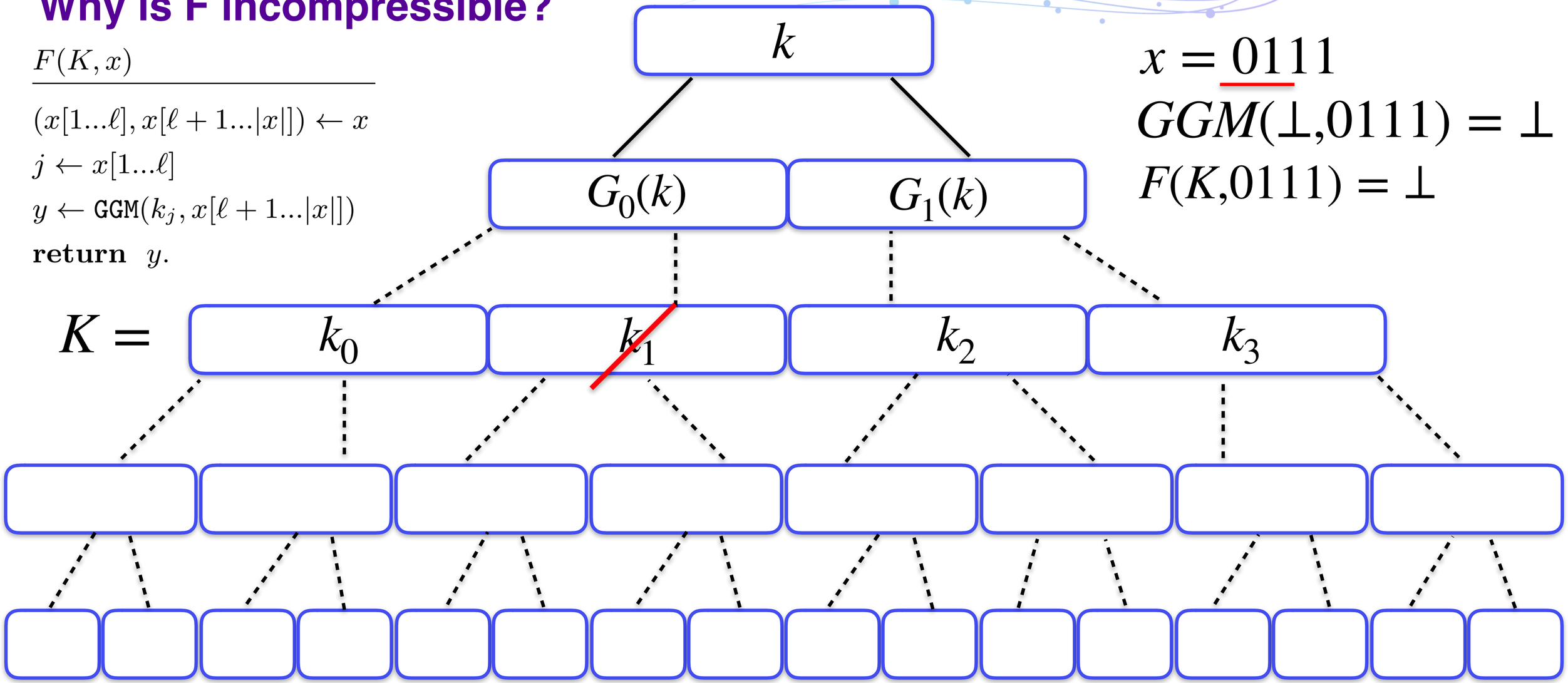
$y \leftarrow \mathtt{GGM}(k_j, x[\ell+1...|x|])$

return y.

$x = 0111$

$GGM(\perp, 0111) = \perp$

$F(K, 0111) = \perp$



Why is F incompressible?

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$\mathtt{Comp}_{\mathtt{PRF}}(k)$

for j **from** 0 **to** $2^{\ell} - 1$

$k_j := \mathtt{GGM}(k, < j >)$

$K \leftarrow k_0 || ... || k_{2^{\ell}-1}$

return K

$F(K, x)$

$(x[1...\ell], x[\ell + 1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

$y \leftarrow \mathtt{GGM}(k_j, x[\ell + 1...|x|])$

return y.

We need the complete key K to achieve $f(k, x) = F(K, x)$ for all $x \in \{0,1\}^*$

However, this might only hold depending on the definition of the PRG used in the GGM tree.

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Theorem 1

IF PRF admits a computationally (σ, λ) – incompressible implementation F, the wb-encryption scheme in Constructino 1 is a $(\sigma, \lambda - n - o(1))$– incompressible wb-encryption scheme.

- Proof sketch via reduction: we reduce the incompressibility of F to the incompressibility of the encryption scheme.
  - Cannot produce a valid MAC without the complete key K

Doubly-half injective PRG

- We define a PRG which is left-half and right-half injective.
- Three properties required:
  - **Length-doubling**: For all $x \in \{0,1\}^* \mid g(x) \mid = 2 \mid x \mid$. $g_0(x)$ is the left haf of g and $g_1(x)$is the right half.
  - **Doubly-half injective**: g_0 and g_1 are injective.
  - **Pseudorandomness**: $g(U_n)$ is computationally indistinguishable fromU_{2n}.

Construction 1 via AE-scheme and F

$\underline{\texttt{Kgen}(1^n)}$

$k' \leftarrow\!\!\$\ \{0,1\}^n$
$k'' \leftarrow\!\!\$\ \{0,1\}^n$
$k \leftarrow k'||k''$
return k

$\underline{\texttt{Enc}(k,m)}$

$k' \leftarrow k[0:n-1]$
$k'' \leftarrow k[n:2n-1]$
$t \leftarrow f(k',m)$
$\tau \leftarrow (m,t)$
$c \leftarrow\!\!\$\ \texttt{AEnc}(k'',\tau)$
return c

$\underline{\texttt{Dec}(k,c)}$

$k' \leftarrow k[0:n-1]$
$k'' \leftarrow k[n:2n-1]$
$\tau \leftarrow \texttt{ADec}(k'',c)$
$(m,t) \leftarrow \tau$
if $t = f(k',m)$ **return** m.
else return $\perp$

$\underline{\texttt{Comp}(k)}$

$k' \leftarrow k[0:n-1]$
$k'' \leftarrow k[n:2n-1]$
$K := \texttt{Comp}_{\texttt{PRF}}(k')$
$\texttt{Enc}_{\texttt{WB}} := C[K,k'](.)$
return $\texttt{Enc}_{\texttt{WB}}$

$\underline{C[K,k''](m)}$

$t \leftarrow F(K,m)$
$\tau \leftarrow (m,t)$
$c \leftarrow\!\!\$\ \texttt{AEnc}(k'',\tau)$
return c

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Use cases of white-box cryptography

- Original concern: Digital Rights Management
  - White-box crypto introduced as a method to mitigate piracy
    - Chow, Eisen Johnson and van Oorschot - A white-box cryptography and an AES implementation

- Recently proposed as a method for protecting cryptographic keys within mobile payment applications implemented in software

Global construction of the scheme

- Key expansion property
- Pseudorandomness property follows from the property of the GGM

$f(k, x)$

$y \leftarrow \texttt{GGM}(k, x)$

return y

$\texttt{Comp}_{\texttt{PRF}}(k)$

for j **from** 0 **to** $2^{\ell} - 1$

$k_j := \texttt{GGM}(k, < j >)$

$K \leftarrow k_0 || ... || k_{2^{\ell}-1}$

return K

$F(K, x)$

$(x[1...\ell], x[\ell + 1...|x|]) \leftarrow x$

$j \leftarrow x[1...\ell]$

$y \leftarrow \texttt{GGM}(k_j, x[\ell + 1...|x|])$

return y.

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Methods for mitigating code-lifting attacks

- Two popular methods have been studied in the literature:
  - Traceability
    Delerablée, Lepoint, Paillier, Rivain: *White-box security notions for symmetric encryption schemes*